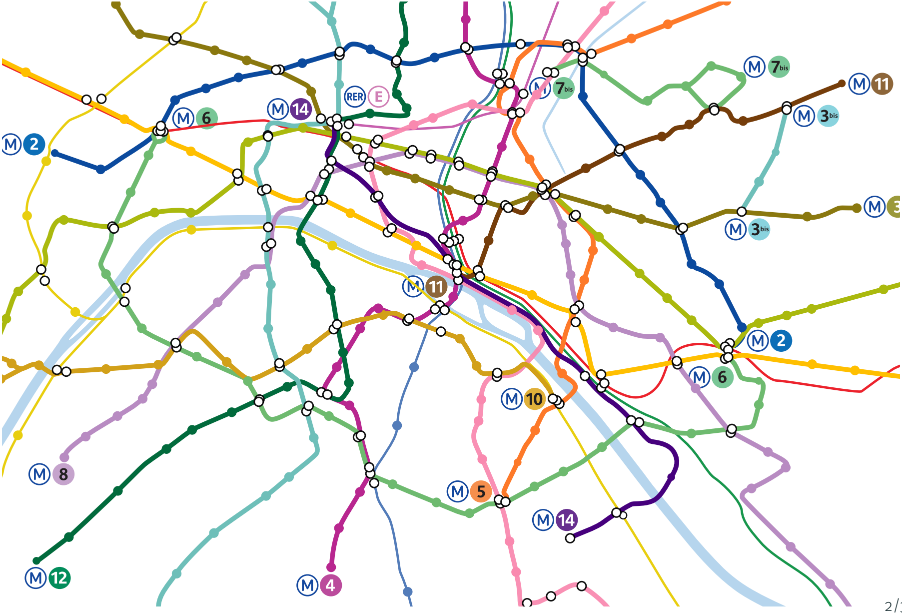
Leveraging the structure of uncertain data

Antoine Amarilli

March 5, 2019

Example application: Subway routing



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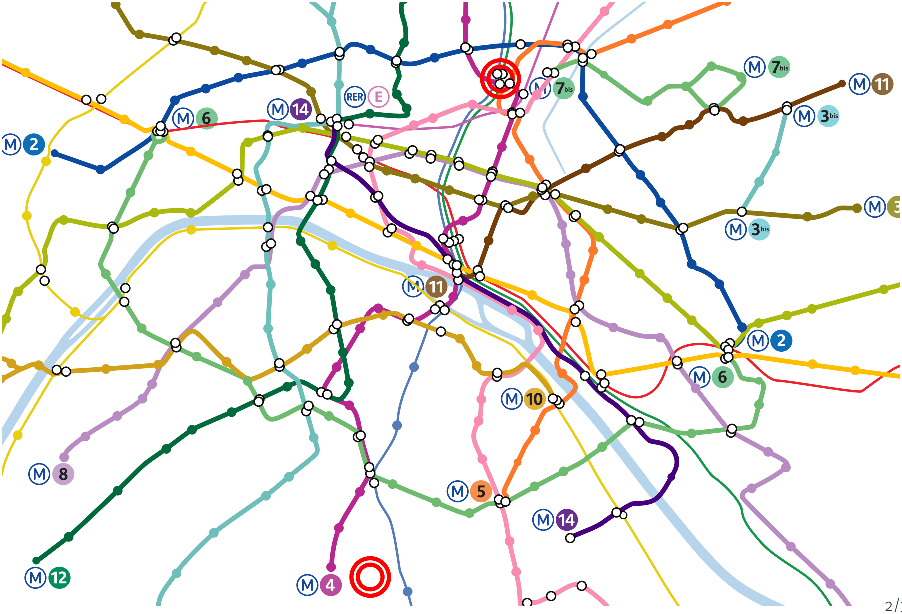

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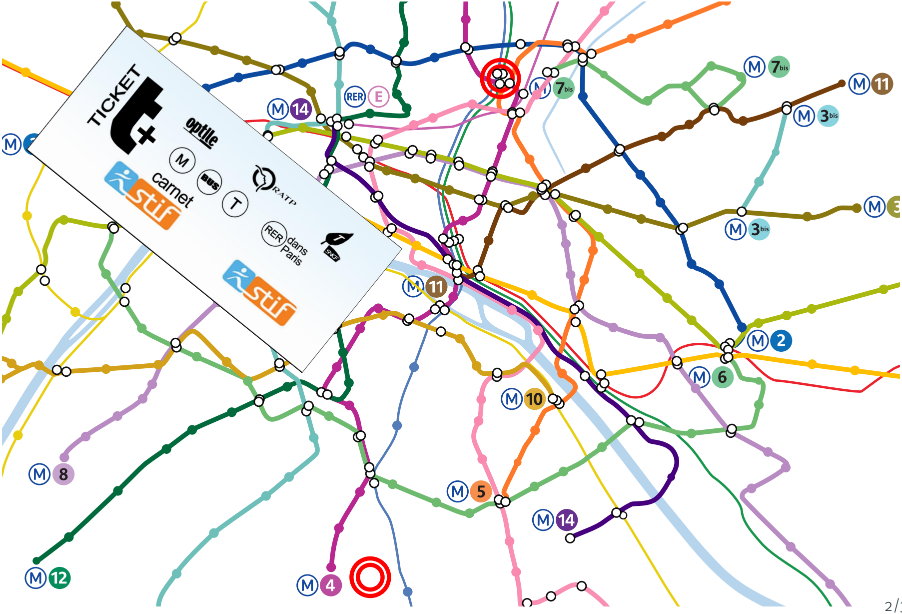

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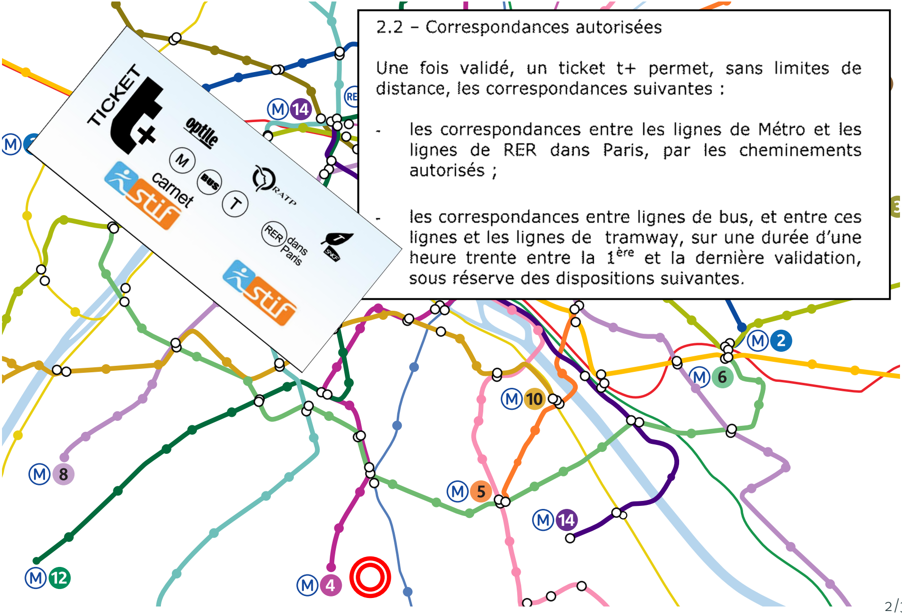

2.2 – Correspondances autorisées

Une fois validé, un ticket t+ permet, sans limites de distance, les correspondances suivantes :

- les correspondances entre les lignes de Métro et les lignes de RER dans Paris, par les cheminements autorisés ;
- les correspondances entre lignes de bus, et entre ces lignes et les lignes de tramway, sur une durée d'une heure trente entre la 1ère et la dernière validation, sous réserve des dispositions suivantes.

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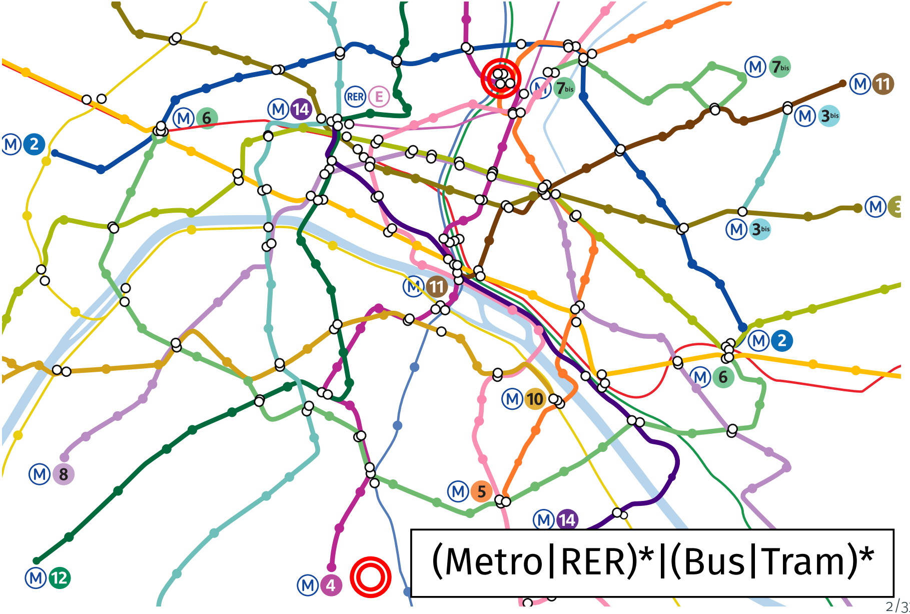
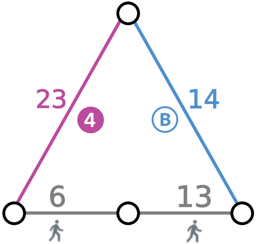


(Metro|RER)*|(Bus|Tram)*

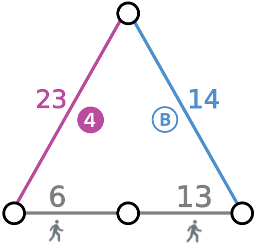
Database



- (Hyper)graph
- Collection of ground facts

$G(aa_1, ab_2), G(ab_2, ac_3),$
$S(aa_1, m_4), S(ab_2, r_B), \ldots$

Database



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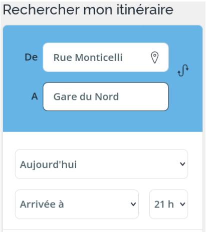
Query



