Combining Existential Rules and Description Logics

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Open-world query answering (QA)

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- We are given:
  -  Relational instance I (ground facts)
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Open-world query answering: – query entailment or containment

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  -  Logical constraints Σ – T-Box
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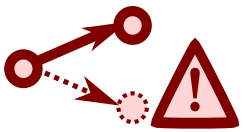
Functionality asserts $\mathsf{Funct}(\mathsf{Mgr}^{-})$ 	n/a

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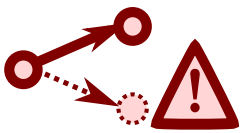
Functionality asserts $\mathsf{Funct}(\mathsf{Mgr}^{-})$ 	n/a

$\rightarrow$ QA is decidable for either language

Our problem

Can we have the best of both worlds?

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We show:

- QA is undecidable for rich DLs and frontier-guarded rules
- QA with rich DLs is decidable for some new rule classes
- Functional dependencies can be added under some conditions

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→ Non-looping rules: prohibit cycles

- $R(x,y)\ S(y,z)\ T(z,x)$
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Head-non-looping frontier-one rules: no cycles in head

→ We can treeify the rules, soundness by unravelling the models

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Theorem

Decidable QA for:

- *Rich DL constraints*
- *Single-head frontier-one rules*
- *Non-conflicting FDs*

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Thanks for your attention!

More details: see poster 76