Enumerating Pattern Matches in Texts and Trees

Antoine Amarilli[1], Pierre Bourhis[2], Stefan Mengel[3], Matthias Niewerth[4]

October 11th, 2018

[1]Télécom ParisTech

[2]CNRS CRIStAL

[3]CNRS CRIL

[4]Universität Bayreuth

Problem: Finding Patterns in Text

- We have a long text T:

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.
French national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP
a3nm@a3nm.net Affiliation Associate professor of computer science (office C201-4) in the DIG team of
Télécom ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France.  Studies PhD in computer science
awarded by Télécom ParisTech on March 14, 2016.  Former student of the École normale supérieure.
More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
```

Problem: Finding Patterns in Text

- We have a long text T:

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.
French national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP
a3nm@a3nm.net Affiliation Associate professor of computer science (office C201-4) in the DIG team of
Télécom ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France.  Studies PhD in computer science
awarded by Télécom ParisTech on March 14, 2016.  Former student of the École normale supérieure.
More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
```

- We want to find a pattern P in the text T:
  - → Example: find email addresses

Problem: Finding Patterns in Text

- We have a long text T:

```
Antoine Amarilli Description Name Antoine Amarilli. Handle: a3nm. Identity Born 1990-02-07.
French national. Appearance as of 2017. Auth OpenPGP. OpenId. Bitcoin. Contact Email and XMPP
a3nm@a3nm.net Affiliation Associate professor of computer science (office C201-4) in the DIG team of
Télécom ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France. Studies PhD in computer science
awarded by Télécom ParisTech on March 14, 2016. Former student of the École normale supérieure.
More Résumé Location Other sites Blogging: a3nm.net/blog Git: a3nm.net/git ...
```

- We want to find a pattern P in the text T:
  - → Example: find email addresses
    - Write the pattern as a regular expression:

$$P := \text{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$

Problem: Finding Patterns in Text

- We have a long text T:

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.
French national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP
a3nm@a3nm.net Affiliation Associate professor of computer science (office C201-4) in the DIG team of
Télécom ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France.  Studies PhD in computer science
awarded by Télécom ParisTech on March 14, 2016.  Former student of the École normale supérieure.
More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
```

- We want to find a pattern P in the text T:
  → Example: find email addresses
  - Write the pattern as a regular expression:

$$P := \text{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$

→ How to find the pattern P efficiently in the text T?

Solution: Automata

- Convert the regular expression P to an automaton A

Solution: Automata

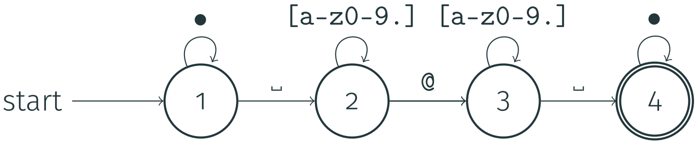
- Convert the regular expression P to an automaton A

$$P := \texttt{␣ [a-z0-9.]}^{*}\ \texttt{@ [a-z0-9.]}^{*}\ \texttt{␣}$$

Solution: Automata

- Convert the regular expression *P* to an automaton *A*

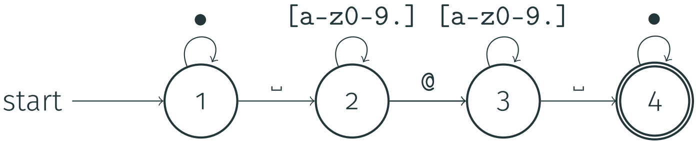
$$P := \texttt{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$



Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \texttt{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$

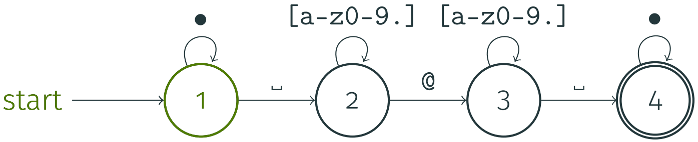


- Then, evaluate the automaton on the text T

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \texttt{␣ [a-z0-9.]}^* \texttt{ @ [a-z0-9.]}^* \texttt{ ␣}$$



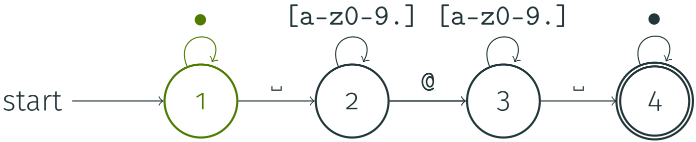
- Then, evaluate the automaton on the text T

E	m	a	i	l	␣	a	3	n	m	@	a	3	n	m	.	n	e	t	␣	A	f	f	i	l	i	a	t	i	o	n

Solution: Automata

- Convert the **regular expression** P to an **automaton** A

$P :=$ `␣ [a-z0-9.]* @ [a-z0-9.]* ␣`



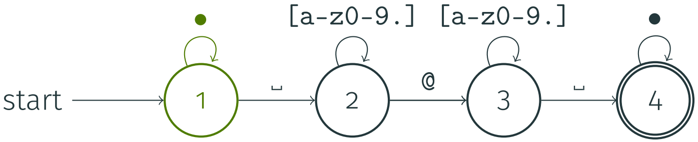
- Then, evaluate the automaton on the **text** T

`Email␣a3nm@a3nm.net␣Affiliation`

Solution: Automata

- Convert the **regular expression** P to an **automaton** A

$P :=$ ␣ [a-z0-9.]* @ [a-z0-9.]* ␣



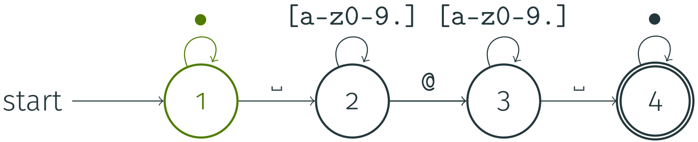
- Then, evaluate the automaton on the **text** T

E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \text{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$



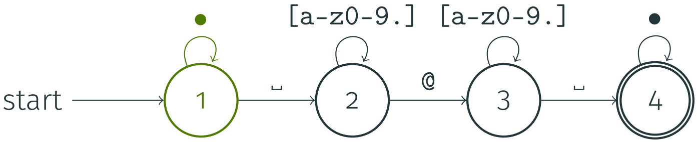
- Then, evaluate the automaton on the text T

E	m	a	i	l	␣	a	3	n	m	@	a	3	n	m	.	n	e	t	␣	A	f	f	i	l	i	a	t	i	o	n

Solution: Automata

- Convert the **regular expression** P to an **automaton** A

$P :=$ `␣ [a-z0-9.]* @ [a-z0-9.]* ␣`



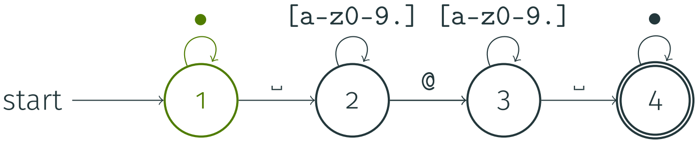
- Then, evaluate the automaton on the **text** T

`Email␣a3nm@a3nm.net␣Affiliation`

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \text{␣ [a-z0-9.]}^* \text{ @ [a-z0-9.]}^* \text{ ␣}$$



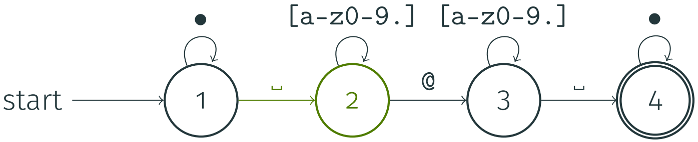
- Then, evaluate the automaton on the text T

`Email␣a3nm@a3nm.net␣Affiliation`

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \texttt{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$



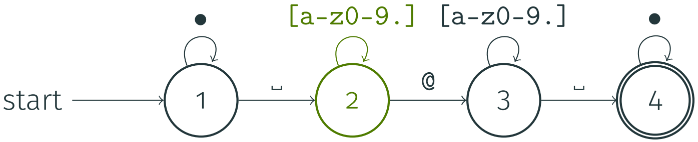
- Then, evaluate the automaton on the text T

`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \texttt{␣ [a-z0-9.]}^* \texttt{ @ [a-z0-9.]}^* \texttt{ ␣}$$



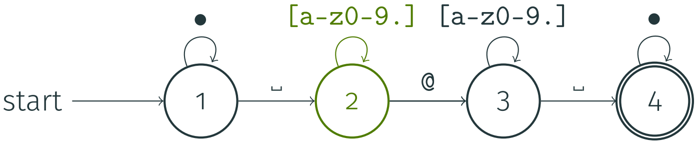
- Then, evaluate the automaton on the text T

`Email␣a3nm@a3nm.net␣Affiliation`

Solution: Automata

- Convert the **regular expression** P to an **automaton** A

$P :=$ `␣ [a-z0-9.]* @ [a-z0-9.]* ␣`



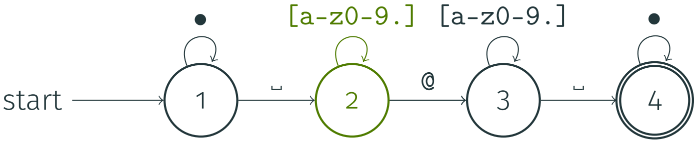
- Then, evaluate the automaton on the **text** T

`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

Solution: Automata

- Convert the **regular expression** P to an **automaton** A

$$P := \texttt{␣ [a-z0-9.]}^* \texttt{ @ [a-z0-9.]}^* \texttt{ ␣}$$



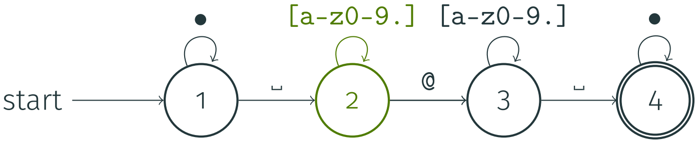
- Then, evaluate the automaton on the **text** T

`Email␣a3nm@a3nm.net␣Affiliation`

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \text{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$



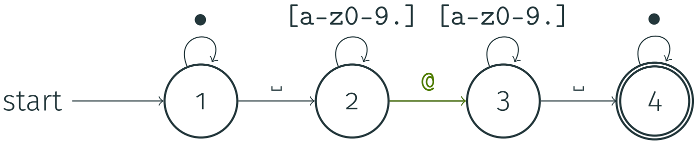
- Then, evaluate the automaton on the text T

`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

Solution: Automata

- Convert the **regular expression** P to an **automaton** A

$P :=$ `␣ [a-z0-9.]* @ [a-z0-9.]* ␣`



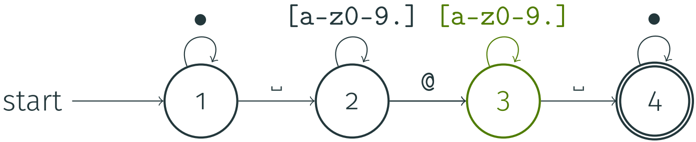
- Then, evaluate the automaton on the **text** T

`Email␣a3nm@a3nm.net␣Affiliation`

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \text{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$



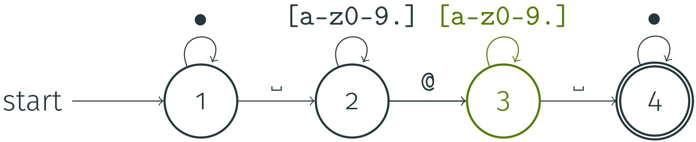
- Then, evaluate the automaton on the text T

`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \texttt{␣ [a-z0-9.]}^* \texttt{ @ [a-z0-9.]}^* \texttt{ ␣}$$



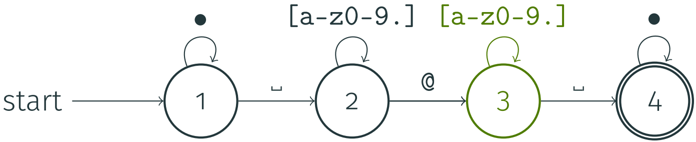
- Then, evaluate the automaton on the text T

`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

Solution: Automata

- Convert the **regular expression** P to an **automaton** A

$$P := \texttt{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$



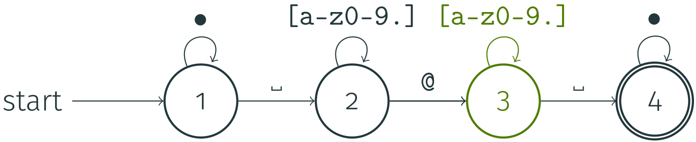
- Then, evaluate the automaton on the **text** T

`Email␣a3nm@a3nm.net␣Affiliation`

Solution: Automata

- Convert the **regular expression** P to an **automaton** A

$P :=$ `␣ [a-z0-9.]* @ [a-z0-9.]* ␣`



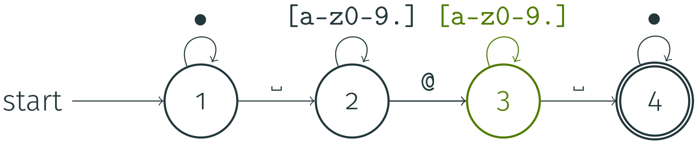
- Then, evaluate the automaton on the **text** T

`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

Solution: Automata

- Convert the **regular expression** *P* to an **automaton** *A*

$$P := \texttt{␣ [a-z0-9.]}^* \texttt{ @ [a-z0-9.]}^* \texttt{ ␣}$$



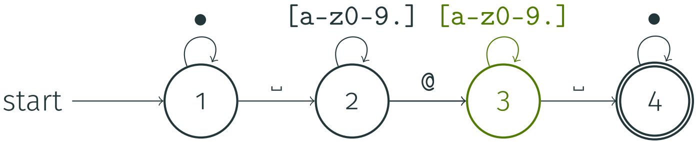
- Then, evaluate the automaton on the **text** *T*

`Email␣a3nm@a3nm.net␣Affiliation`

Solution: Automata

- Convert the **regular expression** P to an **automaton** A

$$P := \texttt{␣ [a-z0-9.]}^* \texttt{ @ [a-z0-9.]}^* \texttt{ ␣}$$



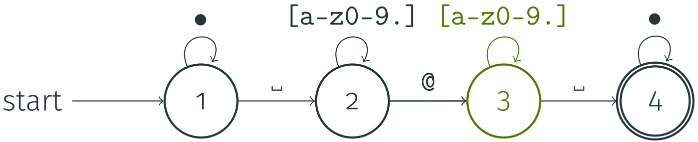
- Then, evaluate the automaton on the **text** T

`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \text{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$



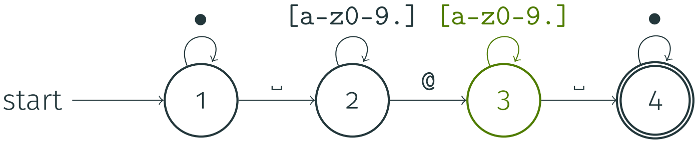
