Efficient Enumeration of Query Answers via Circuits

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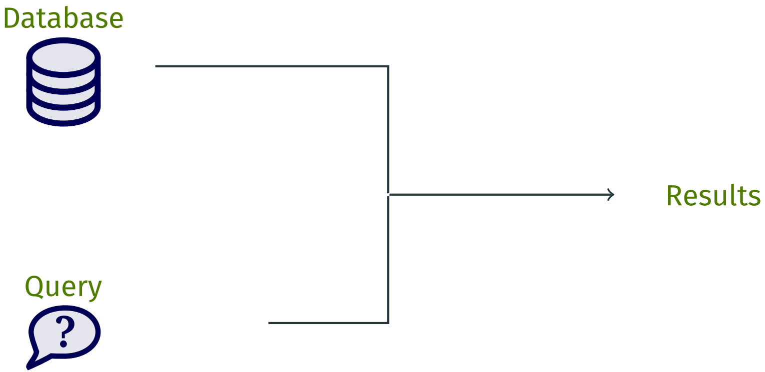
Inria Lille

Query evaluation

Central problem in database theory and practice: query evaluation



Query evaluation

Central problem in database theory and practice: query evaluation

Database

Follows

from	**to**
Alice	Bob
Bob	Carol
Bob	Dave
Carol	Eve

Query

Results

Query evaluation

Central problem in database theory and practice: **query evaluation**

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"Find all pairs of users x and z such that x follows someone who follows z"

→ **Results**

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$$Q(x, z) : \exists y \; F(x, y) \wedge F(y, z)$$

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Combined complexity and data complexity

How to study the computational complexity of query evaluation?

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- Problem of query evaluation (QE)

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  - **Input 1:** query Q
  - **Input 2:** database D

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  $\rightarrow$ Motivation: the data is usually much larger than the query

Data complexity for large output size

- Consider the query Q: *"Find all users* x*,* y*, and* z *such that* x *follows* y *and* y *follows* z*"*

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Step 1:
Preprocessing
in $O(|D|)$

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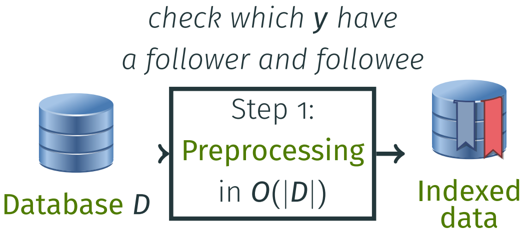

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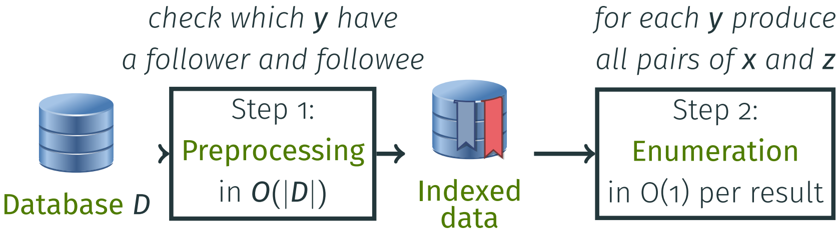

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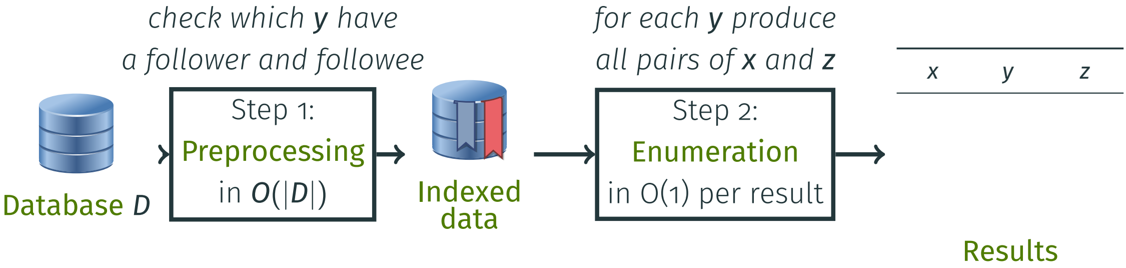

