Query Evaluation:
Enumeration, Maintenance, Reliability

Soutenance d'habilitation à diriger des recherches

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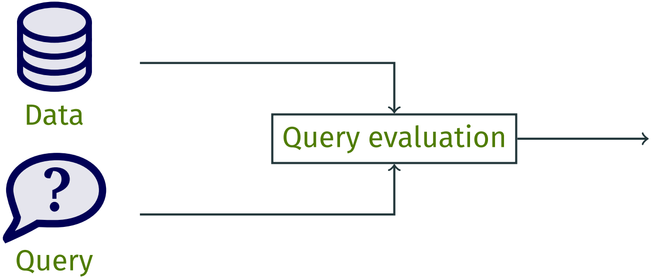
Introduction

Query evaluation

Central question studied in my research: how to efficiently evaluate queries on data?

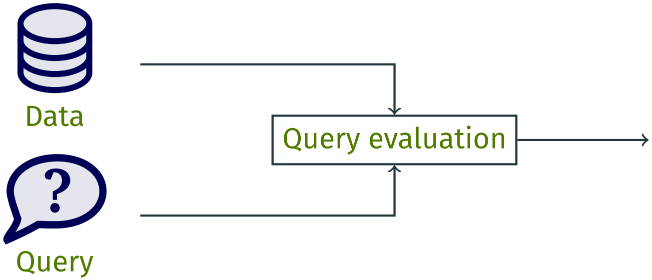


A	B
a_1	b_1
a_2	b_2
a_3	b_3

Results

Query evaluation

Central question studied in my research: how to efficiently evaluate **queries** on **data**?



A	B
a_1	b_1
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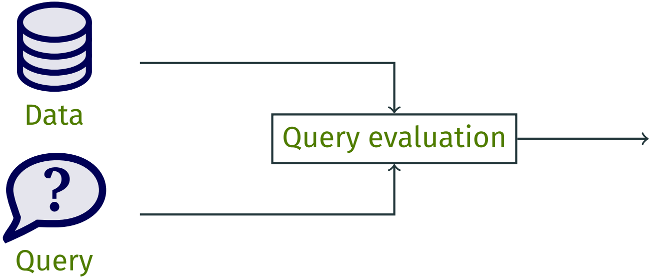
Results

- Measure the **efficiency** of this task

Query evaluation

Central question studied in my research: how to efficiently evaluate **queries** on **data**?



A	B
a_1	b_1
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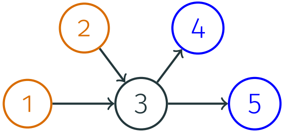
Results

- Measure the **efficiency** of this task
- **Theoretical study** (asymptotic complexity, lower bounds) rather than **practical**

Example: Reachability query



Data: Graph G

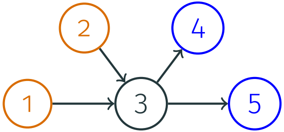
Query $Q(x, y)$: *"Which **orange** nodes x have a directed path to which **blue** nodes y?"*

Query evaluation

x	y
1	4
1	5
2	4
2	5

Results

Example: Reachability query



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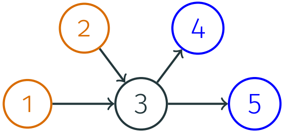
Query $Q(x, y)$: *“Which **orange** nodes **x** have a directed path to which **blue** nodes **y**?”*

Extend to **three tasks**: enumeration, maintenance, and reliability

Enumeration: Producing results in streaming



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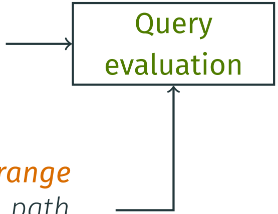

Query $Q(x, y)$: *“Which **orange** nodes **x** have a directed path to which **blue** nodes **y**?”*

- **Usual complexity measure:** time to produce the **entire output**

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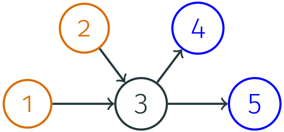

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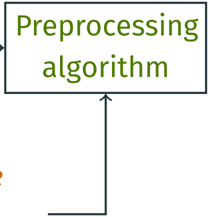

Query $Q(x, y)$: *"Which* ***orange*** *nodes* $\boldsymbol{x}$ *have a directed path to which* ***blue*** *nodes* $\boldsymbol{y}$*?"*

- **Usual complexity measure:** time to produce the **entire output**
- More precise measure: **enumeration algorithms**:

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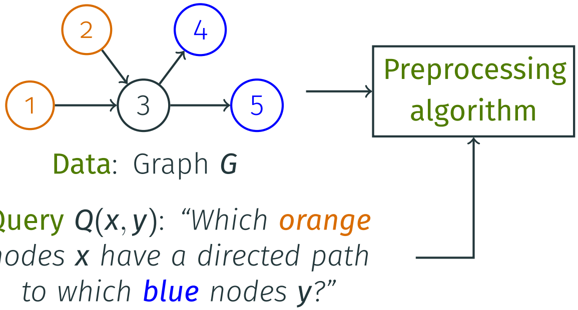

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    - **Preprocessing time**: time to produce **compressed representation**

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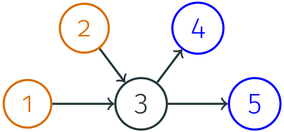

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Preprocessing algorithm

$\{1, 2\} \times \{4, 5\}$

Compressed representation

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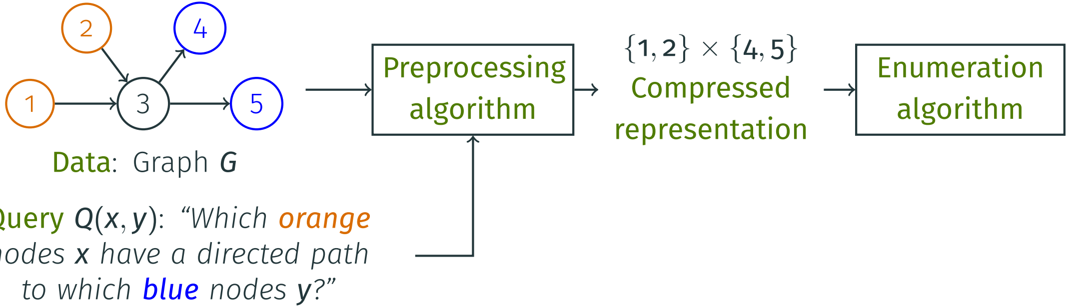

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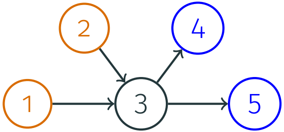

