Probabilities and Provenance on Trees and Treelike Instances

Antoine Amarilli[1], Pierre Bourhis[2], Pierre Senellart[1,3]

[1]Télécom ParisTech

[2]CNRS CRIStAL

[3]National University of Singapore

January 8th, 2016

Query evaluation

- Relational signature σ
  - $\rightarrow$ Example: R (arity 1), S (arity 2), T (arity 1)

Query evaluation

- Relational signature σ
  - $\rightarrow$ Example: R (arity 1), S (arity 2), T (arity 1)
- Fragment $\mathcal{Q}$ of Boolean constant-free queries
  - $\rightarrow$ Example: Boolean conjunctive queries (= existentially quantified conjunction of atoms)
  - $\rightarrow$ Example of CQ: $q : \exists x\, y\ R(x) \wedge S(x, y) \wedge T(y)$

Query evaluation

- Relational signature σ
  - $\rightarrow$ Example: R (arity 1), S (arity 2), T (arity 1)
- Fragment $\mathcal{Q}$ of Boolean constant-free queries
  - $\rightarrow$ Example: Boolean conjunctive queries
    (= existentially quantified conjunction of atoms)
  - $\rightarrow$ Example of CQ: $q : \exists x\, y\ R(x) \land S(x, y) \land T(y)$
- Class $\mathcal{I}$ of instances
  - $\rightarrow$ Example: all instances; acyclic instances; treelike instances; ...

Query evaluation

- Relational signature σ
  - → Example: R (arity 1), S (arity 2), T (arity 1)
- Fragment $\mathcal{Q}$ of Boolean constant-free queries
  - → Example: Boolean conjunctive queries (= existentially quantified conjunction of atoms)
  - → Example of CQ: $q : \exists x\, y\ R(x) \wedge S(x, y) \wedge T(y)$
- Class $\mathcal{I}$ of instances
  - → Example: all instances; acyclic instances; treelike instances; ...

→ Query evaluation problem for $\mathcal{Q}$ and $\mathcal{I}$:

- Fix a query $q \in \mathcal{Q}$
- Given an input instance $I \in \mathcal{I}$
- Determine whether I satisfies q (written $I \models q$)
- Complexity as a function of I, not q (= data complexity)

Query evaluation example

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

Query evaluation example

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\; R(x) \wedge S(x, y) \wedge T(y)$$

Query evaluation example

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\; R(x) \wedge S(x, y) \wedge T(y)$$

R
a
b
c

Query evaluation example

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\; R(x) \land S(x, y) \land T(y)$$

R
a
b
c

S	
a	a
b	v
b	w

Query evaluation example

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\ R(x) \wedge S(x, y) \wedge T(y)$$

R
a
b
c

S	
a	a
b	v
b	w

T
v
w
b

Query evaluation example

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\; R(x) \wedge S(x, y) \wedge T(y)$$

R
a
b
c

S	
a	a
b	v
b	w

T
v
w
b

Probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of queries, class $\mathcal{I}$ of instances.

$\rightarrow$ Probabilistic query evaluation problem for $\mathcal{Q}$ and $\mathcal{I}$:

- Fix a query $q \in \mathcal{Q}$
- Given an input instance $I \in \mathcal{I}$

Probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of queries, class $\mathcal{I}$ of instances.

$\rightarrow$ Probabilistic query evaluation problem for $\mathcal{Q}$ and $\mathcal{I}$:

- Fix a query $q \in \mathcal{Q}$
- Given an input instance $I \in \mathcal{I}$
- And given a probability valuation π
  mapping facts of I to probabilities in $[0, 1]$

Probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of queries, class $\mathcal{I}$ of instances.

$\rightarrow$ Probabilistic query evaluation problem for $\mathcal{Q}$ and $\mathcal{I}$:

- Fix a query $q \in \mathcal{Q}$
- Given an input instance $I \in \mathcal{I}$
- And given a probability valuation π mapping facts of I to probabilities in $[0, 1]$
- Compute the probability that $I \models q$

Probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of queries, class $\mathcal{I}$ of instances.

$\rightarrow$ Probabilistic query evaluation problem for $\mathcal{Q}$ and $\mathcal{I}$:

- Fix a query $q \in \mathcal{Q}$
- Given an input instance $I \in \mathcal{I}$
- And given a probability valuation π mapping facts of I to probabilities in $[0, 1]$
- Compute the probability that $I \models q$
- Data complexity: measured as a function of I and π

Probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of queries, class $\mathcal{I}$ of instances.

$\rightarrow$ Probabilistic query evaluation problem for $\mathcal{Q}$ and $\mathcal{I}$:

- Fix a query $q \in \mathcal{Q}$
- Given an input instance $I \in \mathcal{I}$
- And given a probability valuation π mapping facts of I to probabilities in $[0, 1]$
- Compute the probability that $I \models q$
- Data complexity: measured as a function of I and π

- Semantics: (I, π) represents a probability distribution on $I' \subseteq I$:
  - Each fact $F \in I$ is either present or absent with probability $\pi(F)$
  - Facts are independent

Example of a probabilistic instance

S		
a	a	1
b	v	$.5$
b	w	$.2$

Example of a probabilistic instance

S		
a	a	1
b	v	$.5$
b	w	$.2$

This (I, π) represents the following probability distribution:

Example of a probabilistic instance

S		
a	a	1
b	v	$.5$
b	w	$.2$

This (I, π) represents the following probability distribution:

$.5 \times .2$

S	
a	a
b	v
b	w

Example of a probabilistic instance

S		
a	a	1
b	v	$.5$
b	w	$.2$

This (I, π) represents the following probability distribution:

$.5 \times .2$

S	
a	a
b	v
b	w

$.5 \times (1 - .2)$

S	
a	a
b	v

Example of a probabilistic instance

S		
a	*a*	1
b	*v*	$.5$
b	*w*	$.2$

This (I, π) represents the following probability distribution:

$.5 \times .2$

S	
a	*a*
b	*v*
b	*w*

$.5 \times (1 - .2)$

S	
a	*a*
b	*v*

$(1 - .5) \times .2$

S	
a	*a*
b	*w*

Example of a probabilistic instance

S		
a	a	1
b	v	$.5$
b	w	$.2$

This (I, π) represents the following probability distribution:

$.5 \times .2$

S	
a	a
b	v
b	w

$.5 \times (1 - .2)$

S	
a	a
b	v

$(1 - .5) \times .2$

S	
a	a
b	w

$(1 - .5) \times (1 - .2)$

S	
a	a

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\ R(x) \land S(x, y) \land T(y)$$

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\ R(x) \wedge S(x, y) \wedge T(y)$$

R	
a	1
b	$.4$
c	$.6$

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\; R(x) \land S(x, y) \land T(y)$$

R	
a	1
b	$.4$
c	$.6$

S		
a	a	1
b	v	$.5$
b	w	$.2$

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\ R(x) \wedge S(x, y) \wedge T(y)$$

R	
a	1
b	$.4$
c	$.6$

S		
a	a	1
b	v	$.5$
b	w	$.2$

T	
v	$.3$
w	$.7$
b	1

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\ R(x) \wedge S(x, y) \wedge T(y)$$

R	
a	1
b	$.4$
c	$.6$

S		
a	a	1
b	v	$.5$
b	w	$.2$

T	
v	$.3$
w	$.7$
b	1

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\; R(x) \wedge S(x, y) \wedge T(y)$$

R	
a	1
b	$.4$
c	$.6$

S		
a	a	1
b	v	$.5$
b	w	$.2$

T	
v	$.3$
w	$.7$
b	1

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\; R(x) \wedge S(x, y) \wedge T(y)$$

R	
a	1
b	$.4$
c	$.6$

S		
a	a	1
b	v	$.5$
b	w	$.2$

T	
v	$.3$
w	$.7$
b	1

- The query is true iff $R(b)$ is here and one of:

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\; R(x) \wedge S(x, y) \wedge T(y)$$

R	
a	1
b	$.4$
c	$.6$

S		
a	a	1
b	v	$.5$
b	w	$.2$

T	
v	$.3$
w	$.7$
b	1

- The query is true iff $R(b)$ is here and one of:
  - $S(b, v)$ and $T(v)$ are here

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\ R(x) \land S(x,y) \land T(y)$$

R	
a	1
b	.4
c	.6

S		
a	a	1
b	v	.5
b	w	.2

T	
v	.3
w	.7
b	1

- The query is true iff $R(b)$ is here and one of:
    - $S(b,v)$ and $T(v)$ are here
    - $S(b,w)$ and $T(w)$ are here

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\; R(x) \wedge S(x, y) \wedge T(y)$$

R	
a	1
b	$.4$
c	$.6$

S		
a	a	1
b	v	$.5$
b	w	$.2$

T	
v	$.3$
w	$.7$
b	1

- The query is true iff $R(b)$ is here and one of:
  - $S(b, v)$ and $T(v)$ are here
  - $S(b, w)$ and $T(w)$ are here

$\rightarrow$ Probability:

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\; R(x) \wedge S(x,y) \wedge T(y)$$