- Regular path

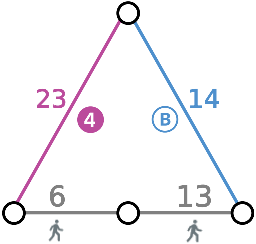
(Metro|RER)*
|(Bus|Tram)*

- Logic formula

$\forall X (rm \in X \land \forall xy$
$(x \in X \land G(x, y) \rightarrow$
$y \in X)) \rightarrow gn \in X$

Database theory and query evaluation

Database



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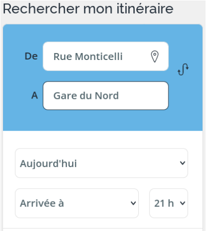

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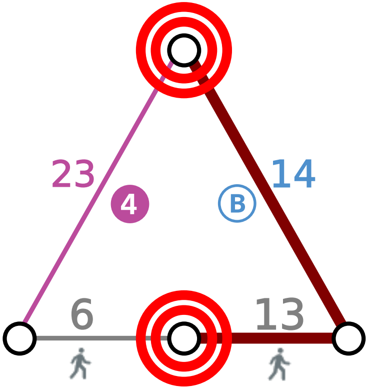
Result

- TRUE/FALSE

↔ Model checking

Probabilistic query evaluation



Panne du RER B : trafic interrompu entre Paris et Roissy, des TGV en renfort

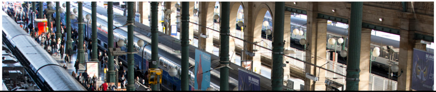
> Transports | 06 décembre 2016, 9h56 | MAJ : 06 décembre 2016, 17h03 |

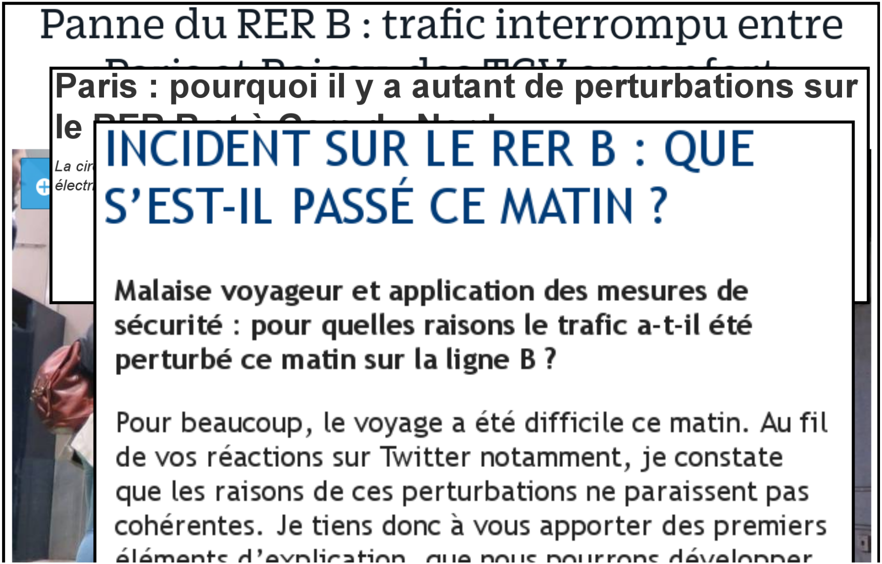
Probabilistic query evaluation

Panne du RER B : trafic interrompu entre

Paris : pourquoi il y a autant de perturbations sur le RER B et à Gare du Nord

La circulation de l'ensemble des trains au départ de gare du Nord est totalement interrompue à la suite d'une panne électrique.





Panne du RER B : trafic interrompu entre

Paris : pourquoi il y a autant de perturbations sur

le

La cir

électr

INCIDENT SUR LE RER B : QUE S'EST-IL PASSÉ CE MATIN ?

Malaise voyageur et application des mesures de sécurité : pour quelles raisons le trafic a-t-il été perturbé ce matin sur la ligne B ?

Pour beaucoup, le voyage a été difficile ce matin. Au fil de vos réactions sur Twitter notamment, je constate que les raisons de ces perturbations ne paraissent pas cohérentes. Je tiens donc à vous apporter des premiers

Probabilistic query evaluation

Panne du RER B : trafic interrompu entre

Paris : pourquoi il y a autant de perturbations sur

le

ACTUALITÉS

Le RER B en panne, les voyageurs n'ont pas eu d'autre choix que de descendre sur les voies

Alors que la circulation alternée a augmenté le nombre de voyageurs dans les transports en commun, le RER B s'est retrouvé à l'arrêt.

06/12/2016 11:57 CET | **Actualisé** 06/12/2016 20:14 CET

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INCIDENT SUR LE RER B : QUE

ACTUALITÉS

Le RER B en panne, les voyageurs n'ont pas eu

RER B et D en panne, gare du Nord paralysée, pollution: deuxième jour de galère

Actualité / Société / Trafic / Par Iris Péron, publié le 07/12/2016 à 13:40 , mis à jour à 16:07

partages

Partager

Tweeter

Partager

0

réaction

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Probabilistic query evaluation

Panne du RER B : trafic interrompu entre

Paris : pourquoi il y a autant de perturbations sur le

ACTUALITÉS

Le RER B

RER B e

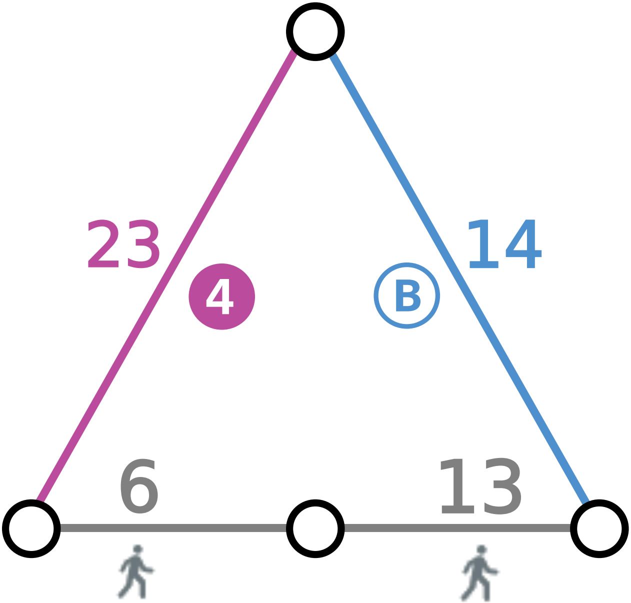
Ile-de-France : le trafic toujours interrompu sur le RER B entre Aulnay-sous-Bois et Roissy

La circulation est arrêtée depuis mardi matin en raison d'une panne de caténaire. Le retour à la normale a été plusieurs fois retardé mais devrait avoir lieu mercredi vers 16 heures, selon la SNCF.

que

cohé

Probabilistic query evaluation



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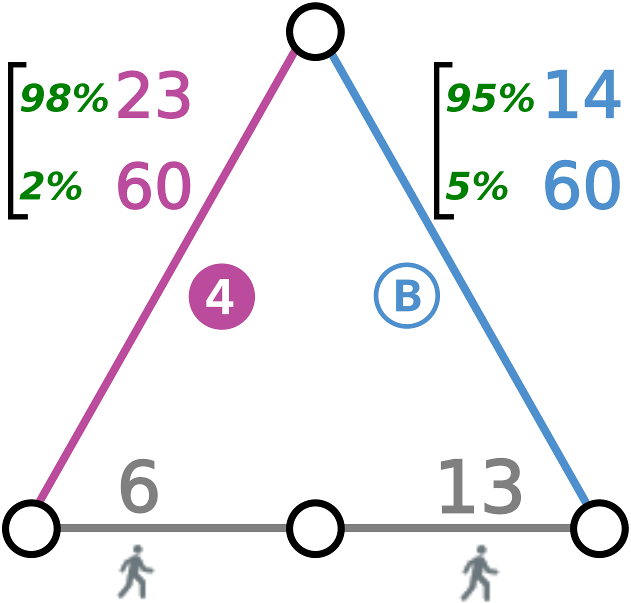
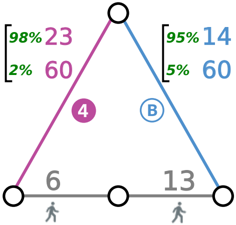


Probabilistic database

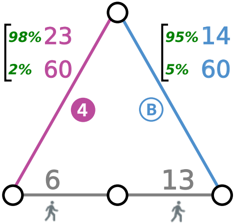


- (Hyper)graph
- Collection of ground facts

+ independent probabilities

Probabilistic query evaluation

Probabilistic database



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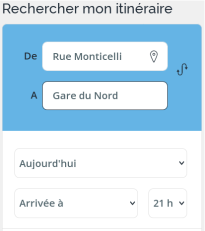
Query



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Probabilistic query evaluation

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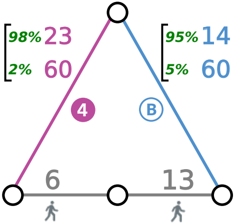

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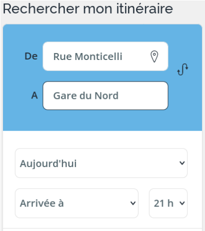

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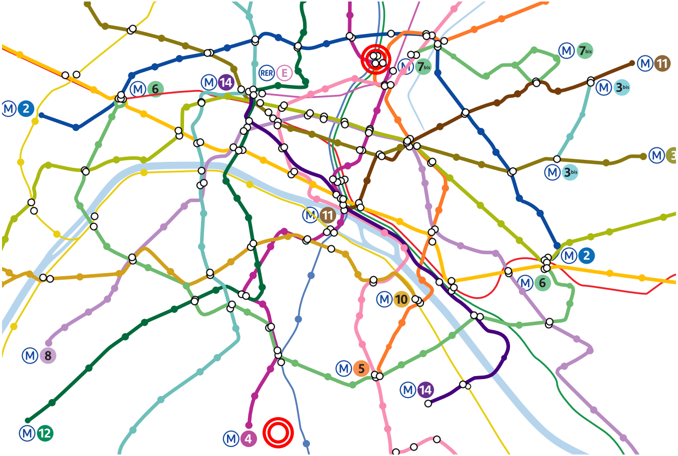
Probabilistic Result

proba to be on time: **98%**

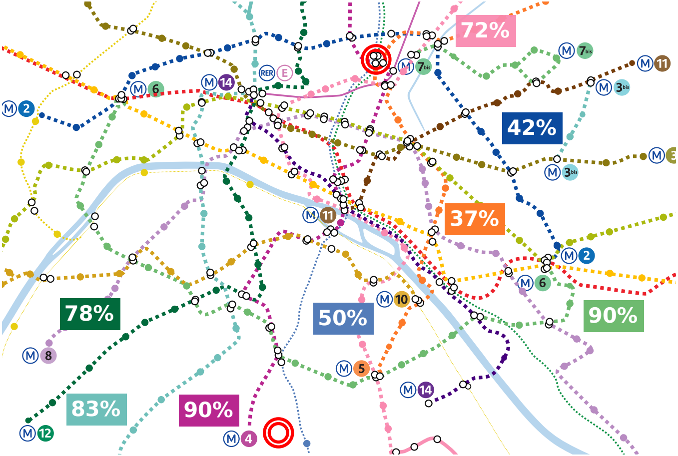
- Probability according to the input distribution