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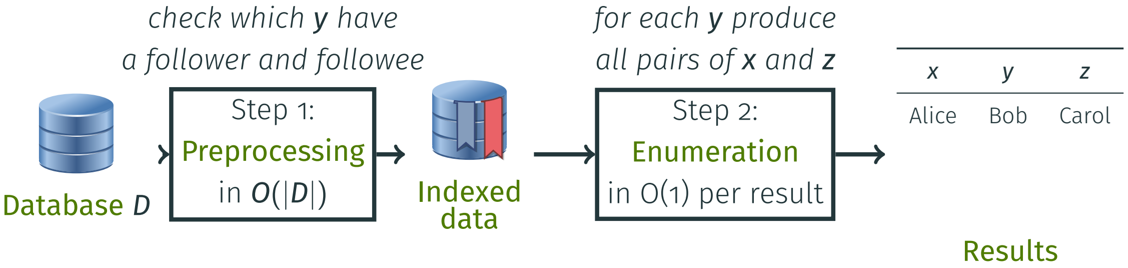

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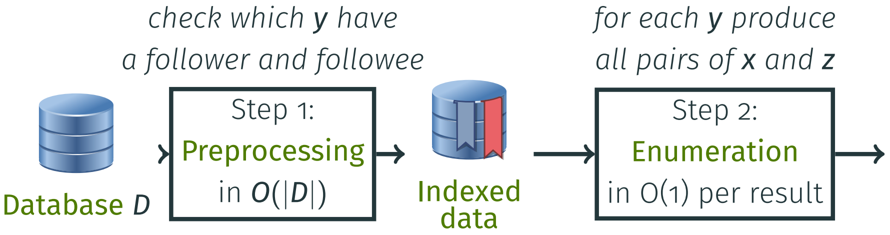

x	y	z
Alice	Bob	Carol
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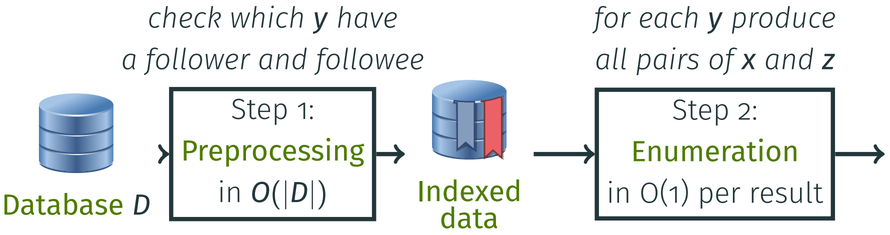

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check which y have a follower and followee

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Database D → Step 1: **Preprocessing** in $O(|D|)$ →

Indexed data → Step 2: **Enumeration** in O(1) per result →

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→ Tests **if there is an answer** in time $O(|D|)$

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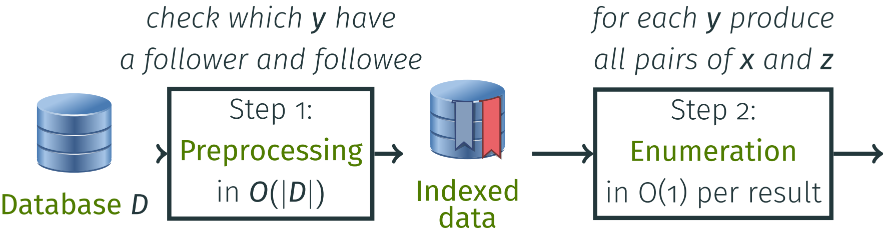

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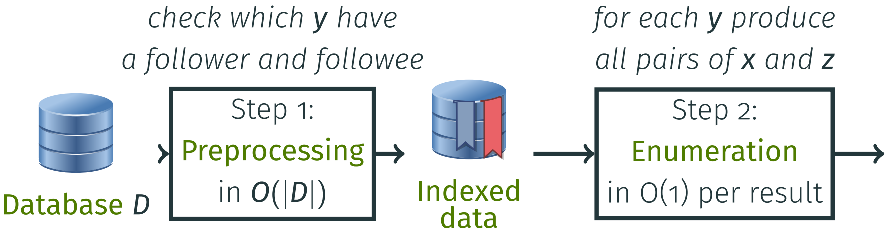