R	
a	1
b	$.4$
c	$.6$

S		
a	a	1
b	v	$.5$
b	w	$.2$

T	
v	$.3$
w	$.7$
b	1

- The query is true iff $R(b)$ is here and one of:
  - $S(b,v)$ and $T(v)$ are here
  - $S(b,w)$ and $T(w)$ are here

$\rightarrow$ Probability: $.4 \times$

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\; R(x) \land S(x,y) \land T(y)$$

R	
a	1
b	$.4$
c	$.6$

S		
a	a	1
b	v	$.5$
b	w	$.2$

T	
v	$.3$
w	$.7$
b	1

- The query is true iff $R(b)$ is here and one of:
  - $S(b,v)$ and $T(v)$ are here
  - $S(b,w)$ and $T(w)$ are here

$\rightarrow$ Probability: $.4 \times (1 -$

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\; R(x) \wedge S(x, y) \wedge T(y)$$

R	
a	1
b	$.4$
c	$.6$

S		
a	a	1
b	v	$.5$
b	w	$.2$

T	
v	$.3$
w	$.7$
b	1

- The query is true iff $R(b)$ is here and one of:
  - $S(b, v)$ and $T(v)$ are here
  - $S(b, w)$ and $T(w)$ are here

$\rightarrow$ Probability: $.4 \times (1 - (1 - .5 \times .3)$

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\ R(x) \wedge S(x, y) \wedge T(y)$$

R	
a	1
b	$.4$
c	$.6$

S		
a	a	1
b	v	$.5$
b	w	$.2$

T	
v	$.3$
w	$.7$
b	1

- The query is true iff $R(b)$ is here and one of:
  - $S(b, v)$ and $T(v)$ are here
  - $S(b, w)$ and $T(w)$ are here

$\rightarrow$ Probability: $.4 \times (1 - (1 - .5 \times .3) \times (1 - .2 \times .7))$

Example of probabilistic query evaluation

Signature σ, class $\mathcal{Q}$ of conjunctive queries, class $\mathcal{I}$ of all instances.

$$q : \exists x\, y\ R(x) \wedge S(x, y) \wedge T(y)$$

R	
a	1
b	$.4$
c	$.6$

S		
a	a	1
b	v	$.5$
b	w	$.2$

T	
v	$.3$
w	$.7$
b	1

- The query is true iff $R(b)$ is here and one of:
  - $S(b, v)$ and $T(v)$ are here
  - $S(b, w)$ and $T(w)$ are here

$\rightarrow$ Probability: $.4 \times (1 - (1 - .5 \times .3) \times (1 - .2 \times .7)) = .1076$

Complexity of probabilistic query evaluation (PQE)

Question: what is the complexity of probabilistic query evaluation depending on the class $\mathcal{Q}$ of queries and class $\mathcal{I}$ of instances?

Complexity of probabilistic query evaluation (PQE)

Question: what is the complexity of probabilistic query evaluation depending on the class $\mathcal{Q}$ of queries and class $\mathcal{I}$ of instances?

- Existing dichotomy result: [Dalvi and Suciu, 2012]
  - $\mathcal{Q}$ are (unions of) conjunctive queries, $\mathcal{I}$ is all instances
  - There is a class $\mathcal{S} \subseteq \mathcal{Q}$ of safe queries

Complexity of probabilistic query evaluation (PQE)

Question: what is the complexity of probabilistic query evaluation depending on the class $\mathcal{Q}$ of queries and class $\mathcal{I}$ of instances?

- Existing dichotomy result: [Dalvi and Suciu, 2012]
  - $\mathcal{Q}$ are (unions of) conjunctive queries, $\mathcal{I}$ is all instances
  - There is a class $\mathcal{S} \subseteq \mathcal{Q}$ of safe queries
  - PQE is PTIME for any $q \in \mathcal{S}$ on all instances

Complexity of probabilistic query evaluation (PQE)

Question: what is the complexity of probabilistic query evaluation depending on the class $\mathcal{Q}$ of queries and class $\mathcal{I}$ of instances?

- Existing dichotomy result: [Dalvi and Suciu, 2012]
  - $\mathcal{Q}$ are (unions of) conjunctive queries, $\mathcal{I}$ is all instances
  - There is a class $\mathcal{S} \subseteq \mathcal{Q}$ of safe queries
  - PQE is PTIME for any $q \in \mathcal{S}$ on all instances
  - PQE is #P-hard for any $q \in \mathcal{Q} \backslash \mathcal{S}$ on all instances

Complexity of probabilistic query evaluation (PQE)

Question: what is the complexity of probabilistic query evaluation depending on the class $\mathcal{Q}$ of queries and class $\mathcal{I}$ of instances?

- Existing dichotomy result: [Dalvi and Suciu, 2012]
  - $\mathcal{Q}$ are (unions of) conjunctive queries, $\mathcal{I}$ is all instances
  - There is a class $\mathcal{S} \subseteq \mathcal{Q}$ of safe queries
  - PQE is PTIME for any $q \in \mathcal{S}$ on all instances
  - PQE is #P-hard for any $q \in \mathcal{Q} \backslash \mathcal{S}$ on all instances
  - $q : \exists x\, y\ R(x) \wedge S(x, y) \wedge T(y)$ is unsafe!

Complexity of probabilistic query evaluation (PQE)

Question: what is the complexity of probabilistic query evaluation depending on the class $\mathcal{Q}$ of queries and class $\mathcal{I}$ of instances?

- Existing dichotomy result: [Dalvi and Suciu, 2012]
  - $\mathcal{Q}$ are (unions of) conjunctive queries, $\mathcal{I}$ is all instances
  - There is a class $\mathcal{S} \subseteq \mathcal{Q}$ of safe queries
  - PQE is PTIME for any $q \in \mathcal{S}$ on all instances
  - PQE is #P-hard for any $q \in \mathcal{Q} \backslash \mathcal{S}$ on all instances
  - $q : \exists x\, y\; R(x) \wedge S(x, y) \wedge T(y)$ is unsafe!

Is there a smaller class $\mathcal{I}$ such that PQE is tractable for a larger $\mathcal{Q}$?

Complexity of probabilistic query evaluation (PQE)

Question: what is the complexity of probabilistic query evaluation depending on the class $\mathcal{Q}$ of queries and class $\mathcal{I}$ of instances?

- Existing dichotomy result: [Dalvi and Suciu, 2012]
  - $\mathcal{Q}$ are (unions of) conjunctive queries, $\mathcal{I}$ is all instances
  - There is a class $\mathcal{S} \subseteq \mathcal{Q}$ of safe queries
  - PQE is PTIME for any $q \in \mathcal{S}$ on all instances
  - PQE is #P-hard for any $q \in \mathcal{Q} \backslash \mathcal{S}$ on all instances
  - $q : \exists x\, y\; R(x) \wedge S(x, y) \wedge T(y)$ is unsafe!

Is there a smaller class $\mathcal{I}$ such that PQE is tractable for a larger $\mathcal{Q}$?

- Probabilistic XML: [Cohen et al., 2009]
  - $\mathcal{Q}$ are tree automata, $\mathcal{I}$ are trees
  - PQE is PTIME

Trees and treelike instances

- Goal: find an instance class $\mathcal{I}$ where PQE is tractable

Trees and treelike instances

- Goal: find an instance class $\mathcal{I}$ where PQE is tractable
- Idea: take $\mathcal{I}$ to be treelike instances
  - Treelike: the treewidth is bounded by a constant

Trees and treelike instances

- Goal: find an instance class $\mathcal{I}$ where PQE is tractable
- Idea: take $\mathcal{I}$ to be treelike instances
  - Treelike: the treewidth is bounded by a constant
    - Trees have treewidth 1
    - Cycles have treewidth 2
    - k-cliques and $(k-1)$-grids have treewidth $k-1$

Trees and treelike instances

- Goal: find an instance class $\mathcal{I}$ where PQE is tractable
- Idea: take $\mathcal{I}$ to be treelike instances
  - Treelike: the treewidth is bounded by a constant
    - Trees have treewidth 1
    - Cycles have treewidth 2
    - k-cliques and $(k-1)$-grids have treewidth $k-1$

→ For non-probabilistic query evaluation [Courcelle, 1990]:

- $\mathcal{I}$: treelike instances; $\mathcal{Q}$: monadic second-order (MSO) queries

→ non-probabilistic QE is in linear time

Trees and treelike instances

- Goal: find an instance class $\mathcal{I}$ where PQE is tractable
- Idea: take $\mathcal{I}$ to be treelike instances
  - Treelike: the treewidth is bounded by a constant
    - Trees have treewidth 1
    - Cycles have treewidth 2
    - k-cliques and $(k-1)$-grids have treewidth $k-1$

$\rightarrow$ For non-probabilistic query evaluation [Courcelle, 1990]:

- $\mathcal{I}$: treelike instances; $\mathcal{Q}$: monadic second-order (MSO) queries

$\rightarrow$ non-probabilistic QE is in linear time

$\rightarrow$ Does this extend to probabilistic QE?