Computational complexity



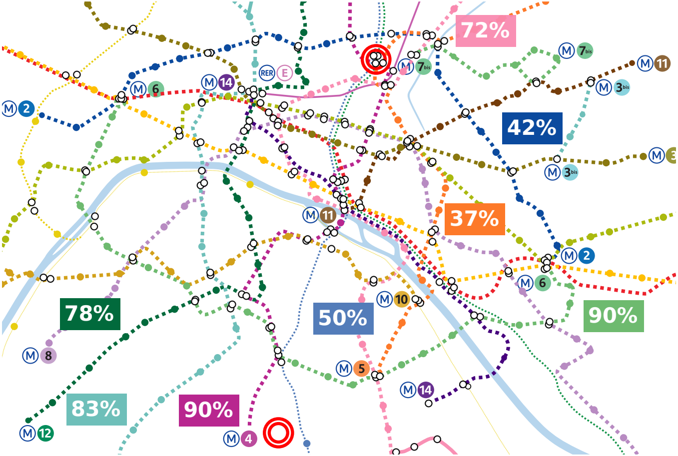
- Computing paths on a large graph:
  → Well-studied problem, efficient algorithms

Computational complexity



- Computing paths on a large probabilistic graph:
  - → ???

Computational complexity



- Computing paths on a large **probabilistic** graph:
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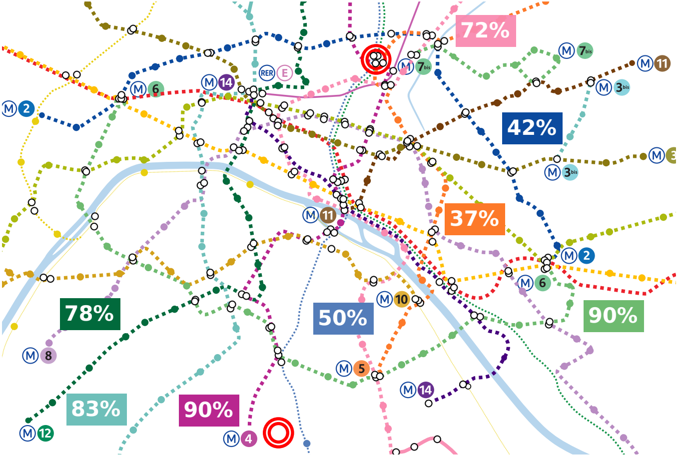
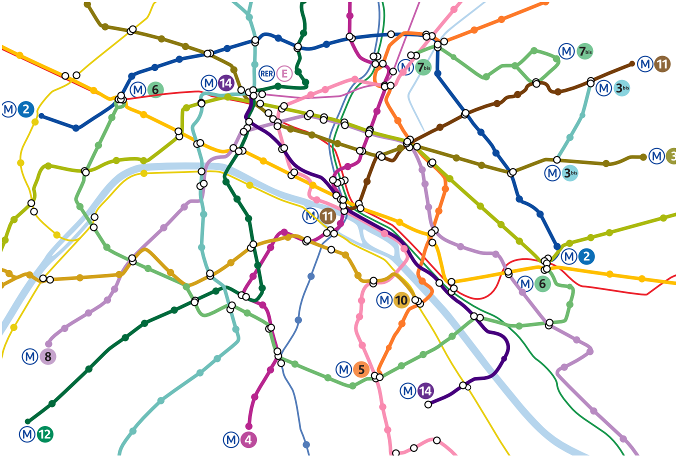
Computational complexity



- Computing paths on a large **probabilistic** graph:
  - → **Exponential** number of possibilities
  - → **#P-hard** computational complexity in the **database**

Idea: use the structure of data



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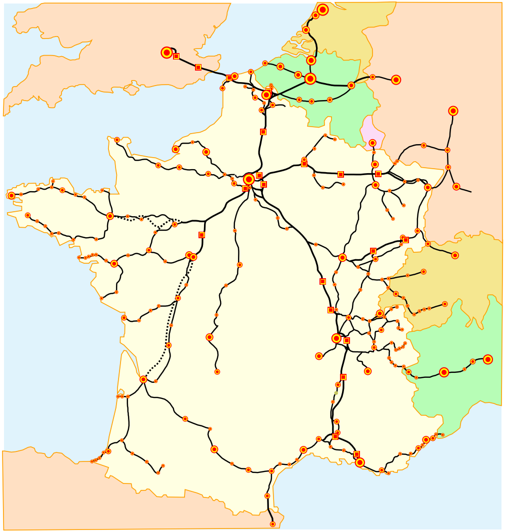

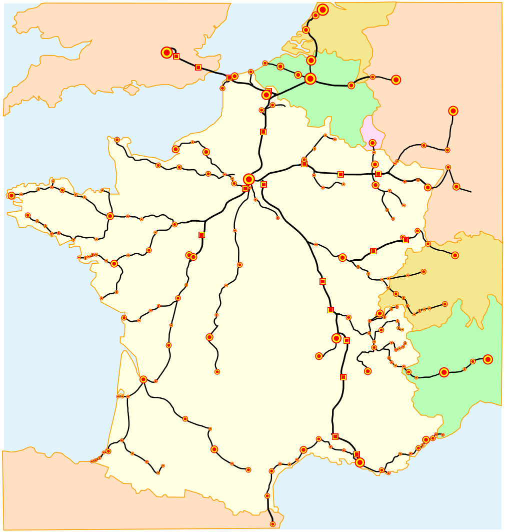

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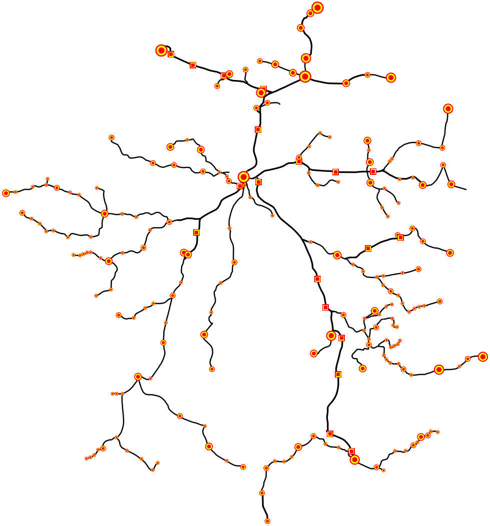

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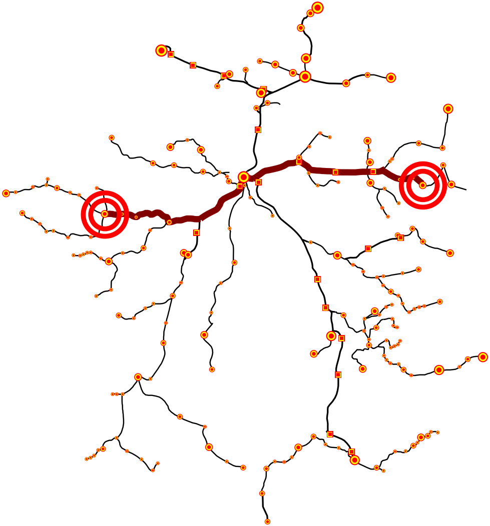

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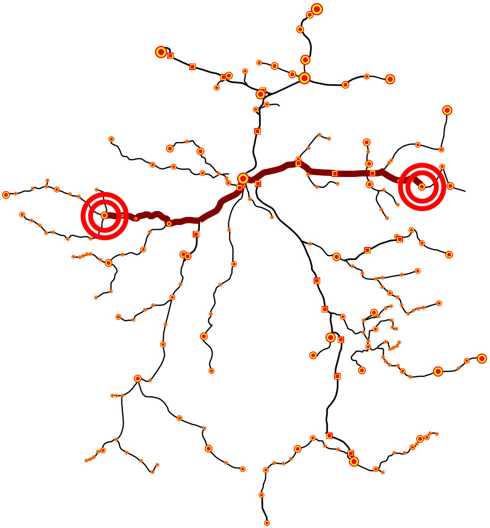

→ Shortest path: very easy on a large tree

Leveraging the structure of uncertain data

*Does query evaluation on probabilistic data have **lower complexity** when the **structure** of the data is **close to a tree?***

Leveraging the structure of uncertain data

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- Other application: enumeration of query results

Table of contents

Query evaluation on words

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"Is there both a pink and a blue node?"

Query evaluation on words

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Result: TRUE/FALSE indicating if the word w satisfies the query Q

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"Is there both a pink and a blue node?"

