Regular Languages: Some New Problems and Algorithms

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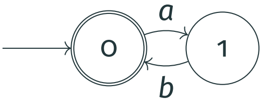
Joint work with: Corentin Barloy, Pierre Bourhis, İsmail İlkan Ceylan, Sven Dziadek, Octave Gaspard, Wolfgang Gatterbauer, Paweł Gawrychowski, Benoît Groz, Santiago Guzman Pro, Louis Jachiet, Sébastien Labbé, Neha Makhija, Kuldeep Meel, Stefan Mengel, Mikaël Monet, Martín Muñoz, Matthias Niewerth, Charles Paperman, Paul Raphaël, Tina Ringleb, Cristian Riveros, Sylvain Salvati, Luc Segoufin, Tim Van Bremen, Nicole Wein

Regular languages

Regular languages are a robust framework for constant-memory computation:

- Correspond to regular expressions, e.g., $(ab^*c|d)^*$
- Correspond to finite automata
  - Can be deterministic (DFA) or nondeterministic (NFA)



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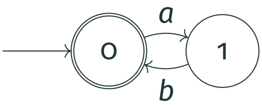

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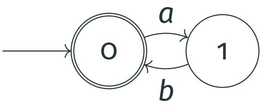

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The computational complexity can be studied in two settings:

- Data complexity: the language L is fixed and the input is w
- Combined complexity: the input is both w and some representation of L

Membership problem

What is the complexity of membership?

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  - $\rightarrow$ $O(1)$ possible for some languages [Aaronson et al., 2019]

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  → Better bounds possible for specific language families, e.g., subword/superword-closed languages [A., Manea, Ringleb, Schmid, 2025]

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  - $\rightarrow$ Better bounds possible for **specific language families**, e.g., subword/superword-closed languages [A., Manea, Ringleb, Schmid, 2025]

This talk: study **extensions** of the membership problem

Roadmap

I will present four extensions of regular language membership…

- Membership for partial words and probabilistic words
- Incremental maintenance of membership
- Enumeration of word factors (beyond Boolean queries)
- Regular language problems on graphs

… and will sketch more directions at the end.

Partial and Probabilistic Words

Partial and probabilistic words

- Partial word: word with holes
  - → e.g., $a\textvisiblespace a\textvisiblespace$ on alphabet $\Sigma = \{a, b\}$
- Possible completions: filling the holes with letters
  - → here, 4 possible completions
- Partial membership to a fixed language L: given a partial word w, how many completions of w belong to L?

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Generalization: probabilistic words

- Each hole specifies a probability distribution over the alphabet
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Given a probabilistic word w, what is the total probability of the completions of w that belong to L?

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In combined complexity:

- It is #P-hard in general but can be approximated (FPRAS) [Arenas et al., 2021]
- It is in PTIME when the input is a DFA or UFA
- Also PTIME for k-ambiguous automata? (following [Stearns and Hunt III, 1985])

Generalizations to context-free grammars: [A., Monet, Raphaël, Salvati, 2025]

Dynamic Membership

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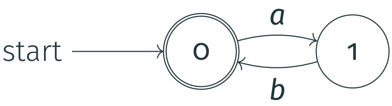
Theorem (General result via balanced trees)

For any regular language L recognized by an NFA A, given a word w, we can maintain dynamic membership of w to L under substitution updates in $O(\mathit{Poly}(|A|) \times \log |w|)$ per update.

Proof sketch of the $O(\log n)$ algorithm

Fix the language $L = (ab)^*$:



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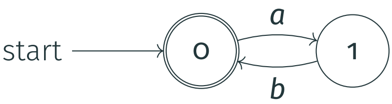

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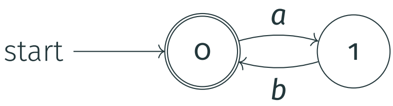

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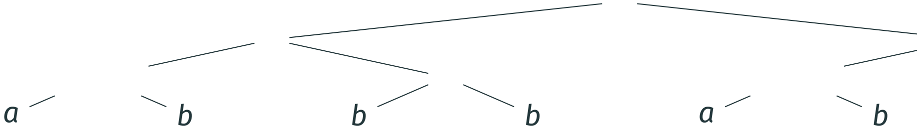

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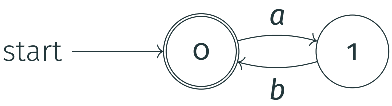

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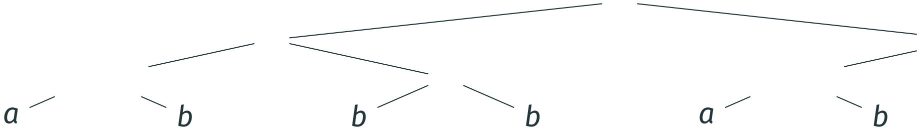

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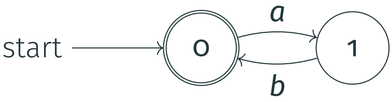

