Probabilities and Provenance on Trees and Treelike Instances

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September 7th, 2016

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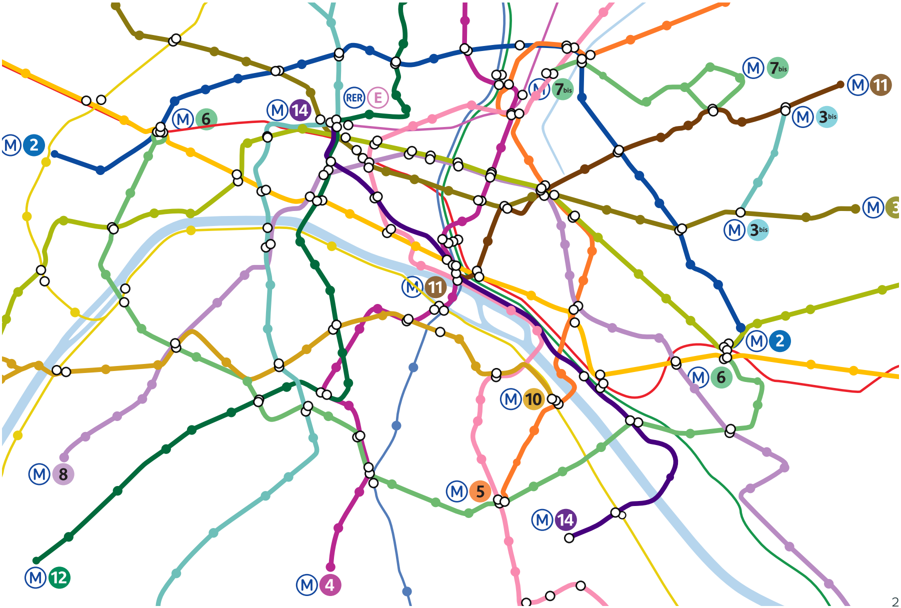
[2]CNRS CRIStAL

[3]National University of Singapore

[4]École normale supérieure de Paris

How to travel to Highlights from Paris?



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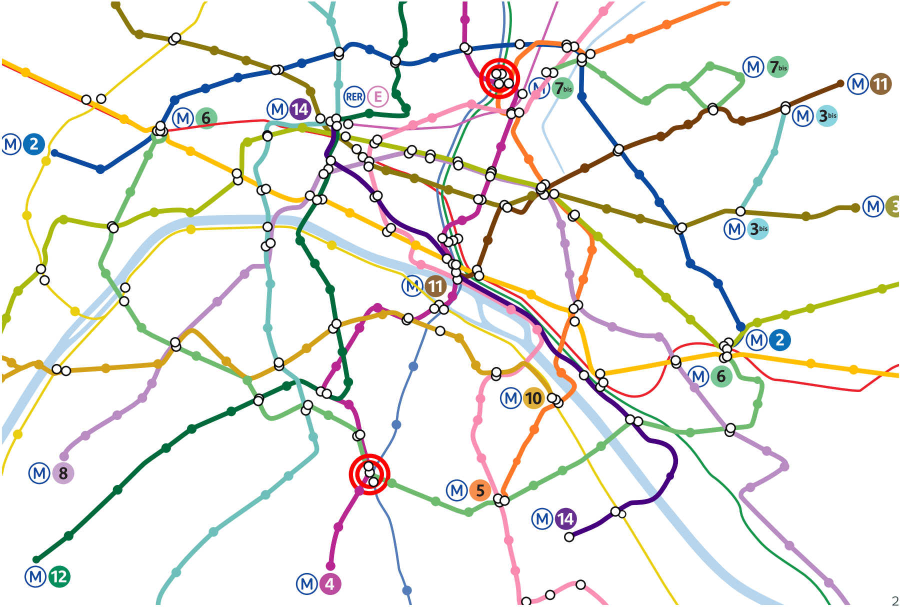

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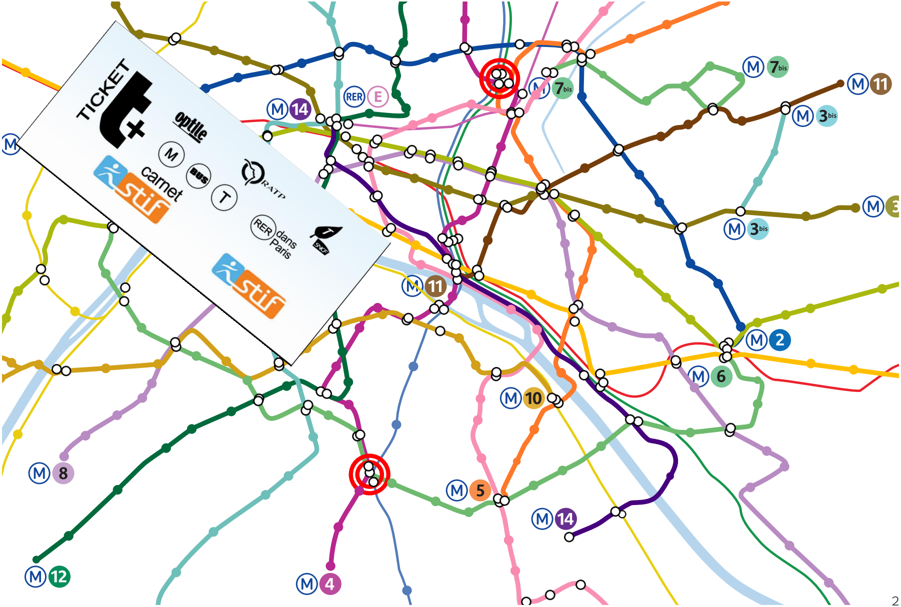