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Computational complexity as a function of w (the query Q is **fixed**)

Monadic second-order logic (MSO)

- $P_{\text{blue}}(x)$ means "x is blue"; also $P_{\text{red}}(x)$, $P_{\bigcirc}(x)$
- $x \to y$ means "x is the predecessor of y"

Monadic second-order logic (MSO)

- $P_{\text{blue}}(x)$ means "x is blue"; also $P_{\text{pink}}(x)$, $P_{\text{white}}(x)$
- $x \to y$ means "x is the predecessor of y"

- Propositional logic: formulas with AND $\wedge$, OR $\vee$, NOT $\neg$
  - $P_{\text{pink}}(x) \wedge P_{\text{blue}}(y)$ means *"Node x is pink and node y is blue"*

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- First-order logic: adds existential quantifier $\exists$ and universal quantifier $\forall$
  - $\exists x\, y\; P_{\color{pink}\bullet}(x) \wedge P_{\color{blue}\bullet}(y)$ means *"There is both a pink and a blue node"*

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- $P_{\color{blue}\bullet}(x)$ means "x is blue"; also $P_{\color{pink}\bullet}(x)$, $P_{\bigcirc}(x)$
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  - $\exists x\, y\; P_{\color{pink}\bullet}(x) \wedge P_{\color{blue}\bullet}(y)$ means *"There is both a pink and a blue node"*

- **Monadic second-order logic (MSO):** adds **quantifiers over sets**
  - $\exists S\; \forall x\; S(x)$ means *"there is a set S containing every element x"*
  - Can express **transitive closure** $x \to^* y$, i.e., *"x is before y"*
  - $\forall x\, P_{\color{pink}\bullet}(x) \Rightarrow \exists y\, P_{\color{blue}\bullet}(y) \wedge x \to^* y$
    means *"There is a blue node after every pink node"*

Translate the query Q to a deterministic word automaton

Alphabet: 

w: 

Q: $\exists x\, y\; P_{\circ}(x) \wedge P_{\circ}(y)$

Word automata

Translate the query Q to a deterministic word automaton

Alphabet:  w:  Q: $\exists x\, y\; P_{\bigcirc}(x) \land P_{\bigcirc}(y)$

- States: $\{\bot, B, P, \top\}$

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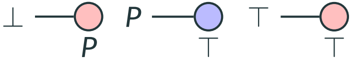
- Transitions (examples): 

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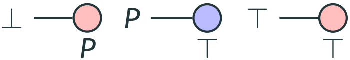
- **Transitions** (examples): 

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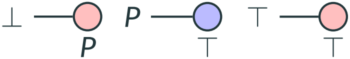
- Transitions (examples): 

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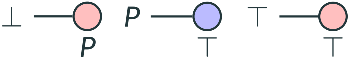
- Transitions (examples): 

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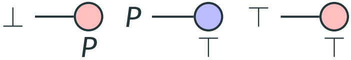
- Transitions (examples): 

Translate the query Q to a deterministic word automaton

Alphabet: 

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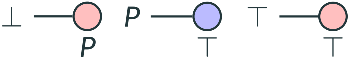
- Transitions (examples): 

Theorem (Büchi, 1960)

MSO and word automata have the same expressive power on words

Word automata

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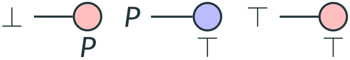
- Transitions (examples): 

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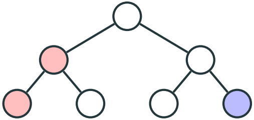
Query evaluation of MSO on words is in linear time.

Query evaluation on trees

Database: a **tree** T where nodes have a color from an alphabet



Query evaluation on trees

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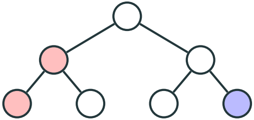

Query Q: a **sentence** in monadic second-order logic (MSO)

- $P_{\text{blue}}(x)$ means "x is blue"
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"Is there both a pink and a blue node?"

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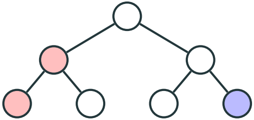

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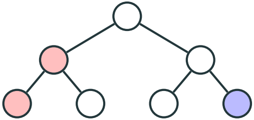

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Tree automata

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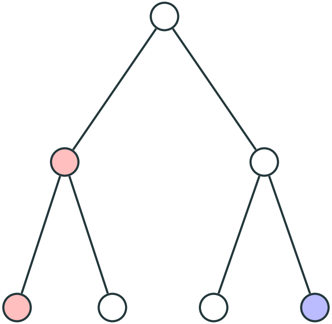

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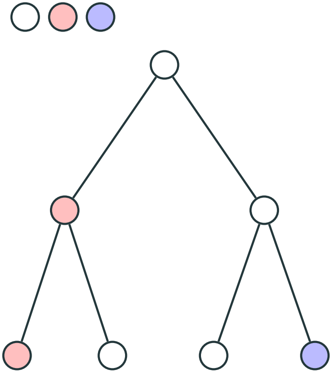

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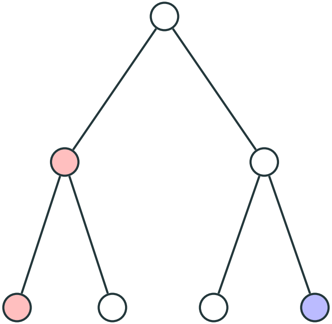

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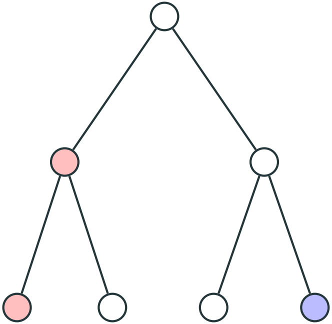

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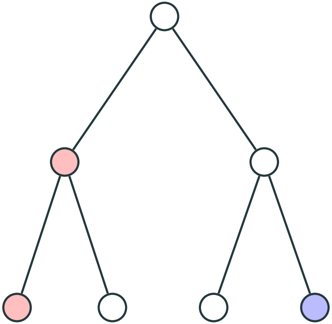

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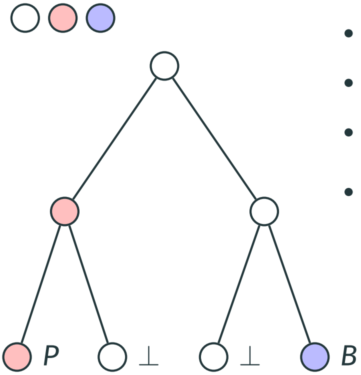

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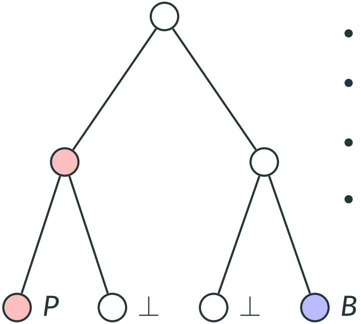

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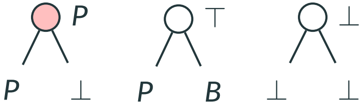

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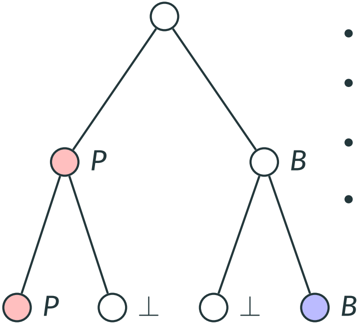

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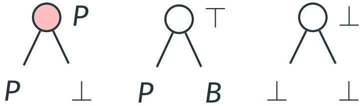

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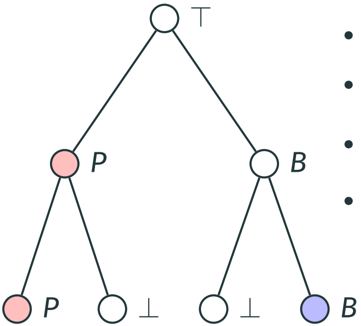

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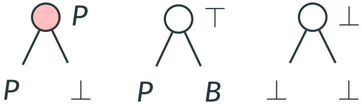

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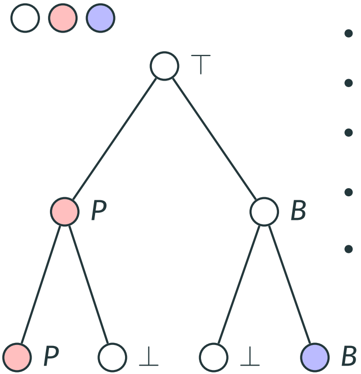

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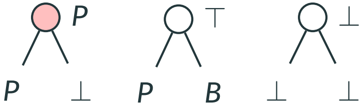

Theorem [Thatcher and Wright, 1968]

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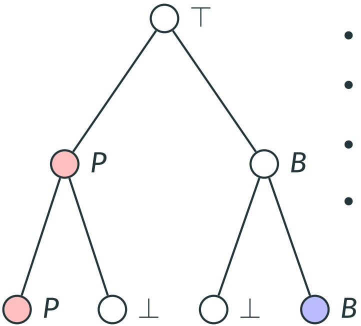

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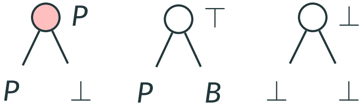

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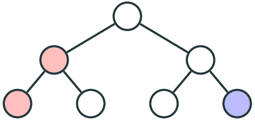

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Result: TRUE/FALSE indicating if T satisfies the query Q

Computational complexity as a function of T

(the query Q is **fixed**)

Query evaluation on treelike data

Database: a **treelike database** T

???

Query Q: a **sentence** in monadic second-order logic (MSO)

(Metro|RER)*
| (Bus|Tram)*

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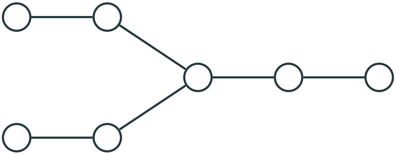

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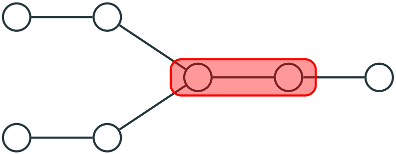

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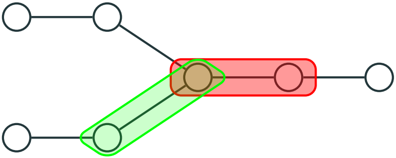

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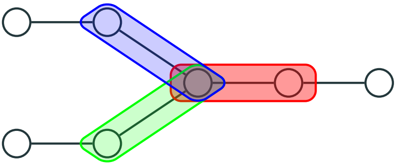

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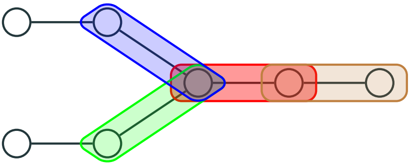

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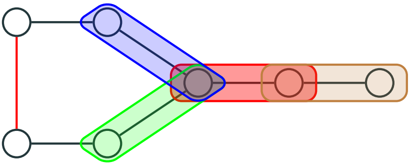

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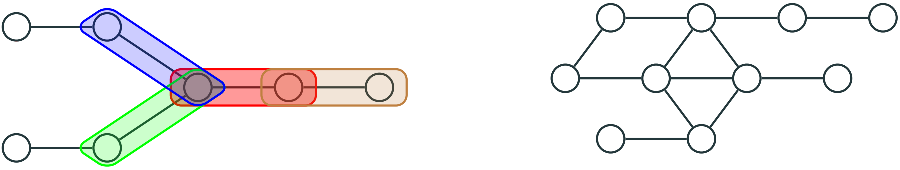

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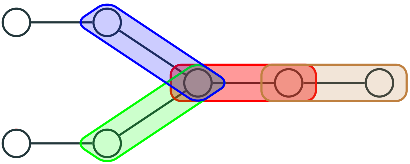