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$\rightarrow$ Computes **all answers** in time $O(|D| + m)$ for m the number of answers

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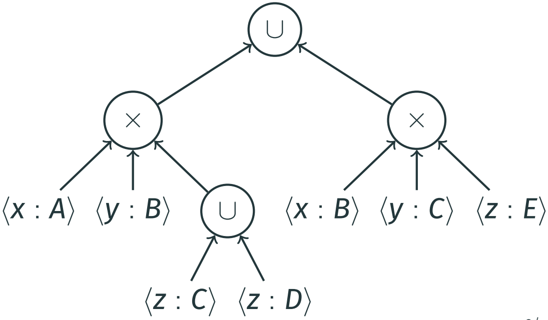
Factorized representation of $Q(D)$



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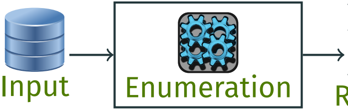

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a	b′	c

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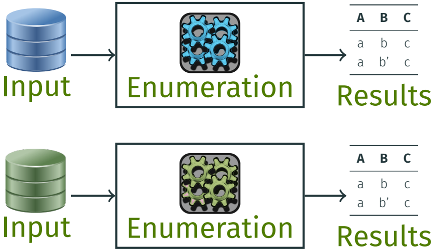

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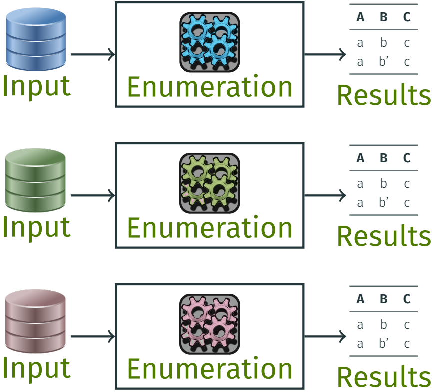

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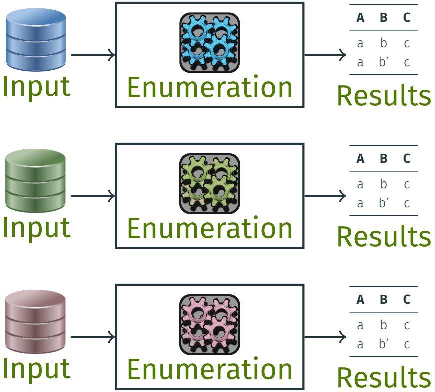

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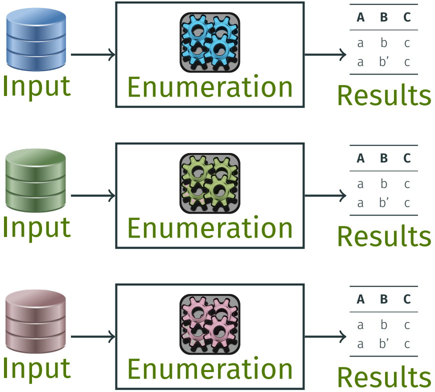

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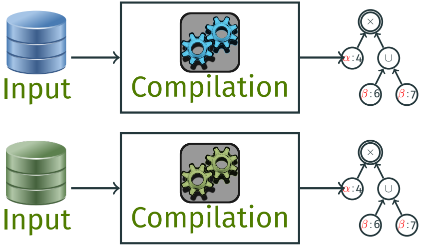

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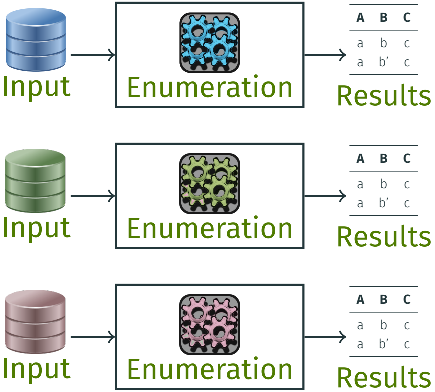