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Preprocessing algorithm → $\{1, 2\} \times \{4, 5\}$ Compressed representation → Enumeration algorithm

x	y

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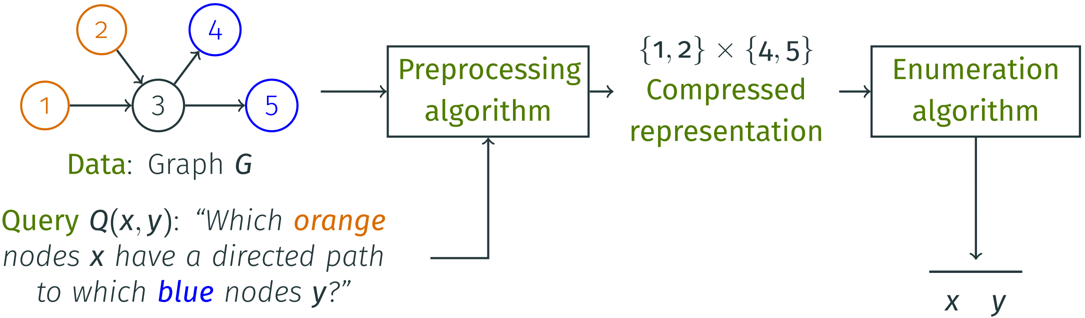
Enumeration: Producing results in streaming



x	y
1	4

Results

- **Usual complexity measure:** time to produce the **entire output**
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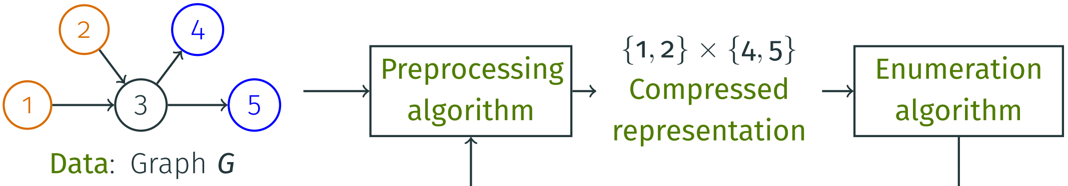
Enumeration: Producing results in streaming



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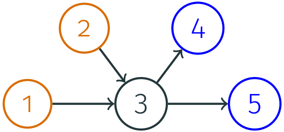

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Preprocessing algorithm → $\{1, 2\} \times \{4, 5\}$ **Compressed representation** → **Enumeration algorithm**

x	y
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2	4

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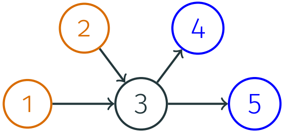

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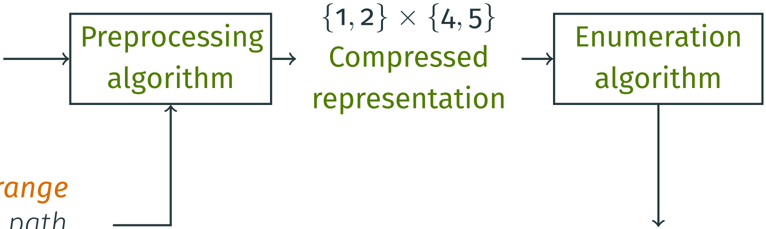

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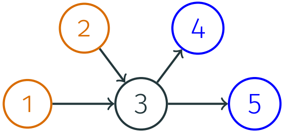

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Results

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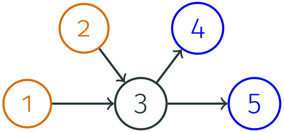
  - **Delay** between each consecutive output

→ Test **existence of a result**, find **some results**, find **all results**...

Maintenance over dynamic data: Adapting to changes



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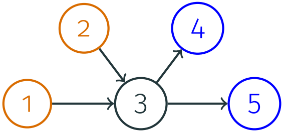
Query $Q(x, y)$: *“Which **orange** nodes **x** have a directed path to which **blue** nodes **y**?”*

Query evaluation

x	y
1	4
1	5
2	4
2	4

Results

Maintenance over dynamic data: Adapting to changes



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Query $Q(x, y)$: *"Which **orange** nodes x have a directed path to which **blue** nodes y?"*

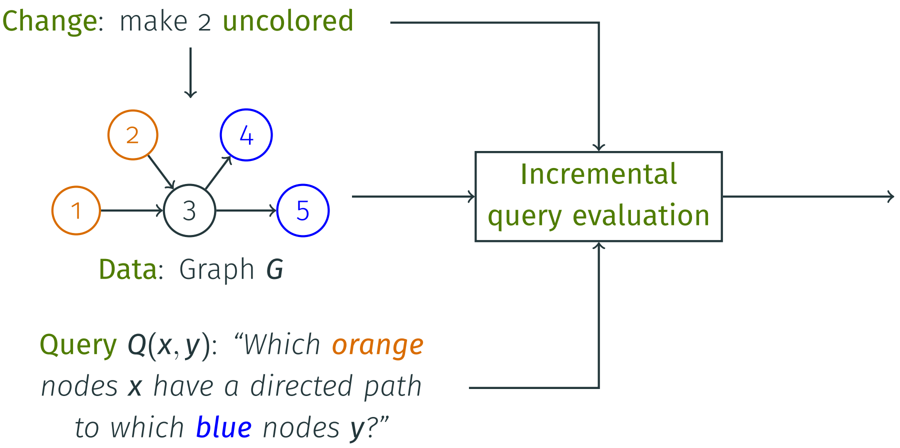
Incremental query evaluation

x	y
1	4
1	5
2	4
2	4

Results

- Whenever the data is **changed**, do not **recompute** the whole result

Maintenance over dynamic data: Adapting to changes



x	y
1	4
1	5
2	4
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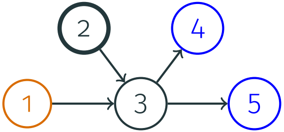
Results

- Whenever the data is changed, do not recompute the whole result

Maintenance over dynamic data: Adapting to changes

Change: make 2 **uncolored**



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1	5
2	4
2	4

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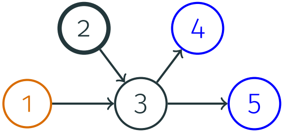

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x	y
1	4
1	5
~~2~~	~~4~~
~~2~~	~~5~~

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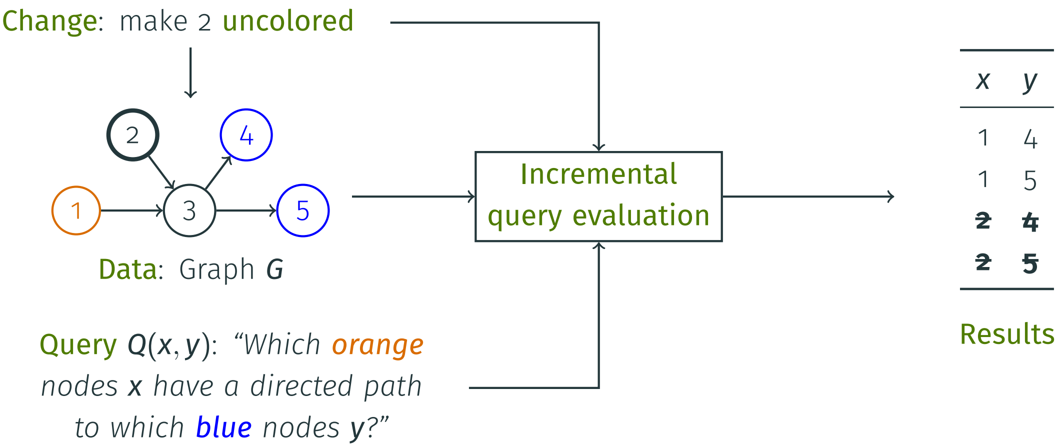
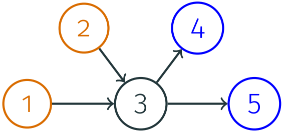
Maintenance over dynamic data: Adapting to changes



