Enumerating Pattern Matches in Texts and Trees

Antoine Amarilli[1], Pierre Bourhis[2], Stefan Mengel[3], Matthias Niewerth[4]

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[1]Télécom ParisTech

[2]CNRS CRIStAL

[3]CNRS CRIL

[4]Universität Bayreuth

Problem: Finding Patterns in Text

- We have a long text T:

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.
French national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP
a3nm@a3nm.net Affiliation Associate professor of computer science (office C201-4) in the DIG team of
Télécom ParisTech, 46 rue Barrault, F-75634 Paris Cedex 13, France.  Studies PhD in computer science
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More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
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  - → Example: find **email addresses**

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    - Write the pattern as a regular expression:

$$P := \text{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$

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→ How to find the pattern P efficiently in the text T?

Solution: Automata

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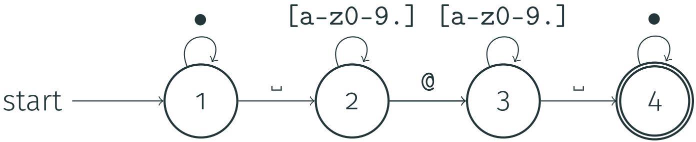

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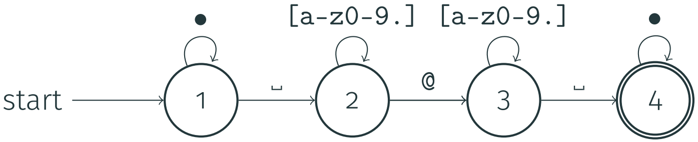

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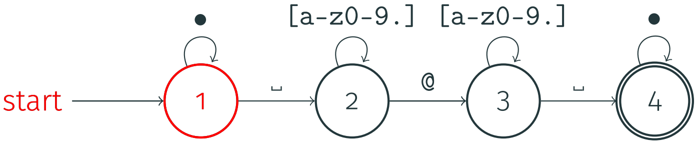

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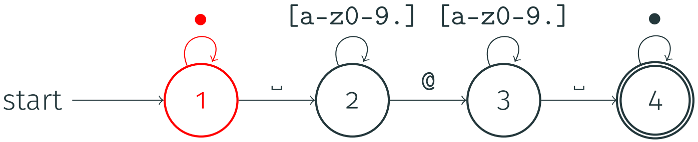

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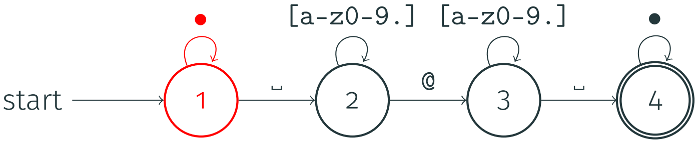

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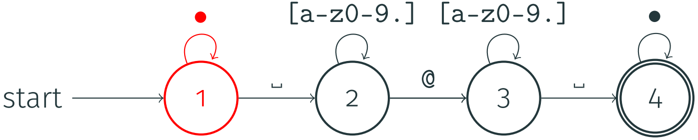

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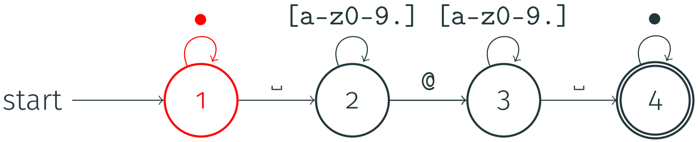

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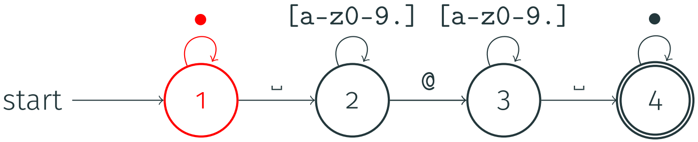

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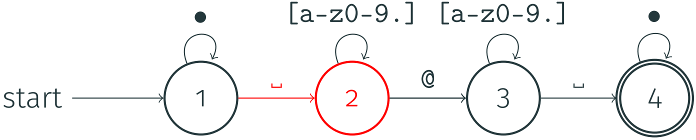

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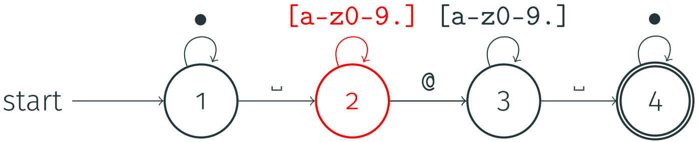

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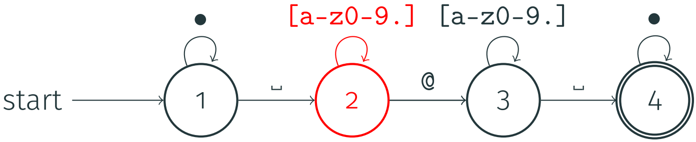

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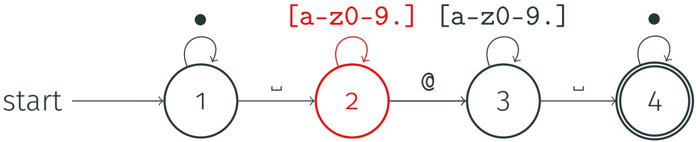

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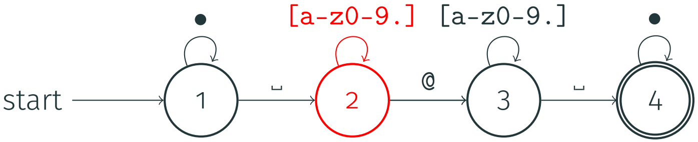

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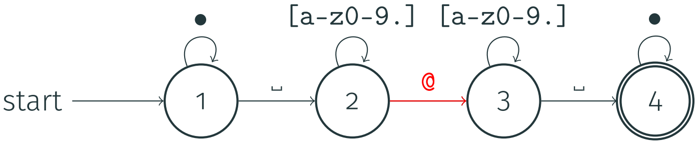

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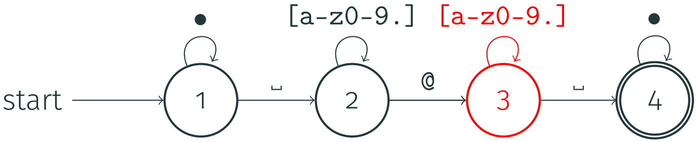

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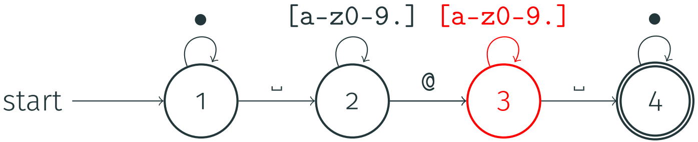

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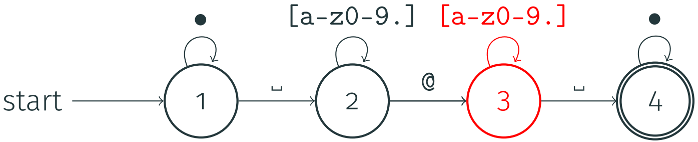

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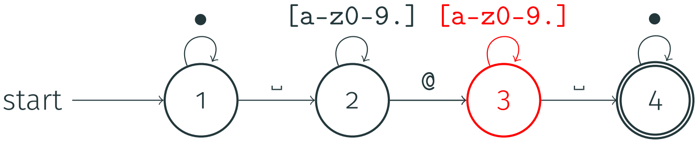

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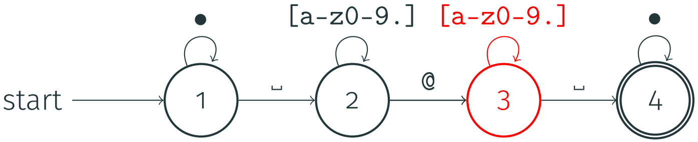

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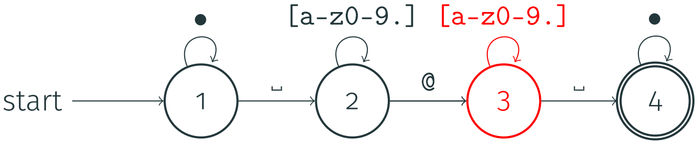

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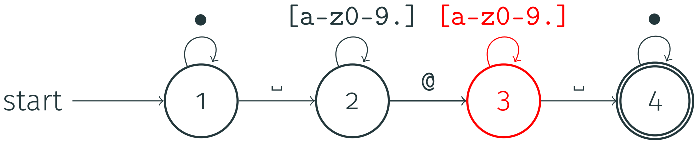

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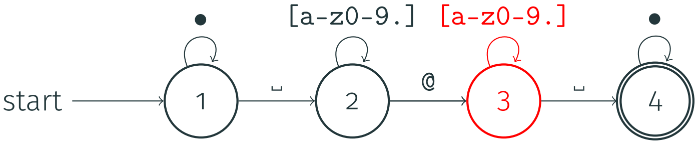