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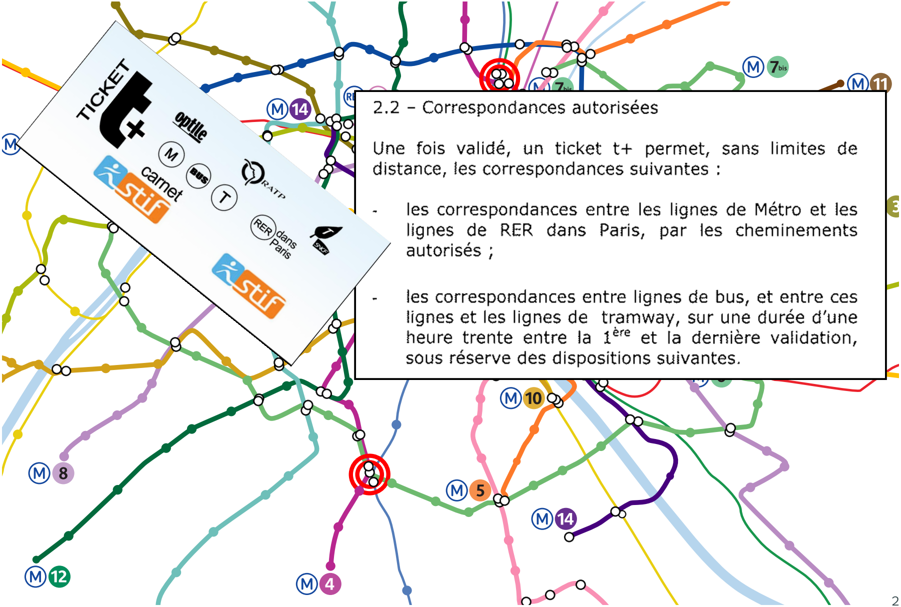

2.2 – Correspondances autorisées

Une fois validé, un ticket t+ permet, sans limites de distance, les correspondances suivantes :

- les correspondances entre les lignes de Métro et les lignes de RER dans Paris, par les cheminements autorisés ;

- les correspondances entre lignes de bus, et entre ces lignes et les lignes de tramway, sur une durée d'une heure trente entre la 1ère et la dernière validation, sous réserve des dispositions suivantes.

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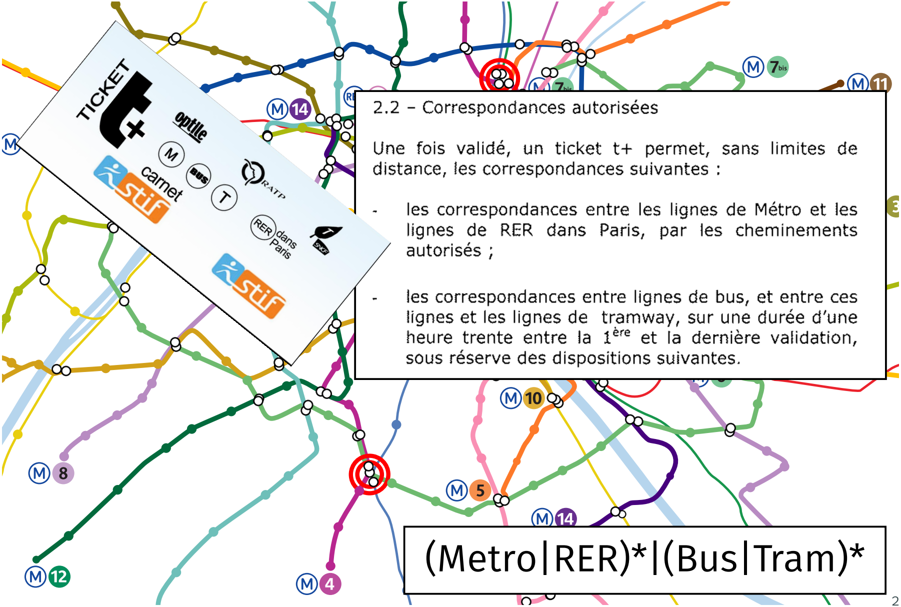