Our results

An instance-based dichotomy result:

Upper bound. For $\mathcal{I}$ the treelike instances and $\mathcal{Q}$ the MSO queries
$\rightarrow$ PQE is in linear time modulo arithmetic costs

Our results

An instance-based dichotomy result:

Upper bound. For $\mathcal{I}$ the treelike instances and $\mathcal{Q}$ the MSO queries

$\rightarrow$ PQE is in linear time modulo arithmetic costs

- Also for expressive provenance representations
- Also with bounded-treewidth correlations

Our results

An instance-based dichotomy result:

Upper bound. For $\mathcal{I}$ the treelike instances and $\mathcal{Q}$ the MSO queries

$\rightarrow$ PQE is in linear time modulo arithmetic costs

- Also for expressive provenance representations
- Also with bounded-treewidth correlations

Lower bound. For any unbounded-tw family $\mathcal{I}$ and $\mathcal{Q}$ FO queries

$\rightarrow$ PQE is #P-hard under RP reductions assuming

- Signature arity is 2 (graphs)
- High-tw instances in $\mathcal{I}$ are easily constructible

Table of contents

Technical tool: provenance

The provenance of a query q on an instance I:

- Boolean function ϕ whose variables are the facts of I
- A subinstance of I satisfies q iff ϕ is true for that valuation

Technical tool: provenance

The provenance of a query q on an instance I:

- Boolean function ϕ whose variables are the facts of I
- A subinstance of I satisfies q iff ϕ is true for that valuation

$\rightarrow$ For all $\nu : I \rightarrow \{0, 1\}$ we have $\nu(\phi) = 1$ iff $\{F \in I \mid \nu(F) = 1\} \models q$

Technical tool: provenance

The provenance of a query q on an instance I:

- Boolean function ϕ whose variables are the facts of I
- A subinstance of I satisfies q iff ϕ is true for that valuation

$\rightarrow$ For all $\nu : I \rightarrow \{0, 1\}$ we have $\nu(\phi) = 1$ iff $\{F \in I \mid \nu(F) = 1\} \models q$

Example query: $\exists x\, y\, z\; R(x, y) \wedge R(y, z)$

Technical tool: provenance

The provenance of a query q on an instance I:

- Boolean function ϕ whose variables are the facts of I
- A subinstance of I satisfies q iff ϕ is true for that valuation

$\rightarrow$ For all $\nu : I \rightarrow \{0, 1\}$ we have $\nu(\phi) = 1$ iff $\{F \in I \mid \nu(F) = 1\} \models q$

Example query: $\exists x\, y\, z\ R(x, y) \wedge R(y, z)$

R		
a	b	f_1
b	c	f_2
d	e	f_3
e	d	f_4
f	f	f_5

Technical tool: provenance

The provenance of a query q on an instance I:

- Boolean function ϕ whose variables are the facts of I
- A subinstance of I satisfies q iff ϕ is true for that valuation

$\rightarrow$ For all $\nu : I \rightarrow \{0, 1\}$ we have $\nu(\phi) = 1$ iff $\{F \in I \mid \nu(F) = 1\} \models q$

Example query: $\exists x\, y\, z\; R(x, y) \wedge R(y, z)$

R		
a	b	f_1
b	c	f_2
d	e	f_3
e	d	f_4
f	f	f_5

$\rightarrow$ Provenance: $(f_1 \wedge f_2)$

Technical tool: provenance

The provenance of a query q on an instance I:

- Boolean function ϕ whose variables are the facts of I
- A subinstance of I satisfies q iff ϕ is true for that valuation

→ For all $\nu : I \to \{0,1\}$ we have $\nu(\phi) = 1$ iff $\{F \in I \mid \nu(F) = 1\} \models q$

Example query: $\exists x\, y\, z\; R(x,y) \wedge R(y,z)$

R		
a	b	f_1
b	c	f_2
d	e	f_3
e	d	f_4
f	f	f_5

→ Provenance: $(f_1 \wedge f_2) \vee (f_3 \wedge f_4)$

Technical tool: provenance

The provenance of a query q on an instance I:

- Boolean function ϕ whose variables are the facts of I
- A subinstance of I satisfies q iff ϕ is true for that valuation

$\rightarrow$ For all $\nu : I \rightarrow \{0, 1\}$ we have $\nu(\phi) = 1$ iff $\{F \in I \mid \nu(F) = 1\} \models q$

Example query: $\exists x\, y\, z\; R(x, y) \wedge R(y, z)$

R		
a	b	f_1
b	c	f_2
d	e	f_3
e	d	f_4
f	f	f_5

$\rightarrow$ Provenance: $(f_1 \wedge f_2) \vee (f_3 \wedge f_4)$

Technical tool: provenance

The provenance of a query q on an instance I:

- Boolean function ϕ whose variables are the facts of I
- A subinstance of I satisfies q iff ϕ is true for that valuation

$\rightarrow$ For all $\nu : I \rightarrow \{0, 1\}$ we have $\nu(\phi) = 1$ iff $\{F \in I \mid \nu(F) = 1\} \models q$

Example query: $\exists x\, y\, z\; R(x, y) \land R(y, z)$

R		
a	b	f_1
b	c	f_2
d	e	f_3
e	d	f_4
f	f	f_5

$\rightarrow$ Provenance: $(f_1 \land f_2) \lor (f_3 \land f_4) \lor f_5$

Technical tool: provenance

The provenance of a query q on an instance I:

- Boolean function ϕ whose variables are the facts of I
- A subinstance of I satisfies q iff ϕ is true for that valuation

$\rightarrow$ For all $\nu : I \rightarrow \{0,1\}$ we have $\nu(\phi) = 1$ iff $\{F \in I \mid \nu(F) = 1\} \models q$

Example query: $\exists x\, y\, z\; R(x,y) \wedge R(y,z)$

R		
a	b	f_1
b	c	f_2
d	e	f_3
e	d	f_4
f	f	f_5

$\rightarrow$ Provenance: $(f_1 \wedge f_2) \vee (f_3 \wedge f_4) \vee f_5$

General roadmap

- Use provenance for probabilistic query evaluation:
  - Compute a provenance representation efficiently

  $\rightarrow$ Probability of the provenance = probability of the query

General roadmap

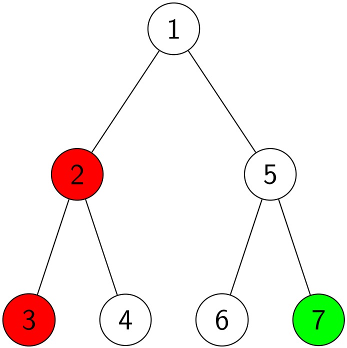
- Use provenance for probabilistic query evaluation:
  - Compute a provenance representation efficiently
  - $\rightarrow$ Probability of the provenance = probability of the query
  - Compute the provenance probability efficiently
    (show it is not #P-hard as in the general case)

General roadmap

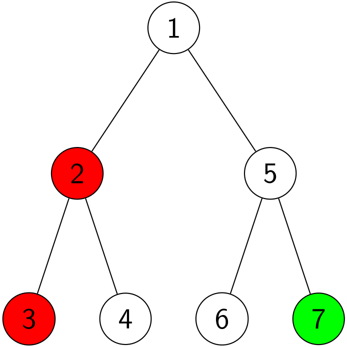
- Use provenance for probabilistic query evaluation:
  - Compute a provenance representation efficiently

  $\rightarrow$ Probability of the provenance $=$ probability of the query
  - Compute the provenance probability efficiently
    (show it is not #P-hard as in the general case)
- To solve the PQE problem on treelike instances for MSO
  - First solve the problem on trees with tree automata
  - Then use the results of [Courcelle, 1990]

Uncertain trees

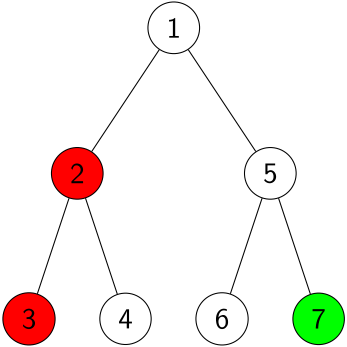


Uncertain trees



A valuation of a tree decides whether to keep or discard node labels.

Uncertain trees



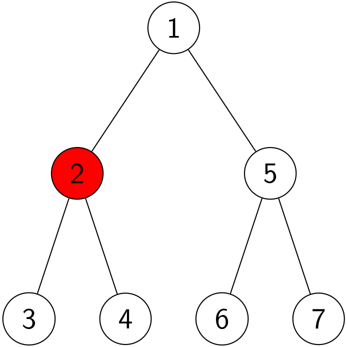
A valuation of a tree decides whether to keep or discard node labels.

Example tree automaton:
"Is there both a red and a green node?"

Valuation: $\{2, 3, 7\}$

The tree automaton accepts

Uncertain trees



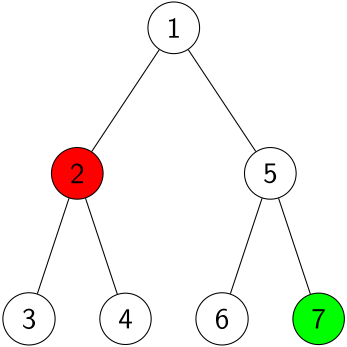
A valuation of a tree decides whether to keep or discard node labels.

Example tree automaton:
"Is there both a red and a green node?"