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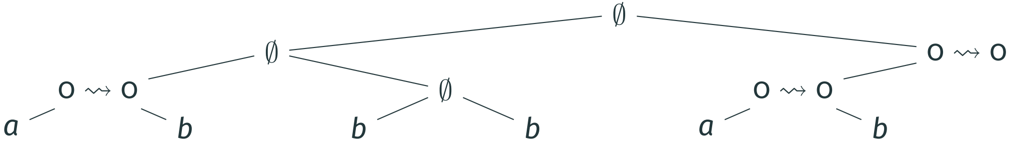

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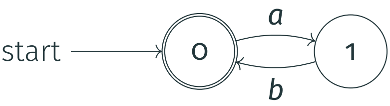

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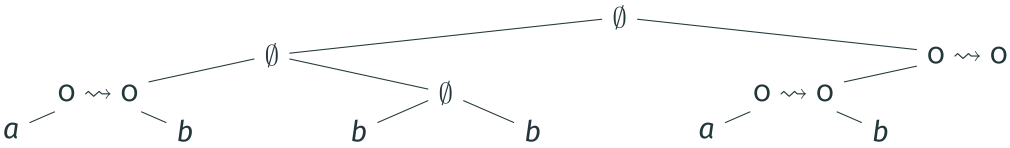

- The **tree root** describes if $w \in L$
- We can update the tree for each substitution **in $O(\log n)$**
- Can be improved to **$O(\log n / \log \log n)$**

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Question:

What is the data complexity of dynamic membership, depending on the fixed regular language L?

Summary of our results [A., Jachiet, Paperman, 2021]

QLZG: in $O(1)$

QSG: in $O(\log \log n)$

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All: in $\Theta(\log n / \log \log n)$

- We identify a class **QLZG** of regular languages:
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Generalizations to **trees**: [A., Barloy, Jachiet, Paperman, 2025] and Labbé's PhD

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There can be $\Theta(n^2)$ results, so we want an **output-sensitive algorithm**

- **Preprocessing:** worst-case running time before the first result
- **Delay** worst-case time between results

Complexity of factor enumeration

Tractable in data complexity or in combined complexity with a DFA:

Theorem (follows from [Florenzano et al., 2018])

Given a word w and a DFA A, we can enumerate the factors in w that match A with preprocessing $O(|w| \times |A|)$ and delay $O(1)$.

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Beyond factor listing: generalizes to automata with capture variables

Generalizations to tree automata [A., Bourhis, Mengel, Niewerth, 2019b] and context-free grammars: [A., Jachiet, Muñoz, Riveros, 2022]

Regular Languages on Graphs

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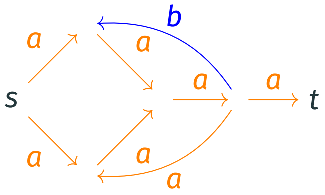

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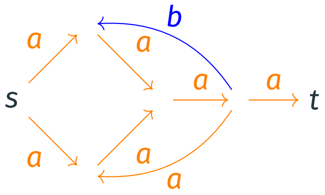

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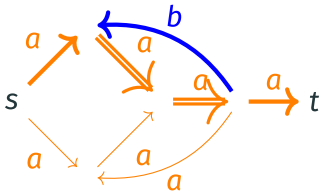

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  - Note: w is not necessarily a simple path

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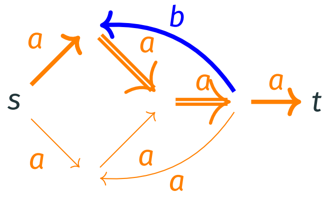

Evaluating RPQs on graph databases

Consider the generalization of the membership problem (in data complexity)

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Many possible generalizations of membership on graph databases:

→ I will focus on one: the smallest witness problem

→ *What is the smallest sub-database of D that satisfies RPQ_L?*

Smallest Witness for RPQs

- Decision problem SW_L for a fixed regular language L:
  - Input: graph database D, vertices s and t, integer $k \in \mathbb{N}$

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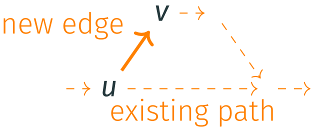

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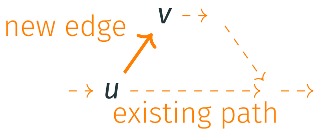

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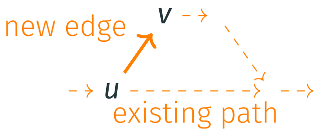

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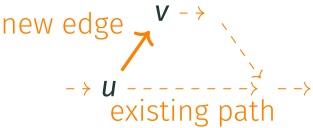

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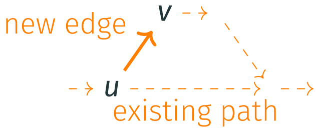

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- Bounded-cutwidth subgraphs can be found by **dynamic programming**

Other Directions and Future Work

Many other directions (talk to me to know more!)

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Thanks for your attention!

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