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(Metro|RER)*|(Bus|Tram)*

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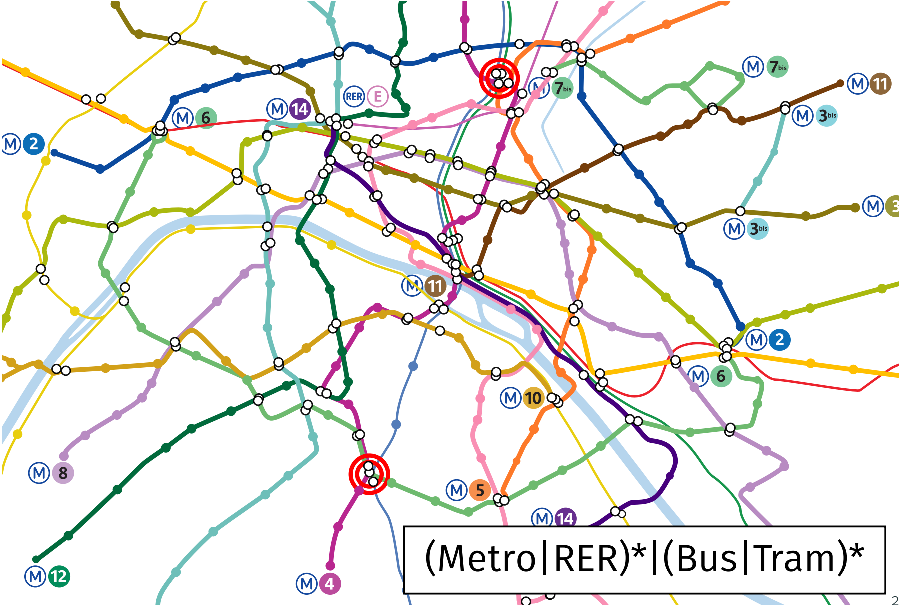

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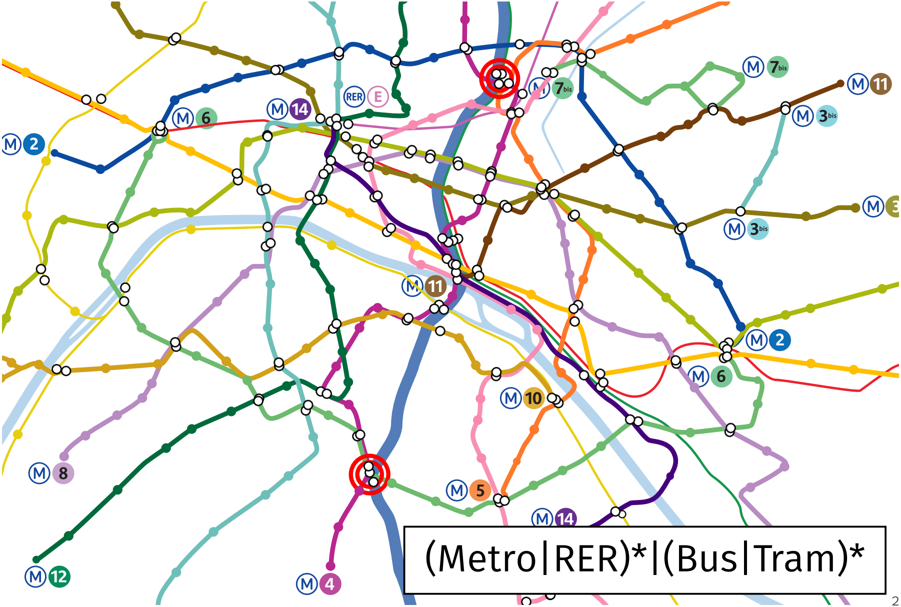

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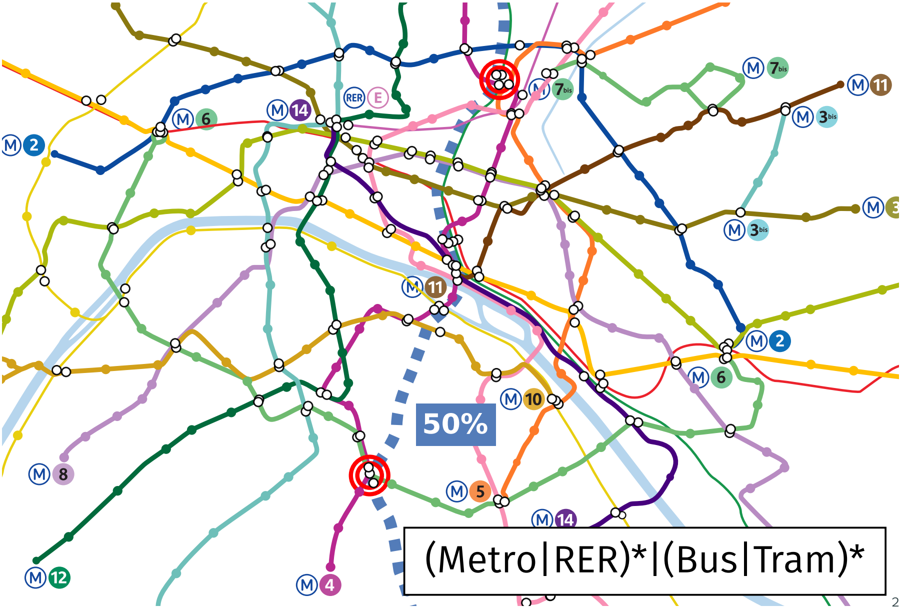