Valuation: $\{2\}$

The tree automaton rejects

Uncertain trees



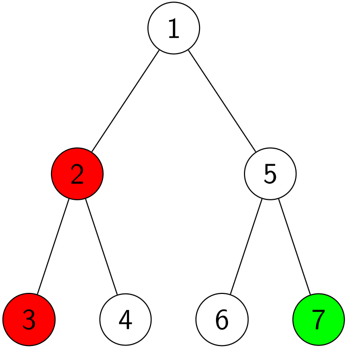
A valuation of a tree decides whether to keep or discard node labels.

Example tree automaton:
"Is there both a red and a green node?"

Valuation: $\{2, 7\}$

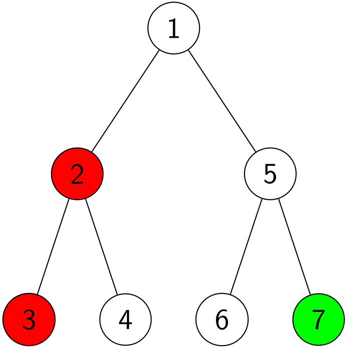
The tree automaton accepts

Provenance formulae and circuits on trees



- Which valuations satisfy the query?

Provenance formulae and circuits on trees



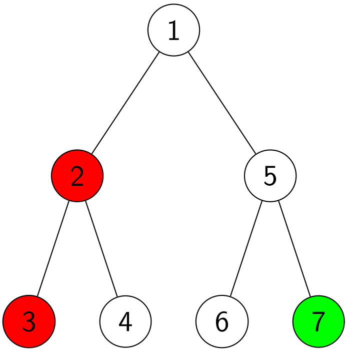
- Which valuations satisfy the query?

→ Provenance of a tree automaton A on an uncertain tree T:

  - Boolean formula ϕ
  - on variables x_2, x_3, x_7

  → A accepts $\nu(T)$ iff $\nu(\phi)$ is true

Provenance formulae and circuits on trees



- Which valuations satisfy the query?

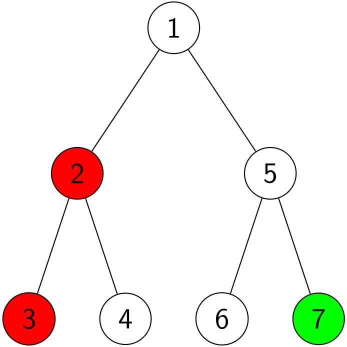
→ Provenance of a tree automaton A on an uncertain tree T:
  - Boolean formula ϕ
  - on variables x_2, x_3, x_7

  → A accepts $\nu(T)$ iff $\nu(\phi)$ is true

- Provenance circuit of A on T [Deutch et al., 2014]
  - Boolean circuit C
  - with input gates g_2, g_3, g_7

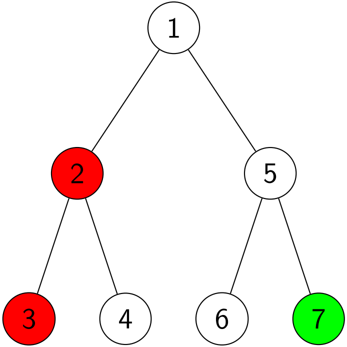
  → A accepts $\nu(T)$ iff $\nu(C)$ is true

Example



Is there both a red and a green node?

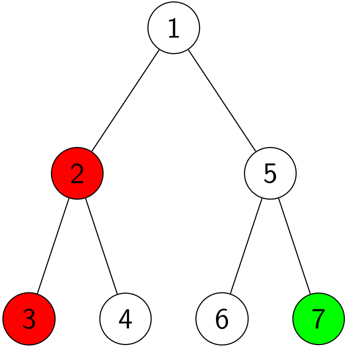
Example



Is there both a red and a green node?

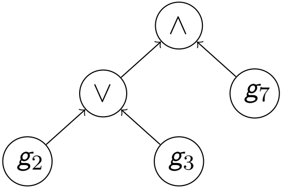
- Provenance formula: $(x_2 \vee x_3) \wedge x_7$

Example



Is there both a red and a green node?

- Provenance formula: $(x_2 \lor x_3) \land x_7$
- Provenance circuit:



Our main result on trees

Theorem

For any bottom-up (nondet) tree automaton A and input tree T, we can build a provenance circuit of A on T in linear time in A and T.

Our main result on trees

Theorem

For any bottom-up (nondet) tree automaton A and input tree T, we can build a provenance circuit of A on T in linear time in A and T.

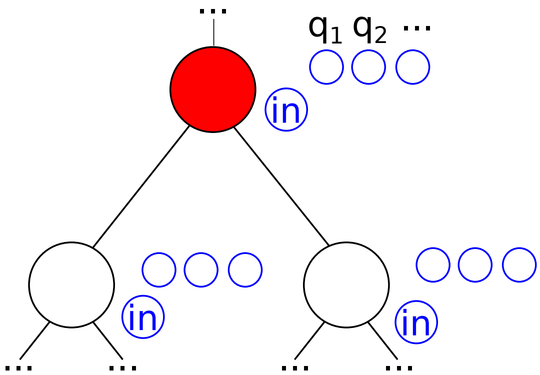
Construct the Boolean provenance circuit bottom-up

Our main result on trees

Theorem

For any bottom-up (nondet) tree automaton A and input tree T, we can build a provenance circuit of A on T in linear time in A and T.

Construct the Boolean provenance circuit bottom-up

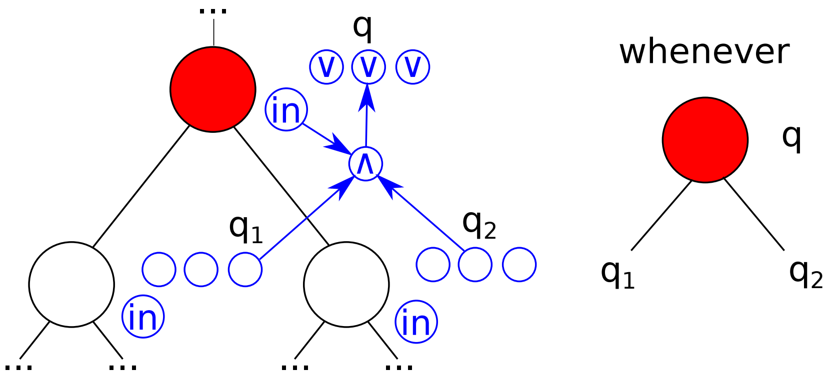


Our main result on trees

Theorem

For any bottom-up (nondet) tree automaton A and input tree T, we can build a provenance circuit of A on T in linear time in A and T.

Construct the Boolean provenance circuit bottom-up

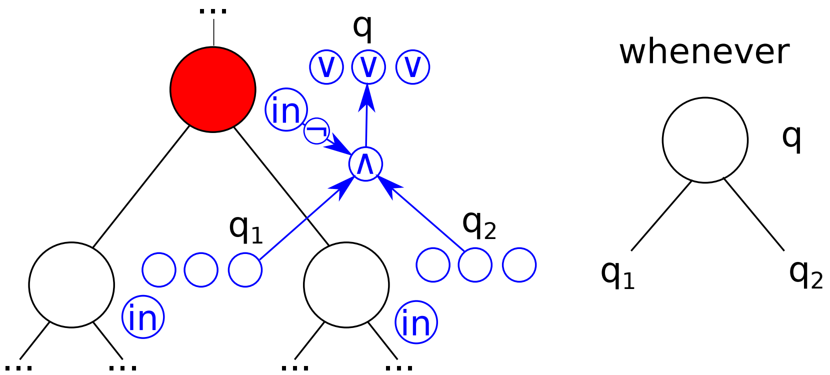


Our main result on trees

Theorem

For any bottom-up (nondet) tree automaton A and input tree T, we can build a provenance circuit of A on T in linear time in A and T.

Construct the Boolean provenance circuit bottom-up



Treelike instances

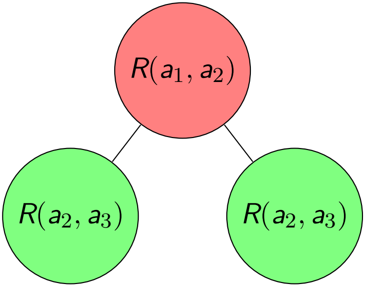
- Treelike instance I
- Tree encoding: tree E on fixed alphabet, represents I
- MSO query on I translates to
  - $\rightarrow$ MSO query on E by [Courcelle, 1990]
  - $\rightarrow$ tree automaton on E by [Thatcher and Wright, 1968]

Treelike instances

- Treelike instance I
- Tree encoding: tree E on fixed alphabet, represents I
- MSO query on I translates to
  - $\rightarrow$ MSO query on E by [Courcelle, 1990]
  - $\rightarrow$ tree automaton on E by [Thatcher and Wright, 1968]
- Uncertain instance: each fact can be present or absent

$\rightarrow$ Possible subinstances are possible valuations of the encoding

R	
a	b
b	c
b	d

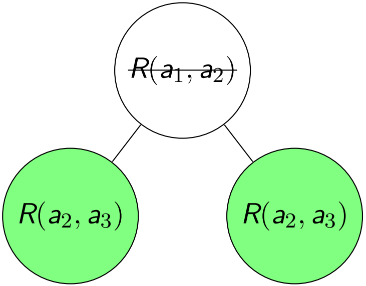


Treelike instances

- Treelike instance I
- Tree encoding: tree E on fixed alphabet, represents I
- MSO query on I translates to
  - $\rightarrow$ MSO query on E by [Courcelle, 1990]
  - $\rightarrow$ tree automaton on E by [Thatcher and Wright, 1968]
- Uncertain instance: each fact can be present or absent