- Whenever the data is **changed**, do not **recompute** the whole result
- **Relabeling updates** vs **more general updates**

Reliability: Probabilistic query evaluation



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Query evaluation

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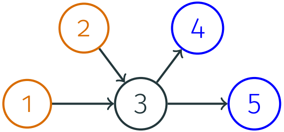
1	5
2	4
2	5

Results

Reliability: Probabilistic query evaluation



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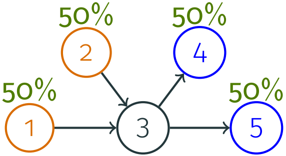
Query $Q(x, y)$: *"Which **orange** nodes x have a directed path to which **blue** nodes y?"*

Probabilistic query evaluation

x	y
1	4
1	5
2	4
2	5

Results

Reliability: Probabilistic query evaluation



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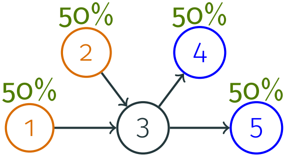
Results

- The color of each node is kept with a given **probability**, assuming **independence**

Reliability: Probabilistic query evaluation



Data: Graph G

Query $Q(x, y)$: *"Which orange nodes* x *have a directed path to which blue nodes* y*?"*

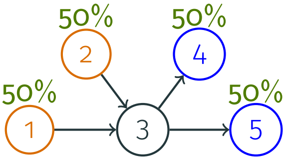
Probabilistic query evaluation

x	y	
1	4	25%
1	5	25%
2	4	25%
2	5	25%

Results

- The color of each node is kept with a given **probability**, assuming **independence**
- We want to know the **probability** of all results

Reliability: Probabilistic query evaluation



Data: Graph G

Probabilistic query evaluation

Query Q: *"Is there an **orange** node having a directed path to a **blue** node?"*

- The color of each node is kept with a given **probability**, assuming **independence**
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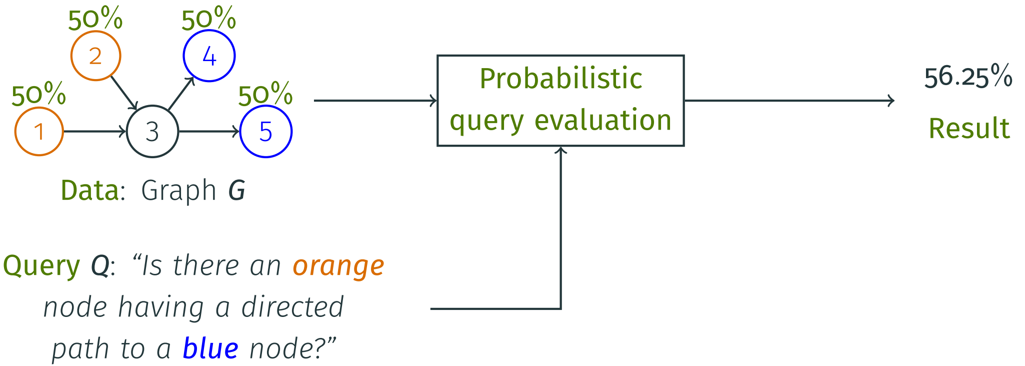
- Here, more interesting: probability of the **Boolean query**

Reliability: Probabilistic query evaluation



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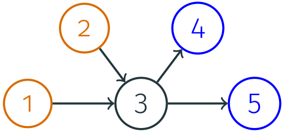
Provenance circuits: A unified approach to these three problems



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Provenance computation

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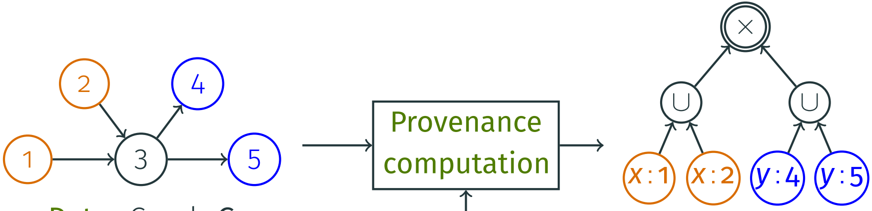

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Provenance circuit

- The **provenance circuit** describes how the query result depends on the data
- Show that it belongs to restricted **circuit classes** from **knowledge compilation**

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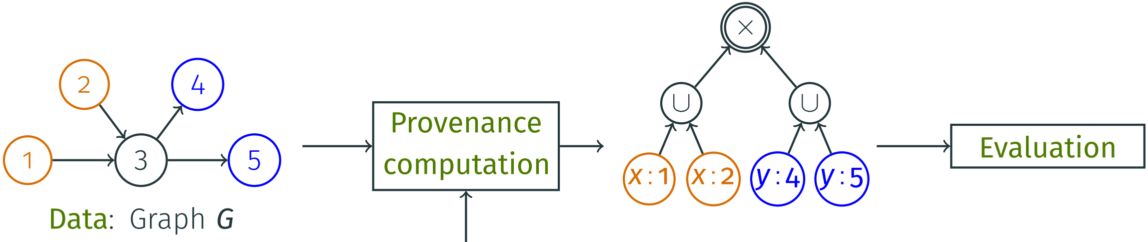

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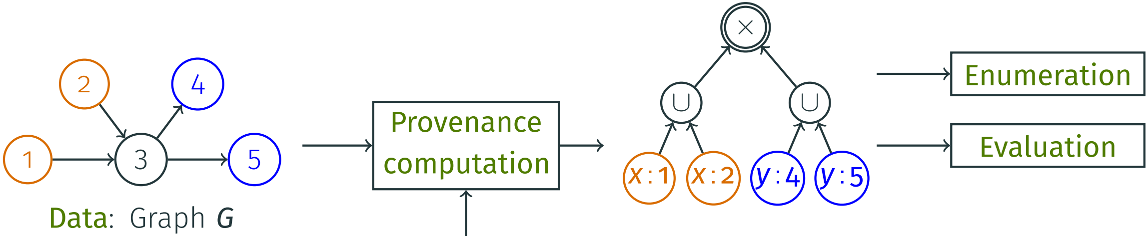