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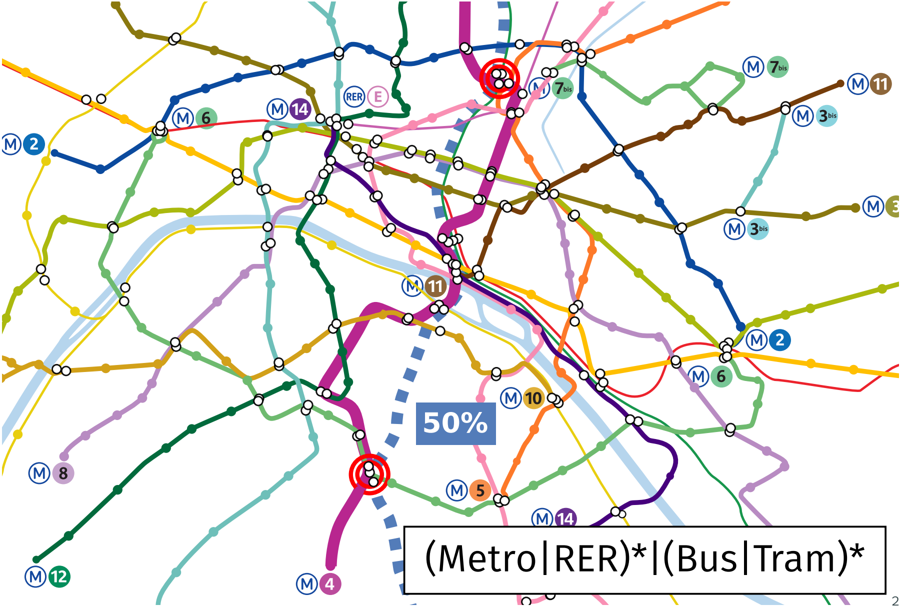

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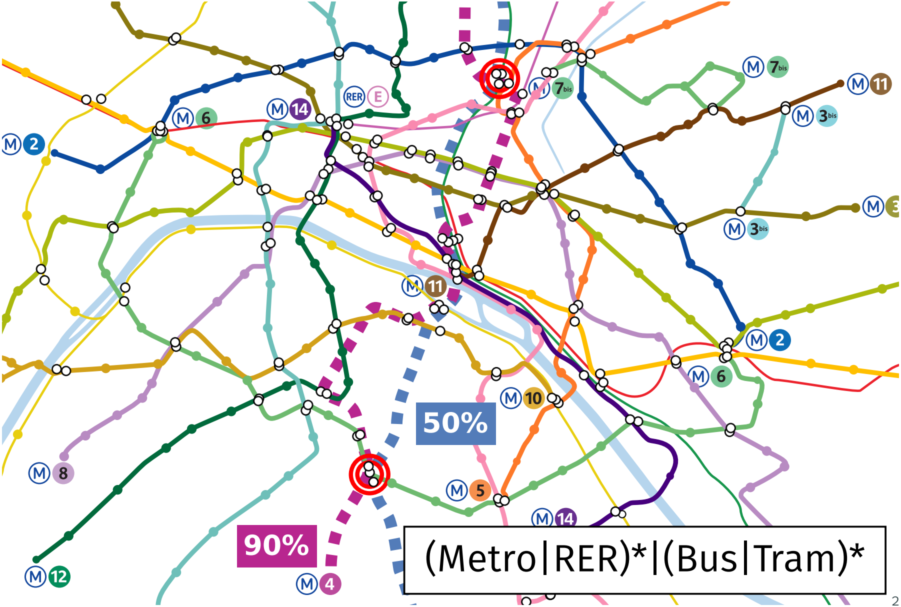

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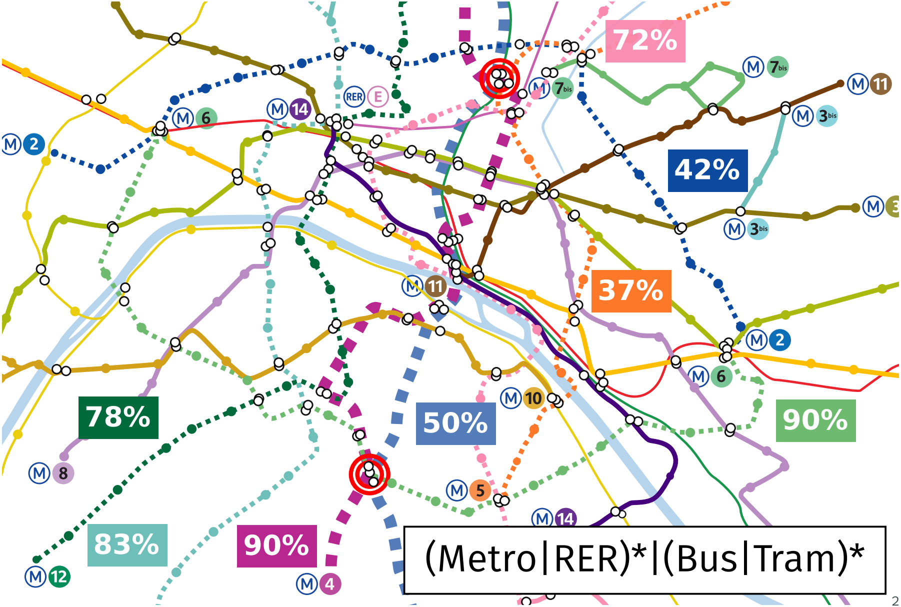