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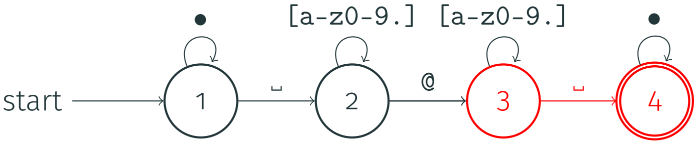

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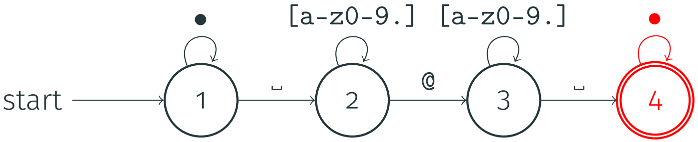

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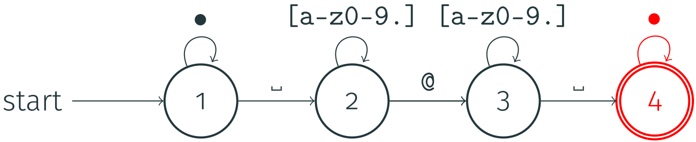

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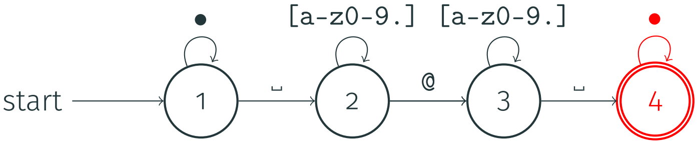

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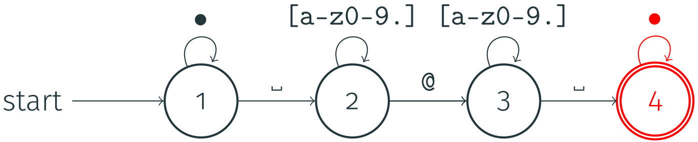

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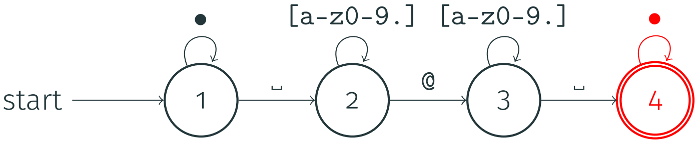

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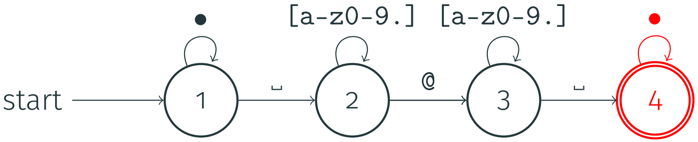

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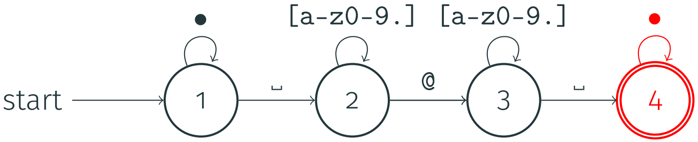

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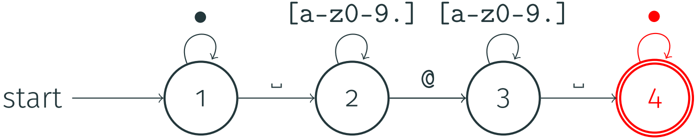

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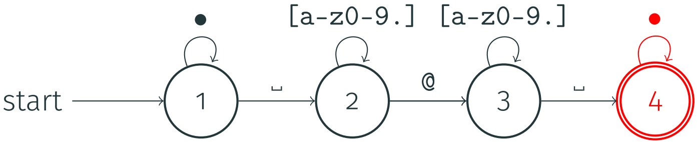

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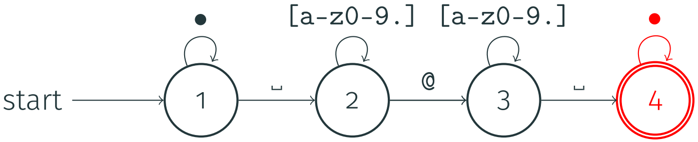

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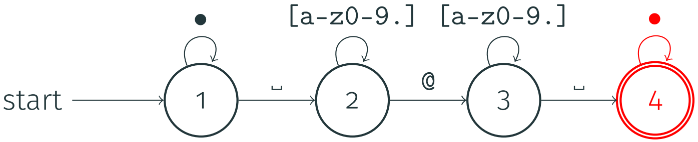

- Then, evaluate the automaton on the **text** *T*

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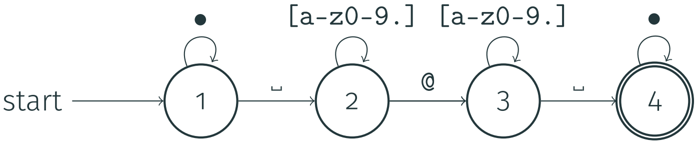

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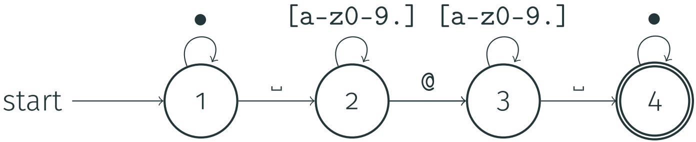

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`Email␣a3nm@a3nm.net␣Affiliation`

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  → This is very efficient in T and reasonably efficient in P

Actual Problem: Extracting all Patterns

- This only tests if the pattern occurs in the text!

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→ We need a **different way** to measure complexity

Enumeration Algorithms

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→ Formalization: **enumeration algorithms**

Formalizing Enumeration Algorithms

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Text *T*

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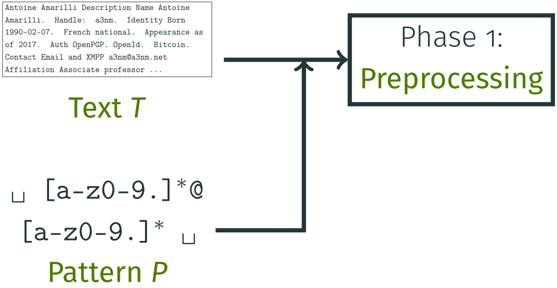

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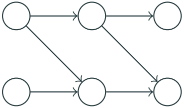
Phase 1:
Preprocessing



Index structure

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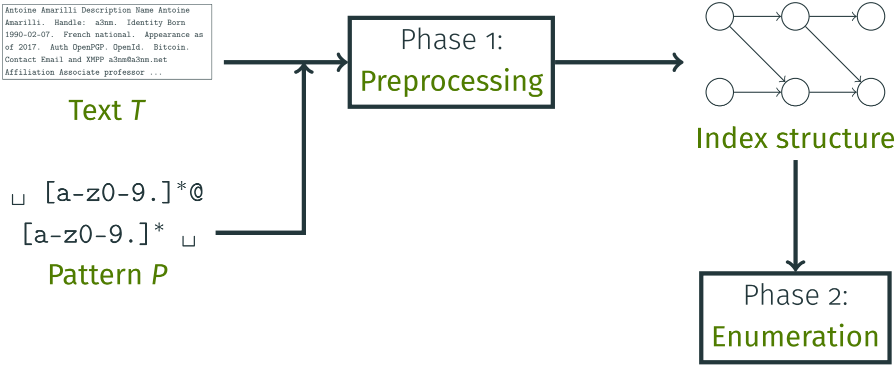

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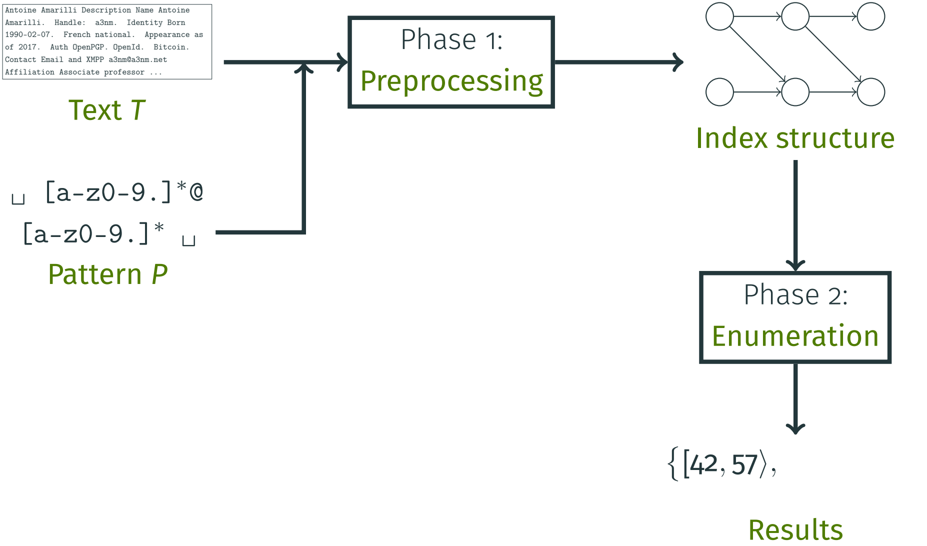

