Provenance Circuits for Trees and Treelike Instances

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General idea

- We consider a query and a relational instance
- Often it is not sufficient to merely evaluate the query:
  - $\rightarrow$ We need quantitative information
  - $\rightarrow$ We need the link from the output to the input data

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$\rightarrow$ Compute query provenance!

Example 1: security for a conjunctive query

Consider the conjunctive query: $\exists xyz\ R(x, y) \wedge R(y, z)$.

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d	e
e	d
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- Add security annotations: Public, Confidential, Secret, Top secret, Never available

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$\rightarrow$ Result: Confidential

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Consider again: $\exists xyz\ R(x, y) \wedge R(y, z)$.

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- Result: true
- Add multiplicity annotations

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- How many query matches?

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$\rightarrow$ Result: 1

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$\rightarrow$ Result: $1 + 1$

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Semiring provenance [Green et al., 2007]

- Semiring $(K, \oplus, \otimes, 0, 1)$
  - $(K, \oplus)$ commutative monoid with identity 0
  - $(K, \otimes)$ commutative monoid with identity 1
  - $\otimes$ distributes over $\oplus$
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- Idea: Maintain annotations on tuples while evaluating:
  - Union: annotation is the sum of union tuples
  - Select: select as usual
  - Project: annotation is the sum of projected tuples
  - Product: annotation is the product

The universal semiring: $\mathbb{N}[X]$

- Consider again: $\exists xyz\, R(x,y) \wedge R(y,z)$.
- Annotate input facts with atomic annotations $X = f_1, \ldots, f_n$
- Most general semiring: $\mathbb{N}[X]$ of polynomials on X

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Specialization and homomorphisms

- The first three examples can be captured using semirings:
  - security semiring $(K, \min, \max, \text{Public}, \text{Never available})$
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$\rightarrow$ Computing $\mathbb{N}[X]$ provenance subsumes all tasks

$\rightarrow$ It can be done in PTIME data complexity for CQs

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  - Counting the query results

Trees and treelike instances

- Idea: restrict the instances to trees and treelike instances
  - Tree decomposition of an instance: cover all facts
  - Treewidth: minimal width (bag size) of a decomposition
    - Trees have treewidth 1
    - Cycles have treewidth 2
    - k-cliques and k-grids have treewidth $k - 1$
  - Treelike: the treewidth is bounded by a constant

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  - Treelike: the treewidth is bounded by a constant
- If the $\mathrm{PosBool}[X]$ provenance is treelike, we can:
  - Compute its probability efficiently (message passing)
  - Count the results by reducing to probability computation

Problem statement

- Many tasks have tractable data complexity on treelike instances:
  - MSO query evaluation is linear [Courcelle et al., 2001]
  - MSO result counting is linear [Arnborg et al., 1991]
  - Probability evaluation is linear for trees [Cohen et al., 2009]

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→ Can we explain this tractability with provenance?

- Idea: queries on treelike instances have treelike provenance?

→ Can we extend tractability to more quantitative tasks?

→ Can we define and compute provenance for MSO?

Table of contents

General idea

- PosBool[X]-provenance on trees and treelike instances
- The world of trees:
  - Query: MSO on trees
  - Encode to a tree automaton
- The world of treelike instances:
  - Query: MSO/GSO on the instance
  - Reduce to trees [Courcelle et al., 2001]

General idea

- $\mathrm{PosBool}[X]$-provenance on trees and treelike instances
- The world of trees:
  - Query: MSO on trees
  - Encode to a tree automaton
- The world of treelike instances:
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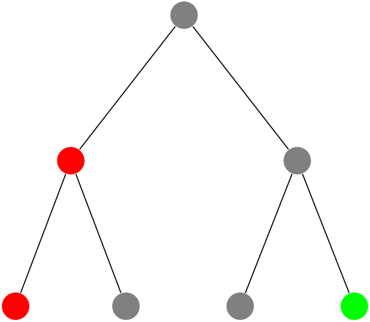
→ Start with $\mathrm{PosBool}[X]$-provenance for tree automata on trees

Tree automata

Tree alphabet:



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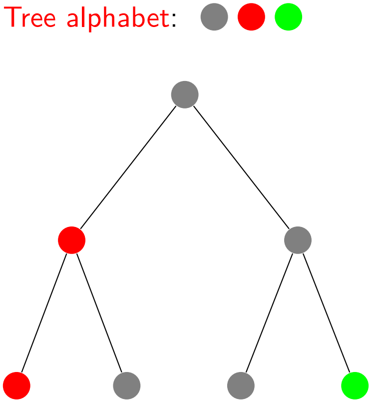

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- "Is there both a red and green node?"

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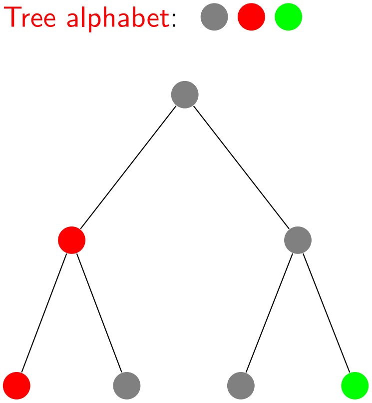

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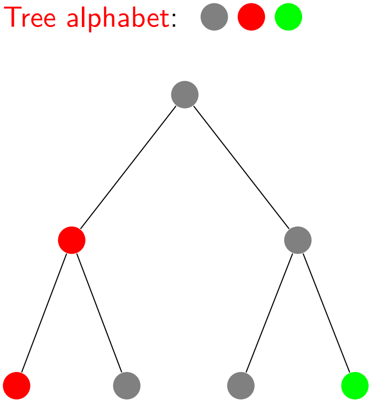

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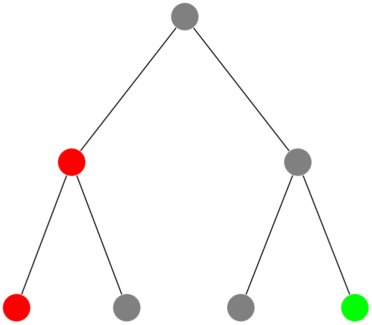

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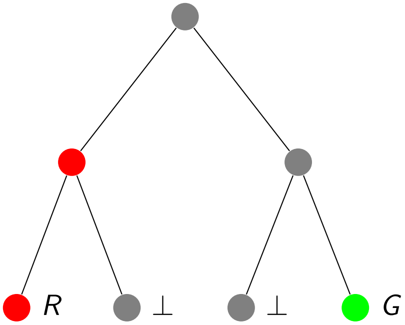

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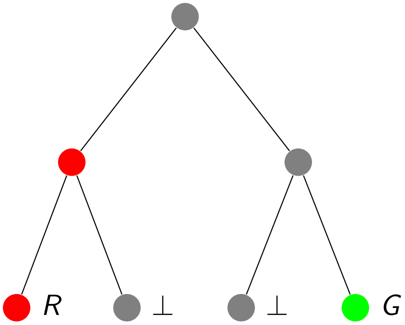

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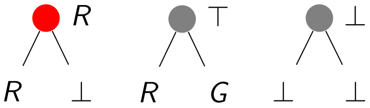

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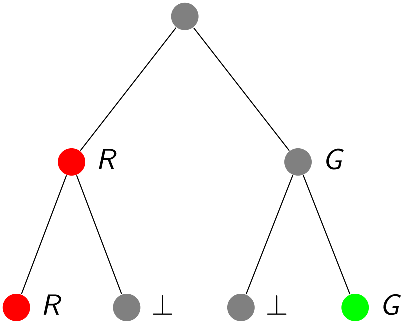

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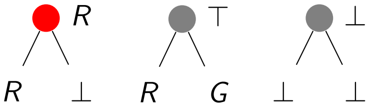

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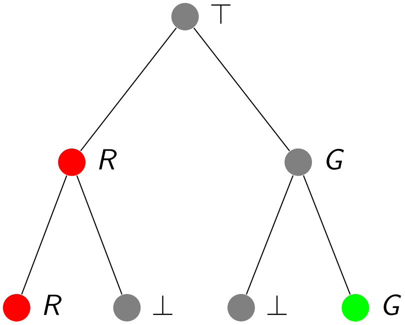

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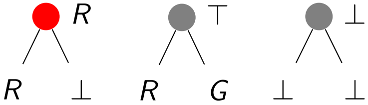

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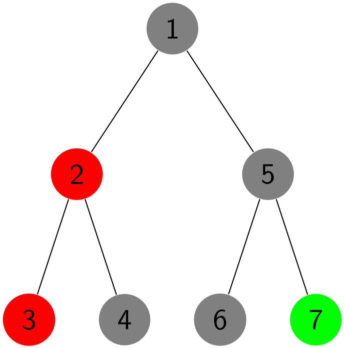
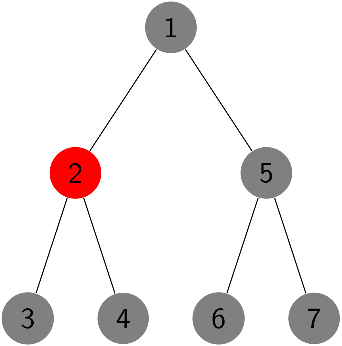


A valuation of a tree decides whether to keep or discard node labels.

Keep: $\{1, 2, 3, 4, 5, 6, 7\}$

The bNTA accepts

Uncertain trees



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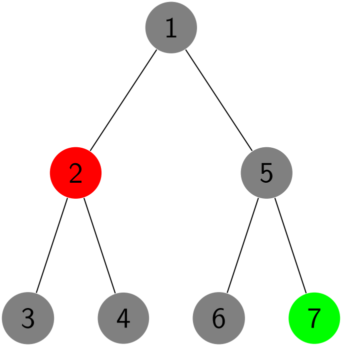

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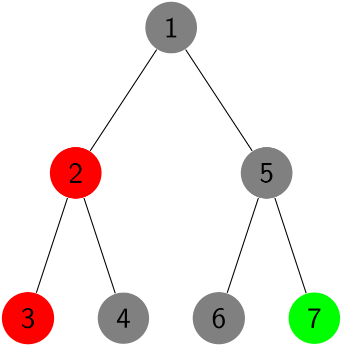
Provenance circuits



- $X = \{g_1, g_2, g_3, g_4, g_5, g_6, g_7\}$
- $\mathrm{PosBool}[X]$-provenance of a bNTA A on tree T:
  - monotone Boolean formula ϕ
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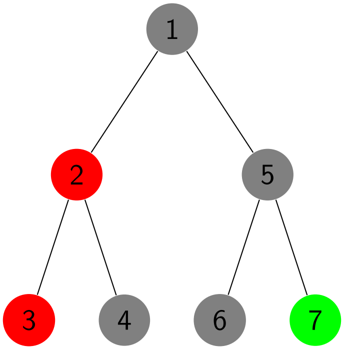

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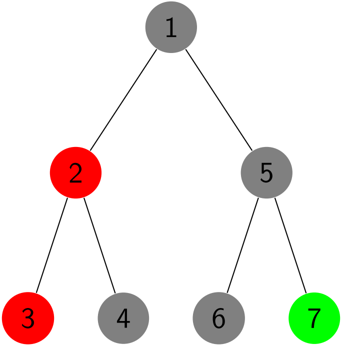
- Represent as a circuit [Deutch et al., 2014]
  - monotone Boolean circuit C
  - with input gates X

  $\rightarrow$ $\nu(T)$ is accepted by A iff $\nu(C)$ is true (output gate)

Example



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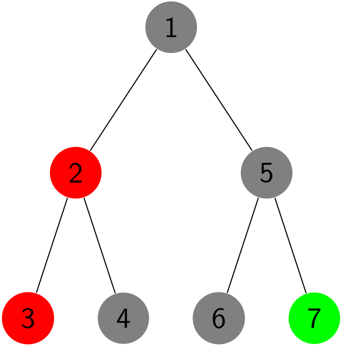

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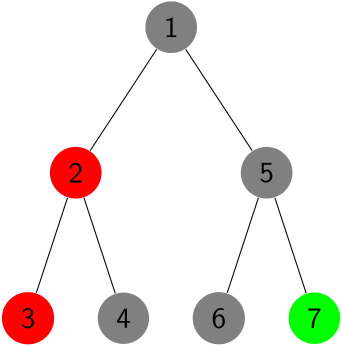

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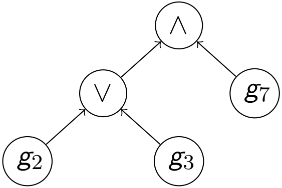
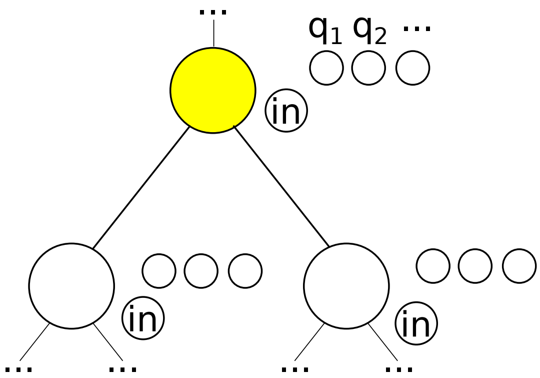


Constructing the provenance circuit

→ Construct a Boolean provenance circuit bottom-up



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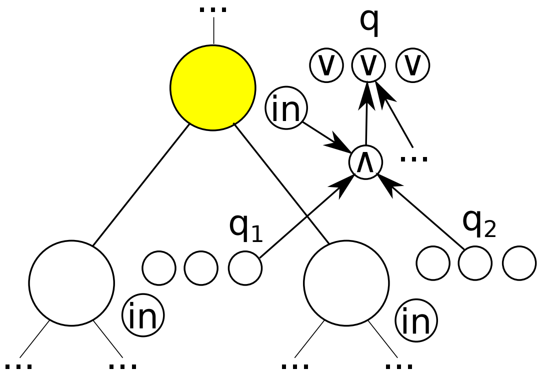

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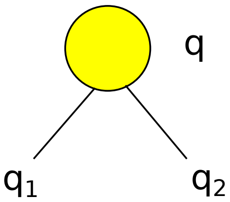

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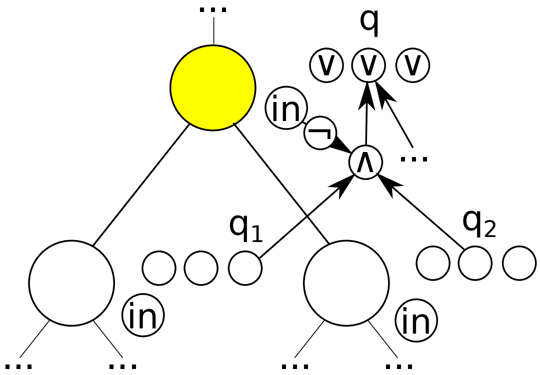

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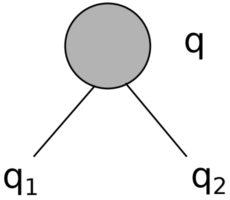

Our results on trees

- A $\mathrm{PosBool}[X]$ provenance circuit of a bNTA on a tree:
  - $\rightarrow$ can be computed in linear time in the bNTA and tree
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$\rightarrow$ Let's extend this to treelike instances!

Encoding treelike instances [Chaudhuri and Vardi, 1992]

Instance:

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a	b
b	c
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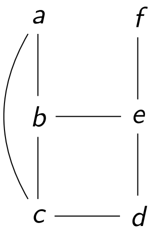

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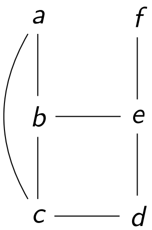

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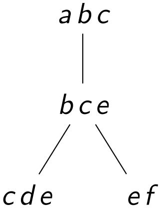

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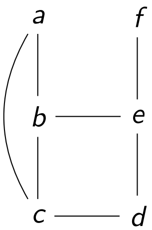

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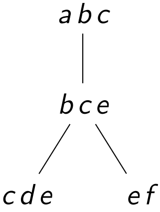

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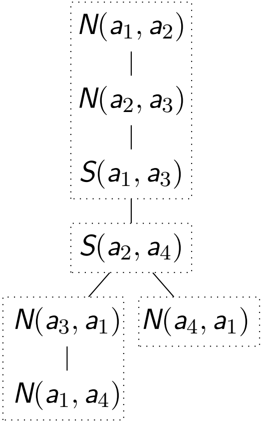

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- Tree encodings: represent treelike instances as trees
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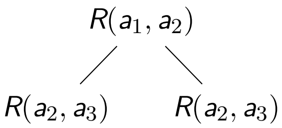

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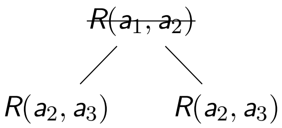

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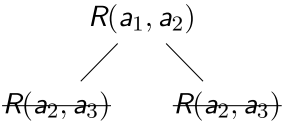

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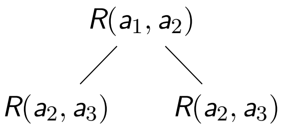

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- We can reduce counting to probabilistic evaluation

$\rightarrow$ Re-proves that MSO counting has linear-time data complexity

Example: block-independent disjoint (BID) instances

name	**city**	**iso**	p
pods	melbourne	au	0.8
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→ For a treelike instance, linear time!

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- We want to generalize from $\mathrm{PosBool}[X]$ to $\mathbb{N}[X]$
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→ We can encode monotone queries to monotone bNTAs

→ Provenance circuits for monotone automata can be monotone

Second problem: intrinsic definition

- Boolean provenance has an intrinsic definition:
  "Characterize which subinstances satisfy the query"
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$\rightarrow$ We restrict to (Boolean) UCQs from now on

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How is $\mathbb{N}[X]$ more expressive than $\mathrm{PosBool}[X]$?

$\rightarrow$ Coefficients: counting multiple derivations

$\rightarrow$ Exponents: using facts multiple times

$\rightarrow$ (Non-absorptivity: $a \oplus (a \otimes b) \neq a$)

Supporting coefficients

- In the world of trees
  - The same valuation can be accepted multiple times

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→ Add assignment facts to represent possible assignments

→ Encode to a bNTA that guesses them

Supporting exponents

- In the world of trees
  - The same fact can be used multiple times
  - Annotate nodes with a multiplicity
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→ Encodes CQs to bNTAs that read multiplicities

- Consider all possible CQ self-homomorphisms
- Count the multiplicities of identical atoms
- Rewrite relations to add multiplicities
- Usual compilation on the modified signature

Our result for $\mathbb{N}[X]$-provenance circuits

We can compute in linear time data complexity a $\mathbb{N}[X]$ provenance circuit (arithmetic circuit) for UCQs.

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→ What fails for MSO/Datalog?

- Unbounded maximal multiplicity
- Logical definition of fact multiplicity?

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- Applications:
  - → Capture counting results (decouple symbolic and numerical computation)
  - → Extend to new applications (probabilities)

Future work

- Monadic Datalog [Gottlob et al., 2010] to avoid high combined complexity
- A neater approach for counting and probabilities
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Thanks for your attention!

References I

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