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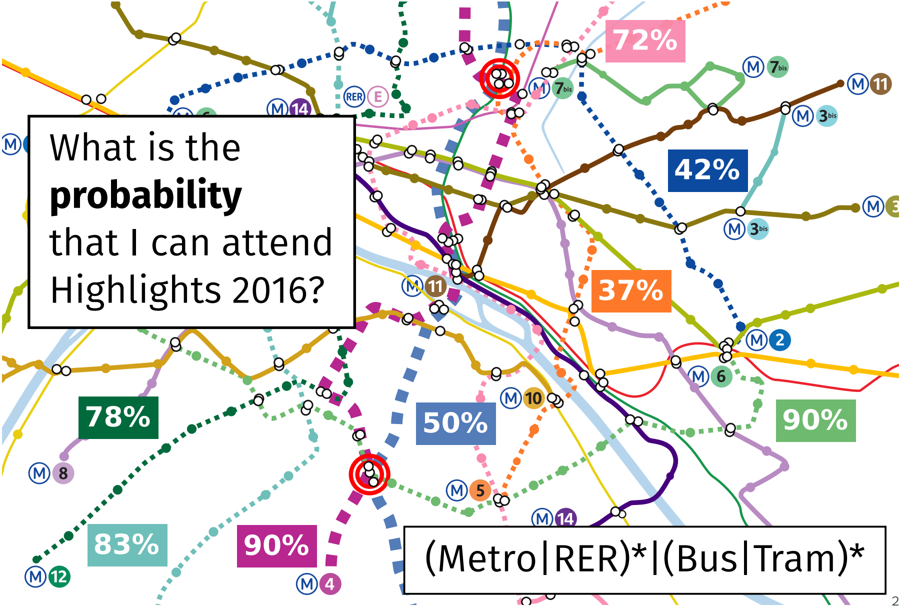

What is the **probability** that I can attend Highlights 2016?

(Metro|RER)*|(Bus|Tram)*

Problem statement

Input:

- Query Q (Metro|RER)*|(Bus|Tram)*

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- Query Q — (Metro|RER)*|(Bus|Tram)*
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- Query *Q* — (Metro|RER)*|(Bus|Tram)*
- Database *D* or graph

- Probabilities on facts or edges — 50%

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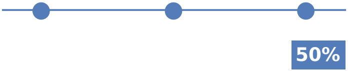

Output: the probability that the query is true under the distribution (assuming independence of all probabilistic events)

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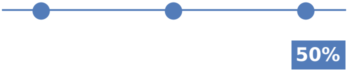

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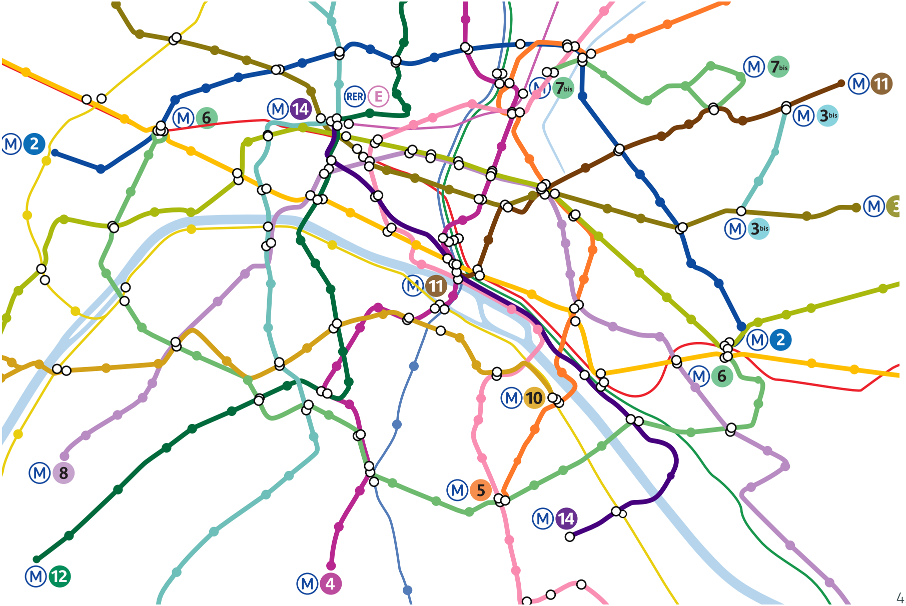
Complexity: already #P-hard in the input database!
(from #MONOTONE-SAT)

Using treewidth to make the problem tractable



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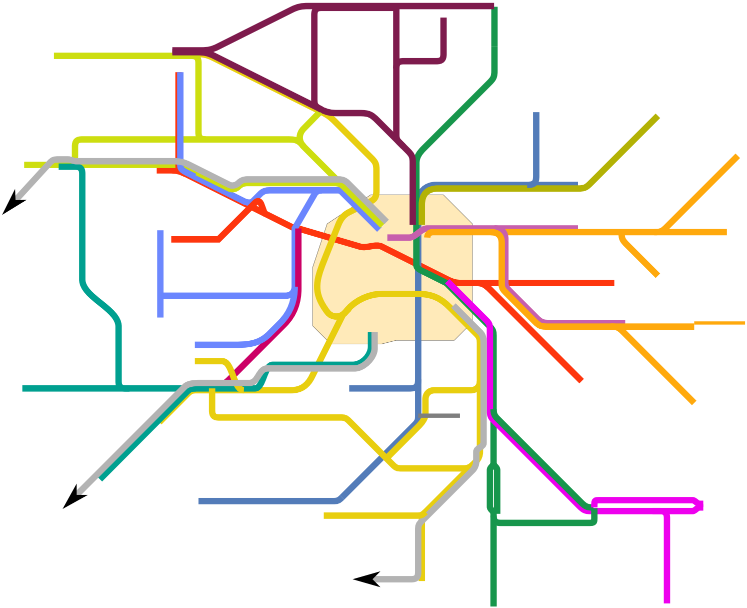

