Enumerating Pattern Matches in Words and Trees

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Problem: Finding patterns in text

- We have a long text T:

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    - Write the pattern as a regular expression:

$$P := \text{␣}^{+}\ \texttt{[a-z0-9.]}^{*}\ \texttt{@}\ \texttt{[a-z0-9.]}^{*}\ \text{␣}^{+}$$

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→ **How to find the pattern P efficiently in the text T?**

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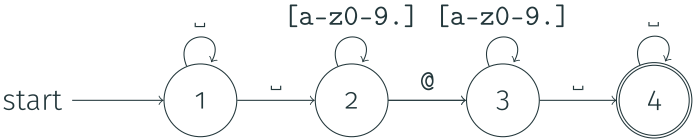

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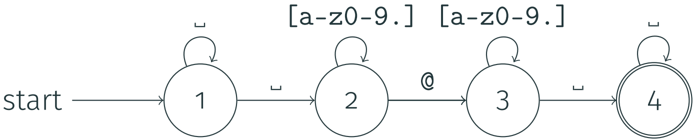

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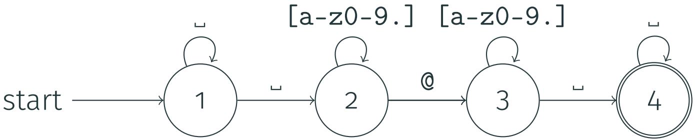

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- How **efficient** is this?

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- **Generalization:** patterns that can capture a **tuple** of positions
  $\rightarrow$ Find the email addresses **without leading/trailing spaces**
  $\rightarrow$ Find all **pairs** of a name followed by an email address

Patterns with capture variables

- Write the pattern P as a regular expression with capture variables

$$P := \bullet^{*} \ \text{␣}^{+} \ \alpha \ \texttt{[a-z0-9.]}^{*} \ \texttt{@} \ \texttt{[a-z0-9.]}^{*} \ \beta \ \text{␣}^{+} \ \bullet^{*}$$

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$\rightarrow$ One match: $\langle \alpha : 20, \beta : 32 \rangle$

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$$\langle \alpha : 187, \beta : 199 \rangle, \ldots$$

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- We measure the **complexity** of the problem:
  - In **data complexity**, as a function of T
  - In **combined complexity**, as a function of P and T

Measuring the complexity

- Naive algorithm: Consider all ways to assign capture variables and test for each of them if it satisfies the pattern

l	o	l

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$\alpha\beta$ `l` `o` `l`

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$\boxed{\quad \beta\ \texttt{l} \qquad \texttt{o}\ \alpha \quad \texttt{l} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$

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    ```
    aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa
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$\rightarrow$ We need a **different way** to measure complexity

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we just need to be able to enumerate answers quickly

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→ Formalization: enumeration algorithms

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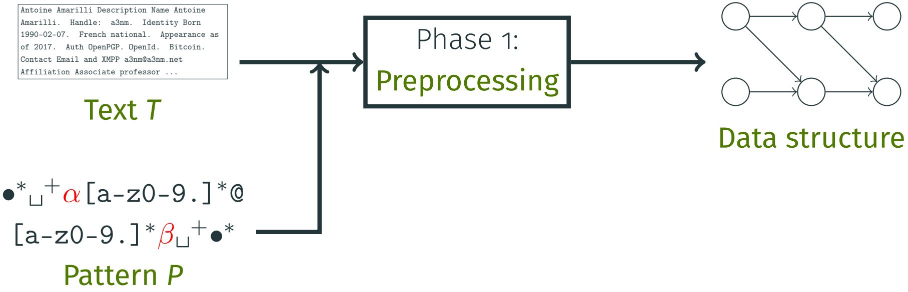
Phase 1:

Preprocessing

Formalizing enumeration algorithms



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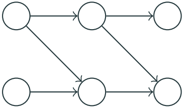
`[a-z0-9.]*` $\beta \sqcup^+ \bullet^*$

