Efficient Enumeration of Query Answers via Circuits

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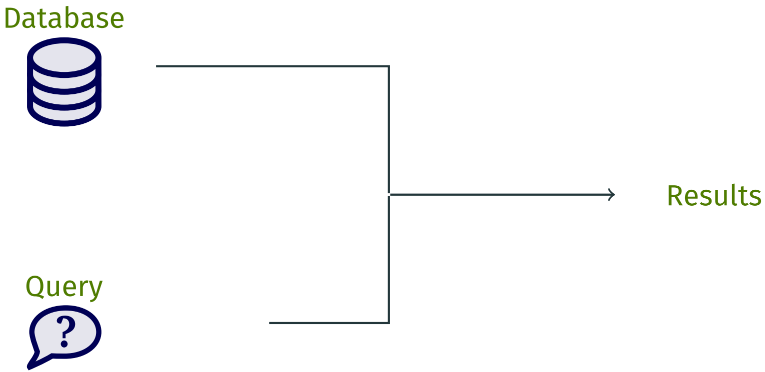
Query evaluation

Central problem in database theory and practice: query evaluation



Query evaluation

Central problem in database theory and practice: query evaluation

Database

Follows

from	to
Alice	Bob
Bob	Carol
Bob	Dave
Carol	Eve

Query

Results

Query evaluation

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“Find all pairs of users $\boldsymbol{x}$ *and* $\boldsymbol{y}$ *such that* $\boldsymbol{x}$ *follows someone who follows* $\boldsymbol{z}$*”*

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*"Find all pairs of users **x** and **z** such that **x** follows someone who follows **z**"*

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Combined complexity and data complexity

How to study the **computational complexity** of query evaluation?

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- **Data complexity:** the query is **fixed**, the input is only the **data**
  - → **Motivation:** the data is usually much larger than the query

Data complexity for large output size

- Consider the query Q: *"Find all users* x*,* y*, and* z *such that* x *follows* y *and* y *follows* z*"*

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→ We need a **better measure of complexity**

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Step 1:
Preprocessing
in $O(|D|)$

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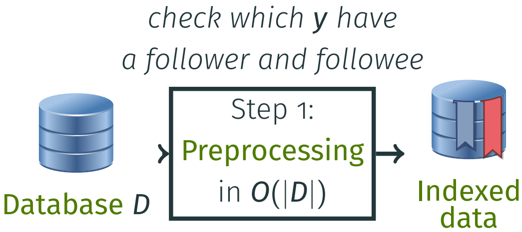

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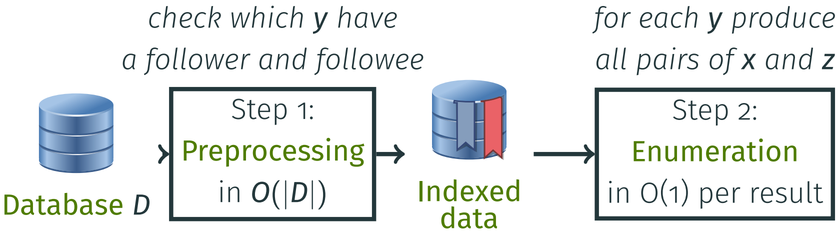

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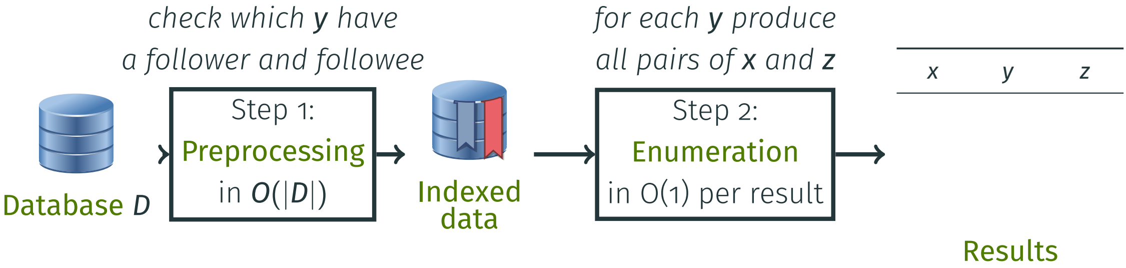

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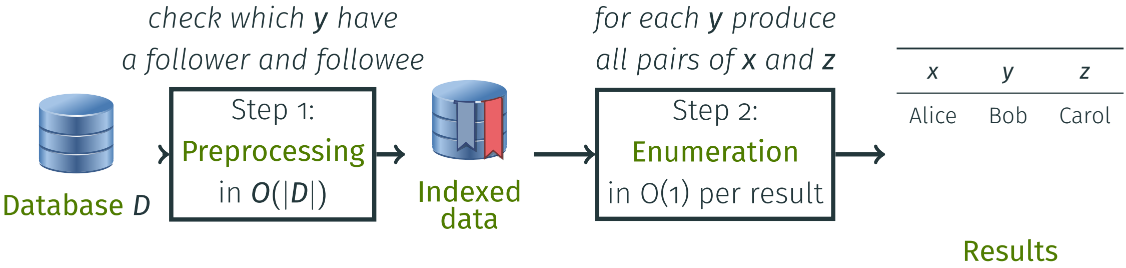