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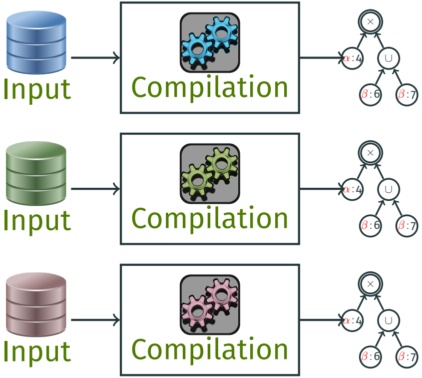

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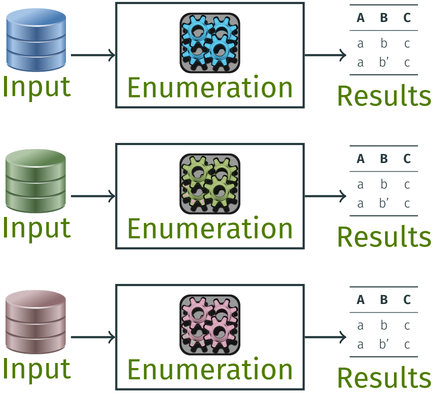

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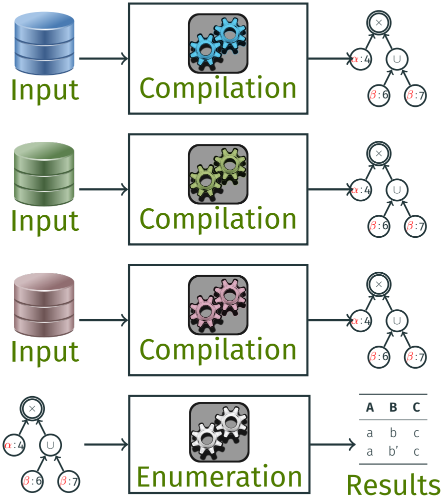

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Results on enumeration for query evaluation, especially via factorized representations

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- **Other settings**: Queries defined by automata / monadic second-order logic

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- Summary and future work

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a''	b''

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- There are **four answers**:

  $(a, b, c), (a, b, c'), (a, b', c'), (a', b', c')$

Cyclic vs acyclic CQs

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Acyclic CQs: the Gaifman graph is acyclic

Cyclic vs acyclic CQs

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Acyclic CQs: the Gaifman graph is acyclic

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We can generalize **acyclic CQs** to arbitrary arity (= α-acyclic Gaifman hypergraph)

Join trees for acyclic CQs

Fact: a CQ is **acyclic** iff it has a **join tree**:

- The vertices are the **atoms** of the query
- For each variable, its occurrences form a **connected subtree**
- (For experts: width-1 generalized hypertree decomposition of the Gaifman hypergraph)

Take the query: $Q(w, x, y, z) : \text{Follows}(w, x) \wedge \text{Subscribed}(x, y) \wedge \text{Follows}(y, z)$

$\text{Subscribed}(x, y)$

$\text{Follows}(w, x)$ $\qquad\qquad$ $\text{Follows}(y, z)$

Yannakakis's algorithm for acyclic CQs

Theorem ([Yannakakis, 1981])

Given an acyclic CQ Q and database D, we can compute $Q(D)$ in time $O(|Q| \times (|D| + m))$, where m is the output size

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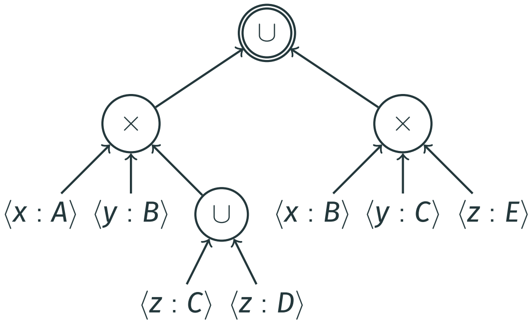
- Join together all relations to get the full result

Factorized representations : d-representations [Olteanu and Závodný, 2015]