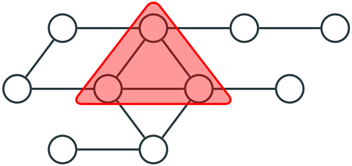

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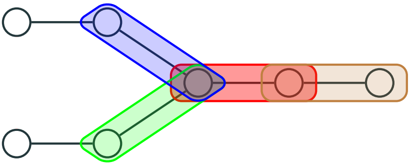

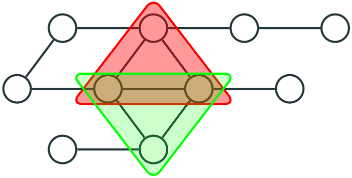

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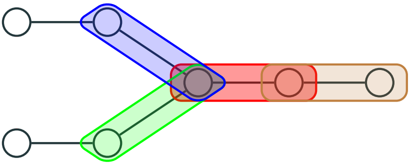

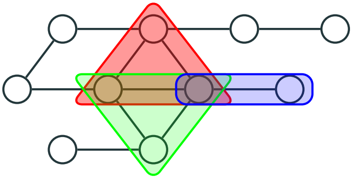

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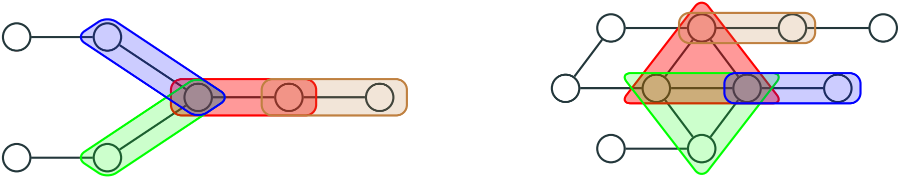

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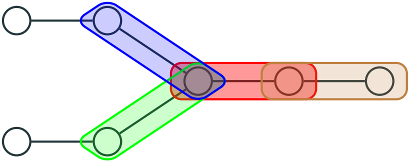

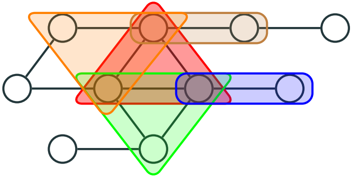

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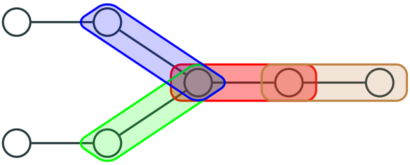

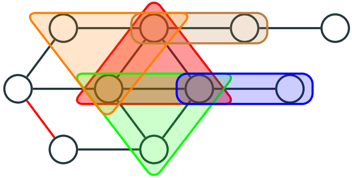

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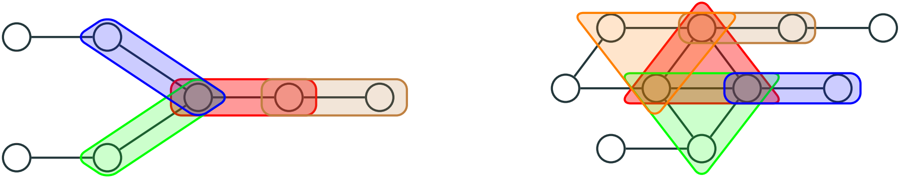

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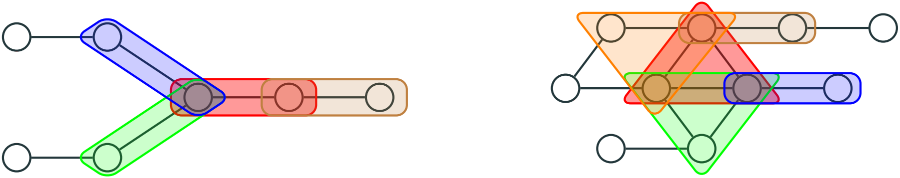

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- **Cycles** have treewidth 2
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Treewidth

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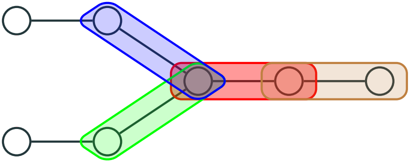

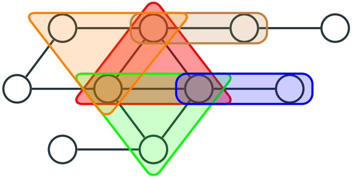

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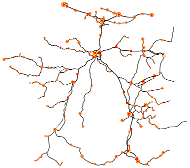
$\rightarrow$ Treelike: the treewidth is bounded by a constant

Courcelle's theorem

Treelike **data**



MSO query

(RER|metro)*
|(bus|tram)*

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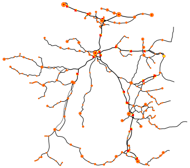

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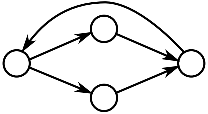
Tree **automaton**



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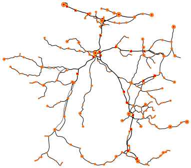
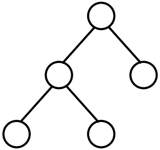


linear

Tree **encoding**



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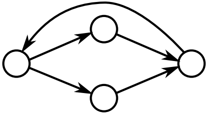

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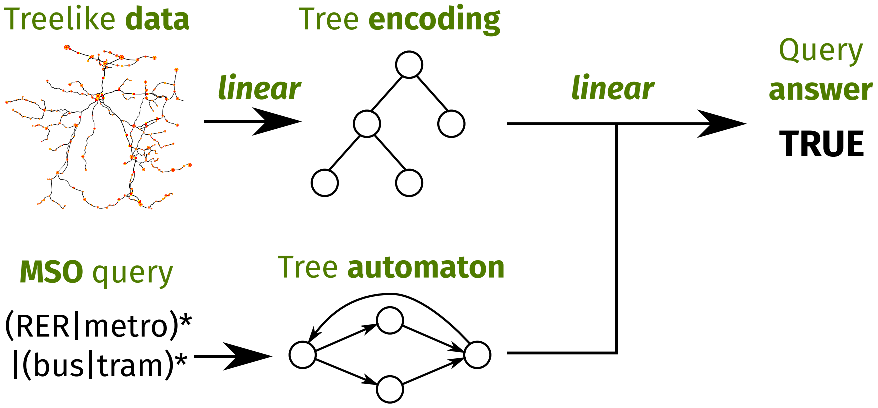

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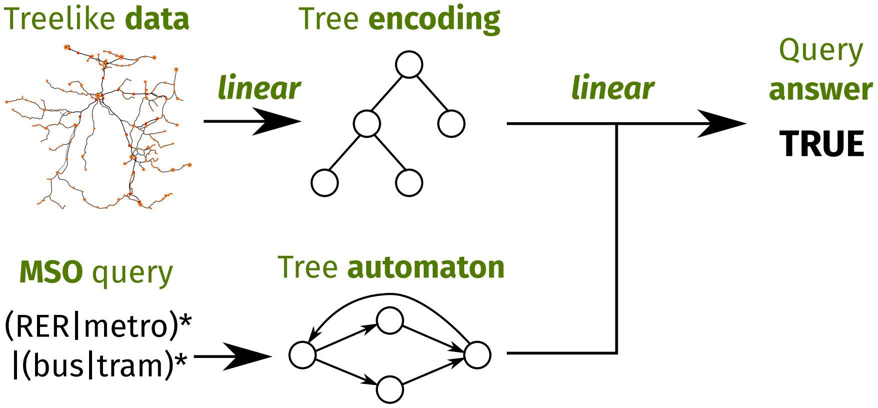

Theorem [Courcelle, 1990]

For any fixed Boolean MSO query ***Q*** *and* $k \in \mathbb{N}$*,*
given a database ***D*** *of treewidth* $\leq k$*,*
we can compute in ***linear time*** *in* ***D*** *whether* ***D*** *satisfies* ***Q***

Table of contents

Probabilistic query evaluation on treelike data

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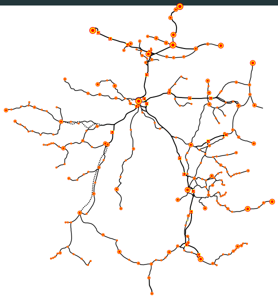
- Probability of each fact of D to be actually present in the data (independently from other facts)



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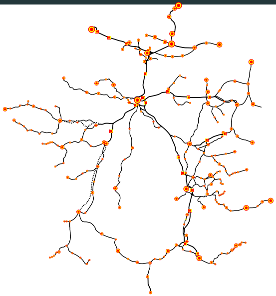

Query Q: a sentence in monadic second-order logic (MSO)

(Metro|RER)*
| (Bus|Tram)*

Probabilistic query evaluation on treelike data

- Database D with treewidth $\leq k$ for some constant k
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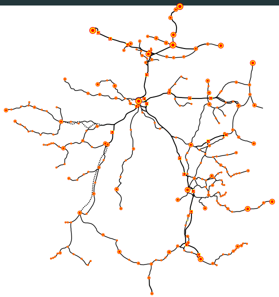

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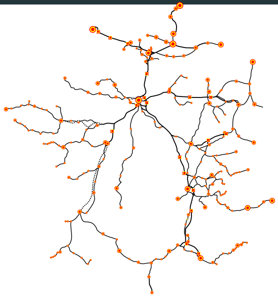

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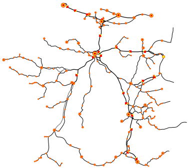
Computational complexity as a function of the **database** D (the query Q is **fixed**)

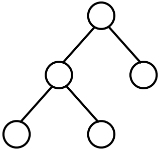
Roadmap

Treelike **data**



linear →

Tree **encoding**



MSO query

(RER|metro)*
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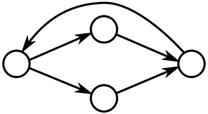
Tree **automaton**