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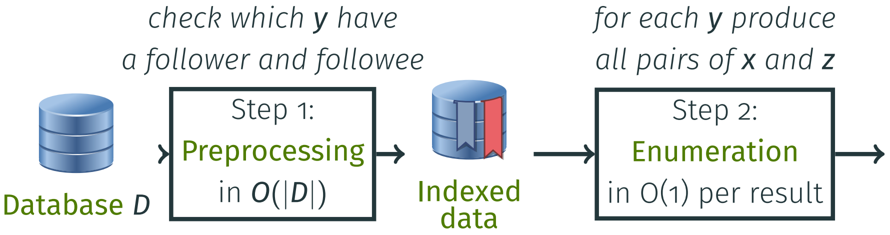

x	*y*	*z*
Alice	Bob	Carol
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Results

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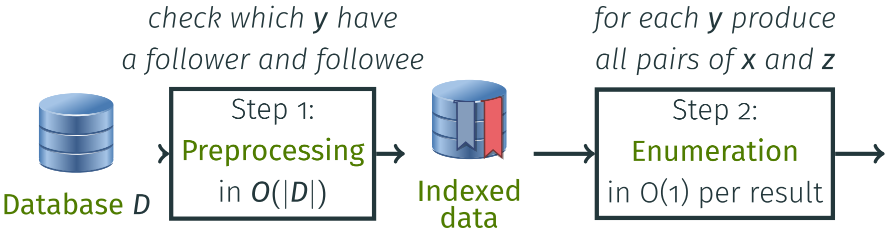

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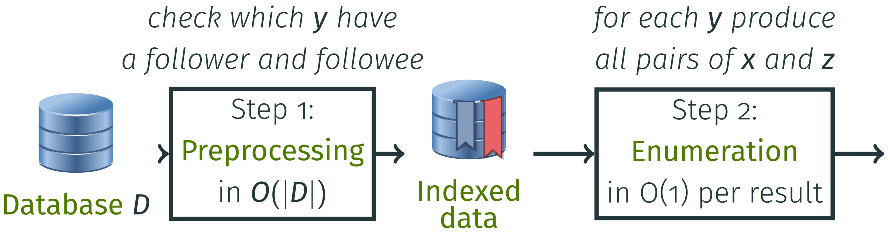

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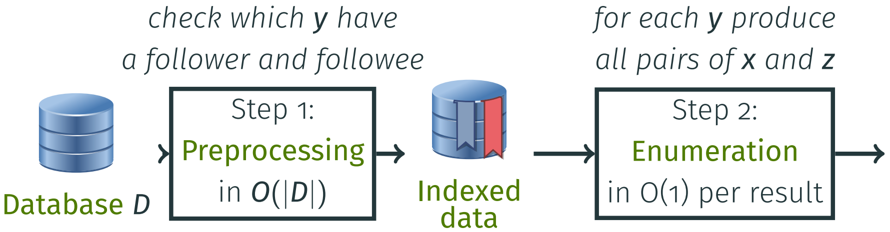

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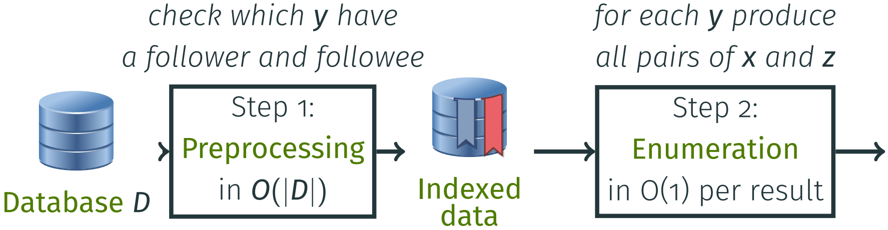

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→ Computes **all answers** in time $O(|D| + m)$ for m the number of answers

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- During enumeration, decompress this factorized representation

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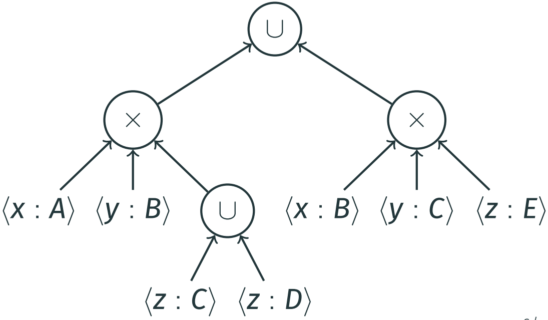
Factorized representation of $Q(D)$



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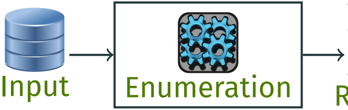

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a	b’	c

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Advantage of factorized representations: Modularity

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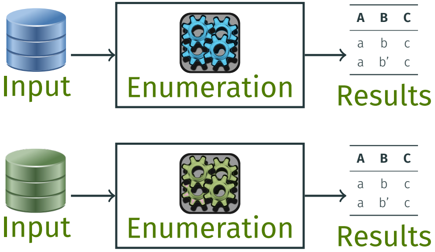

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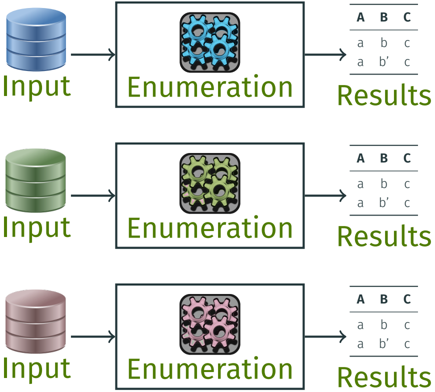