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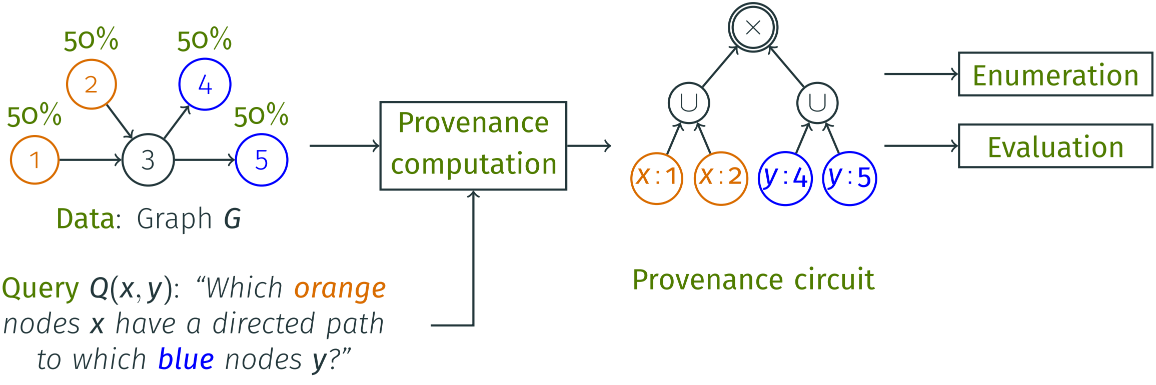

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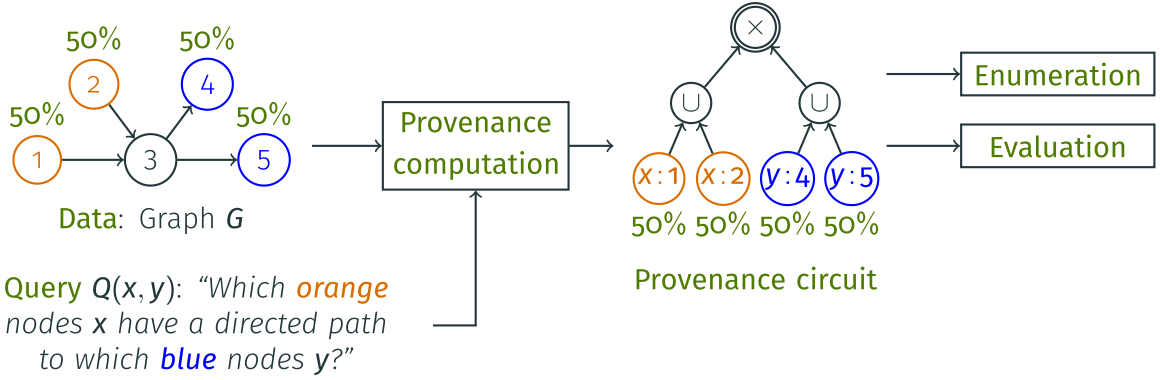

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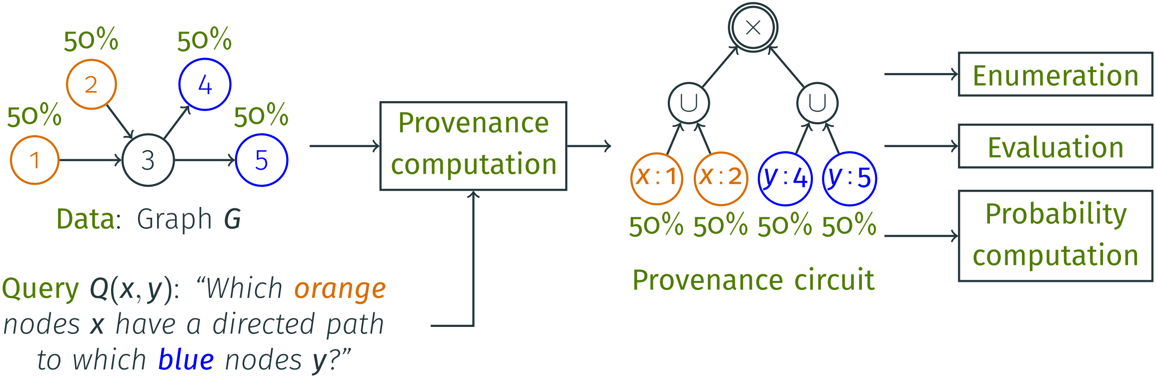

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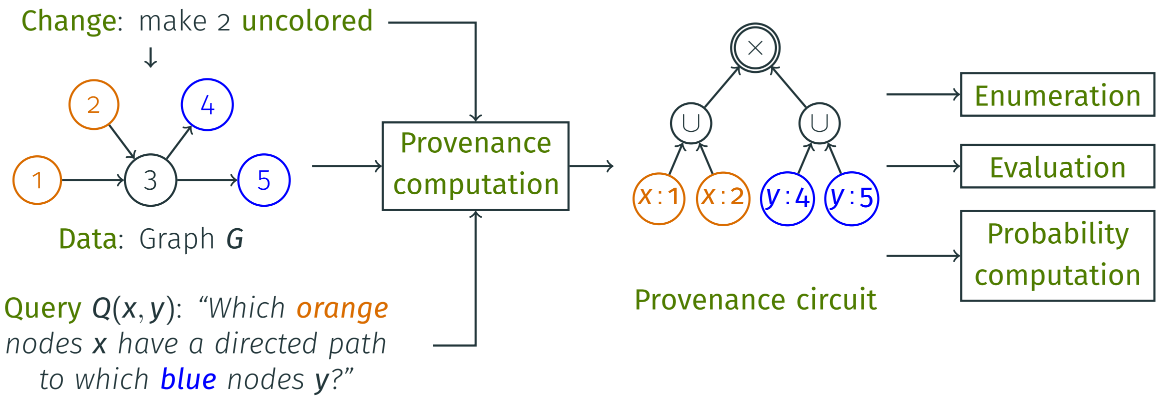

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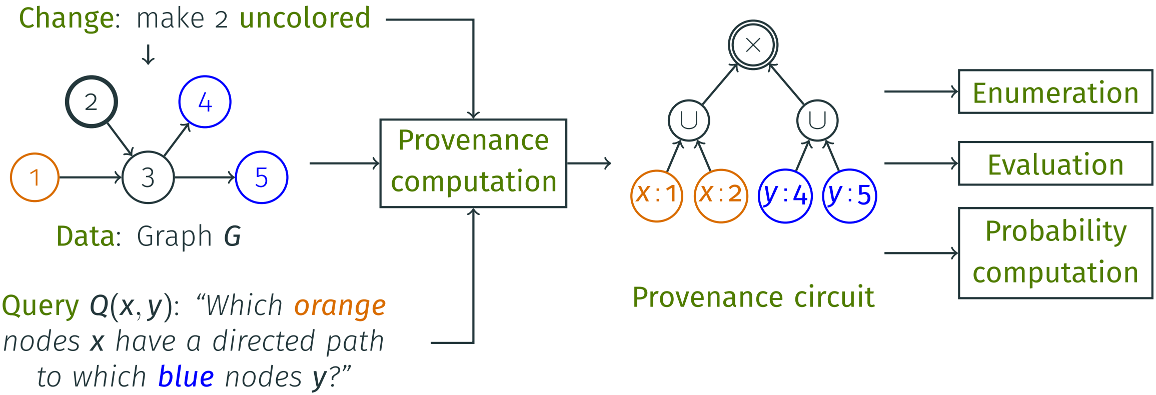