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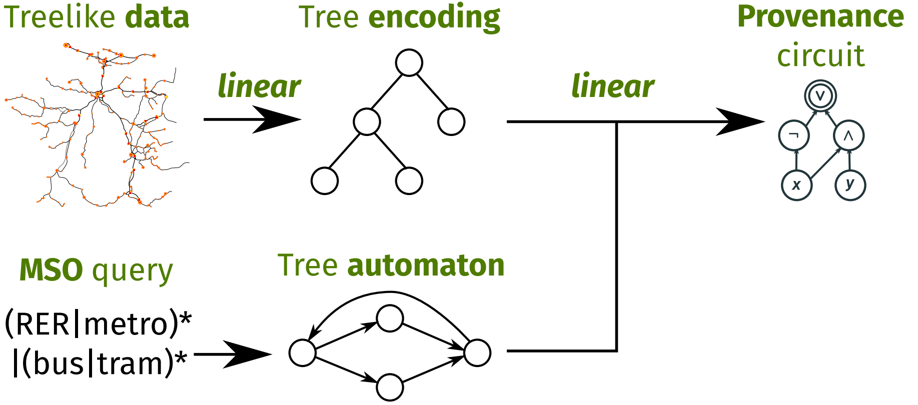
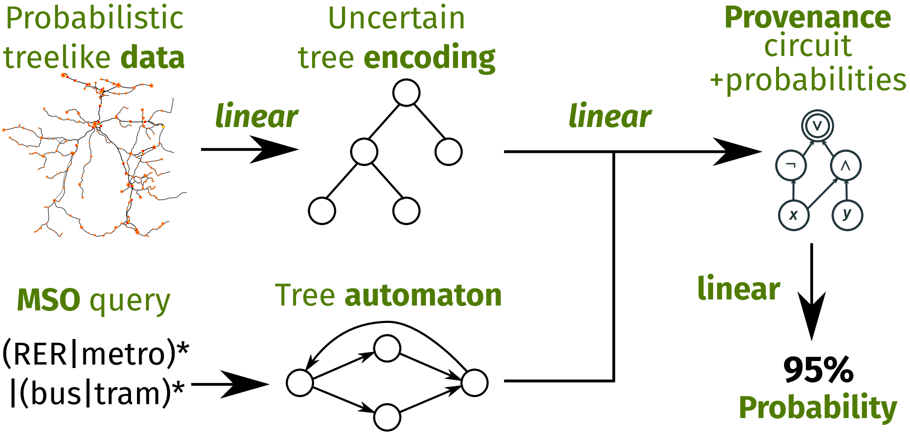


Treelike data
linear
Tree encoding
linear
Provenance circuit
∨
¬
∧
x
y
MSO query
(RER|metro)*
|(bus|tram)*
Tree automaton

Roadmap


Probabilistic treelike **data**
linear
Uncertain tree **encoding**
linear
Provenance circuit +probabilities
MSO query
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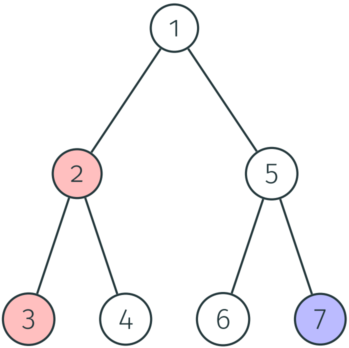
linear
95% Probability

Uncertain trees



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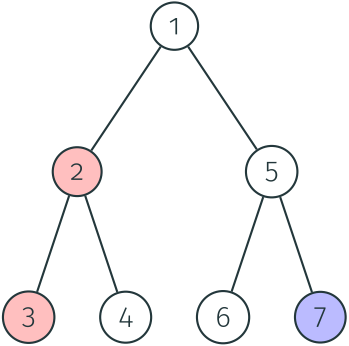

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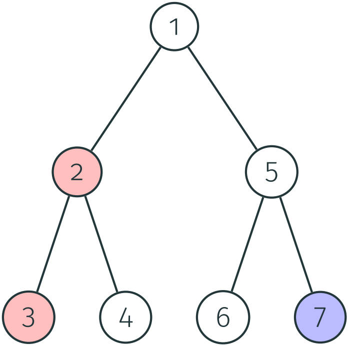

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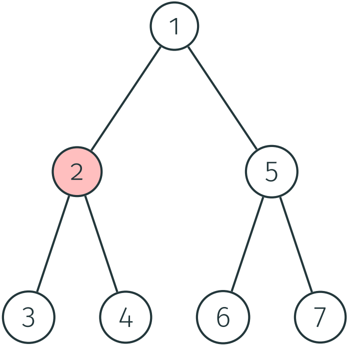

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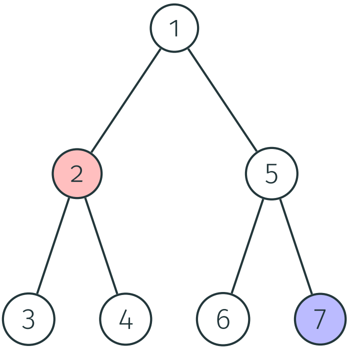

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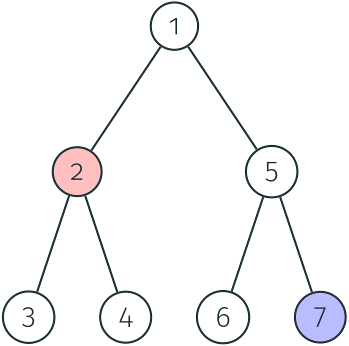

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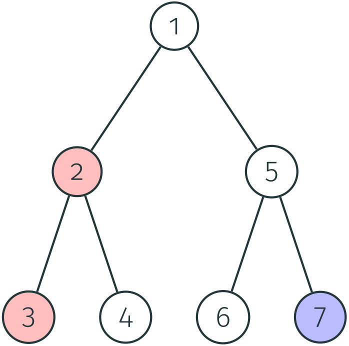

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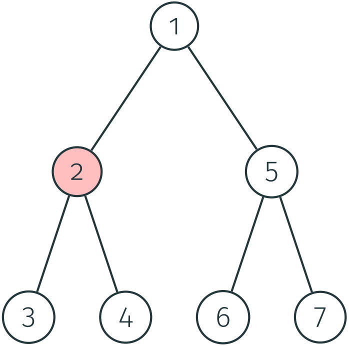

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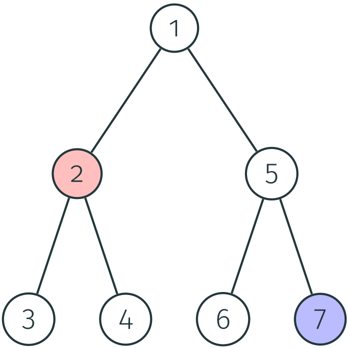

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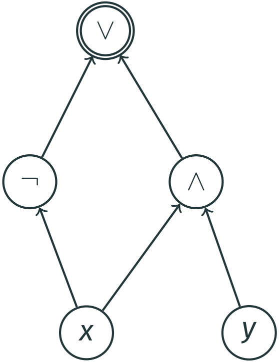
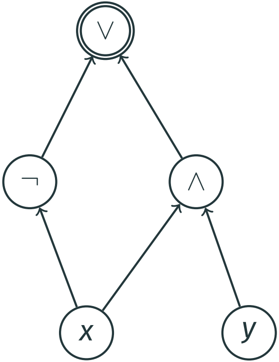


- Directed acyclic graph of gates

Boolean circuit



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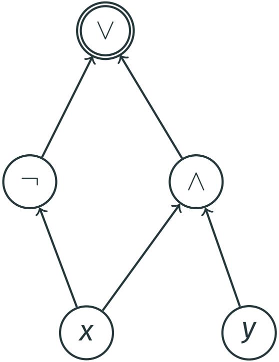
Boolean circuit



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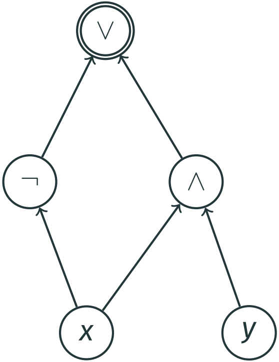
Boolean circuit



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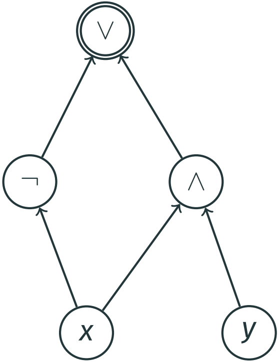
Boolean circuit



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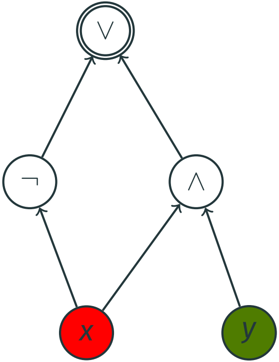

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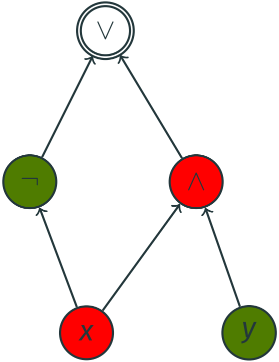

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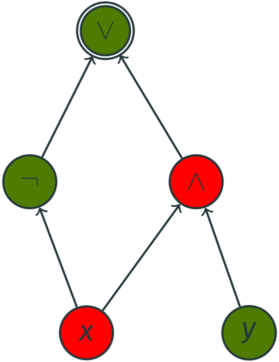

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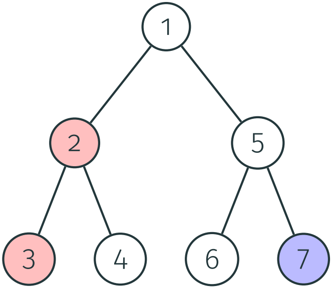
Provenance circuit



Query: *Is there both a pink and a blue node?*

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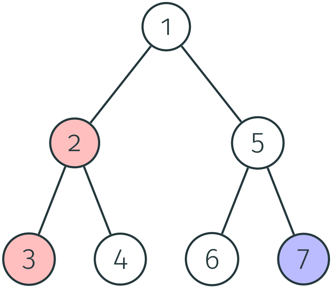

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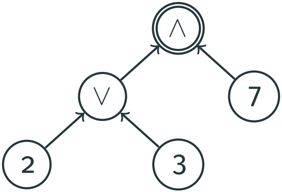

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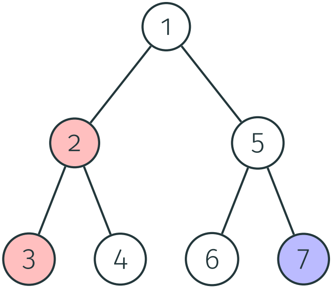

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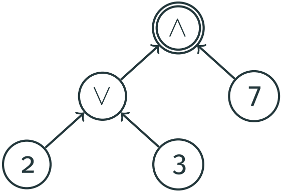

Formally:

- Tree automaton A, uncertain tree T, circuit C
- **Variable gates** of C: nodes of T
- **Condition:** Let ν be a valuation of T, then $\nu(C)$ iff A accepts $\nu(T)$

