Enumerating regular languages with bounded delay

Antoine Amarilli, Mikaël Monet; STACS'23

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Context: Enumeration algorithms

Enumeration algorithms: framework for computation problems producing many results

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Decision problem

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Input $\rightarrow$ YES/NO

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Input $\rightarrow$ YES/NO

Measure: running time

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Input → YES/NO

Measure: running time

Computation problem

Input → { ■ ▼ , ■ ■ ▼ , ■ ▼ ▼ }

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Input $\rightarrow$ YES/NO

Measure: running time

Computation problem

Input $\rightarrow$ $\{ \blacksquare \blacktriangledown, \blacksquare \blacksquare \blacktriangledown, \blacksquare \blacktriangledown \blacktriangledown \}$

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Measure: max delay between two consecutive results

Holy grail: Constant-delay enumeration

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On acyclic conjunctive queries and constant delay enumeration

Guillaume Bagan [*] Arnaud Durand [†] Etienne Grandjean [‡]

Holy grail: Constant-delay enumeration

On acyclic conjunctive queries and constant delay enumeration

Guillau

Enumeration of MSO Queries on Strings with Constant Delay and Logarithmic Updates

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University of Bayreuth

Luc Segoufin
INRIA and ENS Ulm

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Constant delay enumeration for FO queries over databases with local bounded expansion

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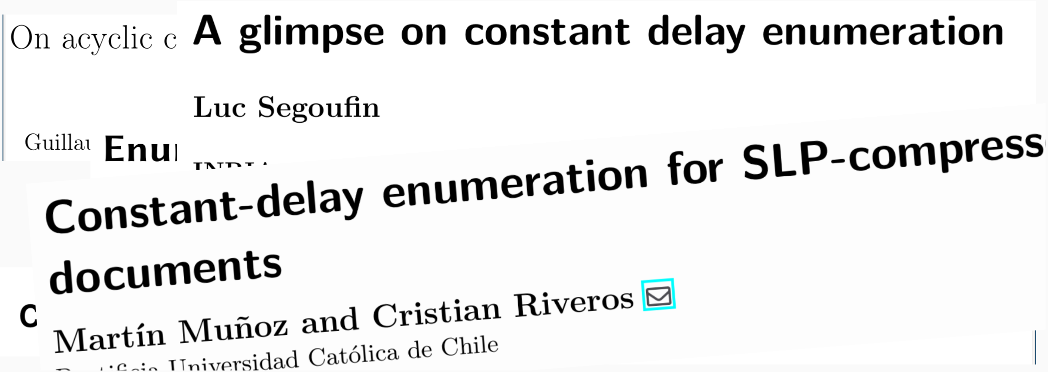
Constant Delay Enumeration for Conjunctive Queries — a Tutorial


On acyclic c

Guillau

Enu

C

A glimpse on constant delay enumeration

Luc Segoufin

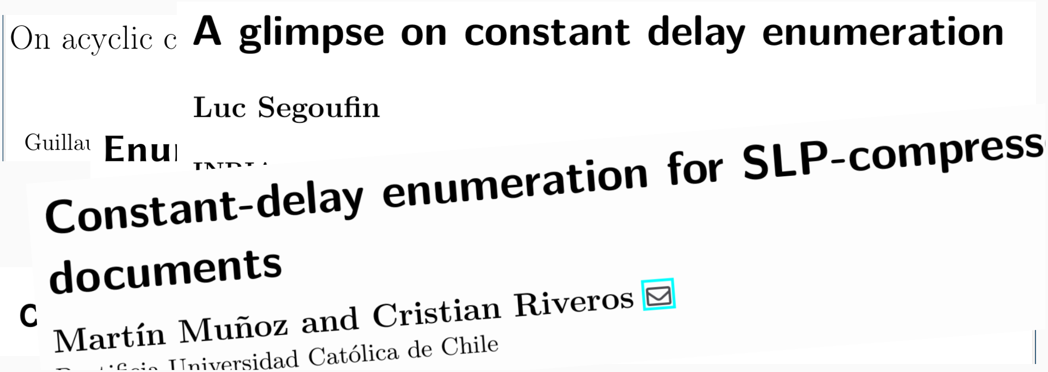
Constant-delay enumeration for SLP-compress