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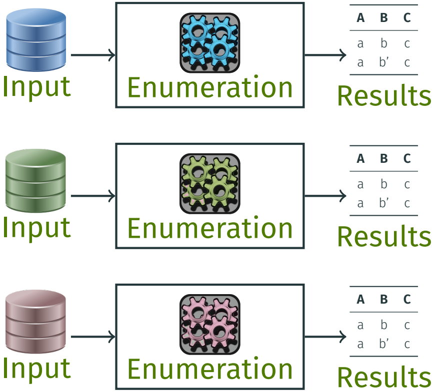

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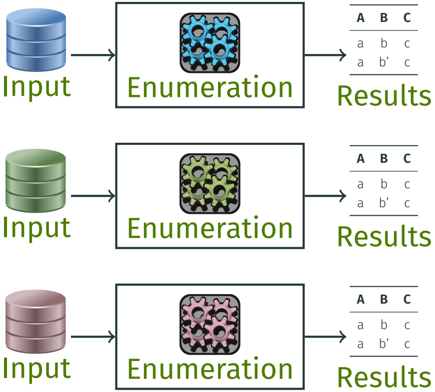

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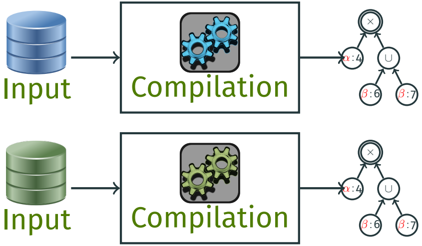

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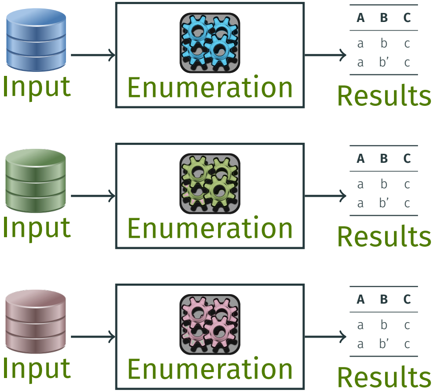

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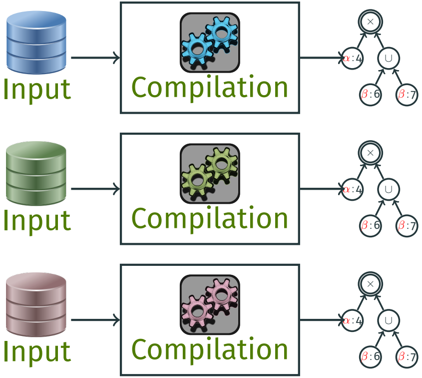

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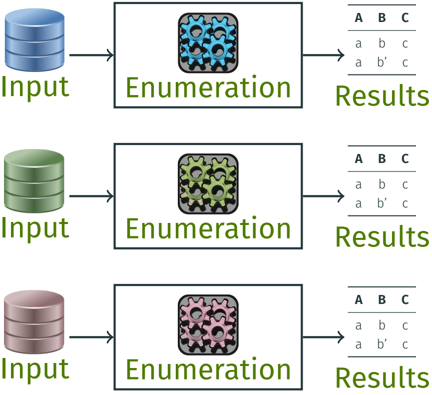

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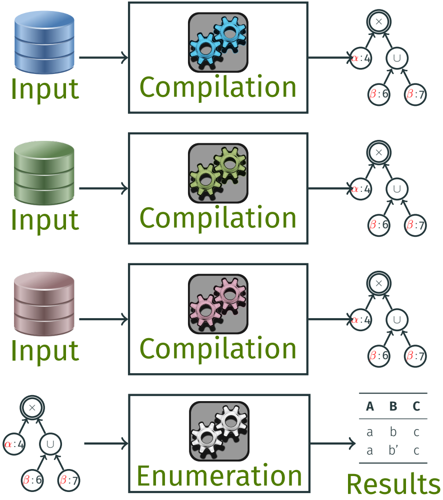

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Results on enumeration for query evaluation, especially via factorized representations

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  - Yannakakis's algorithm for acyclic and free-connex conjunctive queries
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- **Other tasks**: ranked enumeration, direct access, incremental maintenance, etc.

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b	c'
b'	c'

- Query $Q_2(x, y, z) : \text{Follows}(x, y) \wedge \text{Subscribed}(y, z)$
- Database D on the left
- There are **four answers**:

  $(a, b, c), (a, b, c'), (a, b', c'), (a', b', c')$

Cyclic vs acyclic CQs

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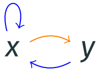

$$x \text{ --- } y$$

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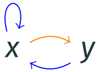

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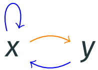

$x — y$

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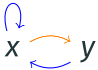

$$x \;—\; y$$

Cyclic CQs:

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Intuition: the **cyclic** queries seem **harder** (e.g., searching for a triangle in an input directed graph)

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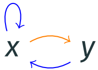

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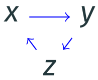
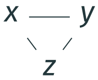




Intuition: the **cyclic** queries seem **harder** (e.g., searching for a triangle in an input directed graph)

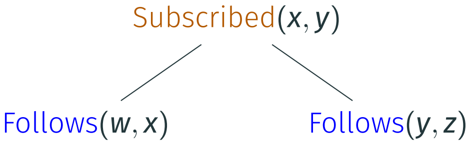
We can generalize **acyclic CQs** to arbitrary arity (= α-acyclic Gaifman hypergraph)