- Then, evaluate the automaton on the text T

`Email␣a3nm@a3nm.net␣Affiliation`

Solution: Automata

- Convert the **regular expression** P to an **automaton** A

$$P := \text{␣ [a-z0-9.]}^* \text{ @ [a-z0-9.]}^* \text{ ␣}$$



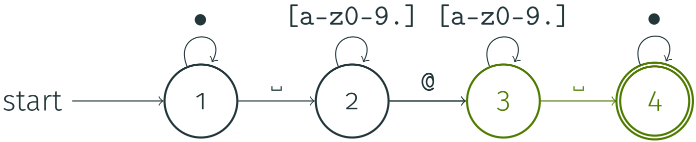
- Then, evaluate the automaton on the **text** T

`Email␣a3nm@a3nm.net␣Affiliation`

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \texttt{␣ [a-z0-9.]}^* \texttt{ @ [a-z0-9.]}^* \texttt{ ␣}$$



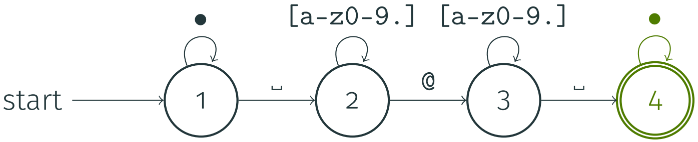
- Then, evaluate the automaton on the text T

`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \text{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$



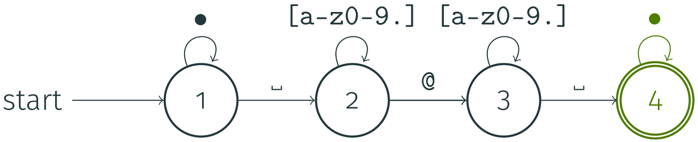
- Then, evaluate the automaton on the text T

E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \texttt{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$



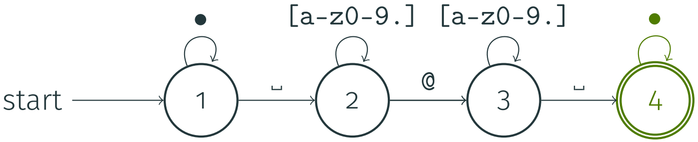
- Then, evaluate the automaton on the text T

`Email␣a3nm@a3nm.net␣Affiliation`

Solution: Automata

- Convert the **regular expression** P to an **automaton** A

$$P := \text{␣ [a-z0-9.]}^{*}\ \text{@ [a-z0-9.]}^{*}\ \text{␣}$$



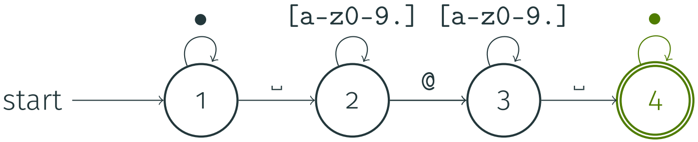
- Then, evaluate the automaton on the **text** T

`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \texttt{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$



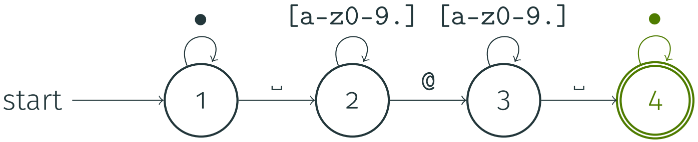
- Then, evaluate the automaton on the text T

`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

Solution: Automata

- Convert the **regular expression** P to an **automaton** A

$$P := \texttt{␣ [a-z0-9.]}^* \texttt{ @ [a-z0-9.]}^* \texttt{ ␣}$$



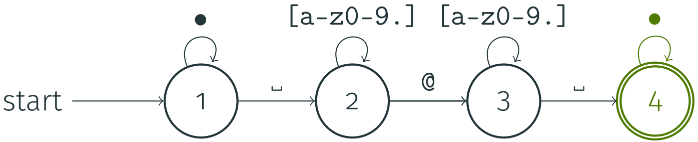
- Then, evaluate the automaton on the **text** T

`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \text{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$



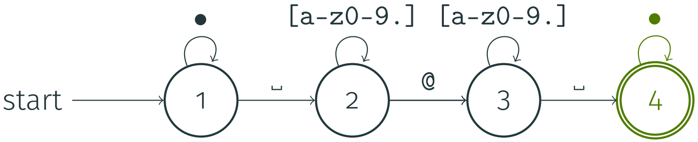
- Then, evaluate the automaton on the text T

E	m	a	i	l	␣	a	3	n	m	@	a	3	n	m	.	n	e	t	␣	A	f	f	i	l	i	a	t	i	o	n

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \text{␣}\ \texttt{[a-z0-9.]}^*\ \texttt{@}\ \texttt{[a-z0-9.]}^*\ \text{␣}$$



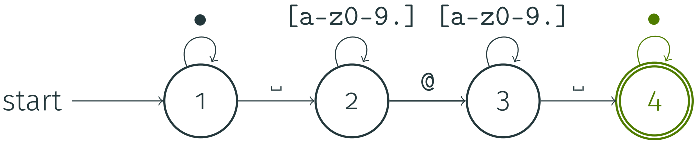
- Then, evaluate the automaton on the text T

`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

Solution: Automata

- Convert the **regular expression** P to an **automaton** A

$P :=$ `␣ [a-z0-9.]* @ [a-z0-9.]* ␣`



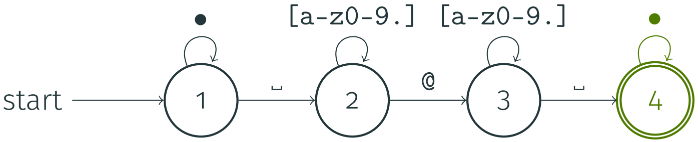
- Then, evaluate the automaton on the **text** T

`Email␣a3nm@a3nm.net␣Affiliation`

Solution: Automata

- Convert the regular expression P to an automaton A

$$P := \texttt{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$



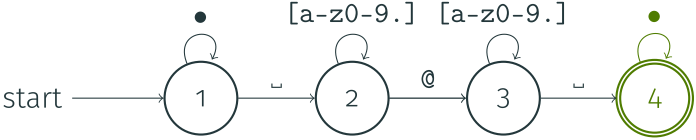
- Then, evaluate the automaton on the text T

`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

Solution: Automata

- Convert the regular expression P to an automaton A

$P :=$ `␣ [a-z0-9.]* @ [a-z0-9.]* ␣`



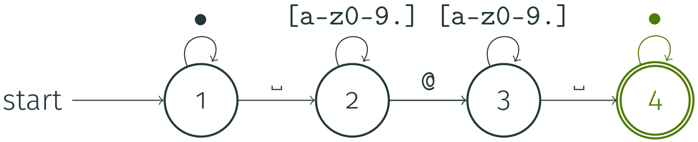
- Then, evaluate the automaton on the text T

`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

Solution: Automata

- Convert the **regular expression** P to an **automaton** A

$$P := \text{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$



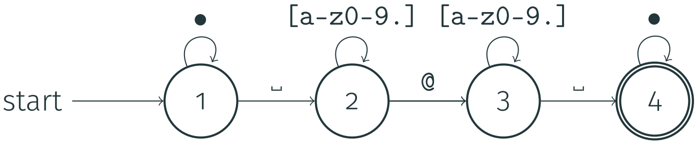
- Then, evaluate the automaton on the **text** T

`Email␣a3nm@a3nm.net␣Affiliation`

Solution: Automata

- Convert the regular expression P to an automaton A

$P :=$ `␣ [a-z0-9.]* @ [a-z0-9.]* ␣`



- Then, evaluate the automaton on the text T

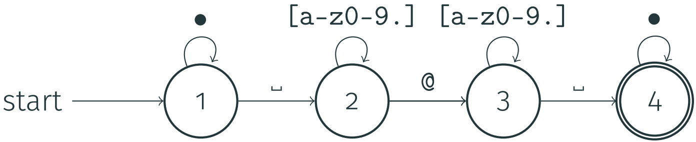
`Email␣a3nm@a3nm.net␣Affiliation`

- The complexity is $O(|A| \times |T|)$, i.e., linear in T and polynomial in P

Solution: Automata

- Convert the **regular expression** P to an **automaton** A

$P :=$ `␣ [a-z0-9.]* @ [a-z0-9.]* ␣`



- Then, evaluate the automaton on the **text** T

`E m a i l ␣ a 3 n m @ a 3 n m . n e t ␣ A f f i l i a t i o n`

- The **complexity** is $O(|A| \times |T|)$, i.e., **linear** in T and **polynomial** in P
  - $\rightarrow$ This is **very efficient** in T and **reasonably efficient** in P

Actual Problem: Extracting all Patterns

- This only tests if the pattern occurs in the text!
  - $\rightarrow$ `''YES''`

Actual Problem: Extracting all Patterns

- This only tests if the pattern occurs in the text!

  $\rightarrow$ ``YES''

- Goal: find all substrings in the text T which match the pattern P

Actual Problem: Extracting all Patterns

- This only tests if the pattern occurs in the text!

  $\rightarrow$ “YES”

- Goal: find all substrings in the text T which match the pattern P

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
E	m	a	i	l	␣	a	3	n	m	@	a	3	n	m	.	n	e	t	␣	A	f	f	i	l	i	a	t	i	o	n

Actual Problem: Extracting all Patterns

- This only tests **if** the pattern **occurs in** the text!

  $\rightarrow$ "YES"

- Goal: find all **substrings** in the text T which match the pattern P

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
E	m	a	i	l	␣	a	3	n	m	@	a	3	n	m	.	n	e	t	␣	A	f	f	i	l	i	a	t	i	o	n

$\rightarrow$ **One match:** $[5, 20\rangle$

Actual Problem: Extracting all Patterns

- This only tests if the pattern occurs in the text!

  → ``YES''

- Goal: find all substrings in the text *T* which match the pattern *P*

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
E	m	a	i	l	␣	a	3	n	m	@	a	3	n	m	.	n	e	t	␣	A	f	f	i	l	i	a	t	i	o	n

→ One match: $[5, 20\rangle$

Formal Problem Statement

- Problem description:

Formal Problem Statement

- Problem description:
  - Input:
    - A text T

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.  French
national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP a3nm@a3nm.net
Affiliation Associate professor of computer science (office C201-4) in the DIG team of Télécom
ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France.  Studies PhD in computer science awarded by
Télécom ParisTech on March 14, 2016.  Former student of the École normale supérieure.  test@example.com
More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
```