documents

Martín Muñoz and Cristian Riveros

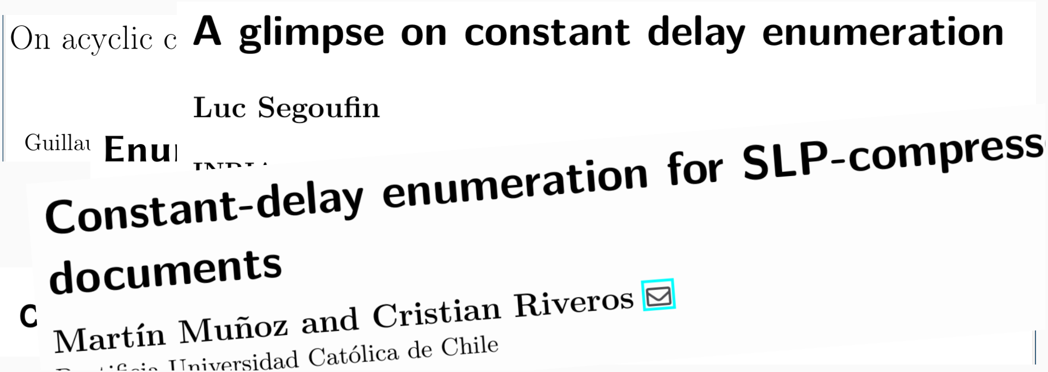
Pontificia Universidad Católica de Chile

Holy grail: Constant-delay enumeration



Problem: assumes that the results to enumerate have constant size

Holy grail: Constant-delay enumeration



Problem: assumes that the results to enumerate have constant size

Ambitious goal

How can we enumerate results of unbounded size in constant delay?

Solution: Cheat!

Do not write each result from scratch, but by editing the previous result!

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____	____	____	____	____	$\cdots$	Results:
1	2	3	4	5		

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1 2 3 4 5

Put(1, );

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Put(1, ■); Put(2, ▼);

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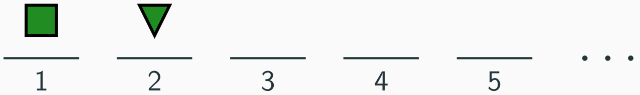

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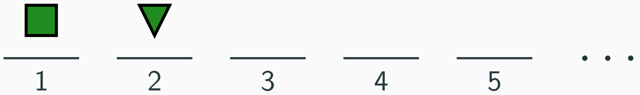

Results:

Put(1, ■); Put(2, ▼); Output();

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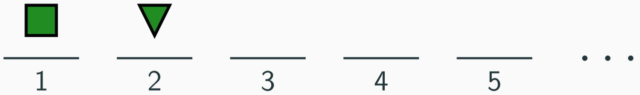

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Remark: Solutions need to be ordered with small distance between consecutive solutions

Enumeration for automata

Problem: enumerate the words accepted by a word automaton

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- Input: Deterministic finite automaton A on alphabet Σ
- Output: The words of its language $\mathrm{L}(A) \subseteq \Sigma^*$

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  - Here, d is the Levenshtein distance
- If yes, can we efficiently produce the sequence of edits?

Examples

a^*

Examples

a^* $\epsilon,\ a,\ aa,\ aaa,\ \ldots$

Examples

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$a^* b^*$

Examples

a^*	$\epsilon, a, aa, aaa, \ldots$
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a^* $\quad$ $\epsilon, a, aa, aaa, \ldots$

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$a^*(c+d)b^*$

Examples

a^*	ϵ, a, aa, aaa, ...
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Can you characterize the orderable languages?