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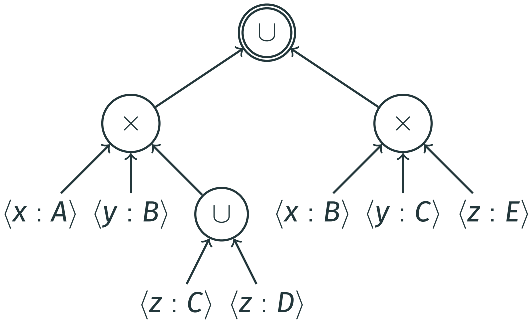

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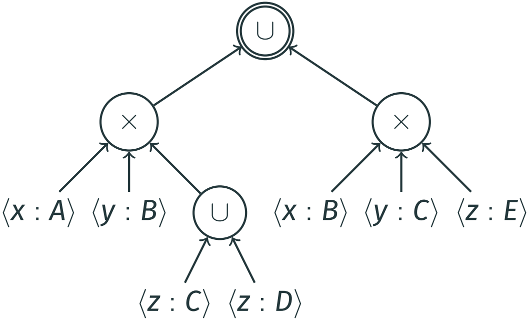

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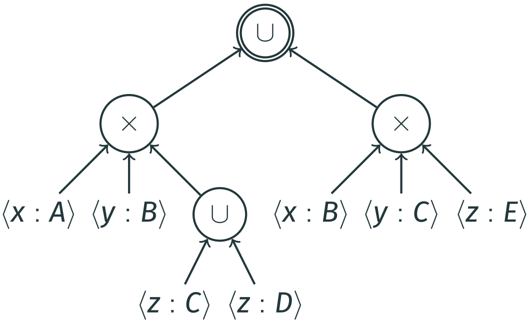

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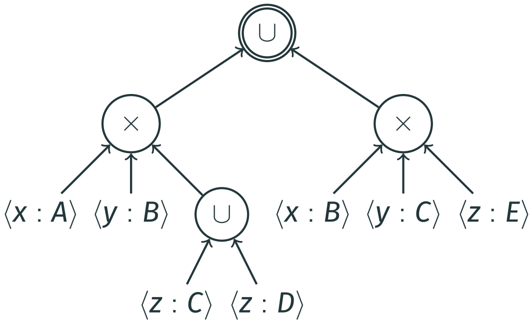

x	y	z
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A	B	D
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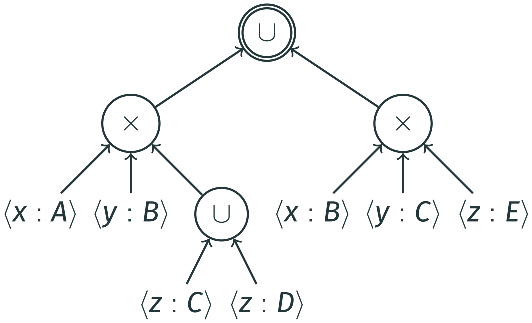

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Conditions on d-representations:

- **Deterministic**: all unions are disjoint
- **Normal**: no union is an input to a union

Enumerating tuples for normal deterministic d-representations

Task: Enumerate the tuples of the relation $R(g)$ captured by a gate g

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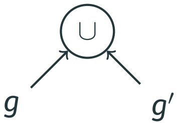
$\cup$-gate



Union: enumerate $R(g)$ and then
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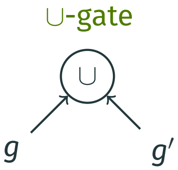

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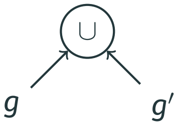

Union: enumerate $R(g)$ and then enumerate $R(g')$

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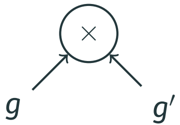
$\times$-gate



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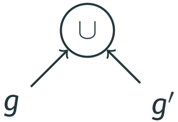

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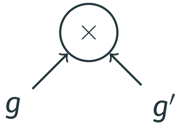

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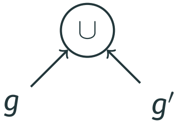