Join trees for acyclic CQs

Fact: a CQ is **acyclic** iff it has a **join tree**:

- The vertices are the **atoms** of the query
- For each variable, its occurrences form a **connected subtree**
- (For experts: width-1 generalized hypertree decomposition of the Gaifman hypergraph)

Take the query: $Q(w, x, y, z) : \text{Follows}(w, x) \land \text{Subscribed}(x, y) \land \text{Follows}(y, z)$



Yannakakis's algorithm for acyclic CQs

Theorem ([Yannakakis, 1981])

Given an acyclic CQ Q and database D, we can compute $Q(D)$ in time $O(|Q| \times (|D| + m))$, where m is the output size

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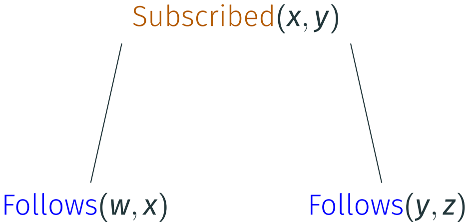

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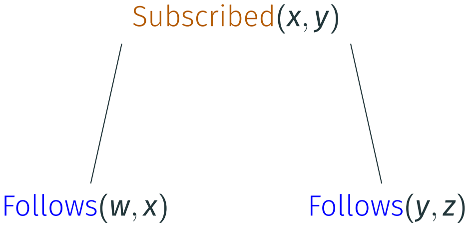

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Subscribed(x, y)

x	y
b	a
b	a'
b	a''
b'	a'

Follows(w, x)

w	x
a	b
a	b''
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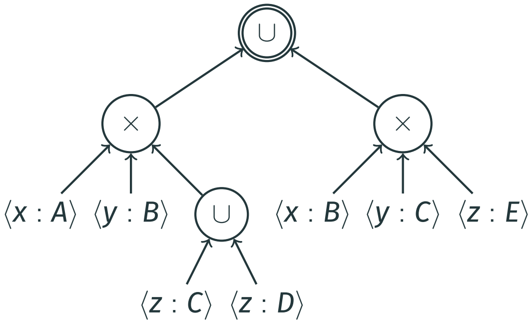
- Do semijoins on the tree top-down
- Join together all relations to get the full result

Factorized representations [Olteanu and Závodný, 2015]



- Directed acyclic graph of **gates**
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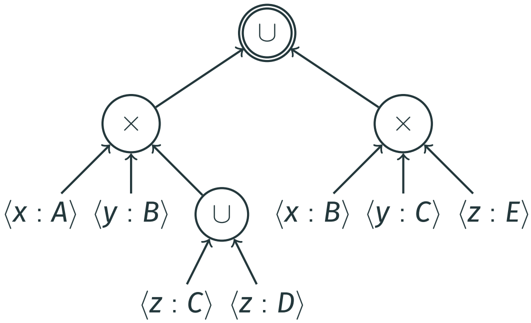
Factorized representations [Olteanu and Závodný, 2015]



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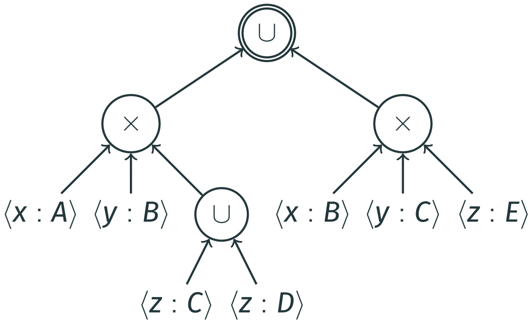

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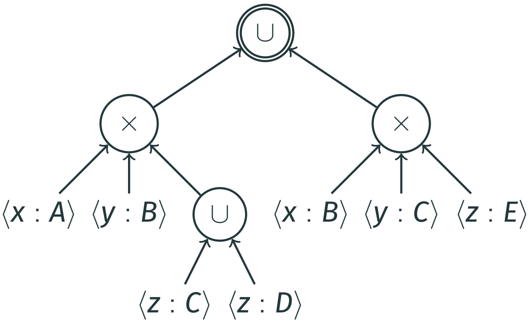

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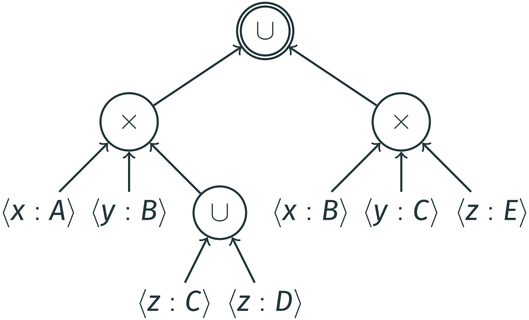

x	y	z
A	B	C
A	B	D
B	C	E

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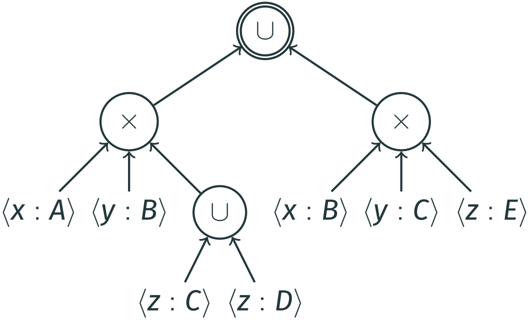

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Conditions on d-representations:

- **Deterministic**: all unions are disjoint
- **Normal**: no union is an input to a union