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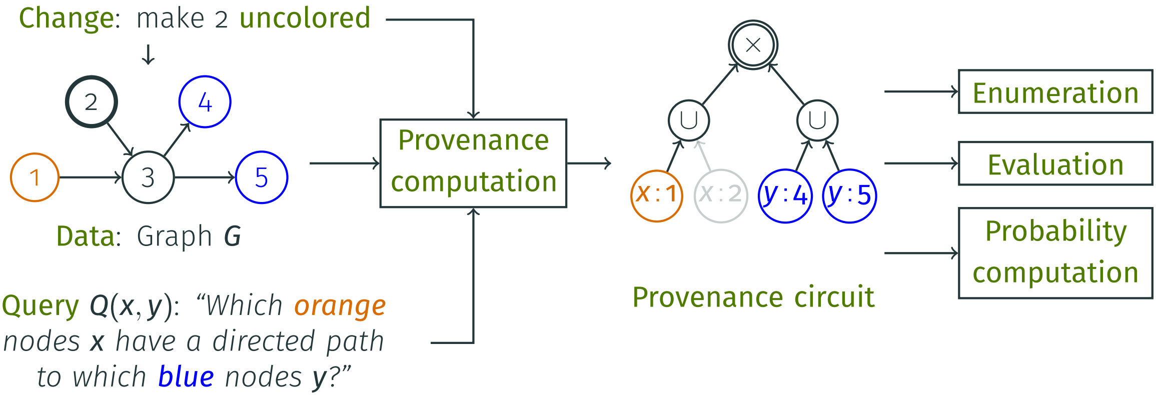

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Roadmap of the presentation

- Present data and query formalisms:
  - $\rightarrow$ Monadic second-order logic (MSO) on words/trees

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- Results on enumeration

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- Results on **enumeration**
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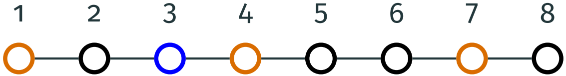
  - $\rightarrow$ **Monadic second-order** logic (MSO) on words/trees
- Results on **enumeration**
- Results on **incremental maintenance**
- Results on **probabilistic query evaluation**

Context

Families of data

- Words:



less expressive

more expressive

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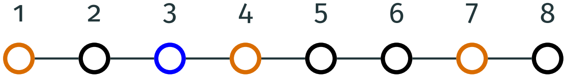
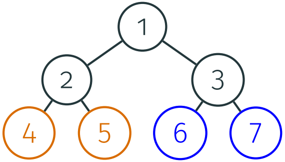
- Words: 

- Trees: 

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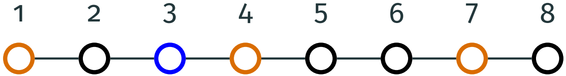
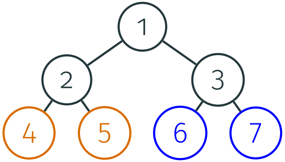
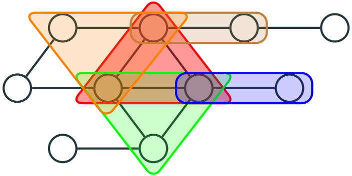
- Words: 

- Trees: 

- Bounded-treewidth graphs: 

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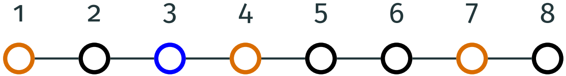

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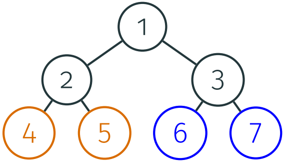

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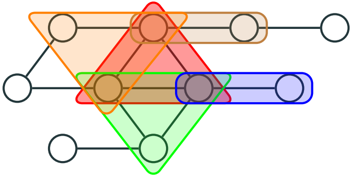
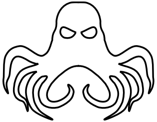


- **Many other classes** of graphs and relational structures:



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Query languages

From **least** to **most** expressive:

- **Conjunctive queries** (CQs): find a pattern
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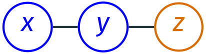
  - $Q(x, y) : \exists z$ 

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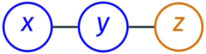
  - $Q(x, y)$: $\exists z$ 

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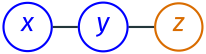
  - $Q(x, y) : \exists z$ 

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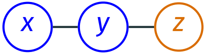
  - $Q(x, y) : \exists z$ 

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- First-order logic (FO):

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- Monadic second-order logic (MSO): extend FO with quantification over sets
  - Equivalent to finite automata on words, trees, tree encodings

Enumeration

Word automata with captures

On words, MSO queries are equivalent to automata

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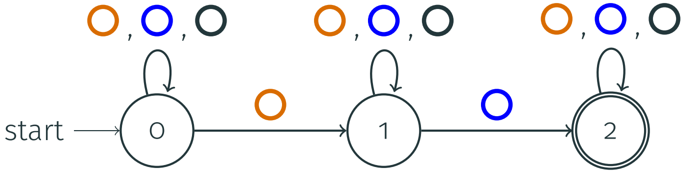

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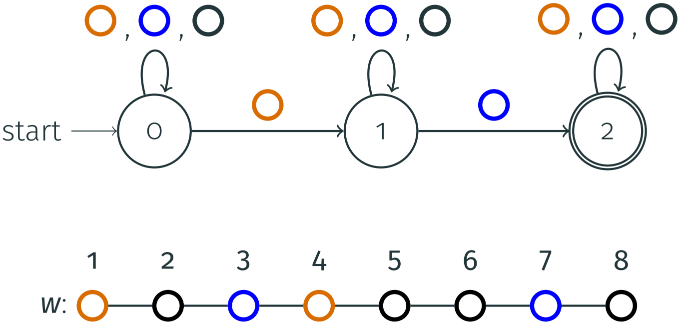

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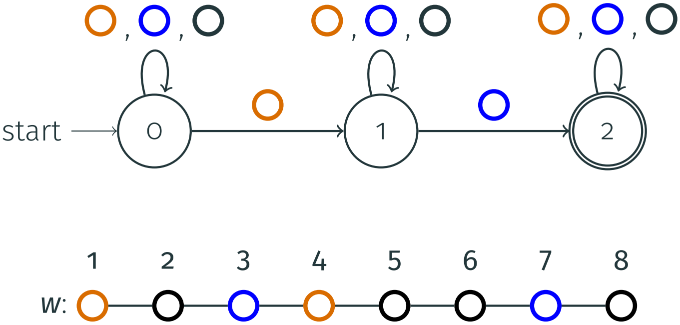