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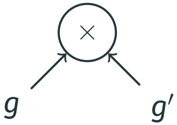

Product: enumerate $R(g)$ and for each result t
  enumerate $R(g')$ and for each result t'
    concatenate t and t'

Enumerating tuples for normal deterministic d-representations (2)

Theorem ([Olteanu and Závodnỳ, 2015], Theorem 4.11)

For any fixed schema $S = (x_1, \ldots, x_k)$, the tuples of a normal deterministic d-representation with schema S can be enumerated in constant delay

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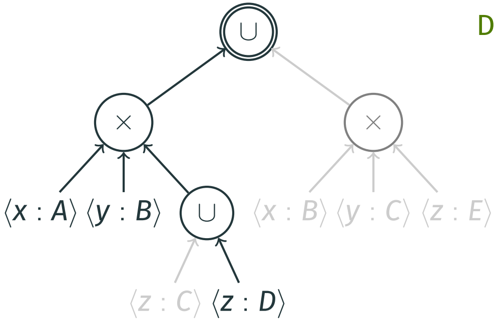

Delay analysis:

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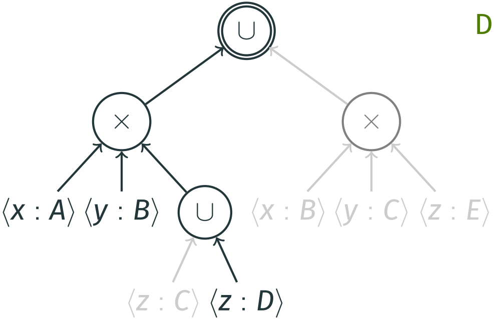

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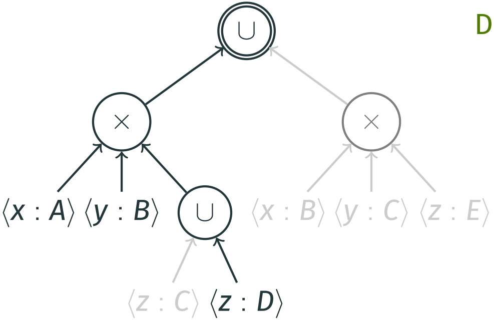

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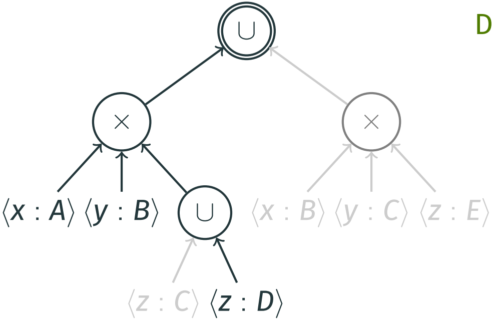

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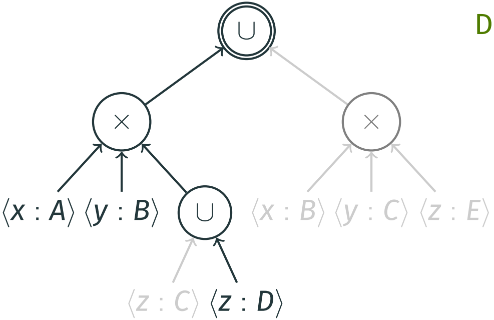

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Note: normal deterministic d-representations also allow us to:

- Count the number of solutions in linear time
- Access the i-th solution, given i, in logarithmic time

Factorized representations for full acyclic CQs

Theorem

*Given an acyclic CQ Q and database D, we can compute a **deterministic normal d-representation of $Q(D)$** in time $O(|Q| \times |D|)$ and hence enumerate $Q(D)$ with **linear preprocessing and constant delay***

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Given an acyclic CQ $\mathbf{Q}$ *and database* $\mathbf{D}$*, we can compute a* ***deterministic normal d-representation of*** $Q(D)$ *in time* $\mathbf{O}(|\mathbf{Q}| \times |\mathbf{D}|)$ *and hence enumerate* $\mathbf{Q}(\mathbf{D})$ *with* ***linear preprocessing and constant delay***

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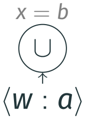