Formal Problem Statement

- Problem description:
  - Input:
    - A text T

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.  French
national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP a3nm@a3nm.net
Affiliation Associate professor of computer science (office C201-4) in the DIG team of Télécom
ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France.  Studies PhD in computer science awarded by
Télécom ParisTech on March 14, 2016.  Former student of the École normale supérieure.  test@example.com
More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
```

    - A pattern P given as a regular expression

$$P := \text{␣ [a-z0-9.]}^* \text{ @ [a-z0-9.]}^* \text{ ␣}$$

Formal Problem Statement

- Problem description:
  - Input:
    - A text T

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.  French
national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP a3nm@a3nm.net
Affiliation Associate professor of computer science (office C201-4) in the DIG team of Télécom
ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France.  Studies PhD in computer science awarded by
Télécom ParisTech on March 14, 2016.  Former student of the École normale supérieure.  test@example.com
More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
```

    - A pattern P given as a regular expression

$$P := \text{␣ [a-z0-9.]}^* \text{ @ [a-z0-9.]}^* \text{ ␣}$$

  - Output: the list of substrings of T that match P:

$$[186, 200\rangle, \quad [483, 500\rangle, \quad \ldots$$

Formal Problem Statement

- Problem description:
  - Input:
    - A text T

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.  French
national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP a3nm@a3nm.net
Affiliation Associate professor of computer science (office C201-4) in the DIG team of Télécom
ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France.  Studies PhD in computer science awarded by
Télécom ParisTech on March 14, 2016.  Former student of the École normale supérieure.  test@example.com
More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
```

    - A pattern P given as a regular expression

$$P := \text{␣}\ \texttt{[a-z0-9.]}^*\ \texttt{@}\ \texttt{[a-z0-9.]}^*\ \text{␣}$$

  - Output: the list of substrings of T that match P:

$$[186, 200\rangle, \quad [483, 500\rangle, \quad \ldots$$

- Goal: be very efficient in T and reasonably efficient in P

Measuring the Complexity

- Naive algorithm: Run the automaton A on each substring of T

`l o l`

Measuring the Complexity

- Naive algorithm: Run the automaton A on each substring of T

$$[\rangle \; \mathtt{l} \quad \mathtt{o} \quad \mathtt{l}$$

Measuring the Complexity

- Naive algorithm: Run the automaton A on each substring of T

$[$ `l` $\rangle$ `o` `l`

Measuring the Complexity

- Naive algorithm: Run the automaton A on each substring of T

$[$ `l` `o` $\rangle$ `l`

Measuring the Complexity

- Naive algorithm: Run the automaton A on each substring of T

$[$ `l o l` $\rangle$

Measuring the Complexity

- Naive algorithm: Run the automaton A on each substring of T

```
l [⟩ o   l
```

Measuring the Complexity

- Naive algorithm: Run the automaton A on each substring of T

`l` $[$ `o` $\rangle$ `l`

Measuring the Complexity

- Naive algorithm: Run the automaton A on each substring of T

l	[	o	l	⟩

Measuring the Complexity

- Naive algorithm: Run the automaton A on each substring of T

l o $[\rangle$ l

Measuring the Complexity

- Naive algorithm: Run the automaton A on each substring of T

l o [l ⟩

Measuring the Complexity

- Naive algorithm: Run the automaton A on each substring of T

`l o l` $[\rangle$

Measuring the Complexity

- **Naive algorithm:** Run the automaton A on **each substring** of T

`l o l`

$\rightarrow$ **Complexity** is $O(|T|^2 \times |A| \times |T|)$

Measuring the Complexity

- **Naive algorithm:** Run the automaton A on **each substring** of T

```
   l   o   l
```

  $\rightarrow$ **Complexity** is $O(|T|^2 \times |A| \times |T|)$

  $\rightarrow$ Can be **optimized** to $O(|T|^2 \times |A|)$

Measuring the Complexity

- **Naive algorithm:** Run the automaton A on **each substring** of T

`l   o   l`

  $\rightarrow$ **Complexity** is $O(|T|^2 \times |A| \times |T|)$

  $\rightarrow$ Can be **optimized** to $O(|T|^2 \times |A|)$

- **Problem:** We may need to output $\Omega(|T|^2)$ matching substrings:

Measuring the Complexity

- **Naive algorithm:** Run the automaton A on **each substring** of T

```
    l   o   l
```

  $\rightarrow$ **Complexity** is $O(|T|^2 \times |A| \times |T|)$

  $\rightarrow$ Can be **optimized** to $O(|T|^2 \times |A|)$

- **Problem:** We may need to output $\Omega(|T|^2)$ matching substrings:
  - Consider the **text** T:

```
aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa
```

Measuring the Complexity

- **Naive algorithm:** Run the automaton A on **each substring** of T