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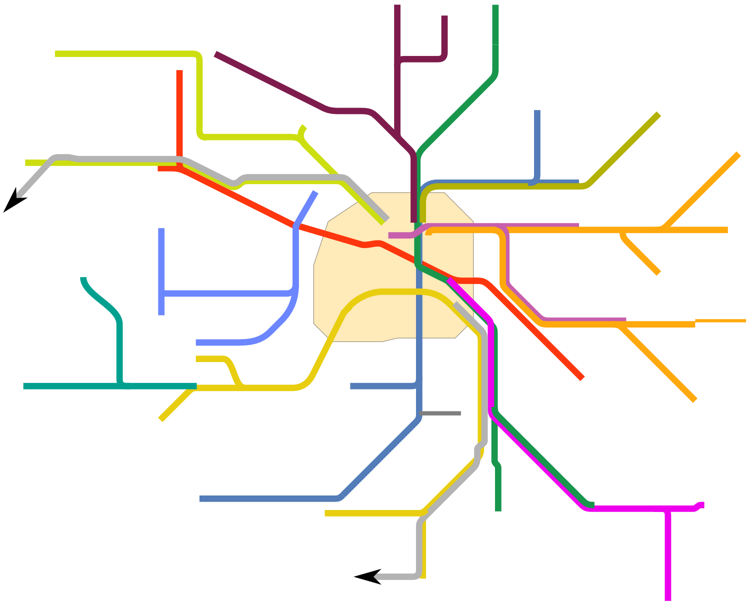

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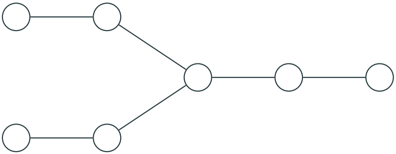

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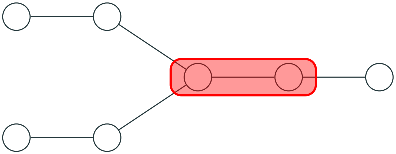

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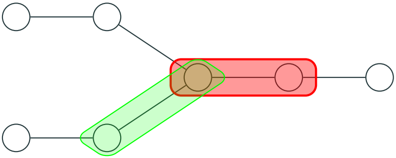

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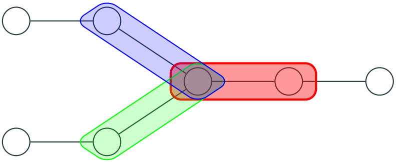

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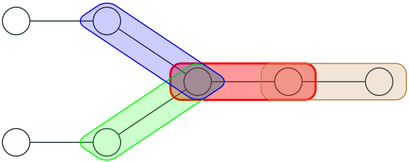

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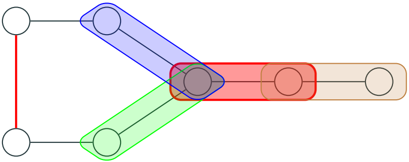

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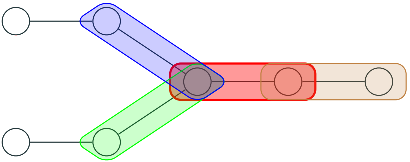

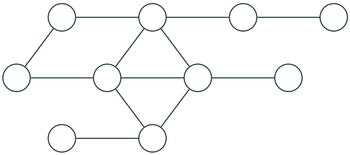

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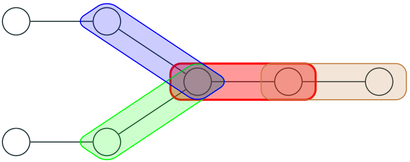

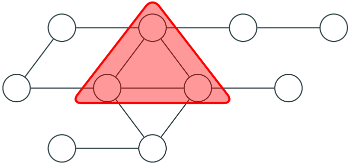

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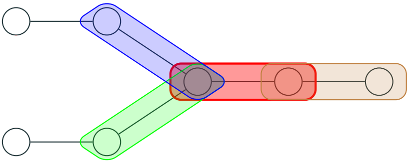

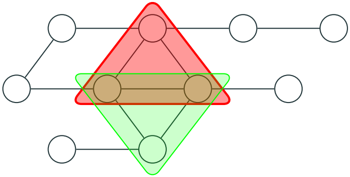

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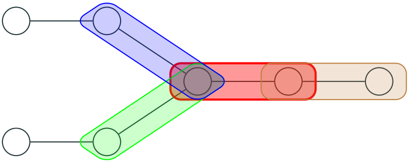

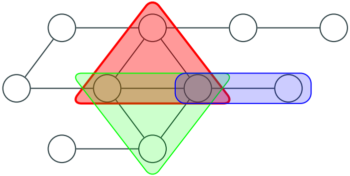

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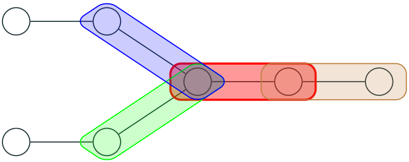