Building provenance circuits on trees

Theorem

*For any bottom-up **tree automaton** A and input **tree** T,
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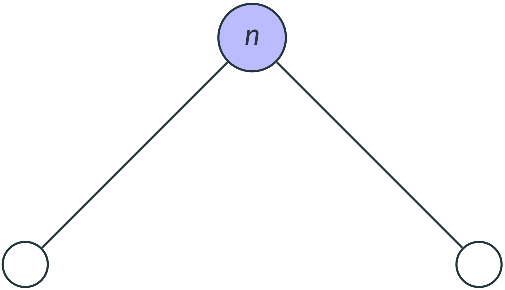

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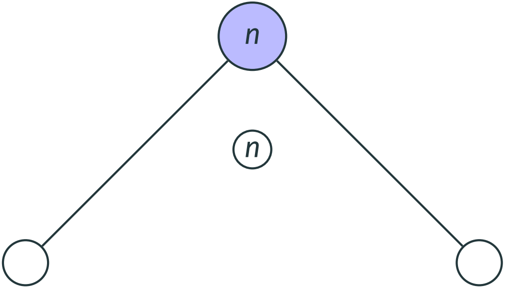

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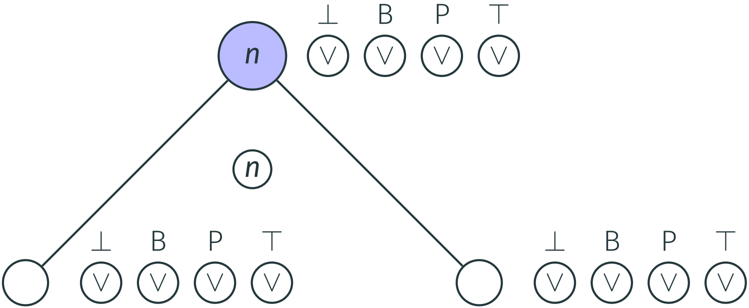

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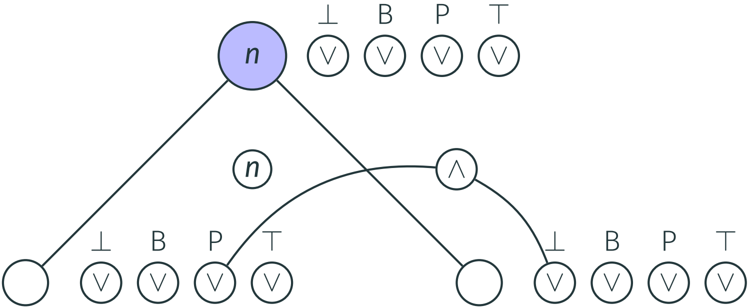

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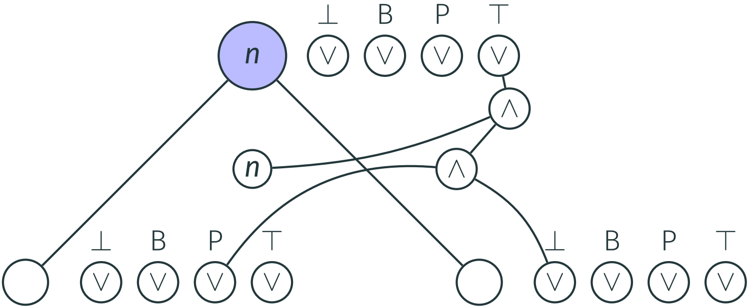

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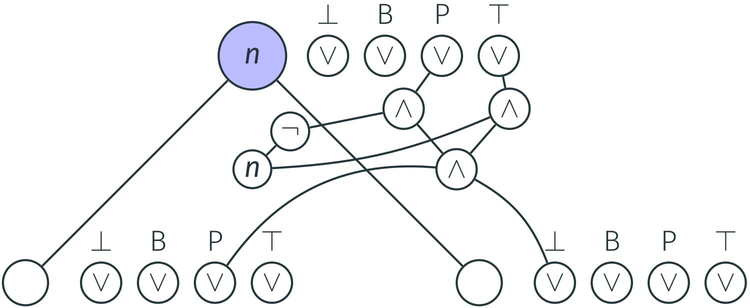
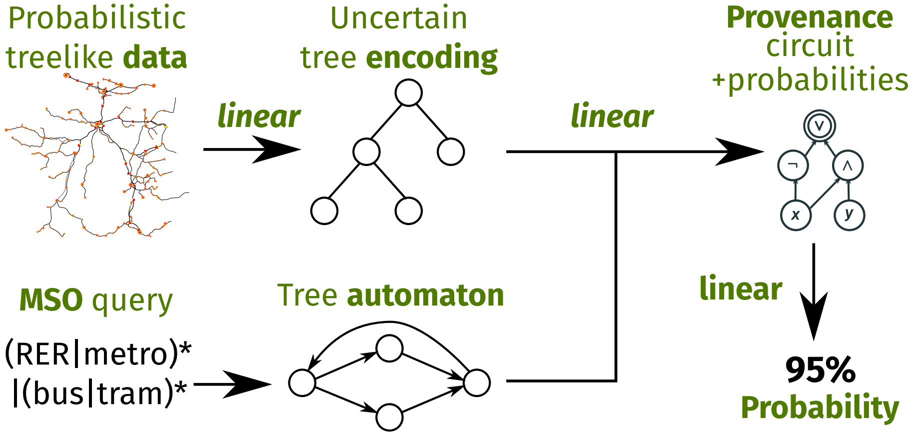
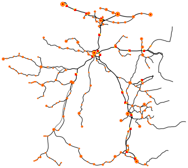


Probabilistic query evaluation



Details of the approach

Probabilistic treelike **data**



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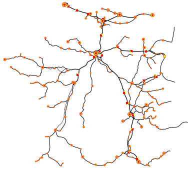

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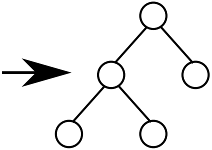

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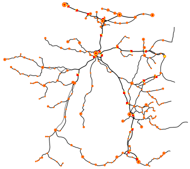

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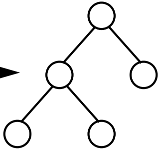

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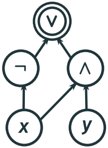
Provenance circuit +probabilities



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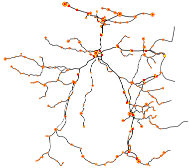

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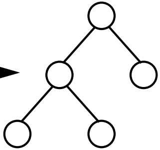

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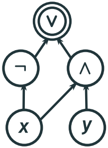

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Probability

95%

*Probability that the **circuit** evaluates to **true***

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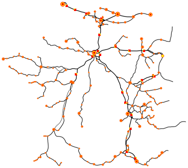

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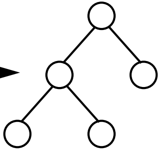

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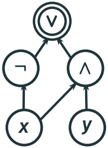

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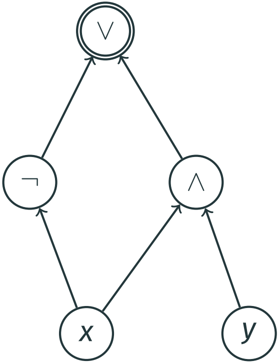
→ How to compute **efficiently** the probability of the circuit?

Computing the probability of a circuit

- We are given a **circuit** and a **probability** P for each variable



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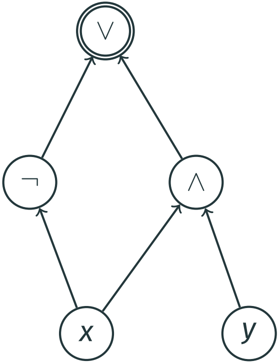

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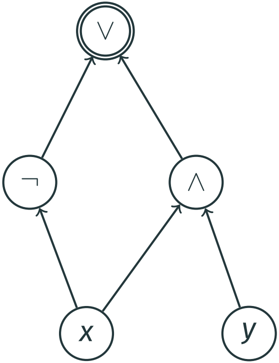

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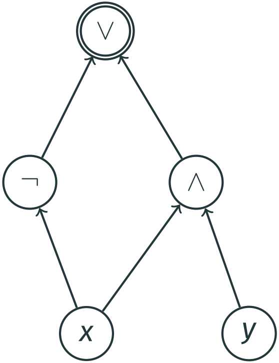

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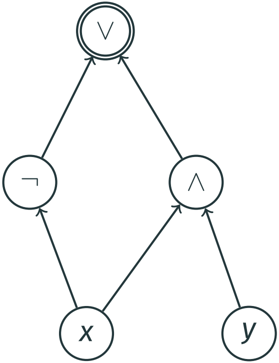

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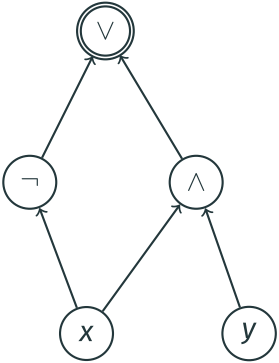

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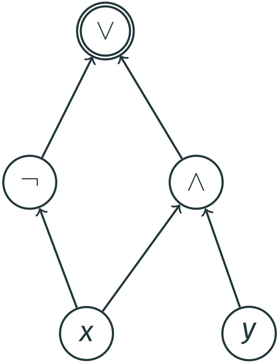

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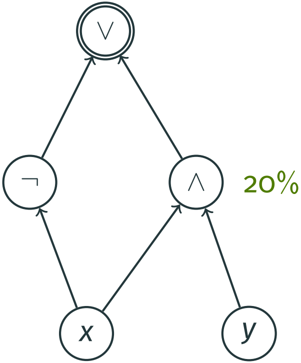

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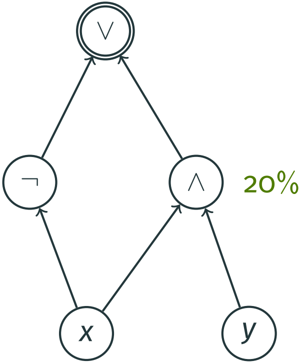

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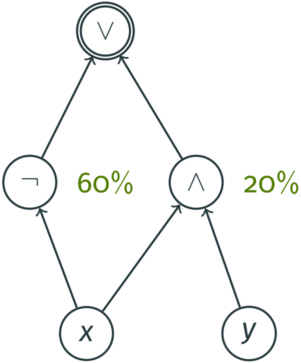

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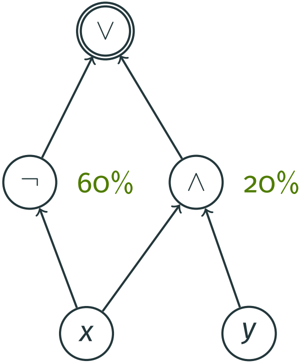