Enumerating tuples for normal deterministic d-representations

Task: Enumerate the tuples of the relation $R(g)$ captured by a gate g

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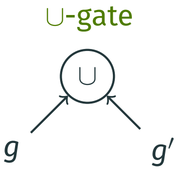

Union: enumerate $R(g)$ and then enumerate $R(g')$

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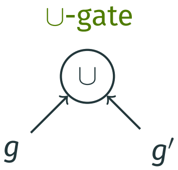

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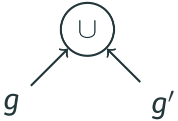
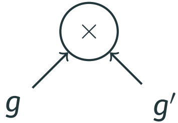
$\cup$-gate



$\times$-gate



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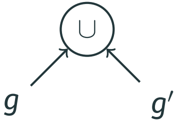

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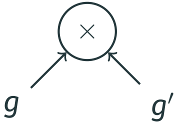

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Product: enumerate $R(g)$ and for each result t enumerate $R(g')$ and for each result t'

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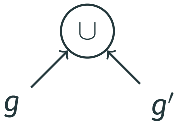

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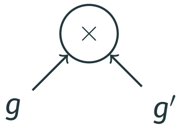

Union: enumerate $R(g)$ and then enumerate $R(g')$

Determinism: no duplicates

Product: enumerate $R(g)$ and for each result t

  enumerate $R(g')$ and for each result t'

    concatenate t and t'

Enumerating tuples for normal deterministic d-representations (2)

Theorem ([Olteanu and Závodný, 2015], Theorem 4.11)

For any fixed schema $S = (x_1, \ldots, x_k)$, the tuples of a normal deterministic d-representation with schema S can be enumerated in constant delay

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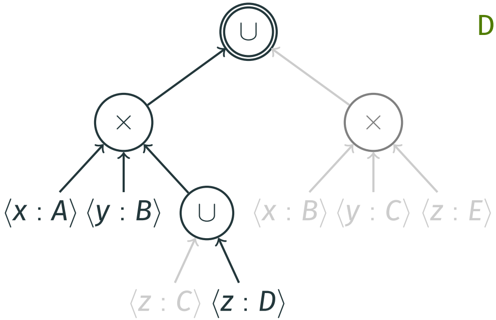

Delay analysis:

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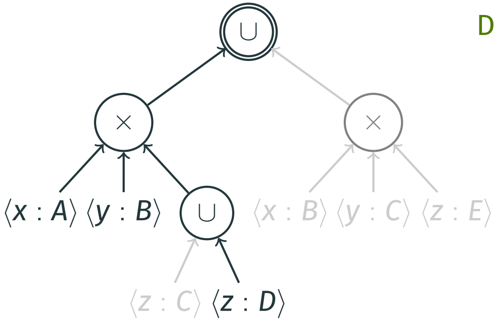

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- Every product gate **nontrivially splits** the assignment to produce

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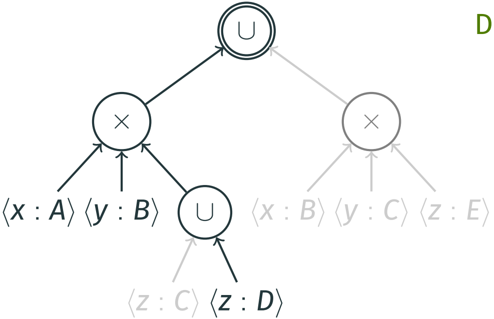

Delay analysis:

- Every product gate nontrivially splits the assignment to produce
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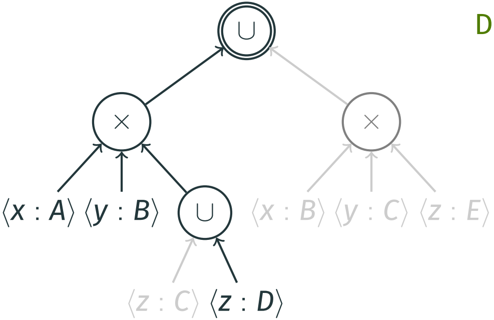


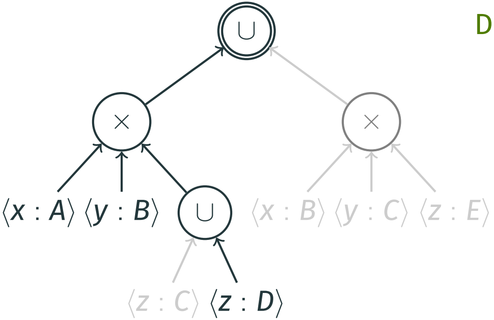
Delay analysis:

- Every product gate **nontrivially splits** the assignment to produce
- The inputs to union gates are **not union gates** (the representation is **normal**)
- Hence, the **trace** (gates visited to get a tuple) has size **linear in the tuple arity**, hence **constant**

Enumerating tuples for normal deterministic d-representations (2)

Theorem ([Olteanu and Závodný, 2015], Theorem 4.11)

For any fixed schema $S = (x_1, \ldots, x_k)$, the tuples of a normal deterministic d-representation with schema S can be enumerated in constant delay



Delay analysis:

- Every product gate **nontrivially splits** the assignment to produce
- The inputs to union gates are **not union gates** (the representation is **normal**)
- Hence, the **trace** (gates visited to get a tuple) has size **linear in the tuple arity**, hence **constant**

Note: normal deterministic d-representations also allow us to:

- **Count** the number of solutions in linear time
- **Access** the i-th solution, given i, in logarithmic time