Pattern P

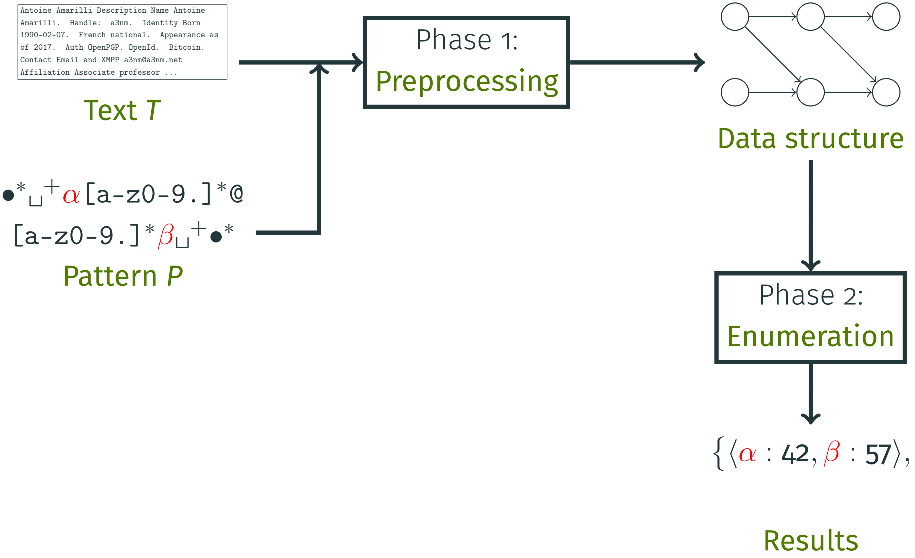
Phase 1:
Preprocessing



Data structure

Phase 2:
Enumeration

Formalizing enumeration algorithms



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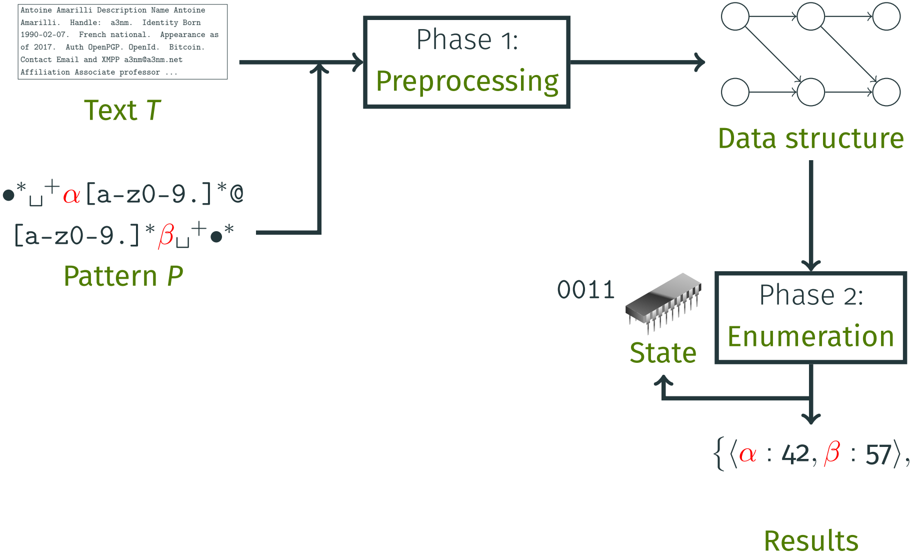

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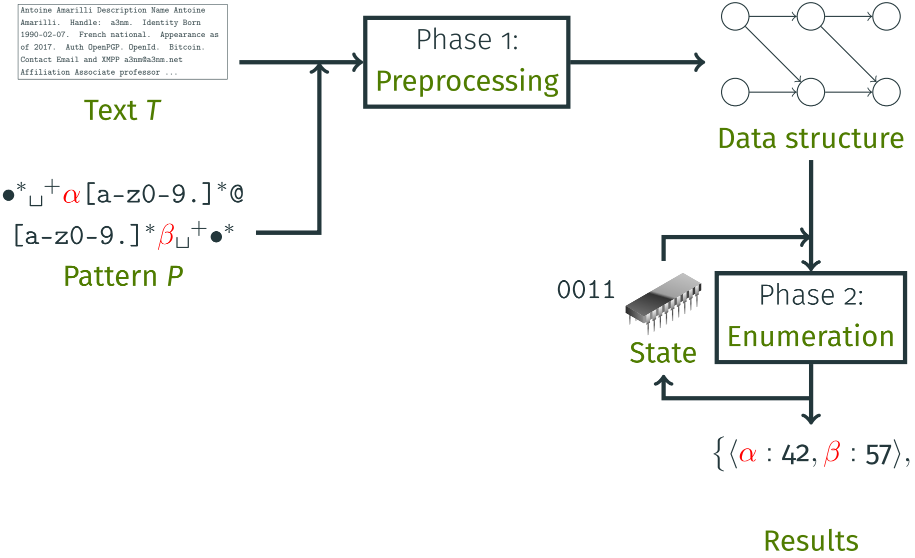