```
    l   o   l
```

  - $\rightarrow$ **Complexity** is $O(|T|^2 \times |A| \times |T|)$
  - $\rightarrow$ Can be **optimized** to $O(|T|^2 \times |A|)$

- **Problem:** We may need to output $\Omega(|T|^2)$ matching substrings:
  - Consider the **text** T:

    ```
    aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa
    ```

  - Consider the **pattern** $P := \texttt{a}^*$

Measuring the Complexity

- **Naive algorithm:** Run the automaton A on **each substring** of T

l o l

  $\rightarrow$ **Complexity** is $O(|T|^2 \times |A| \times |T|)$

  $\rightarrow$ Can be **optimized** to $O(|T|^2 \times |A|)$

- **Problem:** We may need to output $\Omega(|T|^2)$ matching substrings:
  - Consider the **text** T:

`aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa`

  - Consider the **pattern** $P := \texttt{a}^*$
  - The **number of matches** is $\Omega(|T|^2)$

Measuring the Complexity

- **Naive algorithm:** Run the automaton A on **each substring** of T

```
    l   o   l
```

  → **Complexity** is $O(|T|^2 \times |A| \times |T|)$
  → Can be **optimized** to $O(|T|^2 \times |A|)$

- **Problem:** We may need to output $\Omega(|T|^2)$ matching substrings:
  - Consider the **text** T:

```
aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa
```

  - Consider the **pattern** $P :=$ `a`*
  - The **number of matches** is $\Omega(|T|^2)$

→ We need a **different way** to measure complexity

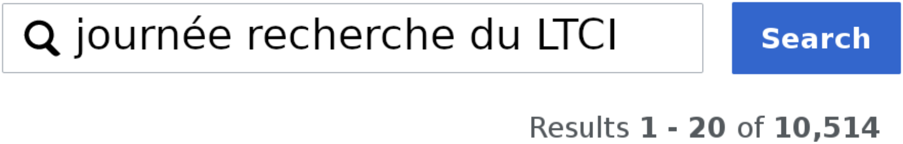
Enumeration Algorithms

Idea: In real life, we do not want to compute all the matches
we just need to be able to enumerate matches quickly

Enumeration Algorithms

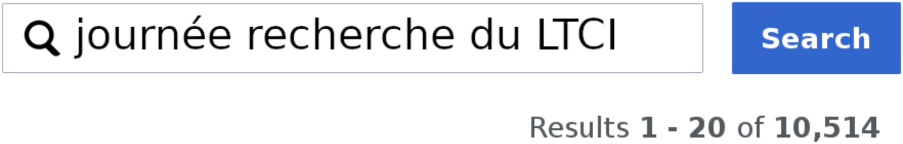
Idea: In real life, we do not want to compute all the matches
we just need to be able to enumerate matches quickly

Idea: In real life, we do not want to compute all the matches
we just need to be able to enumerate matches quickly



Enumeration Algorithms

Idea: In real life, we do not want to compute **all the matches**
we just need to be able to **enumerate** matches quickly



...

Enumeration Algorithms

Idea: In real life, we do not want to compute **all the matches**
we just need to be able to **enumerate** matches quickly

journée recherche du LTCI **Search**

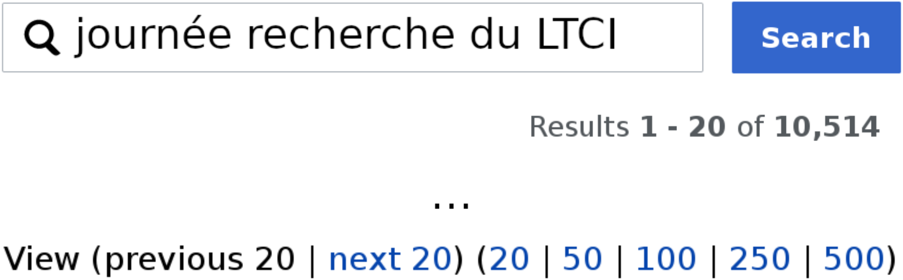
Results **1 - 20** of **10,514**

...

View (previous 20 | next 20) (20 | 50 | 100 | 250 | 500)

Enumeration Algorithms

Idea: In real life, we do not want to compute **all the matches**
we just need to be able to **enumerate** matches quickly



→ Formalization: **enumeration algorithms**

Formalizing Enumeration Algorithms

```
Antoine Amarilli Description Name Antoine
Amarilli.  Handle:  a3nm.  Identity Born
1990-02-07.  French national.  Appearance as
of 2017.  Auth OpenPGP. OpenId.  Bitcoin.
Contact Email and XMPP a3nm@a3nm.net
Affiliation Associate professor ...
```

Text *T*

␣ [a-z0-9.]*@
[a-z0-9.]* ␣

Pattern *P*

Formalizing Enumeration Algorithms