Factorized representations for full acyclic CQs

Theorem

*Given an acyclic CQ Q and database D, we can compute a **deterministic normal d-representation of $Q(D)$** in time $O(|Q| \times |D|)$ and hence enumerate $Q(D)$ with **linear preprocessing and constant delay***

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Subscribed(x, y)

x	y
b	a
b	a'
b	a''
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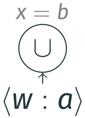

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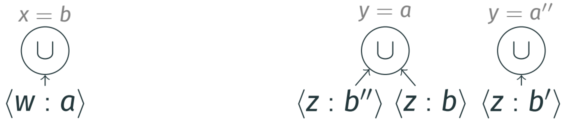

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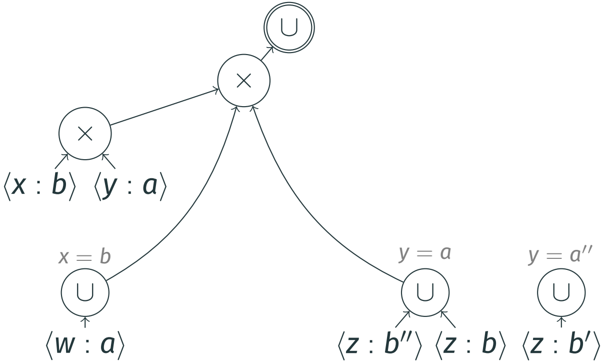

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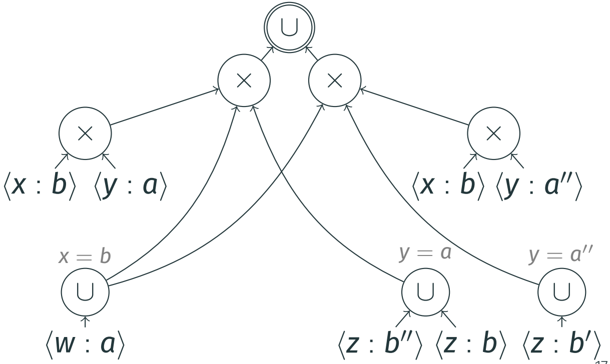

Enumeration for CQs with projections

General CQs extend **full CQs** by making it possible to **project away** some variables:

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A CQ is **free-connex** if it is acyclic and admits a **join tree** which is **free-connex**: there is a **connected subtree** of tree nodes whose union is **exactly** the free variables

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This can also be shown via **deterministic normal d-representations**

Lower bounds for CQ enumeration

What about enumeration for non-free-connex CQs? Let us assume:

- The query is **minimized**: can always be done without loss of generality
- The query is **without self joins**: uses only each relation name once
  - $\rightarrow$ $Q(x, y, z) : \text{Follows}(x, y) \wedge \text{Subscribed}(y, z)$ but not $Q(x, y, z) : \text{Follows}(x, y) \wedge \text{Follows}(y, z)$

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- *If Q is **acyclic but not free-connex**, then we can multiply n-by-n matrices in $O(n^2)$*
  - $\rightarrow$ *we can even do it on sparse matrices*

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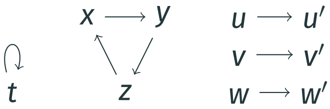

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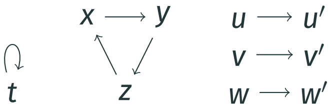

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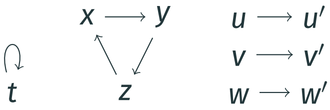

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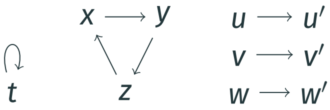

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Open problem: dichotomy on CQs with self-joins? see [Carmeli and Segoufin, 2023]

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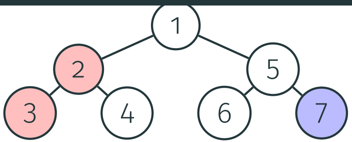
- Now: review enumeration results for MSO, in terms of **factorized representations** (not necessarily normal or deterministic)

MSO query evaluation on trees

Data: a **tree** T where nodes have a color from an alphabet



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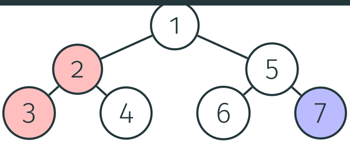

Query Q in monadic second-order logic (MSO)

- $P_{\bigcirc}(x)$ means "x is blue"
- $x \rightarrow y$ means "x is the parent of y"

Equivalent formalism: **tree automata**

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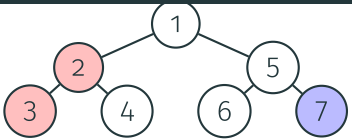

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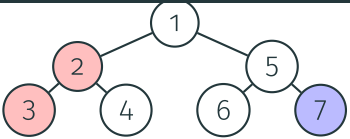

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Data complexity: Measure efficiency as a function of T (the query Q is **fixed**)

Results for MSO on trees

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Note that the d-representation is no longer normal, but we show with some effort:

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Enumerating matches of nondeterministic document spanners

Data: a text T

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.  French national.  Appearance as
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Enumerating matches of nondeterministic document spanners

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Query: a pattern P given as a regular expression

$P :=$ `␣ [a-z0-9.]* @ [a-z0-9.]* ␣`

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Query: a **pattern** P given as a regular expression

$$P := \text{␣}\ \texttt{[a-z0-9.]}^*\ \texttt{@}\ \texttt{[a-z0-9.]}^*\ \text{␣}$$