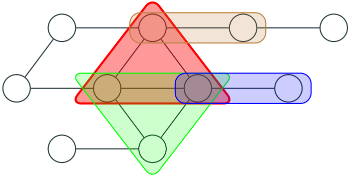

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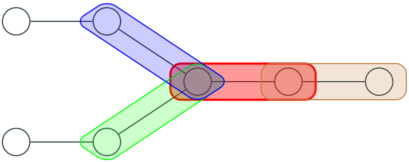

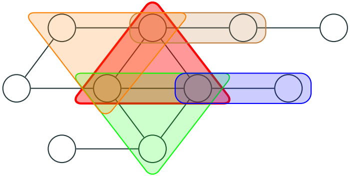

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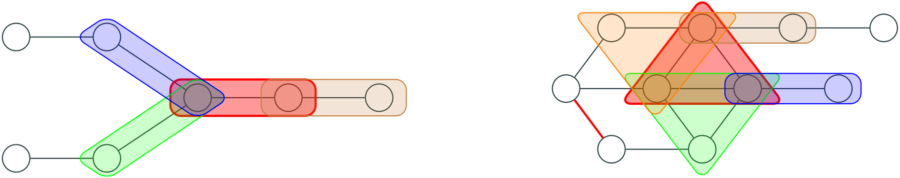

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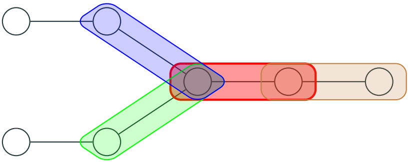

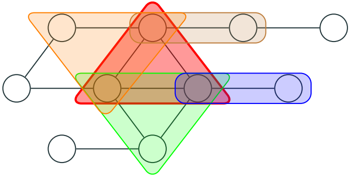

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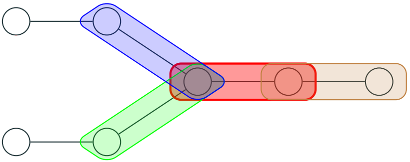

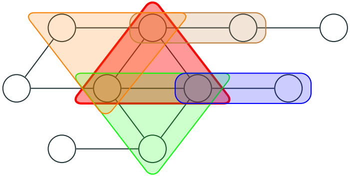

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- Cycles have treewidth 2
- k-cliques and $(k-1)$-grids have treewidth $k-1$

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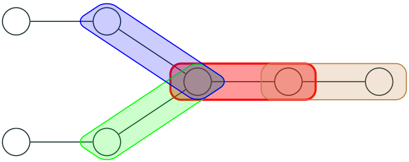

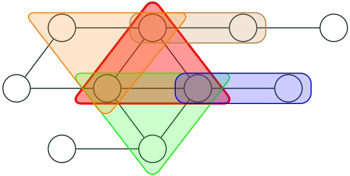

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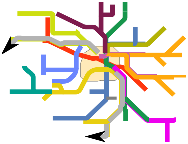
$\rightarrow$ Treelike: the treewidth is bounded by a constant

Tractability on treelike instances

Treelike **data**



MSO query

(RER|metro)*
|(bus|tram)*

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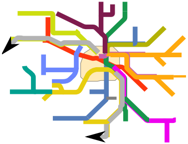

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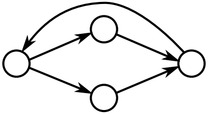
Tree **automaton**



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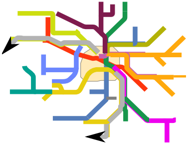
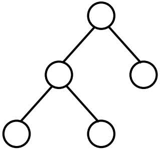


Tree **encoding**



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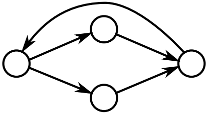

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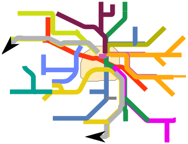

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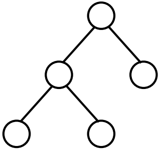

linear
[Courcelle]

Query **answer**

TRUE

MSO query

(RER|metro)*
|(bus|tram)*

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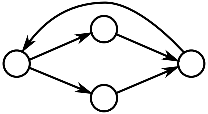

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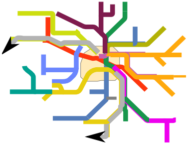

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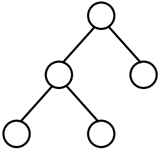

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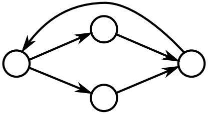

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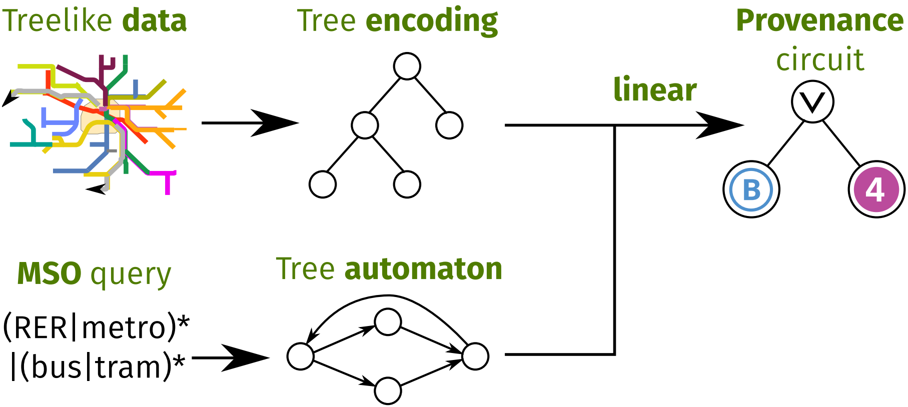