```
Antoine Amarilli Description Name Antoine
Amarilli.  Handle:  a3nm.  Identity Born
1990-02-07.  French national.  Appearance as
of 2017.  Auth OpenPGP. OpenId.  Bitcoin.
Contact Email and XMPP a3nm@a3nm.net
Affiliation Associate professor ...
```

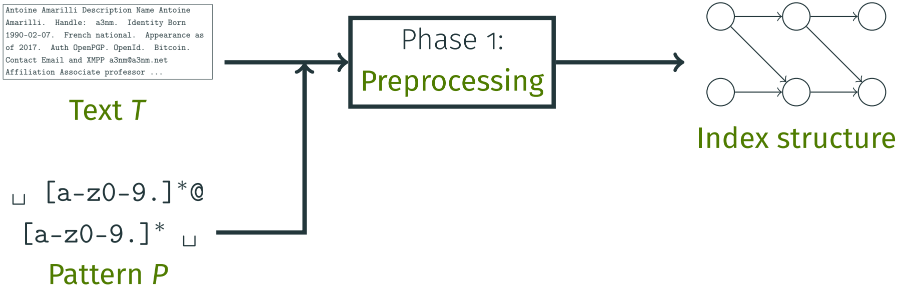
Text T

`␣ [a-z0-9.]*@[a-z0-9.]* ␣`

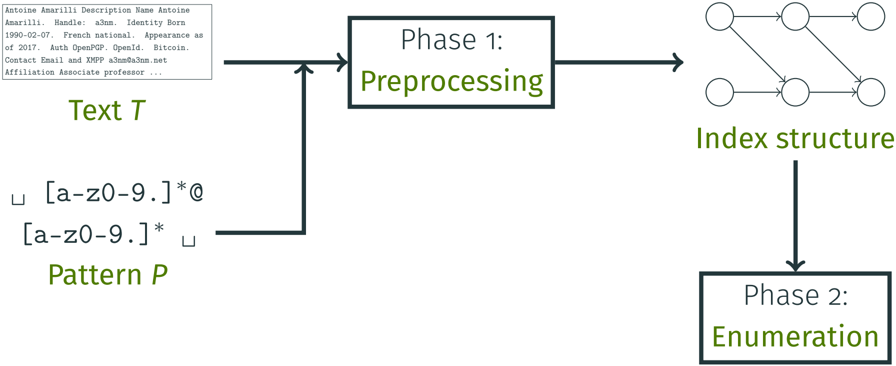
Pattern P

Phase 1:
Preprocessing

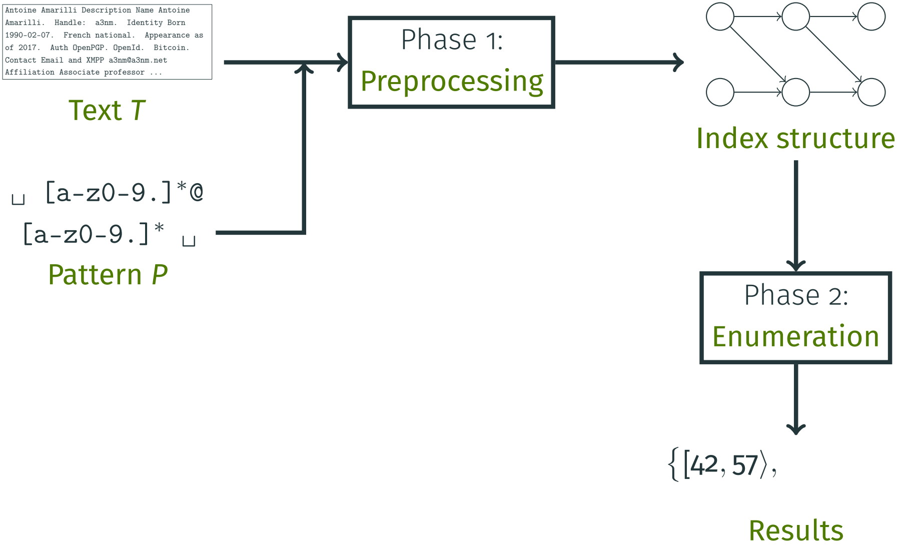
Formalizing Enumeration Algorithms



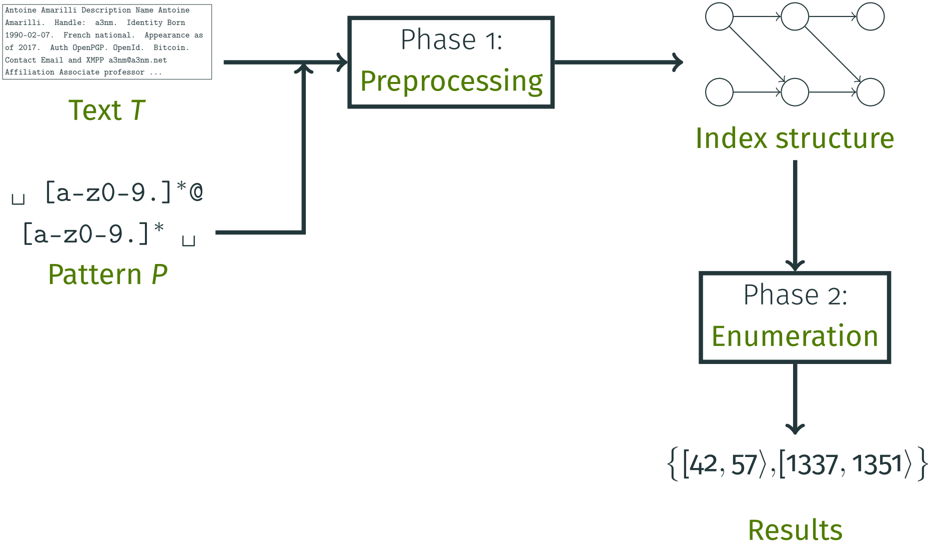
Formalizing Enumeration Algorithms



Formalizing Enumeration Algorithms



Formalizing Enumeration Algorithms



Formalizing Enumeration Algorithms

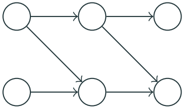
```
Antoine Amarilli Description Name Antoine
Amarilli.  Handle:  a3nm.  Identity Born
1990-02-07.  French national.  Appearance as
of 2017.  Auth OpenPGP. OpenId.  Bitcoin.
Contact Email and XMPP a3nm@a3nm.net
Affiliation Associate professor ...
```

Text T

␣ [a-z0-9.]*@[a-z0-9.]* ␣

Pattern P

Phase 1: Preprocessing



Index structure

Phase 2: Enumeration

$\{[42, 57\rangle, [1337, 1351\rangle\}$

Results

Two ways to measure performance:

- Total time for phase 1
- Delay between two results in phase 2

... as a function of the text and pattern

Complexity of Enumeration Algorithms

- Recall the inputs to our problem:
  - A text T

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.
French national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP
a3nm@a3nm.net Affiliation Associate professor of computer science (office C201-4) in the DIG team of
Télécom ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France.  Studies PhD in computer science
awarded by Télécom ParisTech on March 14, 2016.  Former student of the École normale supérieure.
More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
```

Complexity of Enumeration Algorithms

- Recall the **inputs** to our problem:
  - A **text** T

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.
French national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP
a3nm@a3nm.net Affiliation Associate professor of computer science (office C201-4) in the DIG team of
Télécom ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France.  Studies PhD in computer science
awarded by Télécom ParisTech on March 14, 2016.  Former student of the École normale supérieure.
More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
```

  - A **pattern** P given as a regular expression

$$P := \text{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$

Complexity of Enumeration Algorithms

- Recall the inputs to our problem:
  - A text T

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.
French national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP
a3nm@a3nm.net Affiliation Associate professor of computer science (office C201-4) in the DIG team of
Télécom ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France.  Studies PhD in computer science
awarded by Télécom ParisTech on March 14, 2016.  Former student of the École normale supérieure.
More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
```

  - A pattern P given as a regular expression

$$P := \text{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$

- What is the delay of the naive algorithm?

Complexity of Enumeration Algorithms

- Recall the **inputs** to our problem:
  - A **text** T

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.
French national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP
a3nm@a3nm.net Affiliation Associate professor of computer science (office C201-4) in the DIG team of
Télécom ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France.  Studies PhD in computer science
awarded by Télécom ParisTech on March 14, 2016.  Former student of the École normale supérieure.
More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
```

  - A **pattern** P given as a regular expression

$$P := \text{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$

- What is the **delay** of the **naive algorithm**?

  → it is the **maximal time** to find the next **matching substring**

Complexity of Enumeration Algorithms

- Recall the **inputs** to our problem:
  - A **text** T

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.
French national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP
a3nm@a3nm.net Affiliation Associate professor of computer science (office C201-4) in the DIG team of
Télécom ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France.  Studies PhD in computer science
awarded by Télécom ParisTech on March 14, 2016.  Former student of the École normale supérieure.
More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
```

  - A **pattern** P given as a regular expression

$$P := \text{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$

- What is the **delay** of the **naive algorithm**?

  → it is the **maximal time** to find the next **matching substring**
  → i.e. $O(|T|^2 \times |A|)$, e.g., if only the **beginning** and **end** match