$\rightarrow$ Possible subinstances are possible valuations of the encoding

R	
~~a~~	~~b~~
b	c
b	d

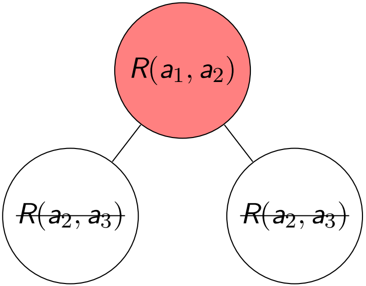


Treelike instances

- Treelike instance I
- Tree encoding: tree E on fixed alphabet, represents I
- MSO query on I translates to
  - $\rightarrow$ MSO query on E by [Courcelle, 1990]
  - $\rightarrow$ tree automaton on E by [Thatcher and Wright, 1968]
- Uncertain instance: each fact can be present or absent

$\rightarrow$ Possible subinstances are possible valuations of the encoding

R	
a	b
~~b~~	~~c~~
~~b~~	~~d~~

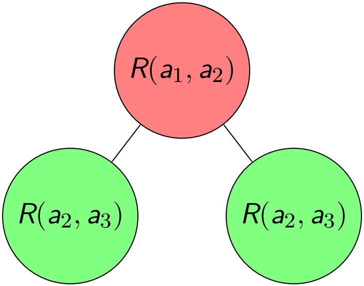


Treelike instances

- Treelike instance I
- Tree encoding: tree E on fixed alphabet, represents I
- MSO query on I translates to
  - $\rightarrow$ MSO query on E by [Courcelle, 1990]
  - $\rightarrow$ tree automaton on E by [Thatcher and Wright, 1968]
- Uncertain instance: each fact can be present or absent

$\rightarrow$ Possible subinstances are possible valuations of the encoding

R	
a	b
b	c
b	d



Our main result on treelike instances

Theorem

For any fixed MSO query q and $k \in \mathbb{N}$,
for any input instance I of treewidth $\leq k$,
we can build in linear time in I a provenance circuit of q on I.

Probability evaluation

Two alternate ways to see why probability evaluation is tractable on our provenance circuits:

Probability evaluation

Two alternate ways to see why probability evaluation is tractable on our provenance circuits:

- They have bounded treewidth themselves
  - Follows the structure of the tree encoding
  - Width only depends on number of automaton states
  - → Apply message passing [Lauritzen and Spiegelhalter, 1988]

Probability evaluation

Two alternate ways to see why probability evaluation is tractable on our provenance circuits:

- They have bounded treewidth themselves
  - Follows the structure of the tree encoding
  - Width only depends on number of automaton states

  $\rightarrow$ Apply message passing [Lauritzen and Spiegelhalter, 1988]

- If the tree automaton is deterministic
  - All conjunctions depend on disjoint sets of input gates
  - All disjunctions are on mutually exclusive outcomes

  $\rightarrow$ Circuit is a d-DNNF [Darwiche, 2001]

Probability evaluation

Two alternate ways to see why probability evaluation is tractable on our provenance circuits:

- They have bounded treewidth themselves
  - Follows the structure of the tree encoding
  - Width only depends on number of automaton states
  - → Apply message passing [Lauritzen and Spiegelhalter, 1988]

- If the tree automaton is deterministic
  - All conjunctions depend on disjoint sets of input gates
  - All disjunctions are on mutually exclusive outcomes
  - → Circuit is a d-DNNF [Darwiche, 2001]

Corollary

Probabilistic query evaluation of MSO queries on treelike instances is in linear time up to arithmetic operations.

Table of contents

Provenance semirings

- Semiring of positive Boolean functions $(\mathrm{PosBool}[X], \vee, \wedge, \mathfrak{f}, \mathfrak{t})$

Provenance semirings

- Semiring of positive Boolean functions $(\mathrm{PosBool}[X], \lor, \land, \mathfrak{f}, \mathfrak{t})$
- Provenance semirings: [Green et al., 2007]
  - Provenance generalized to arbitrary (commutative) semirings
  - For queries in the positive relational algebra and Datalog

Provenance semirings

- Semiring of positive Boolean functions $(\mathrm{PosBool}[X], \vee, \wedge, \mathfrak{f}, \mathfrak{t})$
- Provenance semirings: [Green et al., 2007]
  - Provenance generalized to arbitrary (commutative) semirings
  - For queries in the positive relational algebra and Datalog

$\rightarrow$ Our circuits capture $\mathrm{PosBool}[X]$-provenance in this sense

Provenance semirings

- Semiring of positive Boolean functions $(\mathrm{PosBool}[X], \vee, \wedge, \mathfrak{f}, \mathfrak{t})$
- Provenance semirings: [Green et al., 2007]
  - Provenance generalized to arbitrary (commutative) semirings
  - For queries in the positive relational algebra and Datalog

$\rightarrow$ Our circuits capture $\mathrm{PosBool}[X]$-provenance in this sense
  - The definitions match: all subinstances that satisfy the query

Provenance semirings

- Semiring of positive Boolean functions $(\mathrm{PosBool}[X], \vee, \wedge, \mathfrak{f}, \mathfrak{t})$
- Provenance semirings: [Green et al., 2007]
  - Provenance generalized to arbitrary (commutative) semirings
  - For queries in the positive relational algebra and Datalog

→ Our circuits capture $\mathrm{PosBool}[X]$-provenance in this sense
  - The definitions match: all subinstances that satisfy the query
  - For monotone queries, we can construct positive circuits

Universal provenance

- Universal semiring of polynomials $(\mathbb{N}[X], +, \times, 0, 1)$
  $\rightarrow$ The provenance for $\mathbb{N}[X]$ can be specialized to any $K[X]$

Universal provenance

- Universal semiring of polynomials $(\mathbb{N}[X], +, \times, 0, 1)$
  $\rightarrow$ The provenance for $\mathbb{N}[X]$ can be specialized to any $K[X]$

- Captures many useful semirings:
  - counting the number of matches of a query
  - computing the security level of a query result
  - computing the cost of a query result

$\mathbb{N}[X]$-provenance example

R		
a	b	x_1
b	c	x_2
d	e	x_3
e	d	x_4
f	f	x_5

$\mathbb{N}[X]$-provenance example

R		
a	b	x_1
b	c	x_2
d	e	x_3
e	d	x_4
f	f	x_5

$$\exists x\, y\, z\ R(x, y) \wedge R(y, z)$$

$\rightarrow$ $\mathrm{PosBool}[X]$-provenance:

$\rightarrow$ $\mathbb{N}[X]$-provenance:

- Definition of provenance for conjunctive queries:
  - Sum over query matches
  - Multiply over matched facts

$\mathbb{N}[X]$-provenance example

R		
a	b	x_1
b	c	x_2
d	e	x_3
e	d	x_4
f	f	x_5

$$\exists x\, y\, z\; R(x, y) \wedge R(y, z)$$

$\rightarrow$ $\mathrm{PosBool}[X]$-provenance:

$\rightarrow$ $\mathbb{N}[X]$-provenance:

- Definition of provenance for conjunctive queries:
  - Sum over query matches
  - Multiply over matched facts

$\mathbb{N}[X]$-provenance example

R		
a	b	x_1
b	c	x_2
d	e	x_3
e	d	x_4
f	f	x_5

$$\exists x\, y\, z\; R(x, y) \wedge R(y, z)$$

$\rightarrow$ $\mathrm{PosBool}[X]$-provenance:

$(x_1 \wedge x_2)$

$\rightarrow$ $\mathbb{N}[X]$-provenance:

$(x_1 \times x_2)$

- Definition of provenance for conjunctive queries:
  - Sum over query matches
  - Multiply over matched facts

$\mathbb{N}[X]$-provenance example

R		
a	b	x_1
b	c	x_2
d	e	x_3
e	d	x_4
f	f	x_5

$$\exists x\, y\, z\; R(x, y) \land R(y, z)$$

$\rightarrow$ $\mathrm{PosBool}[X]$-provenance:

$(x_1 \land x_2)$

$\rightarrow$ $\mathbb{N}[X]$-provenance:

$(x_1 \times x_2)$

- Definition of provenance for conjunctive queries:
  - Sum over query matches
  - Multiply over matched facts

$\mathbb{N}[X]$-provenance example

R		
a	b	x_1
b	c	x_2
d	e	x_3
e	d	x_4
f	f	x_5

$$\exists x\, y\, z\ R(x, y) \wedge R(y, z)$$

$\rightarrow$ $\mathrm{PosBool}[X]$-provenance:

$$(x_1 \wedge x_2) \vee (x_3 \wedge x_4)$$

$\rightarrow$ $\mathbb{N}[X]$-provenance:

$$(x_1 \times x_2) + (x_3 \times x_4)$$

- Definition of provenance for conjunctive queries:
  - Sum over query matches
  - Multiply over matched facts