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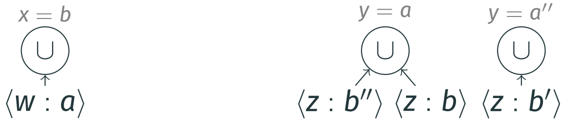

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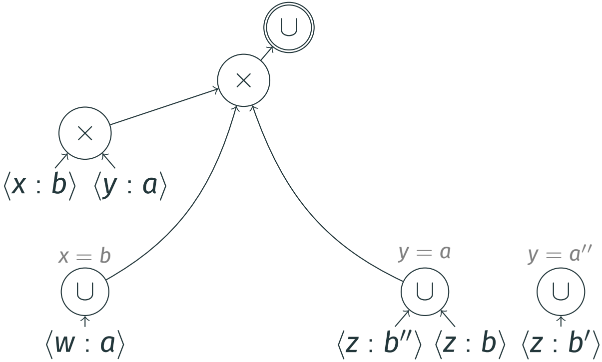

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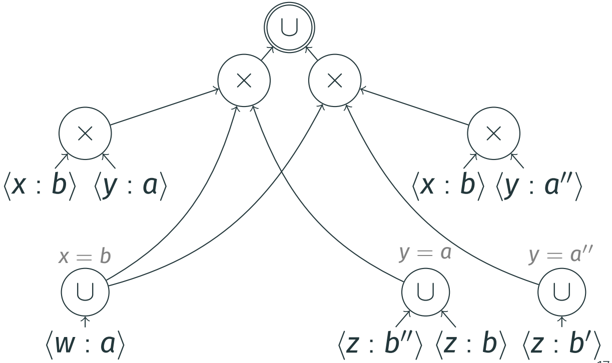

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→ **Ask me** if you want to know more!

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Thanks for your attention!

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What about enumeration for non-free-connex CQs? Let us assume:

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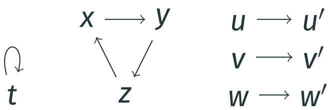

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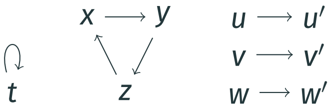

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Open problem: dichotomy on CQs with self-joins? see [Carmeli and Segoufin, 2023]

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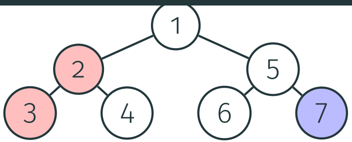
Open problem: dichotomy on UCQs? see [Carmeli and Kröll, 2021]

MSO query evaluation on trees

Data: a **tree** T where nodes have a color from an alphabet ○ ○ ○



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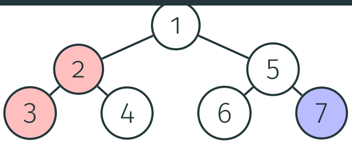

Query Q in monadic second-order logic (MSO)

- $P_{\circ}(x)$ means "x is blue"
- $x \rightarrow y$ means "x is the parent of y"

Equivalent formalism: **tree automata**

"Find the pairs of a pink node and a blue node?"

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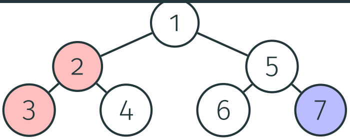

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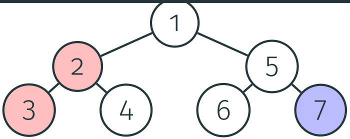

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Data complexity: Measure efficiency as a function of T (the query Q is **fixed**)

Results for MSO on trees

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Note that the d-representation is no longer normal, but we show with some effort:

Theorem (A., Bourhis, Jachiet, Mengel, ICALP'17)

For any fixed schema $S = (x_1, \ldots, x_k)$, the tuples of a deterministic d-representation with schema S can be enumerated with linear preprocessing and constant delay

Enumerating matches of nondeterministic document spanners

Data: a **text** T

```
Antoine Amarilli Description Name Antoine Amarilli. Handle: a3nm. Identity Born 1990-02-07. French national. Appearance as
of 2017. Auth OpenPGP. OpenId. Bitcoin. Contact Email and XMPP a3nm@a3nm.net Affiliation Associate professor of computer
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$$P := \sqcup\ \texttt{[a-z0-9.]}^*\ \texttt{@}\ \texttt{[a-z0-9.]}^*\ \sqcup$$

Output: the list of **substrings** of T that match P:

$$[186, 200\rangle, \quad [483, 500\rangle, \quad \dots$$

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Goal:

- be **very efficient** in T (constant-delay)
- be **reasonably efficient** in P (polynomial-time)

Results for nondeterministic document spanners

Theorem (A., Bourhis, Mengel, Niewerth, ICDT'19; see also PODS'19)

We can enumerate all matches of an input nondeterministic automaton with captures on an input text with

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- Generalizes earlier result on **deterministic automata** [Florenzano et al., 2018]
- To make the algorithm polynomial in the **(nondeterministic) automaton**, we need efficient enumeration for a certain kind of **non-deterministic d-representations**

Other enumeration settings

Efficient enumeration is now being studied in many settings in data management (sometimes with weaker guarantees than linear preprocessing and constant delay):

- For regular path queries [Martens and Trautner, 2018, David et al., 2024]
- For compressed structures:
  - Compressed trees [Lohrey and Schmid, 2024]
  - SLP-compressed documents [Schmid and Schweikardt, 2021, Muñoz and Riveros, 2023]
- For visibly pushdown languages [Muñoz and Riveros, 2022]
- For context-free languages with annotations [Peterfreund, 2021], [A., Jachiet, Muñoz, Riveros, 2023]

There are also software implementations [Riveros et al., 2023]

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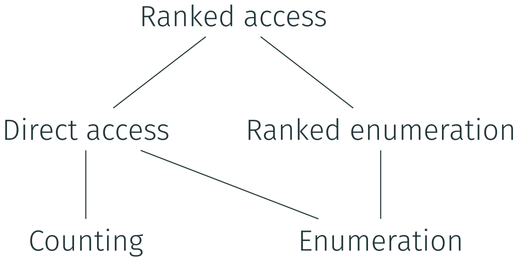

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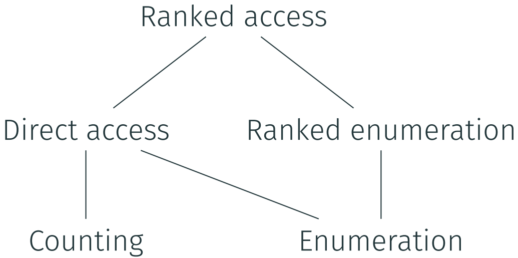

(Adapted from [Carmeli, 2023])

- **Direct access**: getting the i-th answer
- **Counting** the answers
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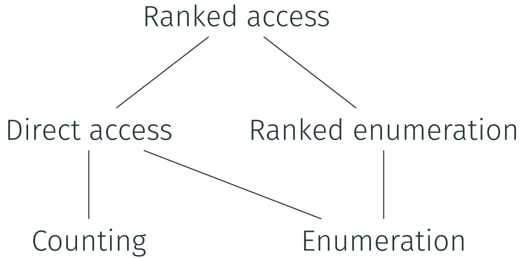

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Another question: **maintain** an enumeration structure under **updates** to the data

Results on ranked enumeration / ranked access

For **CQs and UCQs**:

- Most works study self-join-free CQs under **lexicographic orders** and aim for **logarithmic** access time or delay:
  - Characterization of **tractable orders** for CQs [Carmeli et al., 2023]

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For **MSO** queries on trees:

- **Ranked enumeration** shown with **logarithmic delay** on **words** [Bourhis et al., 2021]
- Recent extension to **trees** [A., Bourhis, Capelli, Monet, 2024]

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For **MSO** queries on trees, aiming for **logarithmic** update time:

- On **words**, linear preprocessing and constant delay enumeration is possible under **insert/delete updates** [Niewerth and Segoufin, 2018]
- On **trees**, linear preprocessing and constant delay enumeration is possible under **substitution updates** [A., Bourhis, Mengel, 2018] and possibly more