Results

Theorem

Given a DFA A, we can determine in PTIME whether its language $L(A)$ is orderable

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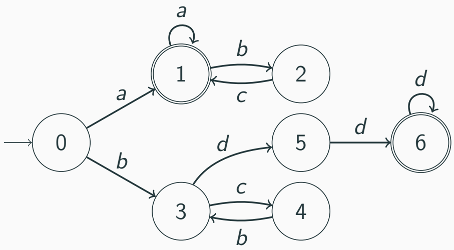
Also:

- Characterization if we only allow edits at the right endpoint (= stack, not deque)
- Finding the minimal distance bound is NP-hard

Proof techniques

Pleasant (and elementary): orderability



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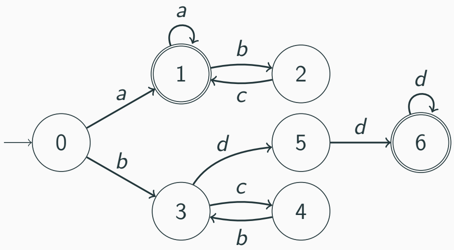

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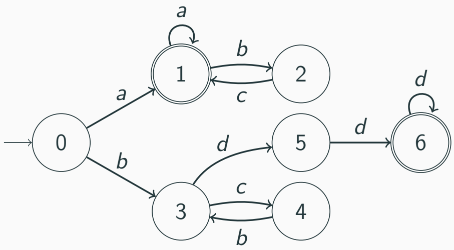

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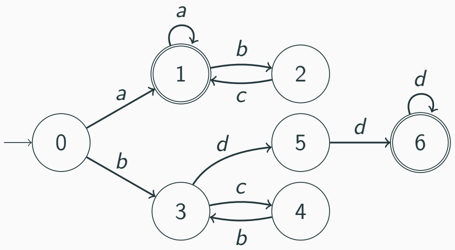

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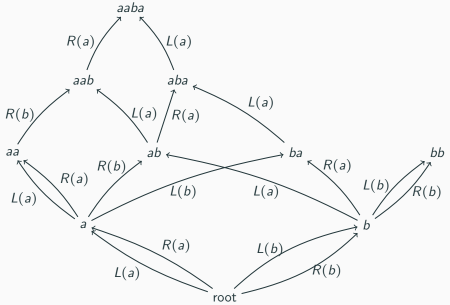

- Pointer machine model because memory usage goes to infinity

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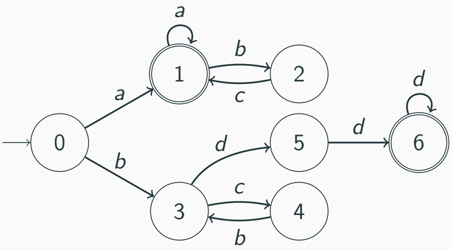

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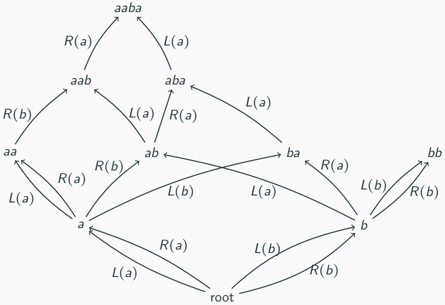

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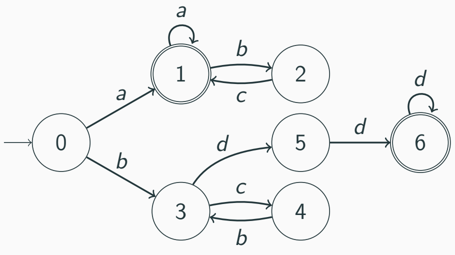

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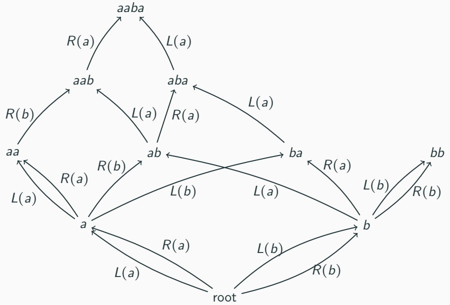

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- Everything is exponential in the DFA
- Probably simplifiable...

Future work

Open questions and future work:

- Make the delay polynomial in $|A|$? (currently it is exponential)
- What about the push-left pop-right distance? the padded Hamming distance?
- What about enumeration in radix order?
- What about regular tree languages?
- Can we go beyond regular languages?
- Other uses of the enumeration model?

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Thanks for your attention!