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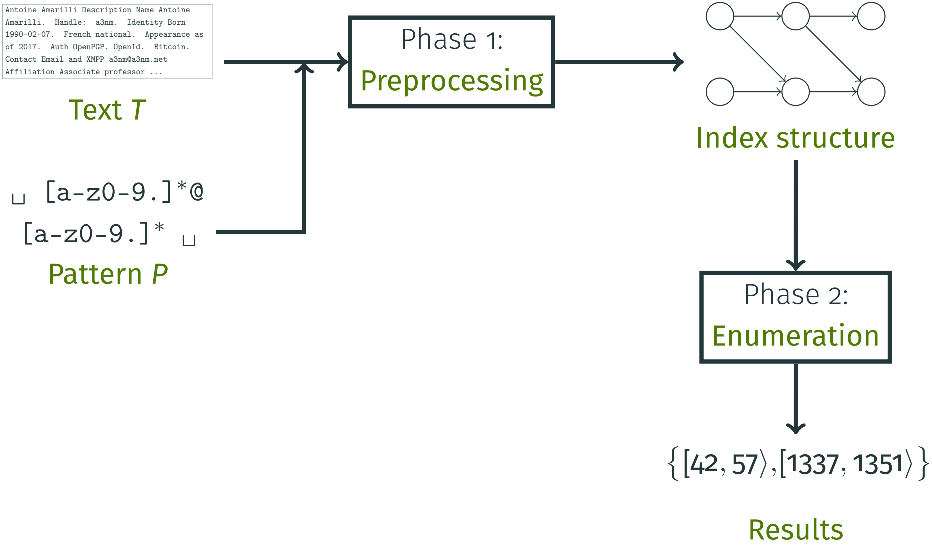

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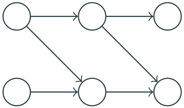

Index structure

Phase 2: **Enumeration**

$\{[42, 57\rangle, [1337, 1351\rangle\}$

Results

Two ways to measure performance:

- **Total time** for phase 1
- **Delay between two results** in phase 2

... as a function of the **text** and **pattern**

Complexity of Enumeration Algorithms

- Recall the inputs to our problem:
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→ Can we do **better**?

Results for Enumerating Pattern Matches

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Theorem [Florenzano et al., 2018]

We can enumerate all matches of a pattern P *on a text* T *with:*

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Theorem

We can enumerate all matches of a pattern P *on a text* T *with:*

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- *Delay polynomial in* P *and independent from* T

Automaton Formalism

- We use automata that read letters and capture variables

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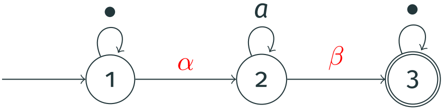

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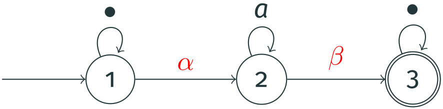

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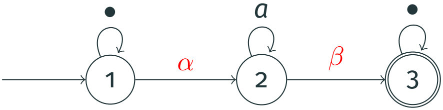

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  → Output: tuples $\langle \alpha : i, \beta : j \rangle$ such that
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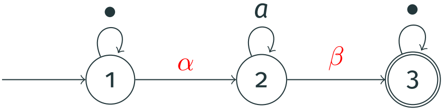
Automaton Formalism

- We use automata that read letters and capture variables
  - → Example: $P := \bullet^* \ \alpha \ a^* \ \beta \ \bullet^*$



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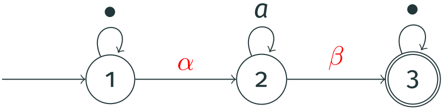

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Proof Idea: Product DAG

Compute a product DAG of the text T and of the automaton A

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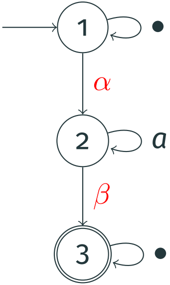

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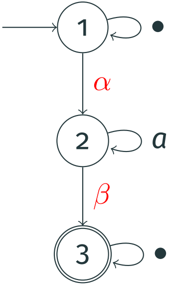
a	a	a	b	a



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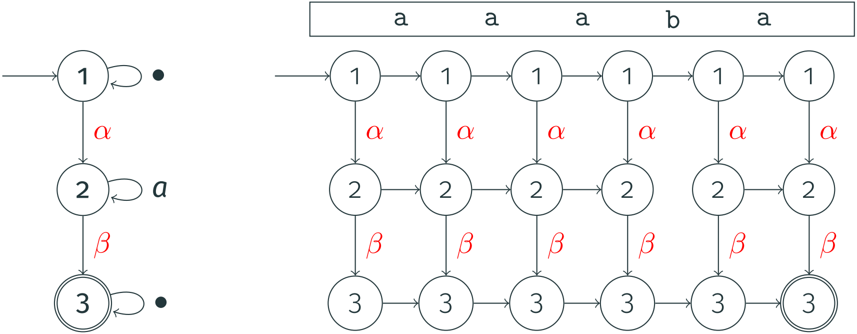

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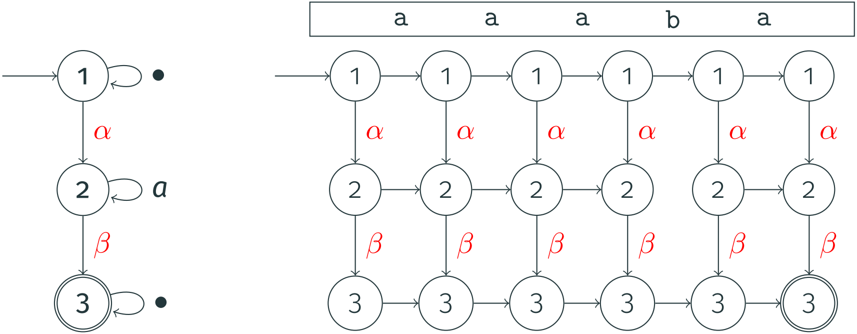

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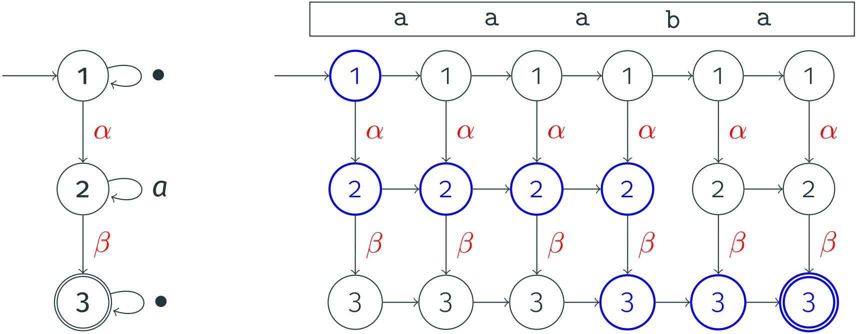

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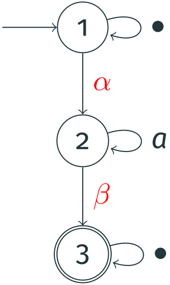

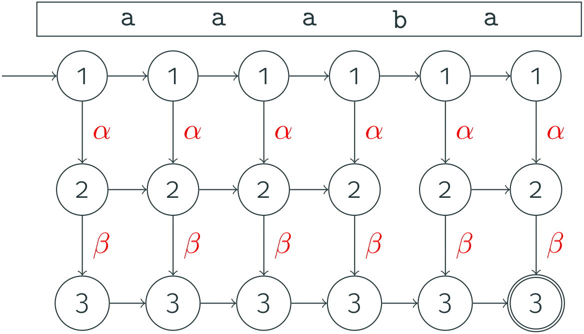

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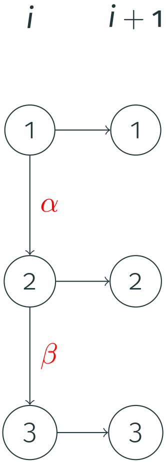
→ **Challenge:** Enumerate paths but avoid **duplicate matches** and do not **waste time** to ensure constant delay

Proof idea: on-the-fly computation to avoid duplicates



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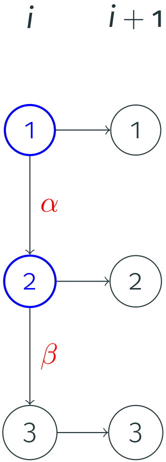

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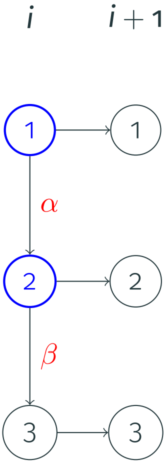