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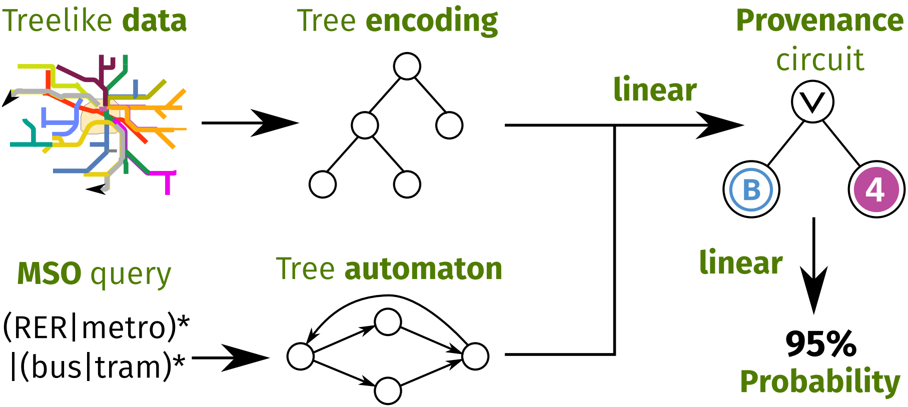

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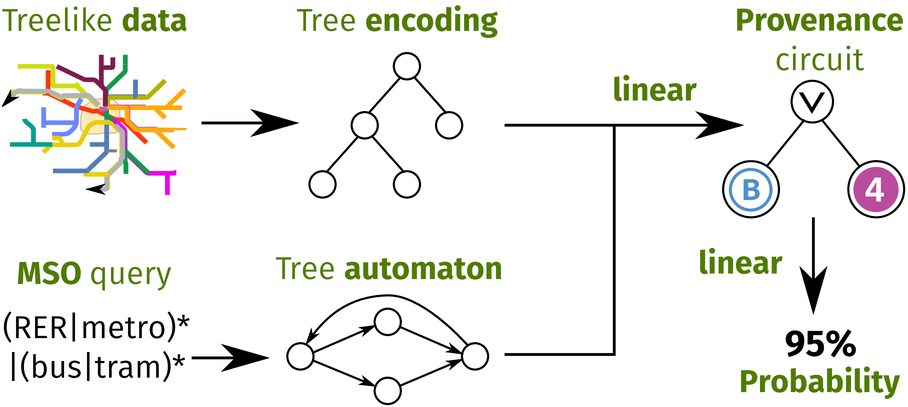

Theorem

For any fixed Boolean MSO query q and $k \in \mathbb{N}$,
given a database D of treewidth $\leq k$ with independent probabilities,
we can compute in linear time the probability that D satisfies q

Lower bound

What can we do for unbounded-treewidth instances?

Lower bound

What can we do for unbounded-treewidth instances? … not much.

Lower bound

Theorem

For any graph signature σ, there is a first-order query $\boldsymbol{q}$ such that for any constructible unbounded-treewidth class $\mathcal{I}$, probability evaluation of $\boldsymbol{q}$ on $\mathcal{I}$ is #P-hard under RP reductions

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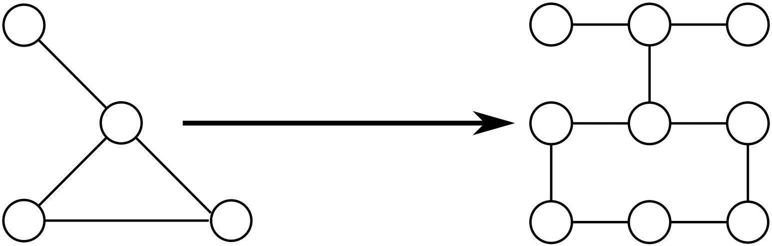

Proof idea: extract instances of a hard problem as topological minors using recent polynomial bounds [Chekuri and Chuzhoy, 2014]

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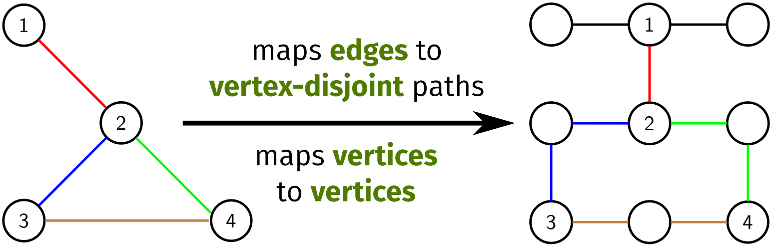

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Future and ongoing work

- Improving the lower bound:
  - From graphs to arbitrary arity databases
  - From FO down to unions of conjunctive queries with $\neq$

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- Complexity in query and database — currently $\Omega\left(2^{2^{\cdot^{\cdot^{\cdot^{2^{|Q|}}}}}} \times |D|\right)$

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Thanks for your attention!

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