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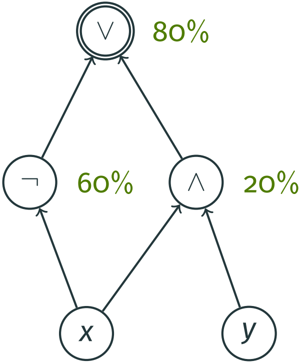

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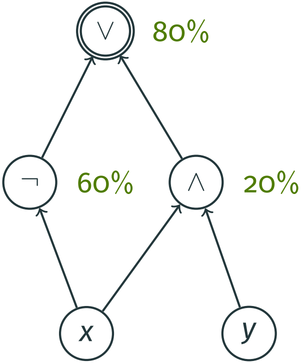

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- Let's focus on a **restricted class** of circuits that satisfies these conditions

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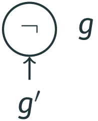

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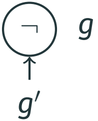

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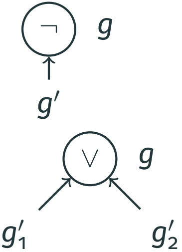

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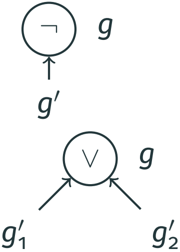

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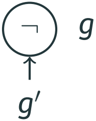

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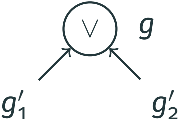

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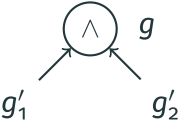

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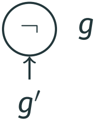

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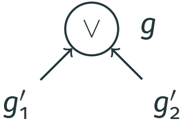

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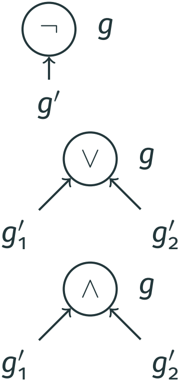

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Lemma

The provenance circuit computed in our construction is a d-DNNF

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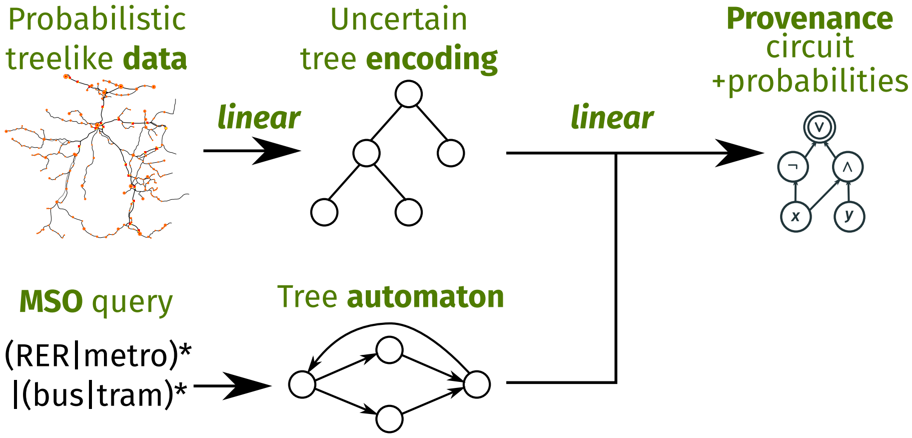

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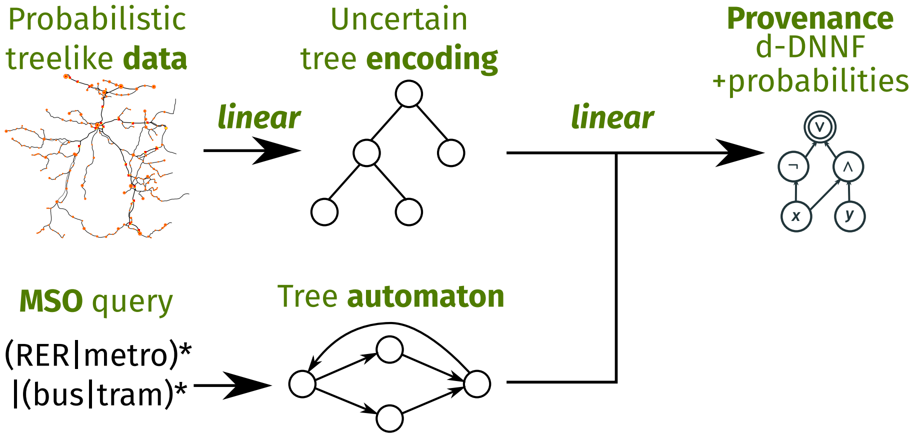

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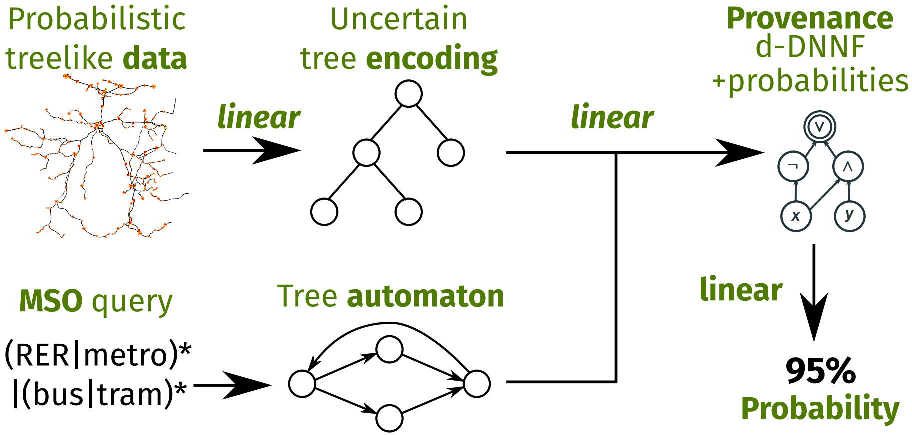

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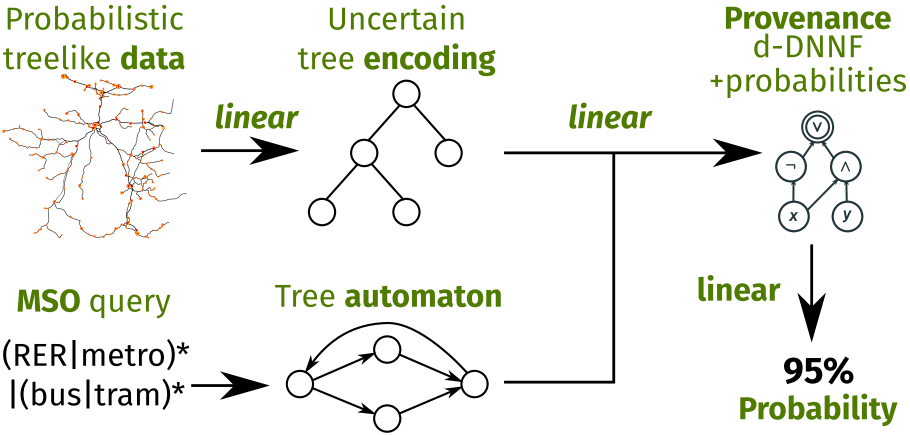

Theorem [Amarilli et al., 2015]

For any ***fixed*** *Boolean MSO query* $\boldsymbol{Q}$ *and* $\boldsymbol{k} \in \mathbb{N}$,
given a database $\boldsymbol{D}$ *of* ***treewidth*** $\leq \boldsymbol{k}$ *with* ***independent probabilities***,
we can compute in ***linear time*** *the probability that* $\boldsymbol{D}$ *satisfies* $\boldsymbol{Q}$

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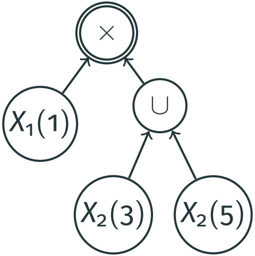

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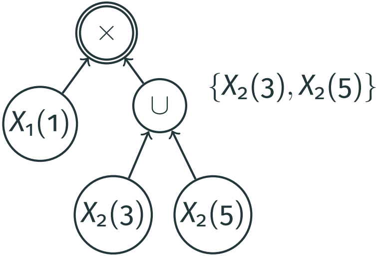

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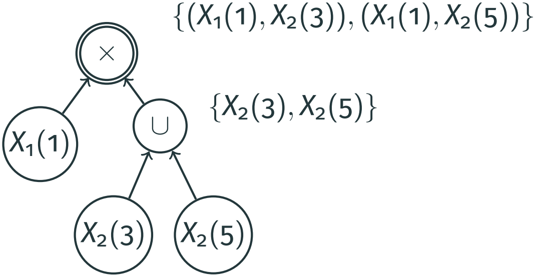

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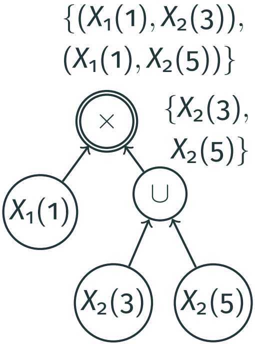

- First, preprocess the circuit in **linear time**
- Then, produce each result in **constant time**

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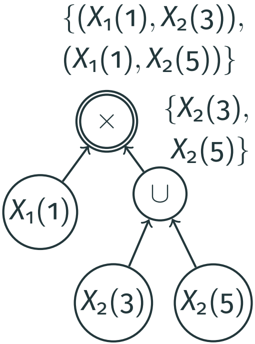

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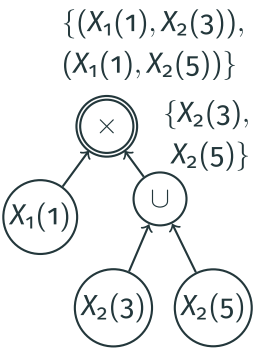

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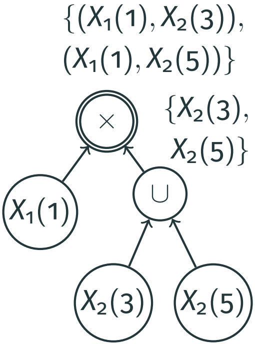

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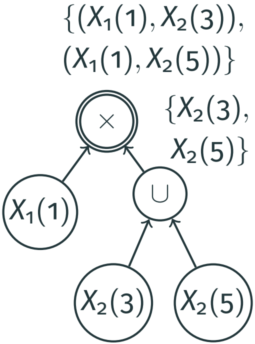

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Conclusion and perspectives

- Other results:
  - **Lower bounds:** probabilistic query evaluation is hard unless treewidth is bounded (modulo assumptions) [Amarilli et al., 2016]
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Thanks for your attention!

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