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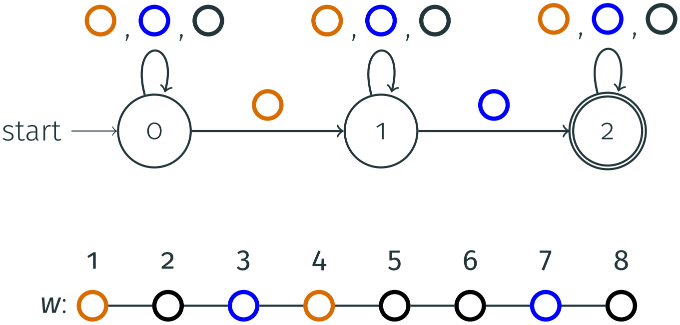

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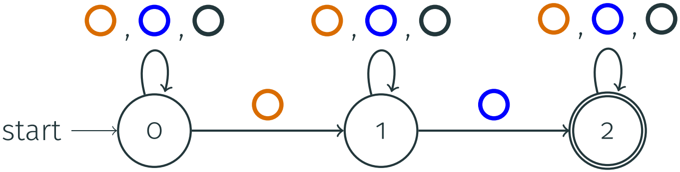

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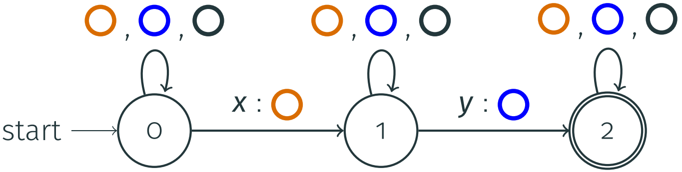

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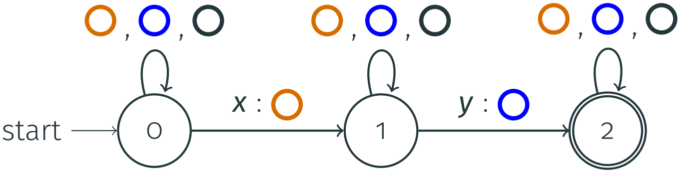
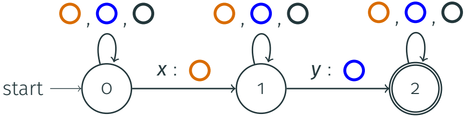


Results: $(x\!:\!1, y\!:\!3), (x\!:\!1, y\!:\!7), (x\!:\!4, y\!:\!7)$

Provenance circuit computation: Product construction



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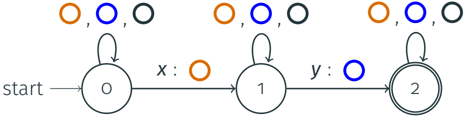

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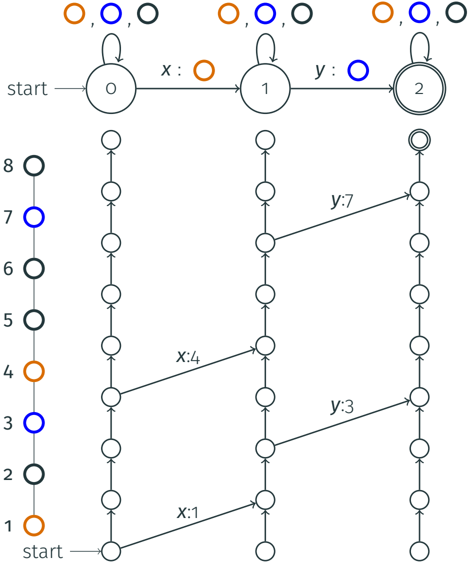

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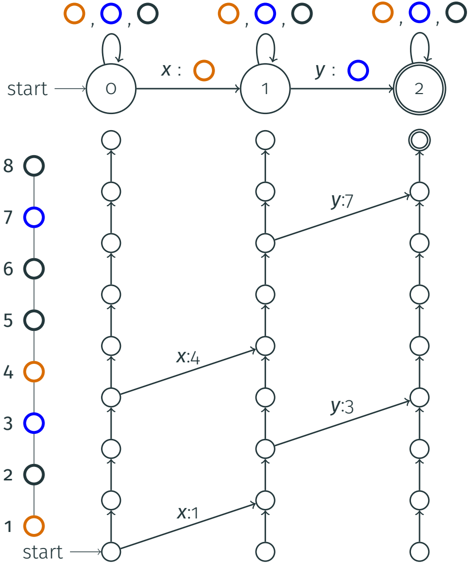

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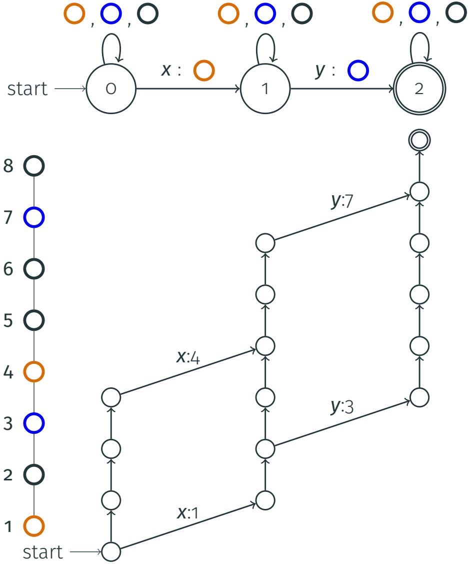

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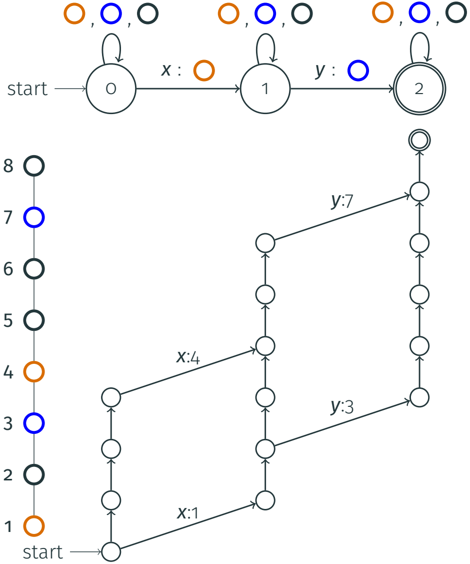