$\mathbb{N}[X]$-provenance example

R		
a	b	x_1
b	c	x_2
d	e	x_3
e	d	x_4
f	f	x_5

$$\exists x\, y\, z\ R(x, y) \wedge R(y, z)$$

$\rightarrow$ $\mathrm{PosBool}[X]$-provenance:

$$(x_1 \wedge x_2) \vee (x_3 \wedge x_4)$$

$\rightarrow$ $\mathbb{N}[X]$-provenance:

$$(x_1 \times x_2) + (x_3 \times x_4)$$

- Definition of provenance for conjunctive queries:
    - Sum over query matches
    - Multiply over matched facts

$\mathbb{N}[X]$-provenance example

R		
a	b	x_1
b	c	x_2
d	e	x_3
e	d	x_4
f	f	x_5

$$\exists x\, y\, z\ R(x, y) \wedge R(y, z)$$

$\rightarrow$ $\mathrm{PosBool}[X]$-provenance:

$$(x_1 \wedge x_2) \vee (x_3 \wedge x_4)$$

$\rightarrow$ $\mathbb{N}[X]$-provenance:

$$(x_1 \times x_2) + (x_3 \times x_4) + (x_4 \times x_3)$$

- Definition of provenance for conjunctive queries:
    - Sum over query matches
    - Multiply over matched facts

$\mathbb{N}[X]$-provenance example

R		
a	b	x_1
b	c	x_2
d	e	x_3
e	d	x_4
f	f	x_5

$$\exists x\, y\, z\ R(x, y) \land R(y, z)$$

$\rightarrow$ $\mathrm{PosBool}[X]$-provenance:

$$(x_1 \land x_2) \lor (x_3 \land x_4)$$

$\rightarrow$ $\mathbb{N}[X]$-provenance:

$$(x_1 \times x_2) + (x_3 \times x_4) + (x_4 \times x_3)$$

- Definition of provenance for conjunctive queries:
  - Sum over query matches
  - Multiply over matched facts

$\mathbb{N}[X]$-provenance example

R		
a	b	x_1
b	c	x_2
d	e	x_3
e	d	x_4
f	f	x_5

$$\exists x\, y\, z\; R(x, y) \wedge R(y, z)$$

→ $\mathrm{PosBool}[X]$-provenance:

$$(x_1 \wedge x_2) \vee (x_3 \wedge x_4) \qquad \vee \quad x_5$$

→ $\mathbb{N}[X]$-provenance:

$$(x_1 \times x_2) + (x_3 \times x_4) + (x_4 \times x_3) + (x_5 \times x_5)$$

- Definition of provenance for conjunctive queries:
  - Sum over query matches
  - Multiply over matched facts

$\mathbb{N}[X]$-provenance example

R		
a	b	x_1
b	c	x_2
d	e	x_3
e	d	x_4
f	f	x_5

$$\exists x\, y\, z\; R(x,y) \wedge R(y,z)$$

$\rightarrow$ $\mathrm{PosBool}[X]$-provenance:

$$(x_1 \wedge x_2) \vee (x_3 \wedge x_4) \qquad \vee \quad x_5$$

$\rightarrow$ $\mathbb{N}[X]$-provenance:

$$(x_1 \times x_2) + (x_3 \times x_4) + (x_4 \times x_3) + (x_5 \times x_5)$$

- Definition of provenance for conjunctive queries:
  - Sum over query matches
  - Multiply over matched facts

$\mathbb{N}[X]$-provenance example

R		
a	b	x_1
b	c	x_2
d	e	x_3
e	d	x_4
f	f	x_5

$$\exists x\, y\, z\; R(x, y) \wedge R(y, z)$$

→ $\mathrm{PosBool}[X]$-provenance:

$$(x_1 \wedge x_2) \vee (x_3 \wedge x_4) \qquad \vee \quad x_5$$

→ $\mathbb{N}[X]$-provenance:

$$(x_1 \times x_2) + (x_3 \times x_4) + (x_4 \times x_3) + (x_5 \times x_5)$$
$$= x_1 x_2 + 2 x_3 x_4 + x_5^2$$

- Definition of provenance for conjunctive queries:
  - Sum over query matches
  - Multiply over matched facts

$\mathbb{N}[X]$-provenance example

R		
a	b	x_1
b	c	x_2
d	e	x_3
e	d	x_4
f	f	x_5

$$\exists x\, y\, z\ R(x, y) \wedge R(y, z)$$

$\rightarrow$ $\mathrm{PosBool}[X]$-provenance:

$$(x_1 \wedge x_2) \vee (x_3 \wedge x_4) \qquad \vee \quad x_5$$

$\rightarrow$ $\mathbb{N}[X]$-provenance:

$$(x_1 \times x_2) + (x_3 \times x_4) + (x_4 \times x_3) + (x_5 \times x_5)$$
$$= x_1 x_2 + 2 x_3 x_4 + x_5^2$$

- Definition of provenance for conjunctive queries:
  - Sum over query matches
  - Multiply over matched facts

How is $\mathbb{N}[X]$ more expressive than $\mathrm{PosBool}[X]$?

$\rightarrow$ Coefficients: counting multiple matches

$\rightarrow$ Exponents: using facts multiple times

Capturing $\mathbb{N}[X]$-provenance

Our construction can be extended to $\mathbb{N}[X]$-provenance
for conjunctive queries and unions of conjunctive queries (UCQ):

Capturing $\mathbb{N}[X]$-provenance

Our construction can be extended to $\mathbb{N}[X]$-provenance
for conjunctive queries and unions of conjunctive queries (UCQ):

Theorem

For any fixed UCQ q and $k \in \mathbb{N}$,
for any input instance I of treewidth $\leq k$,
we can build in linear time a $\mathbb{N}[X]$ provenance circuit of q on I.

Capturing $\mathbb{N}[X]$-provenance

Our construction can be extended to $\mathbb{N}[X]$-provenance for conjunctive queries and unions of conjunctive queries (UCQ):

Theorem

For any fixed UCQ q and $k \in \mathbb{N}$, for any input instance I of treewidth $\leq k$, we can build in linear time a $\mathbb{N}[X]$ provenance circuit of q on I.

$\rightarrow$ What fails for MSO and Datalog?

- Unbounded maximal multiplicity of fact uses

Table of contents

Correlations

- Our probabilistic instances assume independence on all facts
  $\longrightarrow$ Not very expressive!

Correlations

- Our probabilistic instances assume independence on all facts
  $\rightarrow$ Not very expressive!

More expressive formalism: Block-Independent Disjoint instances:

<u>**name**</u>	**city**	**iso**	p
pods	san francisco	us	0.8
pods	los angeles	us	0.2
icalp	rome	it	0.1
icalp	florence	it	0.9

pc-tables

More generally, pc-tables to represent arbitrary correlations

pc-tables

More generally, pc-tables to represent arbitrary correlations

date	teacher	room	
04	John	C42	$\neg x_1$
04	Jane	C42	x_1
11	John	C017	$x_2 \land \neg x_1$
11	Jane	C017	$x_2 \land x_1$
11	John	C47	$\neg x_2 \land \neg x_1$
11	Jane	C47	$\neg x_2 \land x_1$

pc-tables

More generally, pc-tables to represent arbitrary correlations

date	teacher	room	
04	John	C42	$\neg x_1$
04	Jane	C42	x_1
11	John	C017	$x_2 \wedge \neg x_1$
11	Jane	C017	$x_2 \wedge x_1$
11	John	C47	$\neg x_2 \wedge \neg x_1$
11	Jane	C47	$\neg x_2 \wedge x_1$

x_1 John gets sick

$\rightarrow$ Probability 0.1

x_2 Room C017 is available

$\rightarrow$ Probability 0.2

Our results

Probabilistic query evaluation on instances with correlations is tractable if the instance and correlations are bounded-tw:

Our results

Probabilistic query evaluation on instances with correlations is tractable if the instance and correlations are bounded-tw:

Theorem

Probabilistic query evaluation of MSO queries on treelike BID is in linear time up to arithmetic operations.

Our results

Probabilistic query evaluation on instances with correlations
is tractable if the instance and correlations are bounded-tw:

Theorem

Probabilistic query evaluation of MSO queries on treelike BID is in linear time up to arithmetic operations.

"Tree-like" just means the underlying instance (easy correlations)

Our results

Probabilistic query evaluation on instances with correlations is tractable if the instance and correlations are bounded-tw:

Theorem

Probabilistic query evaluation of MSO queries on treelike BID is in linear time up to arithmetic operations.

"Tree-like" just means the underlying instance (easy correlations)

Theorem

Probabilistic query evaluation of MSO queries on treelike pc-tables is in linear time up to arithmetic operations.

Our results

Probabilistic query evaluation on instances with correlations is tractable if the instance and correlations are bounded-tw:

Theorem

Probabilistic query evaluation of MSO queries on treelike BID is in linear time up to arithmetic operations.

"Tree-like" just means the underlying instance (easy correlations)

Theorem

Probabilistic query evaluation of MSO queries on treelike pc-tables is in linear time up to arithmetic operations.