Complexity of Enumeration Algorithms

- Recall the **inputs** to our problem:
  - A **text** T

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.
French national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP
a3nm@a3nm.net Affiliation Associate professor of computer science (office C201-4) in the DIG team of
Télécom ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France.  Studies PhD in computer science
awarded by Télécom ParisTech on March 14, 2016.  Former student of the École normale supérieure.
More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
```

  - A **pattern** P given as a regular expression

$$P := \sqcup\ \texttt{[a-z0-9.]}^*\ \texttt{@}\ \texttt{[a-z0-9.]}^*\ \sqcup$$

- What is the **delay** of the **naive algorithm**?

  → it is the **maximal time** to find the next **matching substring**
  → i.e. $O(|T|^2 \times |A|)$, e.g., if only the **beginning** and **end** match

→ Can we do **better**?

Results for Enumerating Pattern Matches

- Existing work has shown the best possible bounds:

Results for Enumerating Pattern Matches

- Existing work has shown the best possible bounds:

Theorem [Florenzano et al., 2018]

We can enumerate all matches of a pattern P on a text T with:

- *Preprocessing linear in T*
- *Delay constant (independent from T)*

Results for Enumerating Pattern Matches

- Existing work has shown the best possible bounds **in T**:

Theorem [Florenzano et al., 2018]

We can enumerate all matches of a pattern P on a text T with:

- *Preprocessing linear in T and exponential in P*
- *Delay constant (independent from T) and exponential in P*

→ **Problem:** They only measure the complexity as a function of T!

Results for Enumerating Pattern Matches

- Existing work has shown the best possible bounds **in T**:

Theorem [Florenzano et al., 2018]

We can enumerate all matches of a pattern $\mathbf{P}$ on a text $\mathbf{T}$ with:

- *Preprocessing linear in $\mathbf{T}$ and exponential in P*
- *Delay constant (independent from $\mathbf{T}$) and exponential in P*

→ **Problem:** They only measure the complexity as a function of T!

- Our contribution is:

Results for Enumerating Pattern Matches

- Existing work has shown the best possible bounds in T:

Theorem [Florenzano et al., 2018]

We can enumerate all matches of a pattern $\boldsymbol{P}$ *on a text* $\boldsymbol{T}$ *with:*

- *Preprocessing linear in* $\boldsymbol{T}$ *and exponential in* P
- *Delay constant (independent from* $\boldsymbol{T}$*) and exponential in* P

$\rightarrow$ **Problem:** They only measure the complexity as a function of T!

- Our contribution is:

Theorem

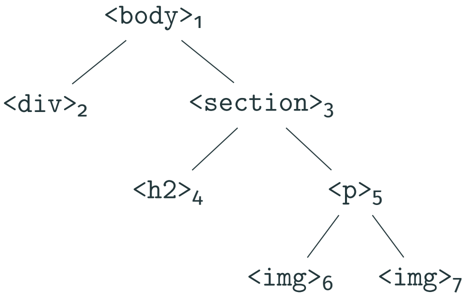
We can enumerate all matches of a pattern $\boldsymbol{P}$ *on a text* $\boldsymbol{T}$ *with:*

- *Preprocessing in* $O(|T| \times Poly(P))$
- *Delay polynomial in* $\boldsymbol{P}$ *and independent from* $\boldsymbol{T}$

Extension: From Text to Trees

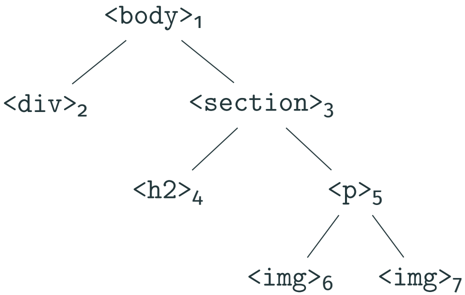
Pattern Matching on Trees

- The data T is no longer text but is now a tree:



Pattern Matching on Trees

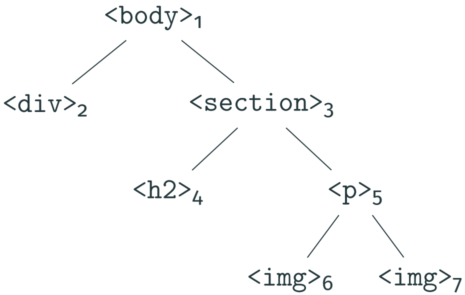
- The **data** *T* is no longer **text** but is now a **tree**:



- The **pattern** *P* asks about the **structure** of the tree:
  *Is there an **h2** header and an **image** in the same section?*

Pattern Matching on Trees

- The **data** T is no longer **text** but is now a **tree**:

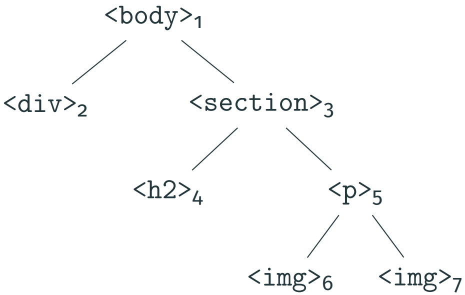


- The **pattern** P asks about the **structure** of the tree:
  *Is there an **h2** header and an **image** in the same section?*

- **Results:**

Pattern Matching on Trees

- The **data** *T* is no longer **text** but is now a **tree**:

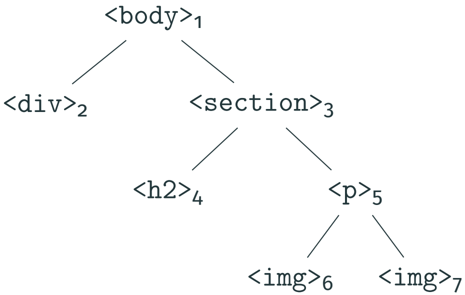


- The **pattern** *P* asks about the **structure** of the tree:
  *Is there α: an **h2** header and β: an **image** in the same section?*

- **Results:**

Pattern Matching on Trees

- The **data** *T* is no longer **text** but is now a **tree**:



- The **pattern** *P* asks about the **structure** of the tree:
  *Is there α: an **h2** header and β: an **image** in the same section?*

- **Results:** $\langle \alpha : 4, \beta : 6 \rangle$, $\langle \alpha : 4, \beta : 7 \rangle$

Definitions and Results on Trees

- Tree patterns P can be written as a kind of tree automaton...

Definitions and Results on Trees

- Tree patterns P can be written as a kind of tree automaton...
- Existing work has studied this problem and shown:

Definitions and Results on Trees

- Tree patterns P can be written as a kind of tree automaton...
- Existing work has studied this problem and shown:

Theorem [Bagan, 2006]

We can find all matches on a tree T of a tree pattern P (with constantly many capture variables) with:

- *Preprocessing linear in T*
- *Delay constant in T*

Definitions and Results on Trees

- Tree patterns P can be written as a kind of tree automaton...
- Existing work has studied this problem and shown:

Theorem [Bagan, 2006]

We can find all matches on a tree T of a tree pattern P (with constantly many capture variables) with:

- *Preprocessing linear in T and exponential in P*
- *Delay constant in T and exponential in P*

- Again, this only measures the complexity in T!

Definitions and Results on Trees

- Tree patterns P can be written as a kind of **tree automaton**...
- Existing work has studied this problem and shown:

Theorem [Bagan, 2006]

We can find all matches on a tree T *of a tree pattern* P *(with constantly many capture variables) with:*

- *Preprocessing linear in* T *and exponential in* P
- *Delay constant in* T *and exponential in* P

- Again, this only measures the **complexity in** T!

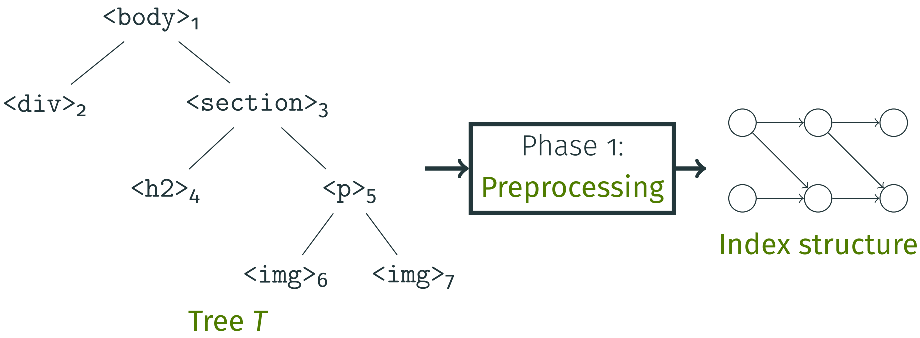
→ We are **working on** proving the following:

Conjecture

- *Preprocessing in* $O(|T| \times Poly(P))$
- *Delay polynomial in* P *and independent from* T

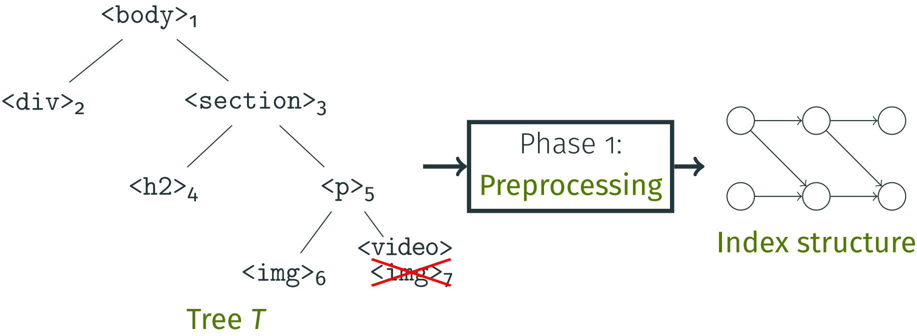
Extension: Supporting Updates

Updates



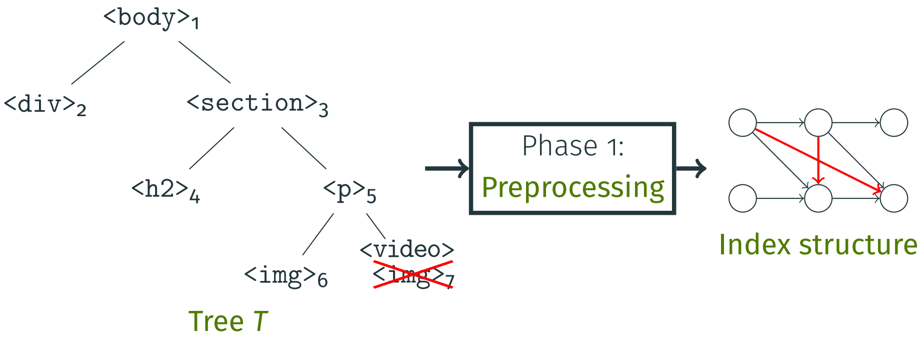
- The input data can be **modified** after the preprocessing

Updates



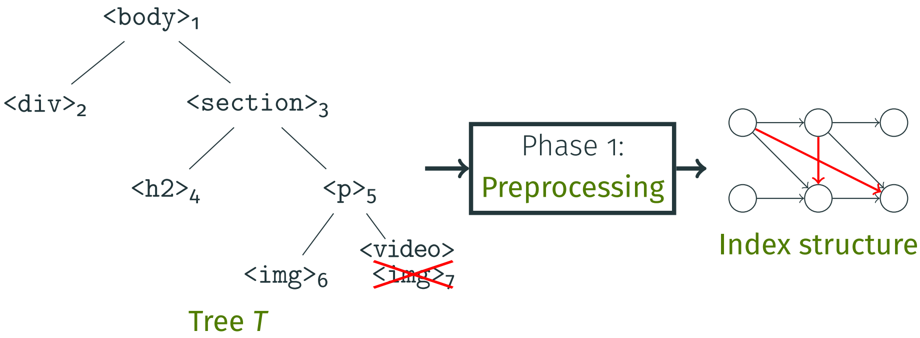
- The input data can be **modified** after the preprocessing

Updates



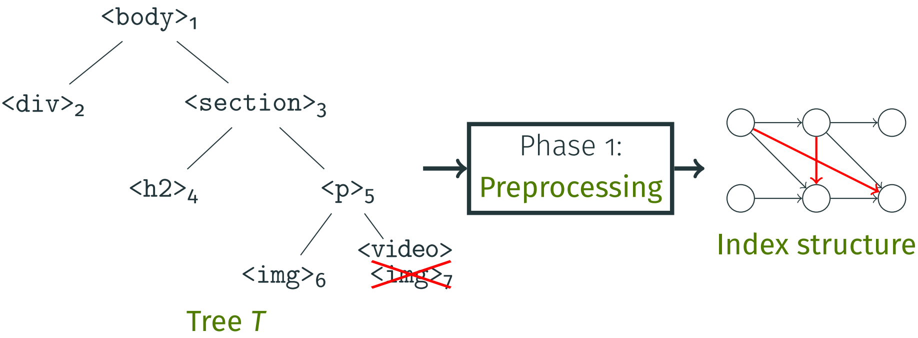
- The input data can be **modified** after the preprocessing

Updates



- The input data can be **modified** after the preprocessing
- If this happen, we must rerun the **preprocessing** from scratch

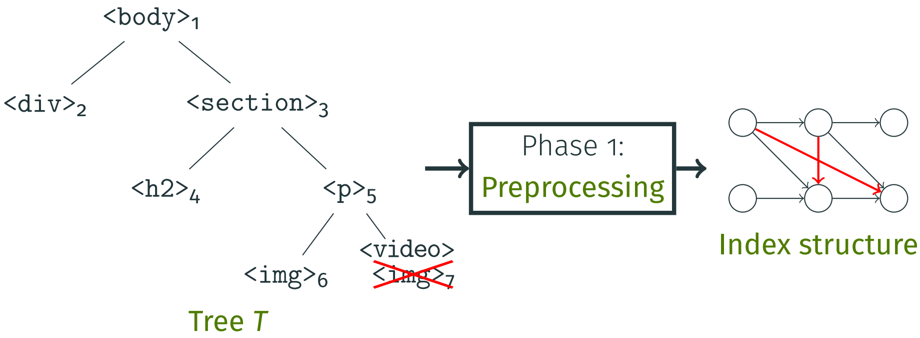
Updates



- The input data can be **modified** after the preprocessing
- If this happen, we must rerun the **preprocessing** from scratch

→ Can we **do better**?

Updates



- The input data can be **modified** after the preprocessing
- If this happen, we must rerun the **preprocessing** from scratch

$\rightarrow$ Can we **do better**?

Conjecture

*When the input data **T** is **updated**, we can update our **index** in time $O(\log |T|)$*

Summary and Future Work

Summary and Future Work

Summary:

- **Problem:** given a text T and a pattern P,
  enumerate efficiently all substrings of T that match P

Summary and Future Work

Summary:

- Problem: given a text T and a pattern P,
  enumerate efficiently all substrings of T that match P
- Result: we can do this with reasonable complexity in P
  and with linear preprocessing and constant delay in T

Summary and Future Work

Summary:

- **Problem:** given a text T and a pattern P,
  enumerate efficiently all substrings of T that match P
- **Result:** we can do this with **reasonable complexity** in P
  and with **linear** preprocessing and **constant** delay in T

Extensions and future work:

- Extending the results from text to **trees**

Summary and Future Work

Summary:

- Problem: given a text T and a pattern P,
  enumerate efficiently all substrings of T that match P
- Result: we can do this with reasonable complexity in P
  and with linear preprocessing and constant delay in T

Extensions and future work:

- Extending the results from text to trees
- Supporting updates on the input data

Summary and Future Work

Summary:

- **Problem:** given a text T and a pattern P,
  enumerate efficiently all substrings of T that match P
- **Result:** we can do this with **reasonable complexity** in P
  and with **linear** preprocessing and **constant** delay in T

Extensions and future work:

- Extending the results from text to **trees**
- Supporting **updates** on the input data
- Testing how well our methods perform in **practice**

Summary and Future Work

Summary:

- **Problem:** given a text T and a pattern P,
  enumerate efficiently all substrings of T that match P
- **Result:** we can do this with **reasonable complexity** in P
  and with **linear** preprocessing and **constant** delay in T

Extensions and future work:

- Extending the results from text to **trees**
- Supporting **updates** on the input data
- Testing how well our methods perform in **practice**

Thanks for your attention!

References i

Bagan, G. (2006).
MSO queries on tree decomposable structures are computable with linear delay.
In *CSL*.

Florenzano, F., Riveros, C., Ugarte, M., Vansummeren, S., and Vrgoc, D. (2018).
Constant delay algorithms for regular document spanners.
In *PODS*.