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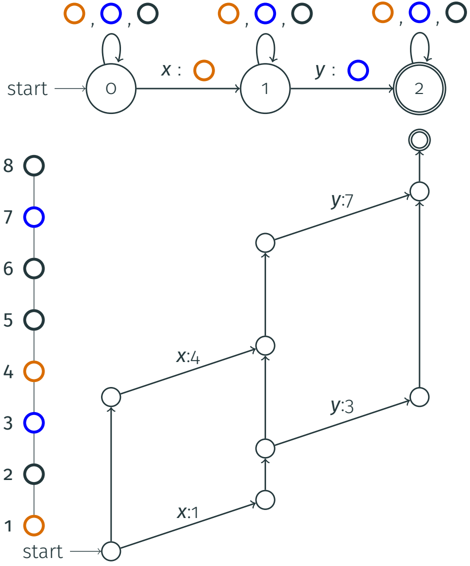

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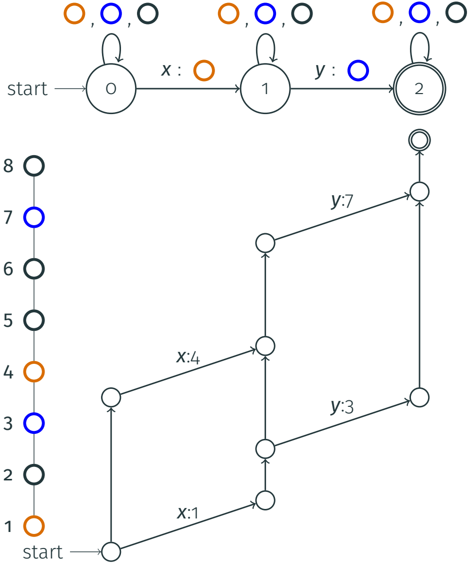

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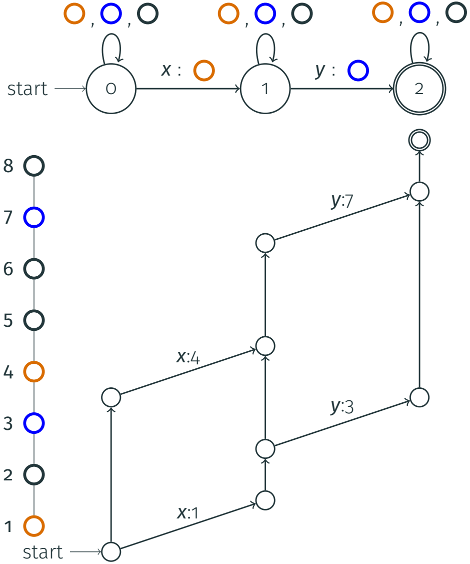

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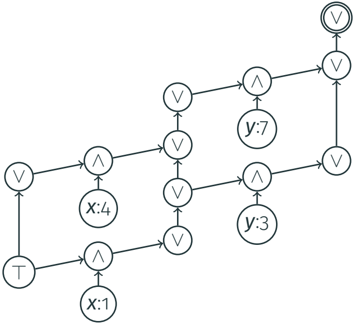
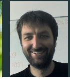


Enumerating the results of MSO queries



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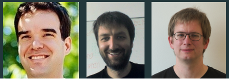

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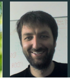


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Given an automaton with captures A with constant number of variables, given a word w, we can enumerate the results of A on w with preprocessing $O(Poly(|A|) \times |w|)$ and delay $O(Poly(|A|))$.

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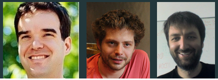
Known result [Bagan, 2006, Kazana and Segoufin, 2013] but polynomial dependency in A

Enumeration for general circuits



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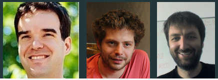

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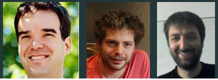

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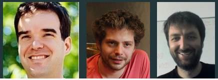

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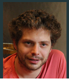


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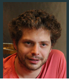


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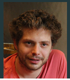
Theorem (ICALP'17; with Bourhis, Jachiet, Mengel)

*Given a d-SDNNF C and a v-tree that structures C, we can enumerate the satisfying assignments of C with **linear preprocessing** and **output-linear delay**.*

Beyond regular languages

Generalize automata with captures into annotation context-free grammars

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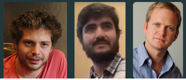

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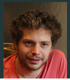

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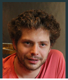

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Given an unambiguous annotation grammar $\mathbf{G}$ *and word* $\mathbf{w}$*, we can enumerate the results of* $\mathbf{G}$ *on* $\mathbf{w}$ *with preprocessing* $O(|G| \times |w|^3)$ *and* ***output-linear*** *delay*

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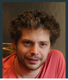

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Better preprocessing time for restricted grammar classes

Maintenance

Maintenance for MSO enumeration on trees

We use provenance circuits for automata on words and trees

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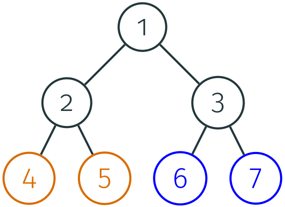

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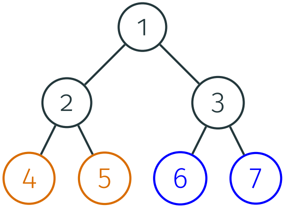

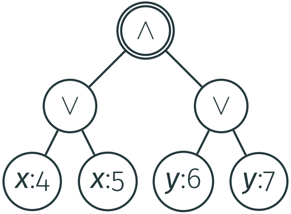

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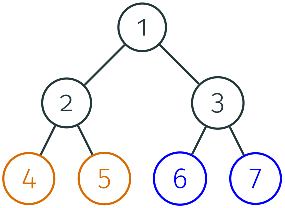

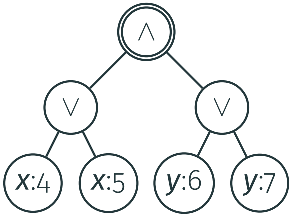

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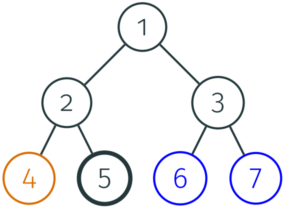

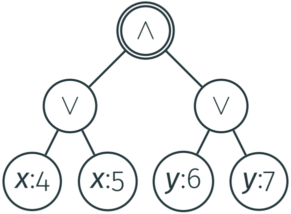

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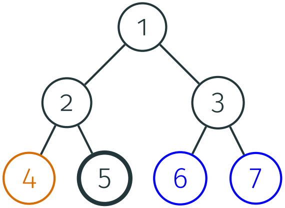

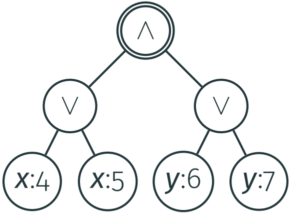

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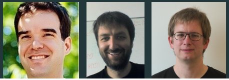
Maintaining MSO enumeration structures under relabelings



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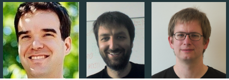

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Same for updates that **change the tree structure** (PODS'19; with Bourhis, Mengel, Niewerth) assuming we have an algorithm to **keep the tree balanced**

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- The update time is $O(\log n)$ and there is a lower bound of $\Omega(\log n / \log\log n)$
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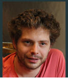
$\rightarrow$ For a fixed language L, given a word w of length n, what is the **best update time** to maintain membership of w to L under relabelings?