"Tree-like" refers to the underlying instance, adding facts to represent variable occurrences and co-occurrences

Table of contents

Lower bound goal

- Class $\mathcal{I}$ of unbounded-treewidth instances, query q in class $\mathcal{Q}$.
- Show that probabilistic query evaluation of q on $\mathcal{I}$ is hard

Lower bound goal

- Class $\mathcal{I}$ of unbounded-treewidth instances, query q in class $\mathcal{Q}$.
- Show that probabilistic query evaluation of q on $\mathcal{I}$ is hard

$\rightarrow$ Restrict to arity-2 ($=$ labeled graphs) for technical reasons

Lower bound goal

- Class $\mathcal{I}$ of unbounded-treewidth instances, query q in class $\mathcal{Q}$.
- Show that probabilistic query evaluation of q on $\mathcal{I}$ is hard

$\rightarrow$ Restrict to arity-2 (= labeled graphs) for technical reasons

$\rightarrow$ Impose that $\mathcal{I}$ is tw-constructible:

Lower bound goal

- Class $\mathcal{I}$ of unbounded-treewidth instances, query q in class $\mathcal{Q}$.
- Show that probabilistic query evaluation of q on $\mathcal{I}$ is hard

$\rightarrow$ Restrict to arity-2 ($=$ labeled graphs) for technical reasons

$\rightarrow$ Impose that $\mathcal{I}$ is tw-constructible:

  - Given $k \in \mathbb{N}$, we can construct in time $\mathrm{Poly}(k)$ an instance of $\mathcal{I}$ of treewidth $\geq k$

Lower bound goal

- Class $\mathcal{I}$ of unbounded-treewidth instances, query q in class $\mathcal{Q}$.
- Show that probabilistic query evaluation of q on $\mathcal{I}$ is hard

→ Restrict to arity-2 (= labeled graphs) for technical reasons

→ Impose that $\mathcal{I}$ is tw-constructible:

  - Given $k \in \mathbb{N}$, we can construct in time $\mathrm{Poly}(k)$ an instance of $\mathcal{I}$ of treewidth $\geq k$

  → Otherwise instances of treewidth k in $\mathcal{I}$ could be very large... see [Makowsky and Marino, 2003]

Our lower bound result

Theorem

There is a first-order query q such that for any unbounded-tw, tw-constructible, arity-2 instance family $\mathcal{I}$, probabilistic query eval for q on $\mathcal{I}$ is #P-hard under RP reductions.

Idea: extracting topological minors

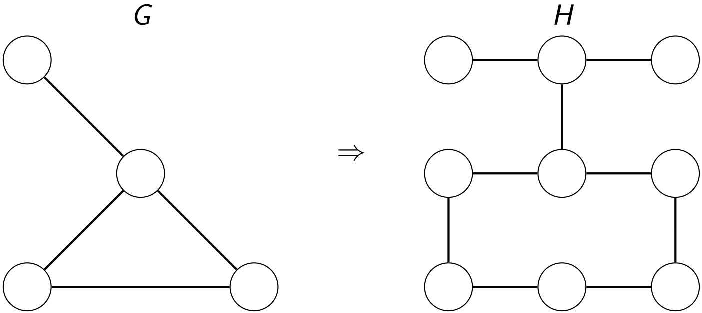
- Let G be a planar graph of degree ≤ 3

Idea: extracting topological minors

- Let G be a planar graph of degree ≤ 3
- G is a topological minor of H if:

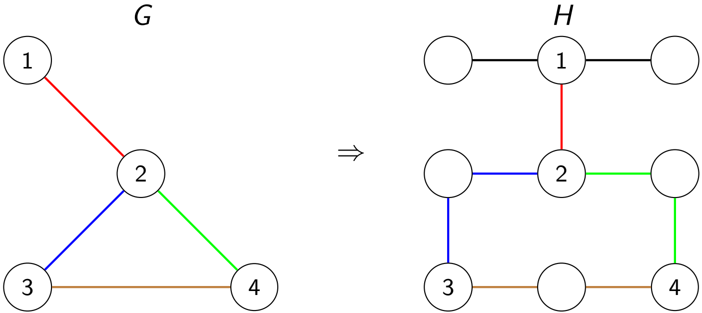
Idea: extracting topological minors

- Let G be a planar graph of degree ≤ 3
- G is a topological minor of H if:



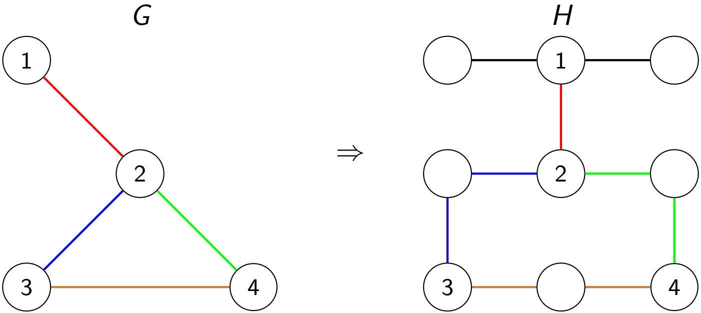
Idea: extracting topological minors

- Let G be a planar graph of degree ≤ 3
- G is a topological minor of H if:



Idea: extracting topological minors

- Let G be a planar graph of degree ≤ 3
- G is a topological minor of H if:



- Map vertices to vertices
- Map edges to vertex-disjoint paths

Topological minor extraction results

Theorem ([Robertson and Seymour, 1986])

For any planar graph G of degree ≤ 3,
for any graph H of sufficiently high treewidth,
G is a topological minor of H.

Topological minor extraction results

Theorem ([Robertson and Seymour, 1986])

For any planar graph G of degree ≤ 3,
for any graph H of sufficiently high treewidth,
G is a topological minor of H.

More recently:

Theorem ([Chekuri and Chuzhoy, 2014])

There is a certain constant $c \in \mathbb{N}$ such that
for any planar graph G of degree ≤ 3,
for any graph H of treewidth $\geq |G|^c$,
G is a topological minor of H and
we can embed G in H (with high probability) in PTIME in $|H|$.

Intuition for our result: reduction

- Choose a problem from which to reduce:
  - Must be #P-hard on planar degree-3 graphs
  - Must be encodable to an FO query q (more later)

$\rightarrow$ We use the problem of counting matchings

Intuition for our result: reduction

- Choose a problem from which to reduce:
  - Must be #P-hard on planar degree-3 graphs
  - Must be encodable to an FO query q (more later)

  $\rightarrow$ We use the problem of counting matchings
- Given an input graph G, compute $k := |G|^c$

Intuition for our result: reduction

- Choose a problem from which to reduce:
  - Must be #P-hard on planar degree-3 graphs
  - Must be encodable to an FO query q (more later)
  - $\rightarrow$ We use the problem of counting matchings
- Given an input graph G, compute $k := |G|^c$
- Compute in PTIME an instance I of $\mathcal{I}$ of treewidth $\geq k$

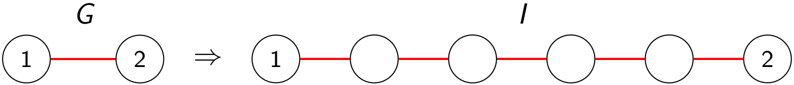
Intuition for our result: reduction

- Choose a problem from which to reduce:
  - Must be #P-hard on planar degree-3 graphs
  - Must be encodable to an FO query q (more later)
  - $\rightarrow$ We use the problem of counting matchings
- Given an input graph G, compute $k := |G|^c$
- Compute in PTIME an instance I of $\mathcal{I}$ of treewidth $\geq k$
- Compute in randomized PTIME an embedding of G in I

Intuition for our result: reduction

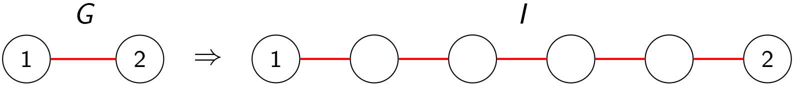
- Choose a problem from which to reduce:
  - Must be #P-hard on planar degree-3 graphs
  - Must be encodable to an FO query q (more later)
  - $\rightarrow$ We use the problem of counting matchings
- Given an input graph G, compute $k := |G|^c$
- Compute in PTIME an instance I of $\mathcal{I}$ of treewidth $\geq k$
- Compute in randomized PTIME an embedding of G in I
- Construct a probability valuation π of I such that:
  - Unneccessary edges of I are removed
  - Probability eval for q gives the answer to the hard problem

Technical issue



- In the embedding, edges of G can become long paths in I
- q must answer the hard problem on G despite subdivisions

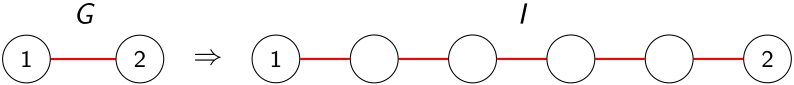
Technical issue



- In the embedding, edges of G can become long paths in I
- q must answer the hard problem on G despite subdivisions

→ Our q restricts to a subset of the worlds of known weight and gives the right answer up to renormalizing

Technical issue



- In the embedding, edges of G can become long paths in I
- q must answer the hard problem on G despite subdivisions

→ Our q restricts to a subset of the worlds of known weight and gives the right answer up to renormalizing

→ For non-probabilistic evaluation, using FO does not work [Frick and Grohe, 2001]

→ Lower bounds for non-probabilistic evaluation are for MSO [Ganian et al., 2014]

Can we do better?