Output: the list of **substrings** of T that match P:

$$[186, 200\rangle,\quad [483, 500\rangle,\quad \ldots$$

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Query: a **pattern** P given as a regular expression

$$P := \sqcup\ \texttt{[a-z0-9.]}^*\ \texttt{@}\ \texttt{[a-z0-9.]}^*\ \sqcup$$

Output: the list of **substrings** of T that match P:

$$[186, 200\rangle, \quad [483, 500\rangle, \quad \ldots$$

Goal:

- be **very efficient** in T (constant-delay)
- be **reasonably efficient** in P (polynomial-time)

Results for nondeterministic document spanners

Theorem (A., Bourhis, Mengel, Niewerth, ICDT'19; see also PODS'19)

*We can enumerate all matches of an input **nondeterministic automaton with captures** on an input **text** with*

- *Preprocessing **linear** in the text and **polynomial** in the automaton*
- *Delay **constant** in the text and **polynomial** in the automaton*

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- *Preprocessing linear in the text and polynomial in the automaton*
- *Delay constant in the text and polynomial in the automaton*

- Generalizes earlier result on **deterministic automata** [Florenzano et al., 2018]
- To make the algorithm polynomial in the **(nondeterministic) automaton**, we need efficient enumeration for a certain kind of **non-deterministic d-representations**

Other enumeration settings

Efficient enumeration is now being studied in **many settings** in data management (sometimes with weaker guarantees than linear preprocessing and constant delay):

- For **regular path queries** [Martens and Trautner, 2018, David et al., 2024]
- For **compressed structures**:
  - Compressed trees [Lohrey and Schmid, 2024]
  - SLP-compressed documents [Schmid and Schweikardt, 2021, Muñoz and Riveros, 2023]
- For **visibly pushdown languages** [Muñoz and Riveros, 2022]
- For **context-free languages** with annotations [Peterfreund, 2021], [A., Jachiet, Muñoz, Riveros, 2023]

There are also **software implementations** [Riveros et al., 2023]

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Introduction: From enumeration to more general tasks

Sometimes, we want **more** than enumerating query results in an unspecified order:

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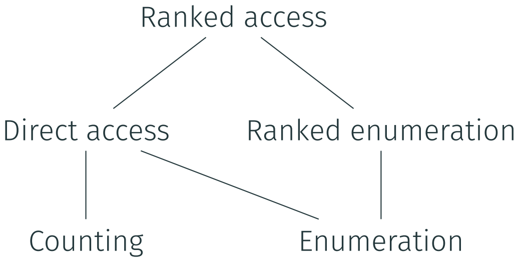

(Adapted from [Carmeli, 2023])

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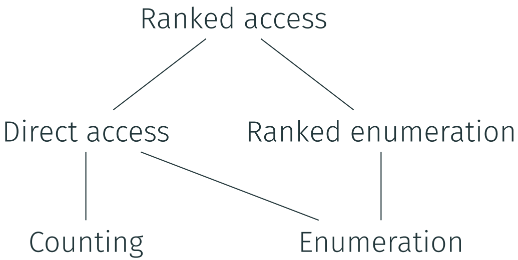

(Adapted from [Carmeli, 2023])

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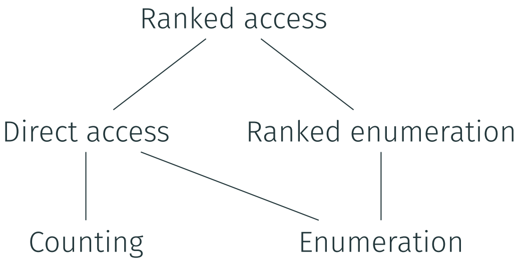

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Another question: **maintain** an enumeration structure under **updates** to the data

Results on ranked enumeration / ranked access

For CQs and UCQs:

- Most works study self-join-free CQs under lexicographic orders and aim for logarithmic access time or delay:
  - Characterization of tractable orders for CQs [Carmeli et al., 2023]

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For MSO queries on trees:

- Ranked enumeration shown with logarithmic delay on words [Bourhis et al., 2021]
- Recent extension to trees [A., Bourhis, Capelli, Monet, 2024]

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For **MSO** queries on trees, aiming for **logarithmic** update time:

- On **words**, linear preprocessing and constant delay enumeration is possible under **insert/delete updates** [Niewerth and Segoufin, 2018]
- On **trees**, linear preprocessing and constant delay enumeration is possible under **substitution updates** [A., Bourhis, Mengel, 2018] and possibly more

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Summary and future work

- We have seen **enumeration algorithms** to produce query answers in streaming
  - $\rightarrow$ Ideally, we want **linear preprocessing** and **constant delay**
- **Modular approach:** compute a factorized representation of the results
- Tractable enumeration is possible for **free-connex CQs** and for **MSO queries on trees**
- **Ongoing research:** ranked enumeration, ranked access, incremental maintenance...

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Thanks for your attention!

References i

Amarilli, A., Bourhis, P., Capelli, F., and Monet, M. (2024).
Ranked enumeration for MSO on trees via knowledge compilation.
In *ICDT*.

Amarilli, A., Bourhis, P., Jachiet, L., and Mengel, S. (2017).
A circuit-based approach to efficient enumeration.
In *ICALP*.

Amarilli, A., Bourhis, P., and Mengel, S. (2018).
Enumeration on trees under relabelings.
In *ICDT*.