Incremental maintenance for regular word languages



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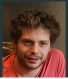

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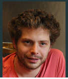 

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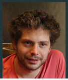


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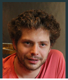

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- **QLZG**: "in all submonoids of the stable semigroup, all subgroup elements are central"
  $\to$ Commutative languages, finite languages, disjoint shuffles, modulo, nearby positions...
- **QSG**: "the stable semigroup satisfies the equation $x^{\omega+1}yx^{\omega} = x^{\omega}yx^{\omega+1}$"
  $\to$ Aperiodic languages, tame combinations of aperiodic and commutative languages...

Reliability

Probabilistic query evaluation (PQE)

Tuple-independent probabilistic data (TID): facts carry independent probabilities

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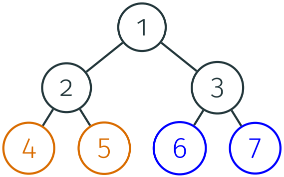

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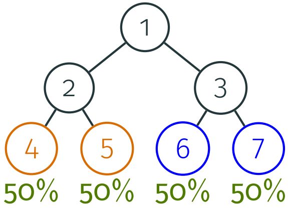

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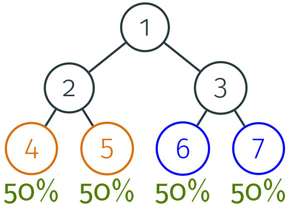

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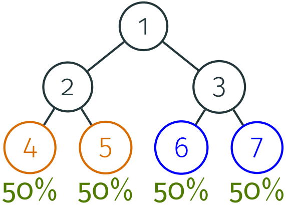

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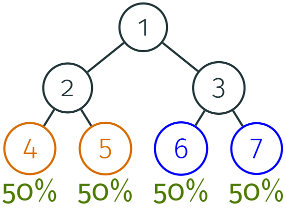

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- Known dichotomy for PQE on **unions of conjunctive queries** (on arbitrary data) [Dalvi and Suciu, 2013]: the problem is either **#P-hard** or **in PTIME**

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For MSO queries on trees, we can solve PQE using d-SDNNF provenance circuits!

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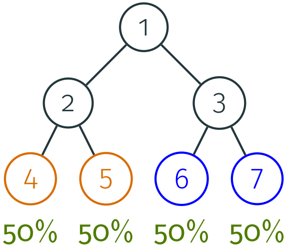

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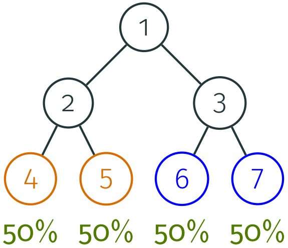

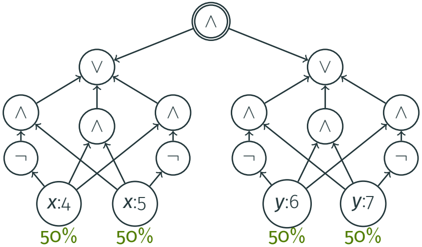

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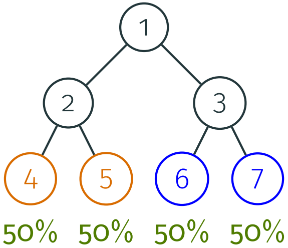

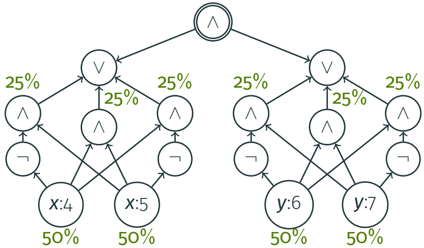

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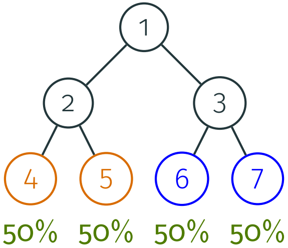

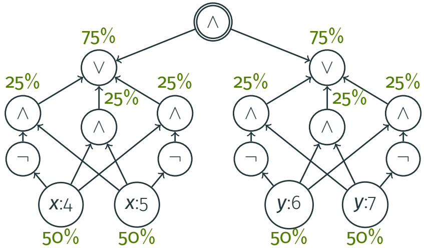

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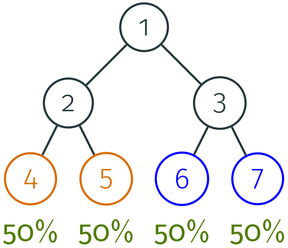

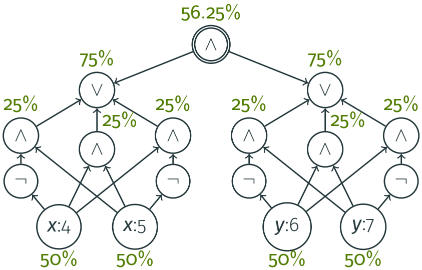

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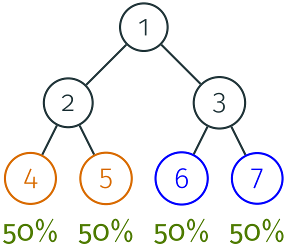

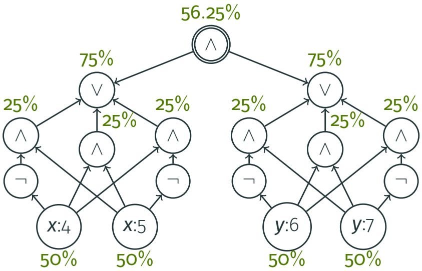

- Probability of $\wedge$ is the **product** of the probabilities (uses decomposability)
- Probability of $\vee$ is the **sum** of the probabilities (uses determinism)

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  - We show the same for all **unbounded homomorphism-closed queries** on graphs

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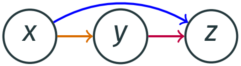
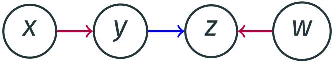
Uses polynomial bounds on the **grid minor theorem** [Chekuri and Chuzhoy, 2016]

Intractability in the uniform setting

A conjunctive query is self-join-free if all edge colors are different

E.g., 

but not 

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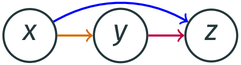
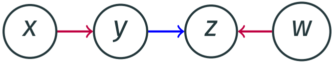
E.g., 

but not 

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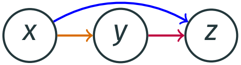
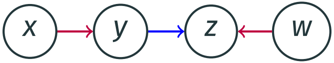
E.g., 

but not 

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Theorem (ICDT'21, LMCS; with Kimelfeld)

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Conclusion

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Thanks for your attention!

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