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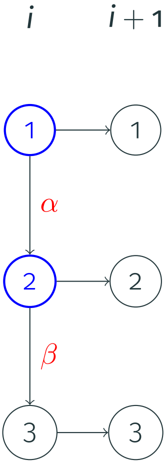

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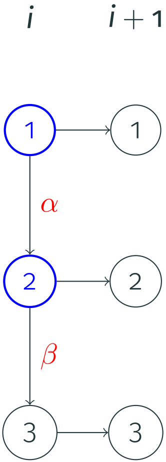

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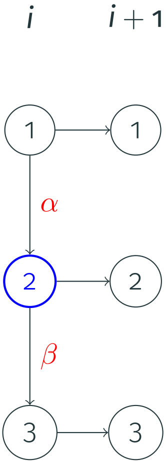

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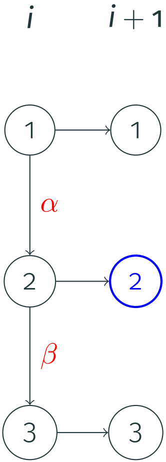

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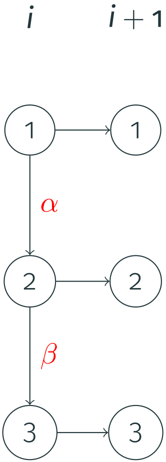

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$\rightarrow$ We must have **preprocessed** the DAG to make sure that we can always finish the run

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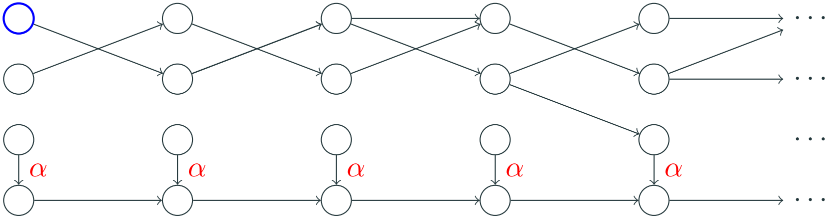

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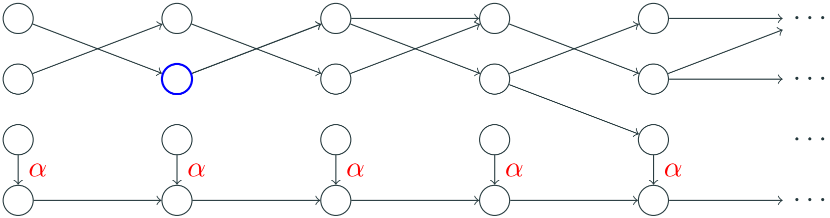

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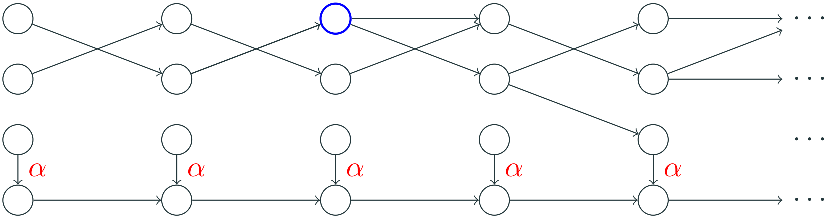

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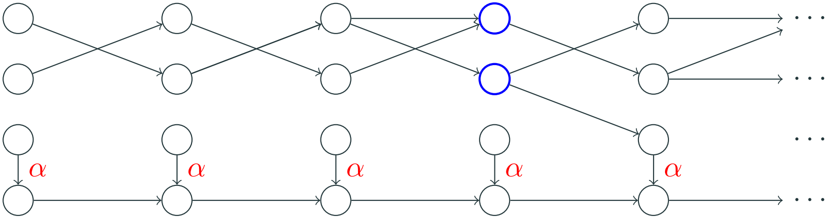

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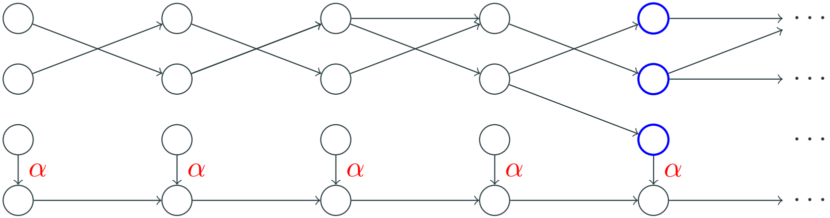

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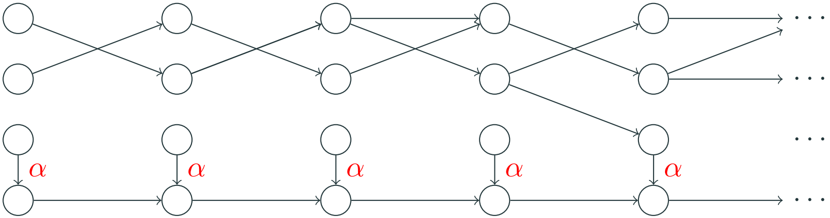

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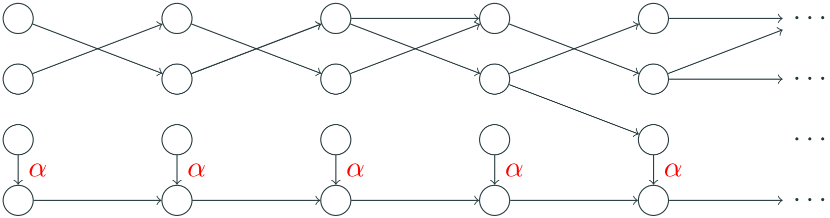

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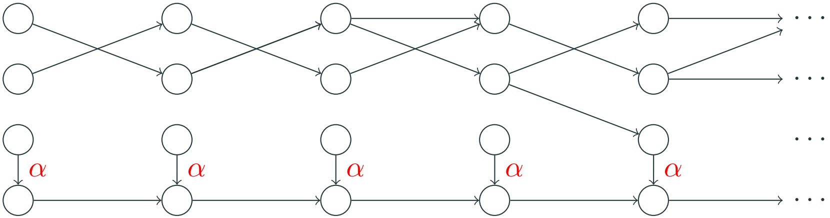

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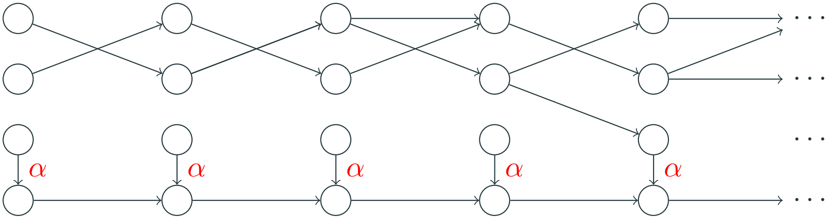

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Proof idea: flashlight search

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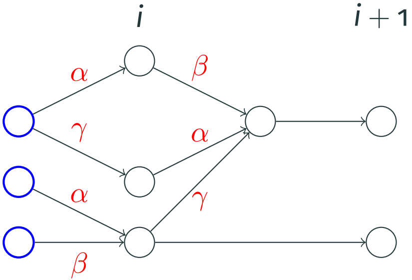

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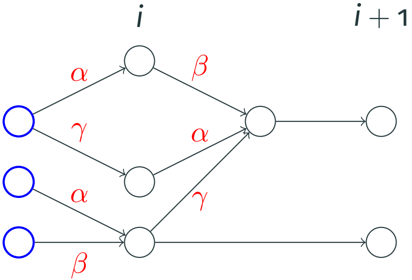

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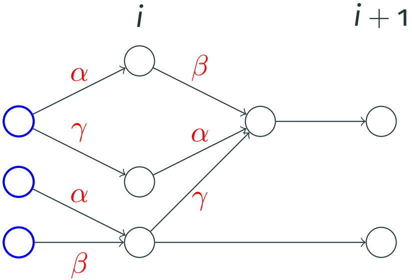

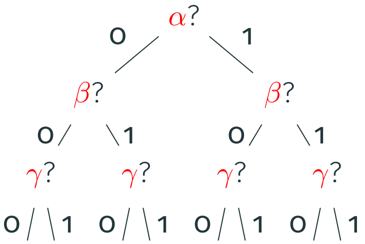

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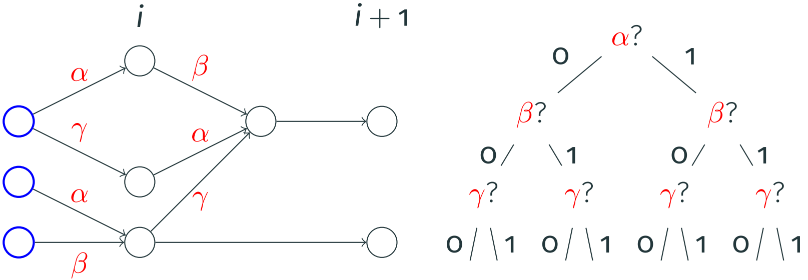

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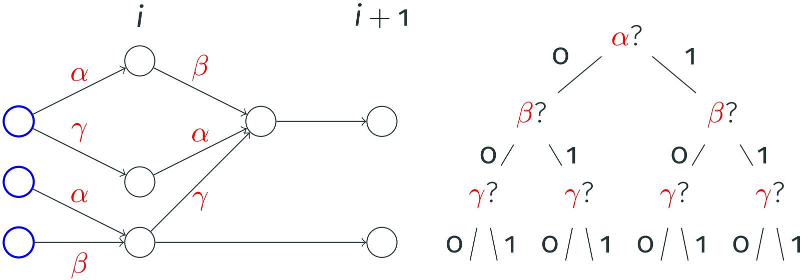

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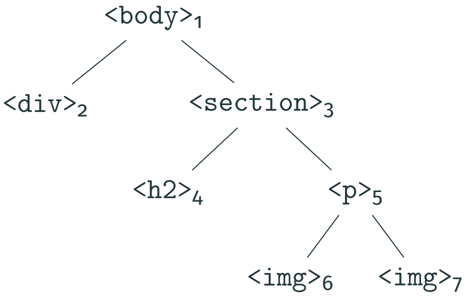
Extension: From Text to Trees

Pattern Matching on Trees

- The data T is no longer text but is now a tree:



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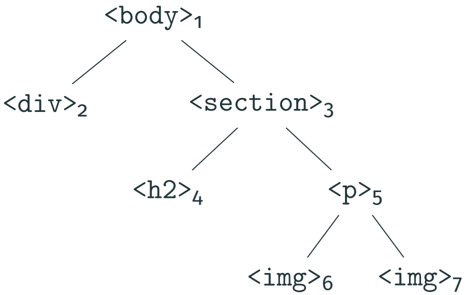

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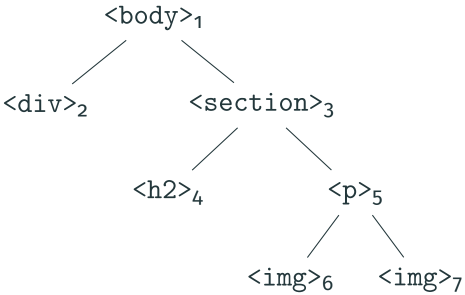

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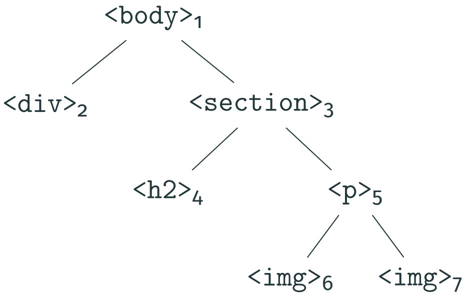

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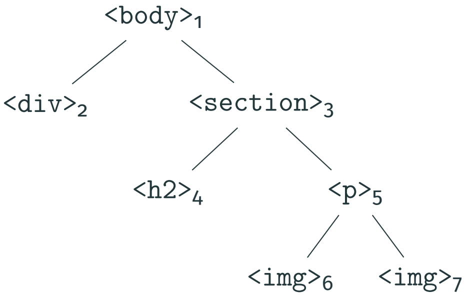

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Definitions and Results on Trees

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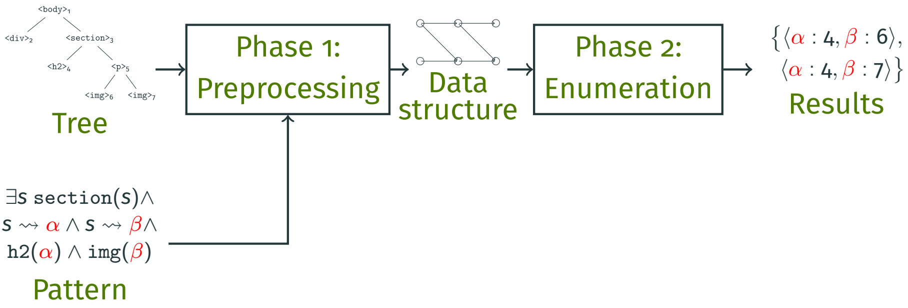
→ We are **working on** proving the following:

Conjecture

- *Preprocessing in $O(|T| \times Poly(P))$*
- *Delay polynomial in P and independent from T*

Proof Idea for Trees: Structure

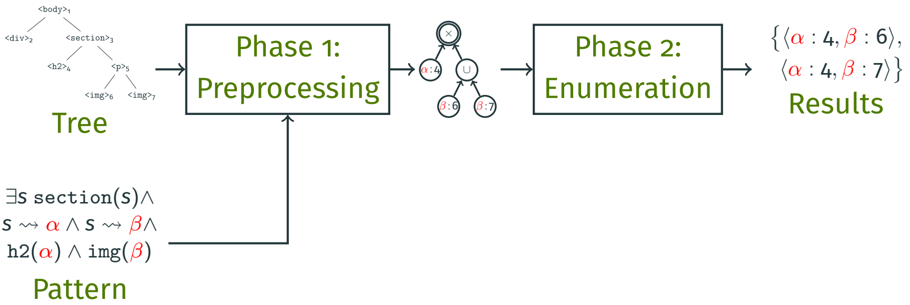
Similar structure to the previous proof, but with a circuit:



Proof Idea for Trees: Structure

Similar structure to the previous proof, but with a **circuit**:

- **Preprocessing:** Compute a **circuit representation** of the answers
- **Enumeration:** Apply a **generic algorithm** on the circuit



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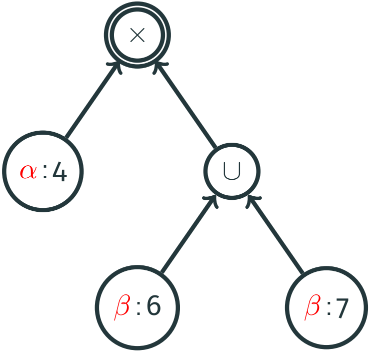

Three kinds of **set-valued gates**:

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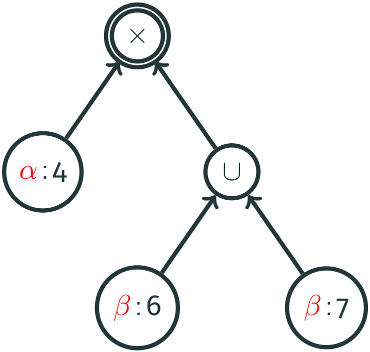

Three kinds of **set-valued gates**:

- **Variable gate** $\alpha : 4$:

  → captures $\{\langle \alpha : 4 \rangle\}$

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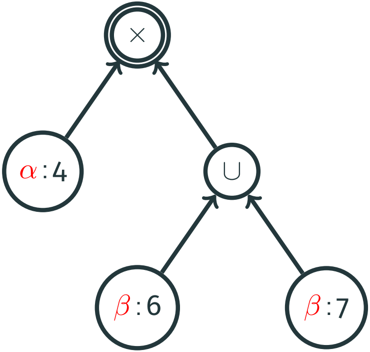

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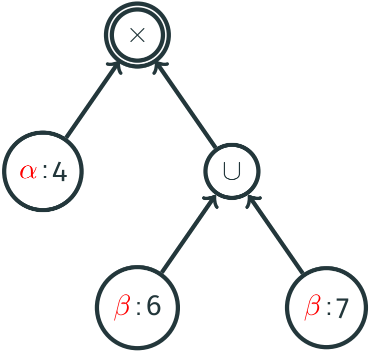

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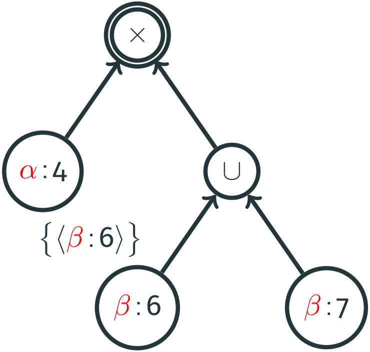

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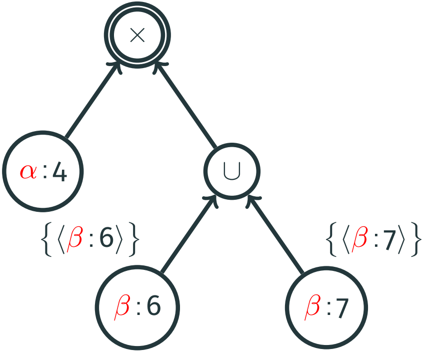

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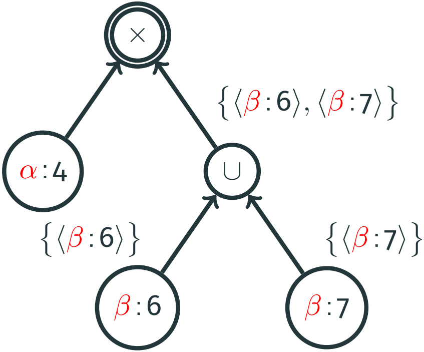

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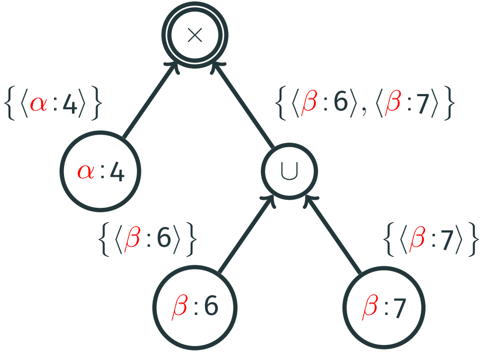

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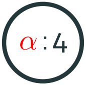
- **Variable gate** 

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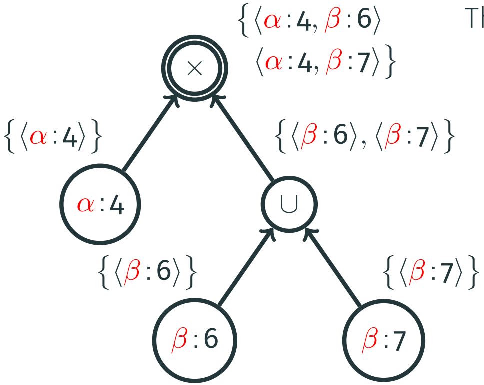

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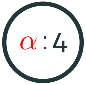
- **Variable gate** 

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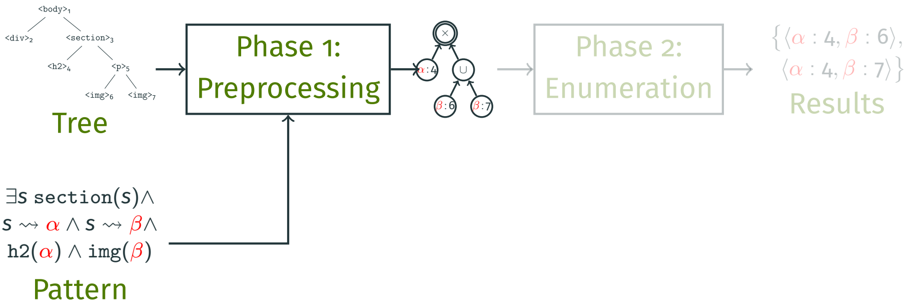

Theorem

For any ***tree automaton*** A *with capture variables* $\alpha_1, \ldots, \alpha_k$*, given a* ***tree*** T*, we can build in* $O(|T| \times |A|)$ *a* ***set circuit*** *capturing exactly the set of tuples* $\{\langle \alpha_1 : n_1, \ldots, \alpha_k : n_k \rangle$ *in the output of* A *on* T

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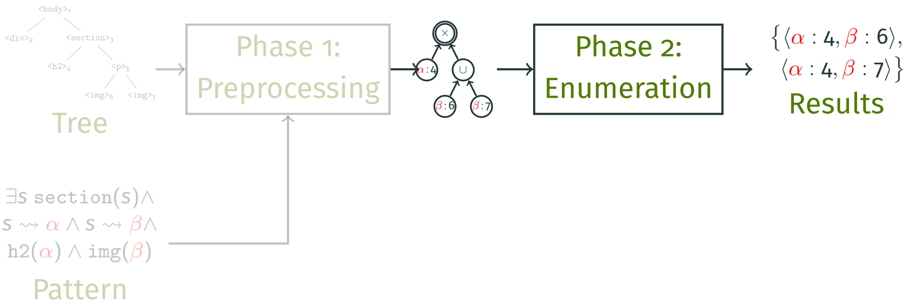


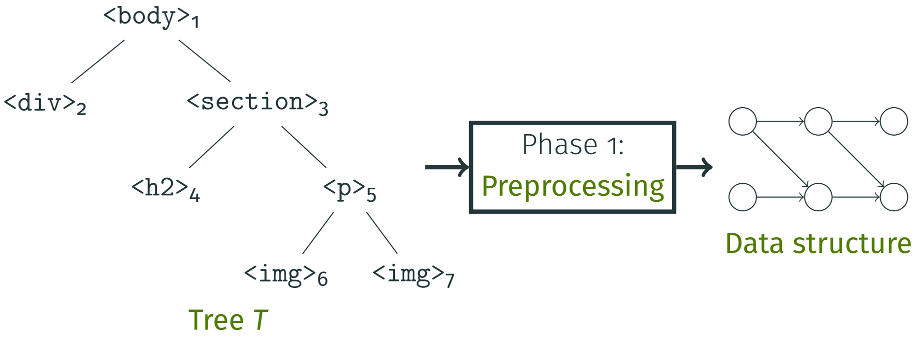
Theorem

*Given a set circuit **satisfying some conditions**, we can enumerate all tuples that it captures with linear preprocessing and constant delay*

E.g., for $\{\langle \alpha : 4, \beta : 6\rangle, \langle \alpha : 4, \beta : 7\rangle\}$: enumerate $\langle \alpha : 4, \beta : 6\rangle$ then $\langle \alpha : 4, \beta : 7\rangle$

Extension: Supporting Updates

Updates



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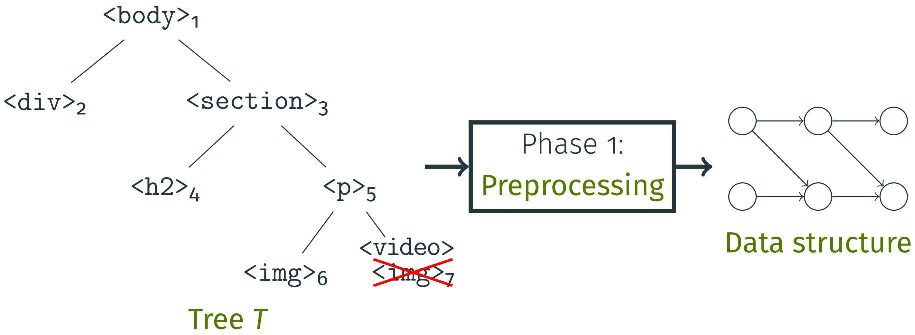

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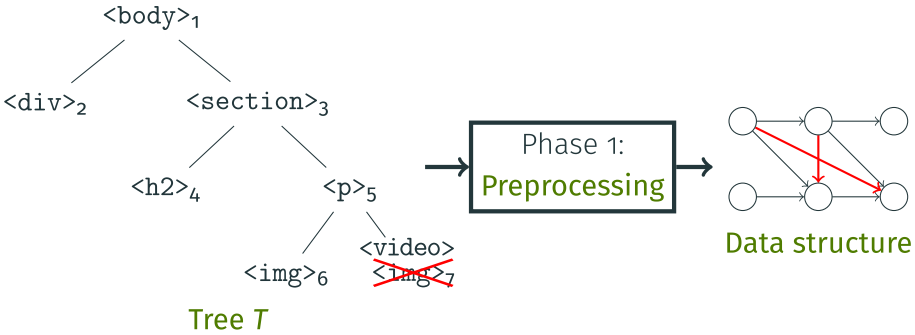

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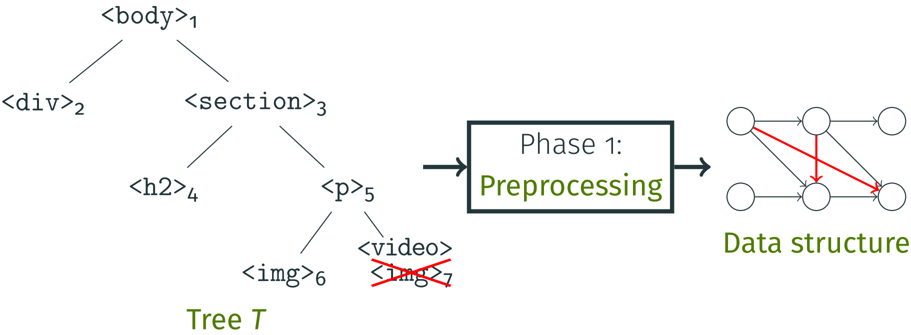

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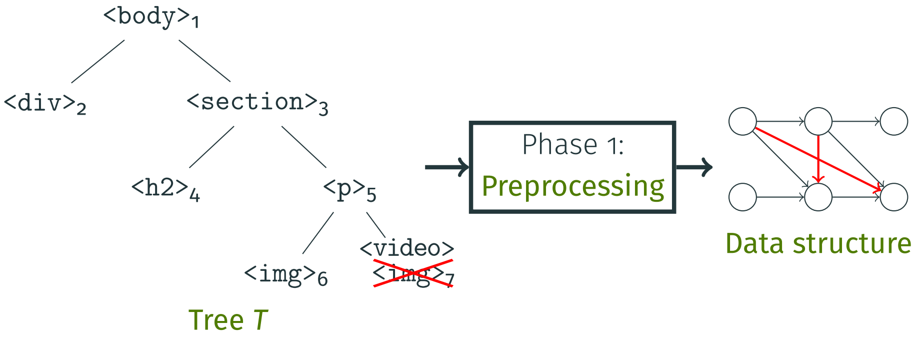

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→ Can we **do better**?

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All these results are on data complexity in T (for a fixed pattern):

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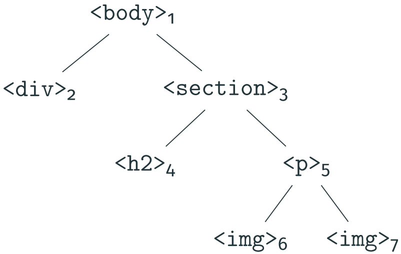

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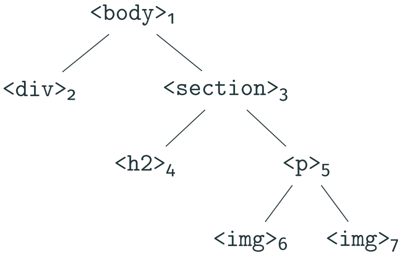

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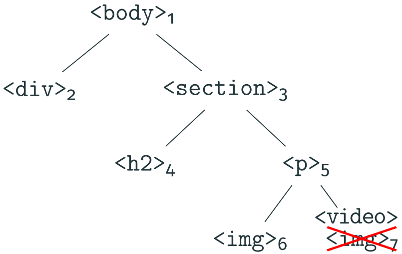

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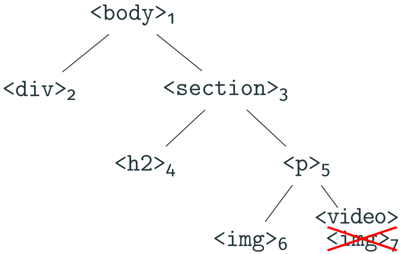

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Thanks for your attention!

References i

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To appear.

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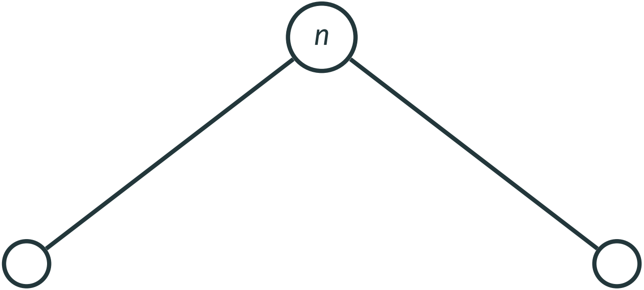

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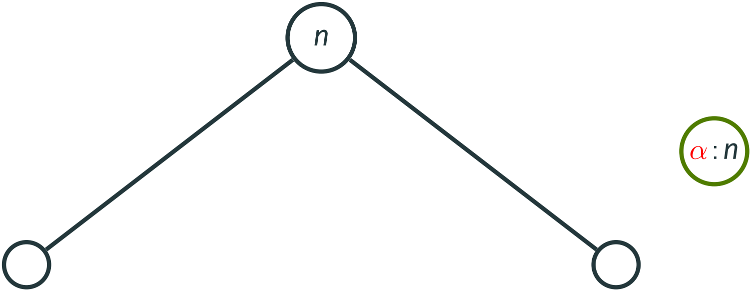

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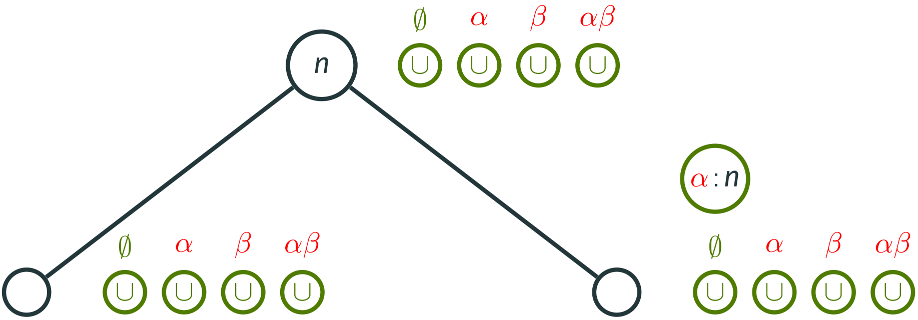

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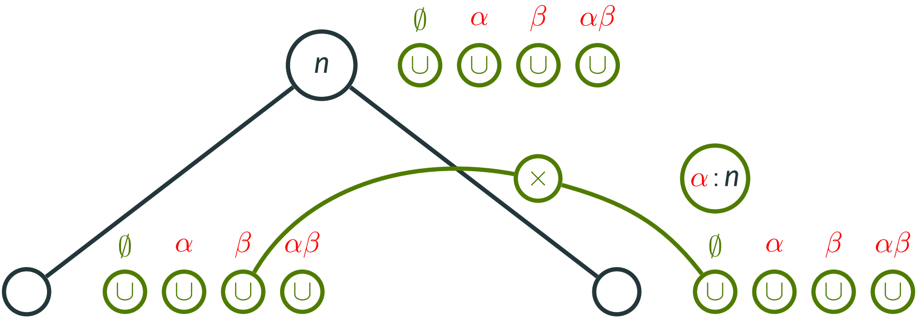

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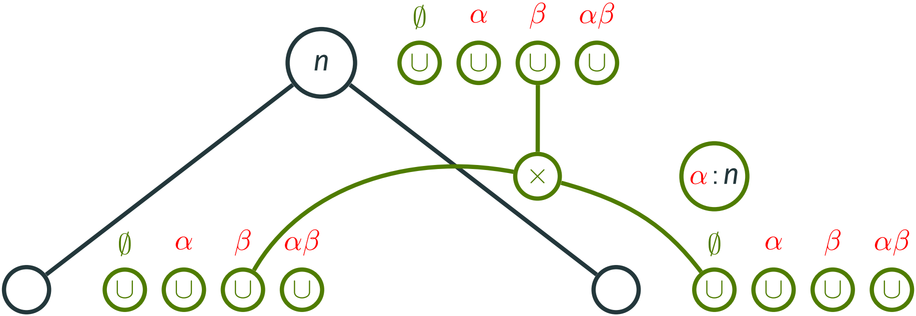

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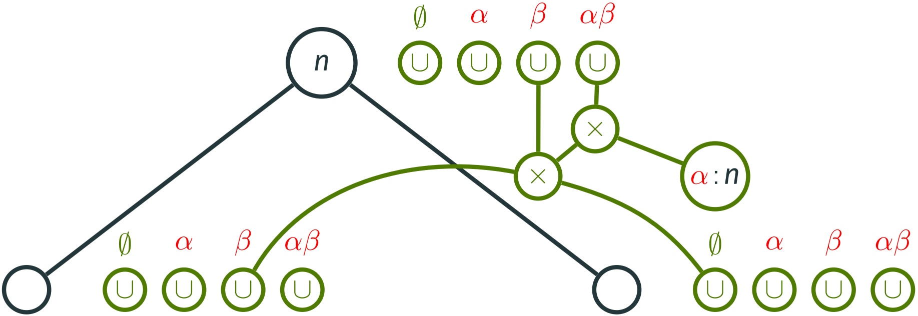

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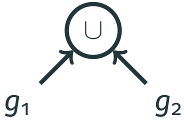

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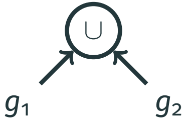

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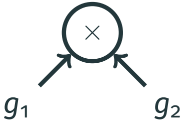

Lexicographic product:

for every t_1 in $T(g_1)$:
  for every t_2 in $T(g_2)$:
    output $t_1 + t_2$

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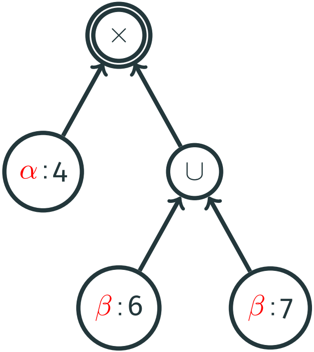
Enumeration relies on some conditions on the input circuit (d-DNNF):



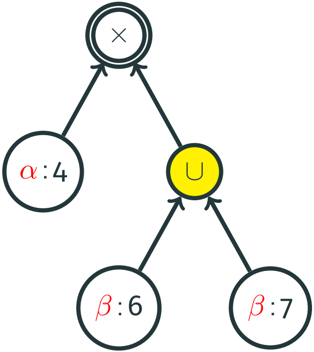
Proof idea for trees: circuit conditions

Enumeration relies on some **conditions** on the input circuit (d-DNNF):

- ($\cup$) are all **deterministic**:

For any two inputs g_1 and g_2 of a $\cup$-gate,
the captured sets $T(g_1)$ and $T(g_2)$ are **disjoint**
(they have no tuple in common)

$\rightarrow$ Avoids **duplicate tuples**



Proof idea for trees: circuit conditions

Enumeration relies on some **conditions** on the input circuit (d-DNNF):

- $\cup$ are all **deterministic**:

For any two inputs g_1 and g_2 of a $\cup$-gate,
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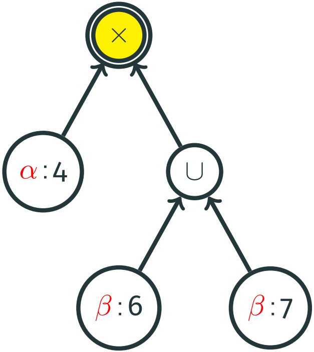

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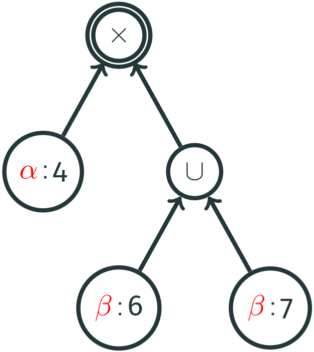

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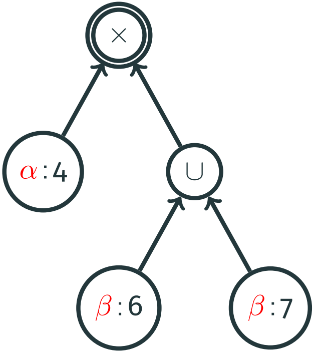

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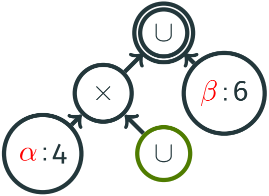
- Our circuit satisfies these thanks to **automaton determinism**

Proof idea for trees: enumeration subtleties



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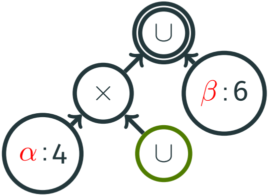

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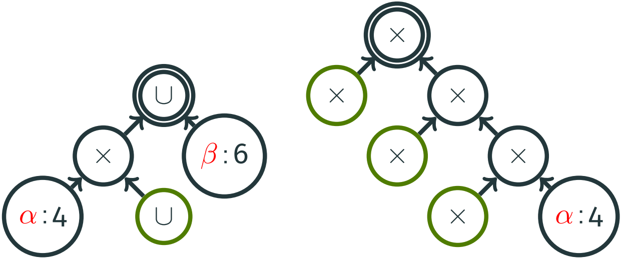

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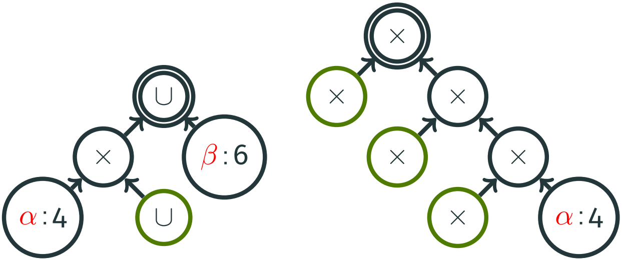

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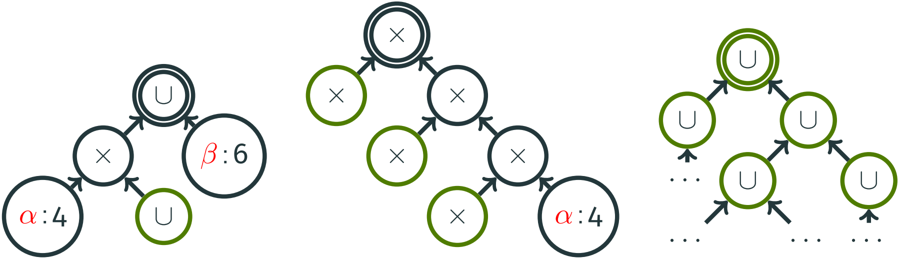

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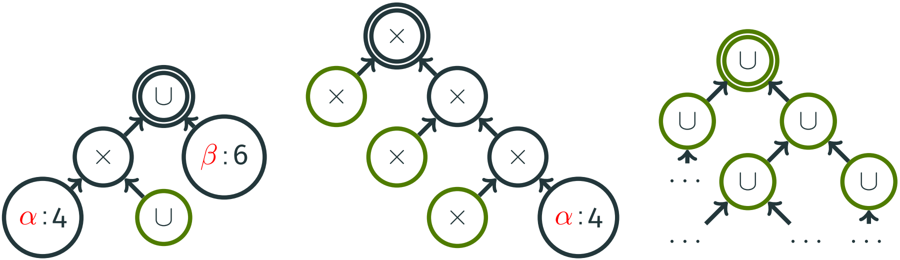

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Connections with Boolean circuits

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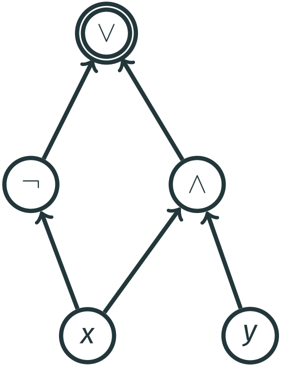
→ Task: Given a Boolean circuit, how to efficiently enumerate its satisfying valuations?

Boolean circuits



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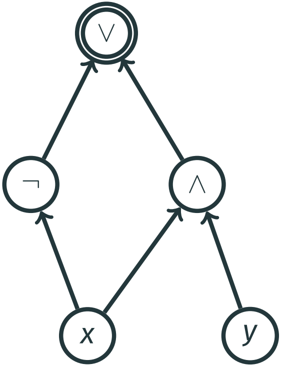
Boolean circuits



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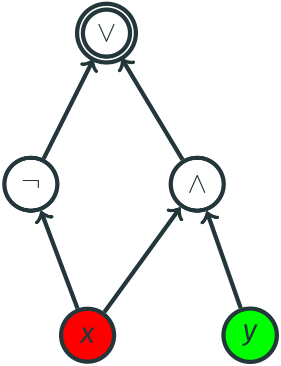

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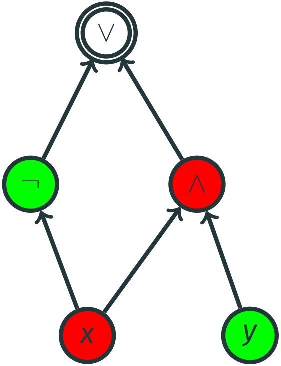

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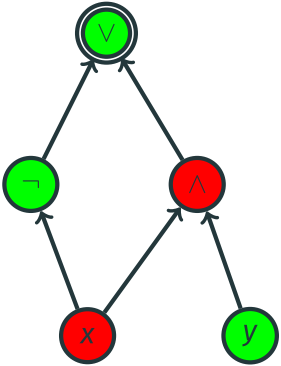

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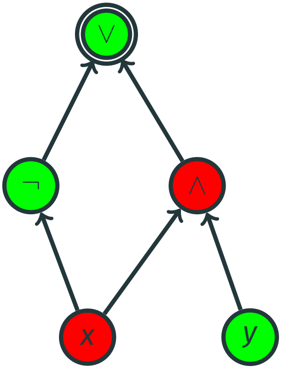

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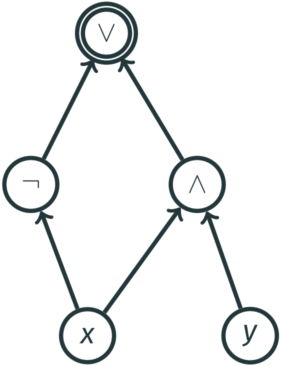

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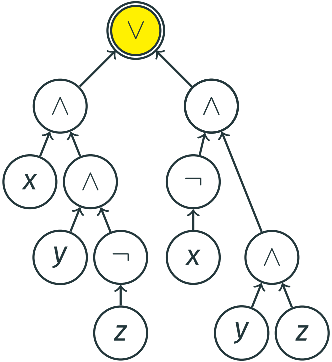
Our task: Enumerate all **satisfying assignments** of an input circuit

Circuit restrictions

d-DNNF:

-  are all **deterministic**:

The inputs are **mutually exclusive** (= no valuation ν makes two inputs simultaneously evaluate to 1)



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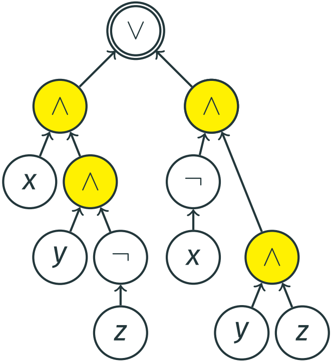

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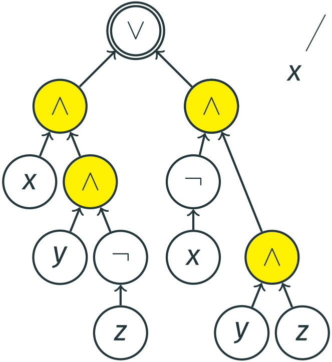
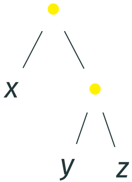
v-tree: $\wedge$-gates follow a **tree** on the variables





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Subtleties: Must **complete** to a set circuit; memory usage problems