Amarilli, A., Bourhis, P., Mengel, S., and Niewerth, M. (2019a).
Constant-delay enumeration for nondeterministic document spanners.
In *ICDT*.

Amarilli, A., Bourhis, P., Mengel, S., and Niewerth, M. (2019b).
Enumeration on trees with tractable combined complexity and efficient updates.
In *PODS*.

Amarilli, A., Jachiet, L., Muñoz, M., and Riveros, C. (2022).
Efficient enumeration for annotated grammars.
In *PODS*.

Bagan, G. (2006).
MSO queries on Tree Decomposable Structures Are Computable with Linear Delay.
In *CSL*.

Bagan, G., Durand, A., and Grandjean, E. (2007).
On acyclic conjunctive queries and constant delay enumeration.
In *CSL*.

Berkholz, C., Gerhardt, F., and Schweikardt, N. (2020).
Constant delay enumeration for conjunctive queries: a tutorial.
ACM SIGLOG News, 7(1).

Berkholz, C., Keppeler, J., and Schweikardt, N. (2017).
Answering conjunctive queries under updates.
In *PODS*.

Bourhis, P., Grez, A., Jachiet, L., and Riveros, C. (2021).
Ranked enumeration of MSO logic on words.
In *ICDT*.

Bringmann, K., Carmeli, N., and Mengel, S. (2022).
Tight fine-grained bounds for direct access on join queries.
In *PODS*.

Capelli, F. and Irwin, O. (2024).
Direct access for conjunctive queries with negation.
In *ICDT*.

Carmeli, N. (2022).
Answering unions of conjunctive queries with ideal time guarantees (invited talk).
In Olteanu, D. and Vortmeier, N., editors, *ICDT*, volume 220 of *LIPIcs*. Schloss Dagstuhl - Leibniz-Zentrum für Informatik.

Carmeli, N. (2023).
Accessing answers to conjunctive queries with ideal time guarantees (abstract of invited talk).
In Kutz, O., Lutz, C., and Ozaki, A., editors, *DL*, volume 3515 of *CEUR Workshop Proceedings*. CEUR-WS.org.

Carmeli, N. and Kröll, M. (2021).
On the enumeration complexity of unions of conjunctive queries.
TODS, 46(2).

Carmeli, N. and Segoufin, L. (2023).
Conjunctive queries with self-joins, towards a fine-grained enumeration complexity analysis.
In *PODS*.

Carmeli, N., Tziavelis, N., Gatterbauer, W., Kimelfeld, B., and Riedewald, M. (2023).
Tractable orders for direct access to ranked answers of conjunctive queries.
TODS, 48(1).

Carmeli, N., Zeevi, S., Berkholz, C., Conte, A., Kimelfeld, B., and Schweikardt, N. (2022).
Answering (unions of) conjunctive queries using random access and random-order enumeration.
TODS, 47(3).

David, C., Francis, N., and Marsault, V. (2024).
Distinct shortest walk enumeration for rpqs.
In *PODS*.

Durand, A. and Grandjean, E. (2007).
First-order queries on structures of bounded degree are computable with constant delay.
TOCL, 8(4).

Eldar, I., Carmeli, N., and Kimelfeld, B. (2024).
Direct access for answers to conjunctive queries with aggregation.
In *ICDT*.

Florenzano, F., Riveros, C., Ugarte, M., Vansummeren, S., and Vrgoc, D. (2018).
Constant delay algorithms for regular document spanners.
In *PODS*.

Kara, A., Nikolic, M., Olteanu, D., and Zhang, H. (2023).
Conjunctive queries with free access patterns under updates.
In *ICDT*.

Kazana, W. and Segoufin, L. (2011).
First-order query evaluation on structures of bounded degree.
Logical Methods in Computer Science, 7.

References x

Kazana, W. and Segoufin, L. (2013).
Enumeration of monadic second-order queries on trees.
TOCL, 14(4).

Lohrey, M. and Schmid, M. L. (2024).
Enumeration for MSO-queries on compressed trees.
In *PODS*.
To appear. arXiv preprint arXiv:2403.03067.

Martens, W. and Trautner, T. (2018).
Evaluation and enumeration problems for regular path queries.
In *ICDT*.

Muñoz, M. and Riveros, C. (2022).
Streaming enumeration on nested documents.
In *ICDT*.

Muñoz, M. and Riveros, C. (2023).
Constant-delay enumeration for SLP-compressed documents.
In *ICDT*.

Niewerth, M. and Segoufin, L. (2018).
Enumeration of MSO queries on strings with constant delay and logarithmic updates.
In *PODS*.

Olteanu, D. and Závodnỳ, J. (2015).
Size bounds for factorised representations of query results.
TODS, 40(1).

Peterfreund, L. (2021).
Grammars for document spanners.
In *ICDT*.

Riveros, C., Van Sint Jan, N., and Vrgoč, D. (2023).
Rematch: A novel regex engine for finding all matches.
PVLDB, 16(11).

Schmid, M. L. and Schweikardt, N. (2021).
Spanner evaluation over SLP-compressed documents.
In *PODS*.

Schweikardt, N., Segoufin, L., and Vigny, A. (2022).
Enumeration for FO queries over nowhere dense graphs.
JACM, 69(3).

Tziavelis, N., Ajwani, D., Gatterbauer, W., Riedewald, M., and Yang, X. (2020).
Optimal algorithms for ranked enumeration of answers to full conjunctive queries.
PVLDB, 13(9).

Yannakakis, M. (1981).
Algorithms for acyclic database schemes.
In *VLDB*, volume 81.