- We can use a non-monotone FO or a monotone MSO query
- Can we use a weaker query language? (e.g., monotone FO)

Can we do better?

- We can use a non-monotone FO or a monotone MSO query
- Can we use a weaker query language? (e.g., monotone FO)

→ We cannot use a connected CQ even with inequalities

→ We cannot use a query closed under homomorphisms

Can we do better?

- We can use a non-monotone FO or a monotone MSO query
- Can we use a weaker query language? (e.g., monotone FO)

→ We cannot use a connected CQ even with inequalities

→ We cannot use a query closed under homomorphisms

- A good candidate query:

$$q : (E(x, y) \lor E(y, x)) \land (E(y, z) \land E(z, y)) \land x \neq z$$

Can we do better?

- We can use a non-monotone FO or a monotone MSO query
- Can we use a weaker query language? (e.g., monotone FO)

→ We cannot use a connected CQ even with inequalities
→ We cannot use a query closed under homomorphisms

- A good candidate query:

$$q : (E(x, y) \lor E(y, x)) \land (E(y, z) \land E(z, y)) \land x \neq z$$

→ This UCQ with inequalities is hard in a weaker sense (no polynomial-size OBDD representations of provenance)
→ We don't know whether it's #P-hard (because of subdivisions)

Table of contents

Summary of our results

Upper. Probabilistic query eval. for MSO on treelike instances has linear data complexity up to arithmetic costs

Summary of our results

Upper. Probabilistic query eval. for MSO on treelike instances has linear data complexity up to arithmetic costs

$\rightarrow$ Also for bounded-treewidth correlations

Summary of our results

Upper. Probabilistic query eval. for MSO on treelike instances has linear data complexity up to arithmetic costs

- → Also for bounded-treewidth correlations
- → Can compute a provenance circuit in linear time

Summary of our results

Upper. Probabilistic query eval. for MSO on treelike instances has linear data complexity up to arithmetic costs

- → Also for bounded-treewidth correlations
- → Can compute a provenance circuit in linear time
  - → Also $\mathbb{N}[X]$-provenance circuits for UCQ queries

Summary of our results

Upper. Probabilistic query eval. for MSO on treelike instances has linear data complexity up to arithmetic costs

- → Also for bounded-treewidth correlations
- → Can compute a provenance circuit in linear time
  - → Also $\mathbb{N}[X]$-provenance circuits for UCQ queries

Lower. PQE for FO on any tw-constructible, arity-2, unbounded-tw instance family is #P-hard under RP reductions

Summary of our results

Upper. Probabilistic query eval. for MSO on treelike instances has linear data complexity up to arithmetic costs

- → Also for bounded-treewidth correlations
- → Can compute a provenance circuit in linear time
  - → Also $\mathbb{N}[X]$-provenance circuits for UCQ queries

Lower. PQE for FO on any tw-constructible, arity-2, unbounded-tw instance family is #P-hard under RP reductions

→ Bounded treewidth is the right notion for tractability of PQE?

Future work (upper bound)

Two promising directions:

- Restricting both instances and queries
  - Hard query on unbounded-treewidth instances may be easy!
  - Query-specific tree decomposition or instance simplification?
  - Tractability criterion based on the instance and query?
  - Understand the connection to the query-based dichotomy?

Future work (upper bound)

Two promising directions:

- Restricting both instances and queries
  - Hard query on unbounded-treewidth instances may be easy!
  - Query-specific tree decomposition or instance simplification?
  - Tractability criterion based on the instance and query?
  - Understand the connection to the query-based dichotomy?

- Combined complexity: tractability in the query and data
  - Cost in the MSO query is nonelementary in general
  - Lower for some query languages? (... on some instances?)
  - Monadic Datalog approaches? [Gottlob et al., 2010]

Future work (lower bounds)

- Can we show #P-hardness under usual P reductions?
  - $\rightarrow$ Depends on [Chekuri and Chuzhoy, 2014]

Future work (lower bounds)

- Can we show #P-hardness under usual P reductions?
  - → Depends on [Chekuri and Chuzhoy, 2014]

- Can we make this work for arbitrary arity signatures?
  - → Problem: correlations between Gaifman graph edges
  - → Extracting minors with non-overlapping edges from bounded-arity hypergraphs?

Future work (lower bounds)

- Can we show #P-hardness under usual P reductions?
  → Depends on [Chekuri and Chuzhoy, 2014]
- Can we make this work for arbitrary arity signatures?
  → Problem: correlations between Gaifman graph edges
  → Extracting minors with non-overlapping edges from bounded-arity hypergraphs?
- What about simpler query languages?
  → Is there a monotone FO query where probability evaluation is hard on any constructible unbounded-treewidth family? (= under arbitrary subdivisions?)

Future work (lower bounds)

- Can we show #P-hardness under usual P reductions?
  $\rightarrow$ Depends on [Chekuri and Chuzhoy, 2014]

- Can we make this work for arbitrary arity signatures?
  $\rightarrow$ Problem: correlations between Gaifman graph edges
  $\rightarrow$ Extracting minors with non-overlapping edges from bounded-arity hypergraphs?

- What about simpler query languages?
  $\rightarrow$ Is there a monotone FO query where probability evaluation is hard on any constructible unbounded-treewidth family? (= under arbitrary subdivisions?)

Thanks for your attention!

References I

Chaudhuri, S. and Vardi, M. Y. (1992).
On the equivalence of recursive and nonrecursive Datalog programs.
In *PODS*.

Chekuri, C. and Chuzhoy, J. (2014).
Polynomial bounds for the grid-minor theorem.
In *STOC*.

Cohen, S., Kimelfeld, B., and Sagiv, Y. (2009).
Running tree automata on probabilistic XML.
In *PODS*.

References II

Courcelle, B. (1990).
The monadic second-order logic of graphs. I. Recognizable sets of finite graphs.
Inf. Comput., 85(1).

Dalvi, N. and Suciu, D. (2012).
The dichotomy of probabilistic inference for unions of conjunctive queries.
JACM, 59(6):30.

Darwiche, A. (2001).
On the tractable counting of theory models and its application to truth maintenance and belief revision.
J. Applied Non-Classical Logics, 11(1-2).

References III

Deutch, D., Milo, T., Roy, S., and Tannen, V. (2014).
Circuits for datalog provenance.
In *ICDT*.

Frick, M. and Grohe, M. (2001).
Deciding first-order properties of locally tree-decomposable structures.
JACM, 48(6).

Ganian, R., Hliněný, P., Langer, A., Obdržálek, J., Rossmanith, P., and Sikdar, S. (2014).
Lower bounds on the complexity of MSO1 model-checking.
JCSS, 1(80).

References IV

Gottlob, G., Pichler, R., and Wei, F. (2010).
Monadic datalog over finite structures of bounded treewidth.
TOCL, 12(1):3.

Green, T. J., Karvounarakis, G., and Tannen, V. (2007).
Provenance semirings.
In *PODS*.

Lauritzen, S. L. and Spiegelhalter, D. J. (1988).
Local computations with probabilities on graphical structures and their application to expert systems.
J. Royal Statistical Society. Series B.

Makowsky, J. A. and Marino, J. (2003).
Tree-width and the monadic quantifier hierarchy.
Theor. Comput. Sci., 303(1).

References V

Robertson, N. and Seymour, P. D. (1986).
Graph minors. V. Excluding a planar graph.
J. Comb. Theory, Ser. B, 41(1).

Thatcher, J. W. and Wright, J. B. (1968).
Generalized finite automata theory with an application to a decision problem of second-order logic.
Mathematical systems theory, 2(1):57–81.

Encoding treelike instances [Chaudhuri and Vardi, 1992]

Instance:

N	
a	*b*
b	*c*
c	*d*
d	*e*
e	*f*

S	
a	*c*
b	*e*

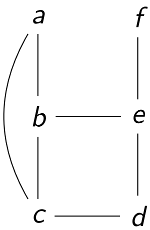
Encoding treelike instances [Chaudhuri and Vardi, 1992]

Instance:

N	
a	*b*
b	*c*
c	*d*
d	*e*
e	*f*

S	
a	*c*
b	*e*

Gaifman graph:



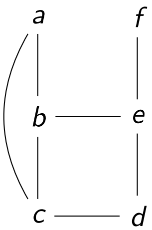
Encoding treelike instances [Chaudhuri and Vardi, 1992]

Instance:

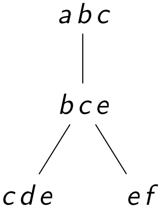
N	
a	b
b	c
c	d
d	e
e	f

S	
a	c
b	e

Gaifman graph:



Tree decomp.:



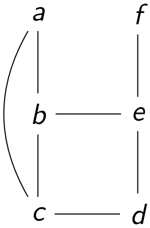
Encoding treelike instances [Chaudhuri and Vardi, 1992]

Instance:

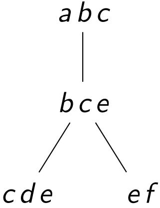
N	
a	*b*
b	*c*
c	*d*
d	*e*
e	*f*

S	
a	*c*
b	*e*

Gaifman graph:



Tree decomp.:



Tree encoding: