Skyline Operators for Document Spanners

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Document spanners

We use document spanners, a declarative formalism for information extraction tasks

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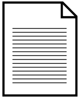

Text document

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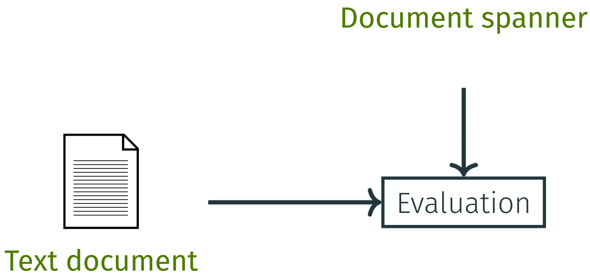

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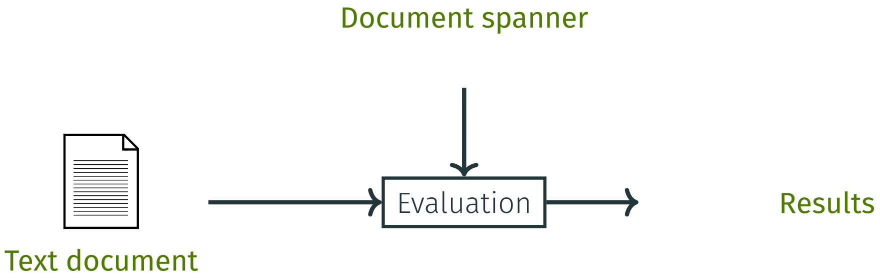

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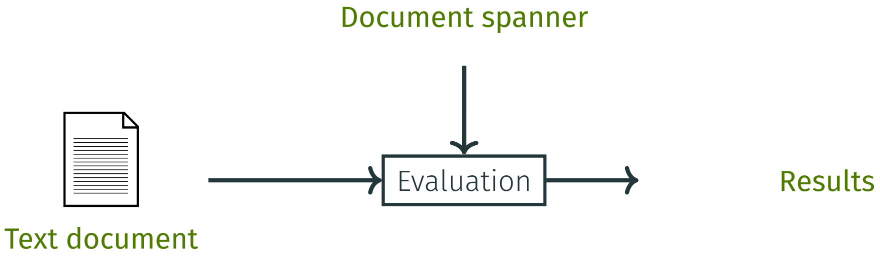
“Extract all email addresses in the document”



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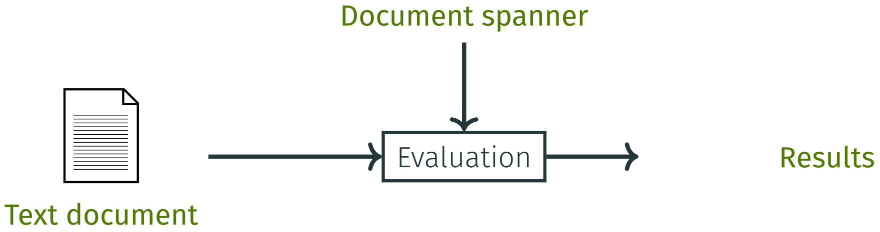

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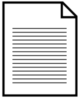

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$\longrightarrow$ $\boxed{\text{Evaluation}}$ $\longrightarrow$ Results

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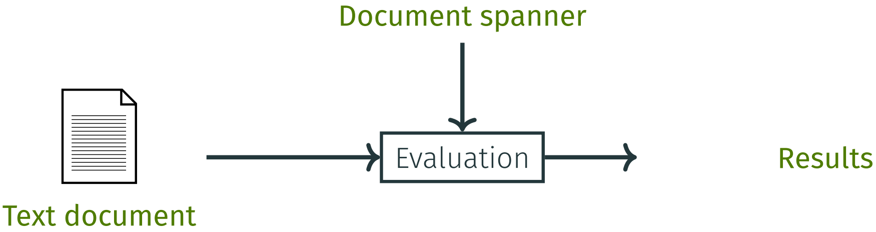

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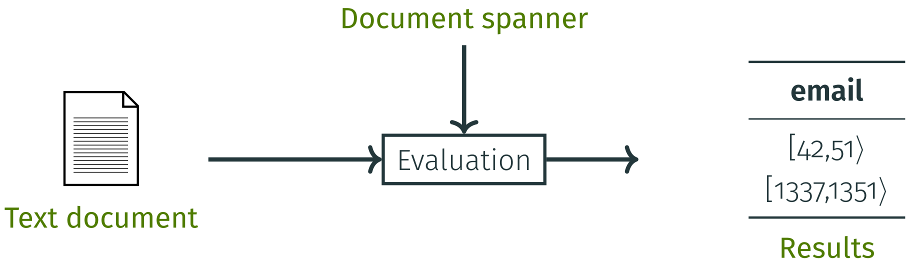

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"Extract all last names with possibly a phone number"

Document spanner

Results

Text document

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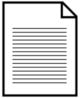

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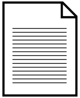

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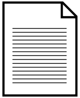

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name	phone
$[145,152\rangle$	–
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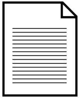

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- Several **formalisms** to express document spanners
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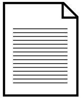

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Results

- Several **formalisms** to express document spanners
  - → Focus: **regular spanners** expressed as **Variable-Set Automata** (VAs) or regex-formulas
- Well-studied task: **efficient evaluation**, including enumeration algorithms

Maximal matches

Standard semantics: extract all mappings of the spanner variables. But...

Maximal matches

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A3 **You**

How can I obtain all matches of a regular expression in a string?

ChatGPT

To obtain all matches of a regular expression in a string, you can use the appropriate function provided by your programming language or library.

python Copy code

```python
pattern = r'your_pattern_here'
input_string = "your_input_string_here"
matches = re.findall(pattern, input_string)
```

Maximal matches

Standard semantics: extract **all mappings** of the spanner variables. But…

`re.findall(pattern, string, flags=0)`

Return all non-overlapping matches of *pattern* in *string*, as a list of strings or tuples. The *string* is scanned left-to-right, and matches are returned in the order found. Empty matches are included in the result.

The result depends on the number of capturing groups in the pattern. If there are no groups, return a list of strings matching the whole pattern. If there is exactly one group, return a list of strings matching that group. If multiple groups are present, return a list of tuples of strings matching the groups. Non-capturing groups do not affect the form of the result.

```
>>> re.findall(r'\bf[a-z]*', 'which foot or hand fell fastest')
['foot', 'fell', 'fastest']
```

python — Copy code

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Maximal matches

Specifically, we may want:

- *"Extract all email addresses"*

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  $\rightarrow$ If the number is given, do not extract a match without it

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Skyline under a domination relation: the results which are **maximal**, i.e., not dominated

Naive skyline computation

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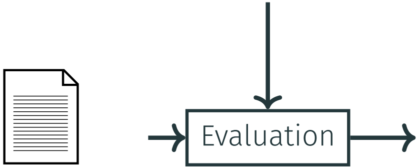
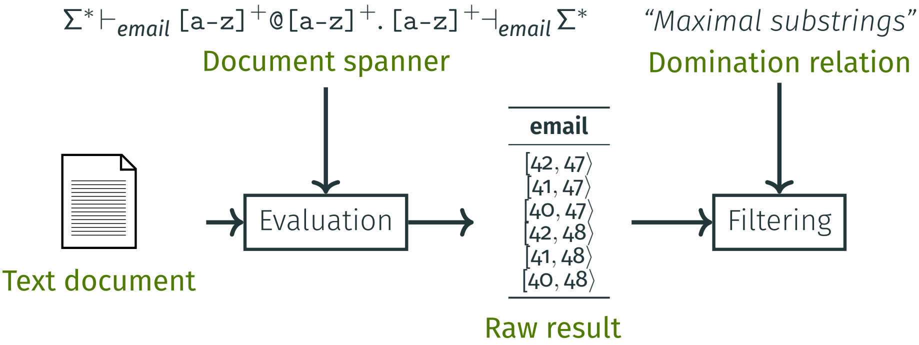


Text document

email
$[42, 47\rangle$
$[41, 47\rangle$
$[40, 47\rangle$
$[42, 48\rangle$
$[41, 48\rangle$
$[40, 48\rangle$

Raw result

Naive skyline computation



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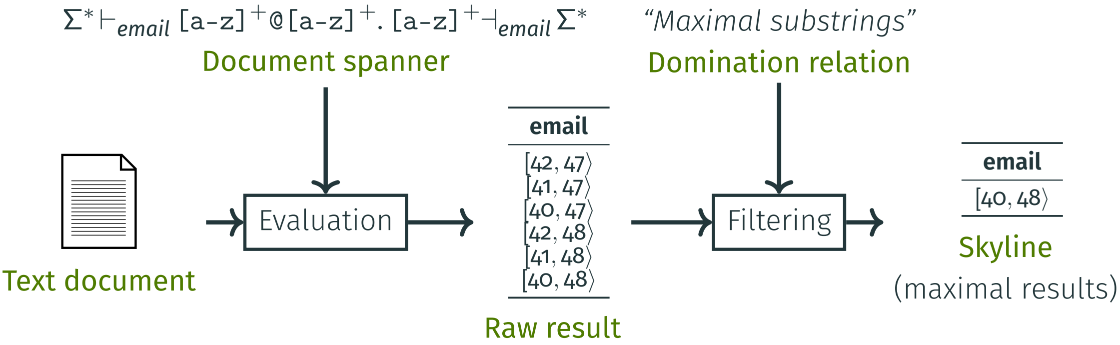

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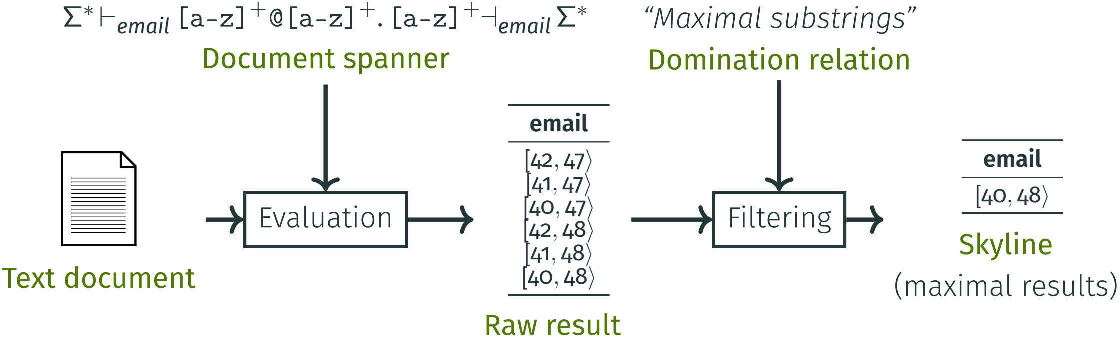

- Can we be **more efficient**, i.e., avoiding materializing the raw result?
- Can we **merge** both steps, i.e., compile the domination relation in the spanner?

Paper contributions and talk outline

- Introduce and formalize the skyline problem for regular spanners

  $\rightarrow$ Propose a general framework to express domination relations

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- Study if we can **compile** the skyline operator **into the spanner**
  $\rightarrow$ **Expressiveness:** is it **possible**?
  $\rightarrow$ **State complexity:** does it **blow up** the spanner representation?

Paper contributions and talk outline

- Introduce and formalize the skyline problem for regular spanners
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- Study if we can compile the skyline operator into the spanner
  $\rightarrow$ Expressiveness: is it possible?
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- Study the problem of efficiently evaluating the skyline operator
  $\rightarrow$ In data complexity and combined complexity

Table of contents

Basics of spanners

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$$d = \begin{array}{cccccccccccc} \mathtt{J} & \mathtt{o} & \mathtt{h} & \mathtt{n} & \sqcup & \mathtt{4} & \mathtt{5} & \mathtt{6} & \mathtt{1} & \mathtt{2} & \mathtt{3} & \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \end{array}$$

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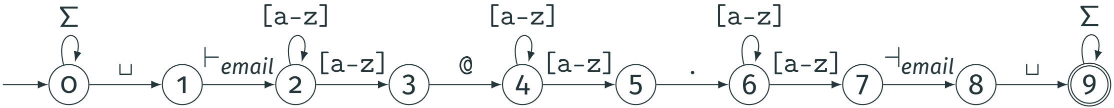
  - $\rightarrow$ ex: for $X = \{x, y, z\}$, map x to $[0, 4\rangle$ and leave y and z unassigned
- **Spanner**: function that maps each document to a set of mappings

Defining spanners

Regular spanners: those that can be expressed as variable-set automata (VAs; always assumed to be sequential)



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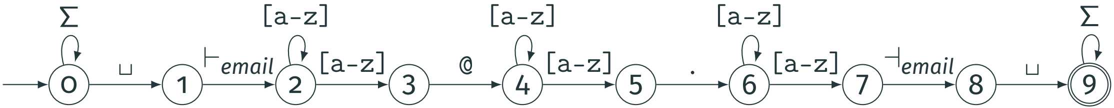

In practice, often more convenient to write in the subclass of **regex-formulas**:

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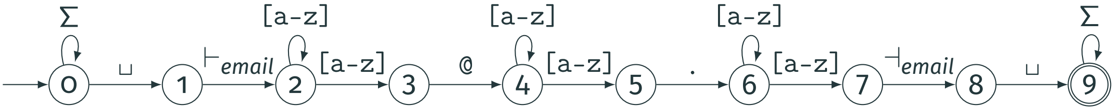

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Other **more general classes**:

- **Core spanners**: featuring **string equality selection**
- **Generalized core spanners**: featuring **difference**

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Can we have a **unified framework** covering those?

Examples of domination relations

Extracted mappings:

x	y
$[1, 2\rangle$	$[2, 3\rangle$
—	$[2, 3\rangle$
$[0, 2\rangle$	$[2, 3\rangle$
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$[0, 2\rangle$	$[2, 3\rangle$
$[4, 6\rangle$	$[4, 10\rangle$

Skyline under span length:

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- We want to express a domination relation: a partial order on mappings
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  - Mapping m maps x to $[42,51\rangle$ and does not map y
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$\rightarrow$ We can define the domination relation as a spanner D, called a domination rule:
  - $\rightarrow$ Definition of spanner D: given d, extract all mappings μ that code a domination pair

Expressing domination relations via domination rules

Consider the example domination relations on a single variable x

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  $\rightarrow$ Not expressible as a regular spanner

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- $\to$ Covers all examples so far

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- Evaluation: from A and D and d, compute the skyline directly

Table of contents

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For regular spanners, this is possible:

Theorem

Given a VA $\boldsymbol{A}$ and a domination rule expressed as a VA $\boldsymbol{D}$, we can compute a VA $\boldsymbol{A'}$ extracting the skyline $\eta_{\boldsymbol{D}}(\boldsymbol{A})$

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For **core spanners**: not possible!

$\rightarrow$ Already in the case where D is the **span inclusion** or **variable inclusion** rule

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Theorem

Given a VA A with n states, a VA A' computing the skyline $\eta(A)$ under variable inclusion needs $2^{\Omega(n)}$ states in general

Proof technique via nFBDDs

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- Combined complexity for fixed rule: fix D, the input is the VA A and the document d

Evaluation in data complexity

The skyline is always tractable to compute in data complexity:

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For any fixed VA A and domination rule D, given a document d, we can compute the skyline of A on d under D in PTIME in d

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  - Then, simply **run A' on d** to compute the maximal mappings

Evaluation in combined complexity

Computing the skyline is **intractable** even under the variable inclusion rule

Theorem

The following problem is NP-hard: given $\boldsymbol{n} \in \mathbb{N}$, a VA $\boldsymbol{A}$, and document $\boldsymbol{d}$, decide if $\boldsymbol{A}$ has more than n mappings on d that are maximal for variable inclusion

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- Hardness also holds for the **span inclusion** rule
- Hardness also holds if the **input document is fixed**

Which domination rules are hard?

- **Skyline computation** is intractable in combined complexity for the **variable inclusion** and **span inclusion** rules
- Of course it is tractable for the **trivial domination rule** (= no skyline)

$\rightarrow$ Can we get a **dichotomy?**

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→ Can we get a **dichotomy?**

In the paper:

- **Sufficient condition for hardness:** whenever a rule captures unboundedly many comparable pairs that are "disjoint", then skyline computation is hard
- **Dichotomy** on a subset of domination rules based on a variant of this condition
- **Troubling asymmetry:** there is a domination rule $\leq$ such that:
  - Computing the skyline under $\leq$ is **easy**
  - Computing the skyline under $\geq$ is **hard**!

Table of contents

Summary and further work

- We have studied skyline computation for document spanners, with a spanner-based framework to express domination rules
- Regular spanners are closed under skyline, but unavoidable exponential blowup
- Evaluation tractable in data complexity but hard in combined complexity

Summary and further work

- We have studied **skyline computation for document spanners**, with a **spanner-based framework** to express domination rules
- Regular spanners are **closed** under skyline, but unavoidable exponential blowup
- Evaluation tractable in data complexity but **hard in combined complexity**

Main open questions:

- Are there **other applications** of the nFBDD correspondence?
  - $\rightarrow$ In the paper: exponential blowup for the **join** of **schemaless regex formulas**
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- Same question for the **state complexity blowup**?
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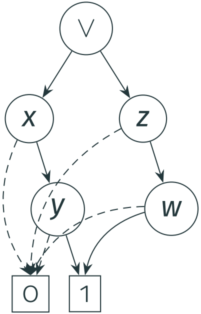
Thanks for your attention!

Proof technique: nFBDDs (aka NROBPs)



(Courtesy of Tim Van Bremen)

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  - Intuitively, OBDDs with nondeterminism and without variable order

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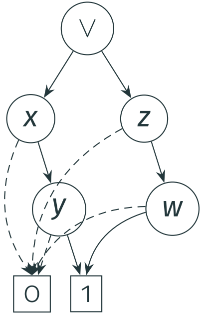

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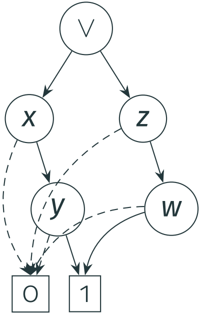

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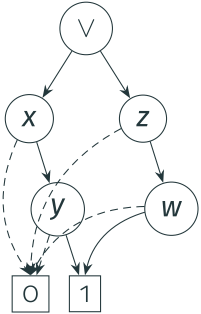

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- But nFBDDs are exponentially less concise than other representations (read-3 monotone 2-CNF formulas)

- For such a formula ψ, we can build a VA A_ψ whose skyline under variable inclusion corresponds to the satisfying assignments of ψ

  $\rightarrow$ not expressible as a small nFBDD, hence not expressible by a small VA

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What is the **skyline** of $r \cup r'$?

- The m mappings of m' are **maximal** (incomparable and not covered by m)
- Other than that:
  - If there is a **maximal mapping from m**, then it is not covered by r' so contains **one literal per clause**: Φ is **satisfiable**
  - Otherwise, all assignments violate some clause: Φ is **unsatisfiable**