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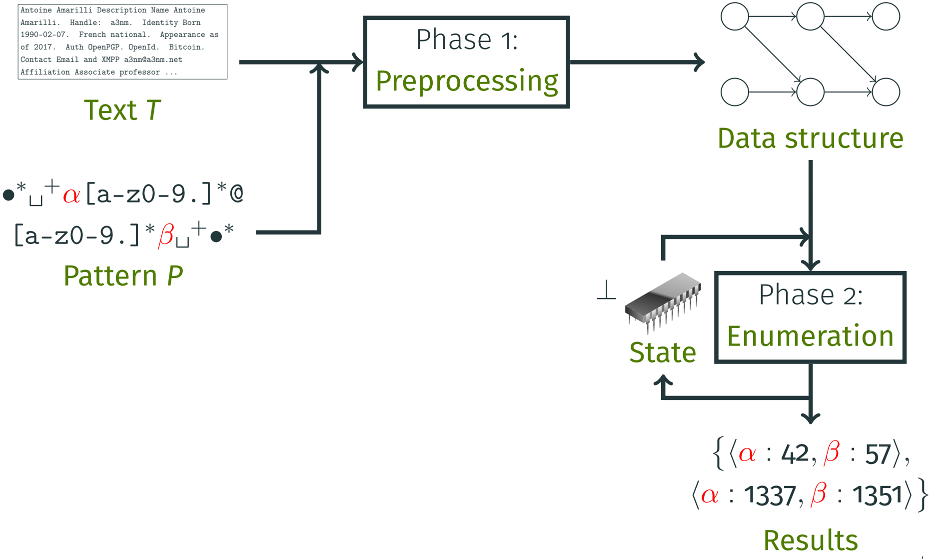

Formalizing enumeration algorithms

```
Antoine Amarilli Description Name Antoine
Amarilli.  Handle:  a3nm.  Identity Born
1990-02-07.  French national.  Appearance as
of 2017.  Auth OpenPGP. OpenId.  Bitcoin.
Contact Email and XMPP a3nm@a3nm.net
Affiliation Associate professor ...
```

Text T

$\bullet^* \sqcup^+ \alpha$ `[a-z0-9.]*@[a-z0-9.]*` $\beta \sqcup^+ \bullet^*$

Pattern P

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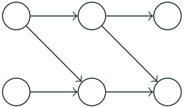

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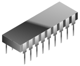
$\perp$



State

$\{\langle \alpha : 42, \beta : 57 \rangle, \langle \alpha : 1337, \beta : 1351 \rangle\}$

Results

Two ways to measure performance:

- **Total time** for phase 1
- **Delay between two results** in phase 2

... in **combined** and **data complexity**

Complexity of enumeration algorithms

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  - The regexp with captures P

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→ Can we do better?

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- Our contribution is:

Theorem

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- *Preprocessing linear in* T *(data) and polynomial in* T *and* P *(combined)*
- *Delay constant in* T *(data) and polynomial in* T *and* P *(combined)*

Automaton formalism

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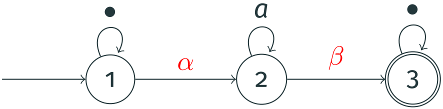

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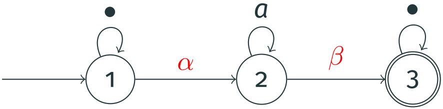

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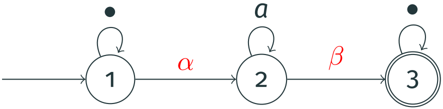

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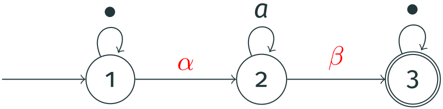

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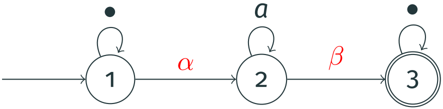

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- **Challenge:** Because of **nondeterminism** we can have many different runs of A producing the same tuple!

Proof idea: product DAG

Compute a product DAG of the text T and of the automaton A

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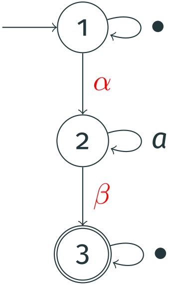

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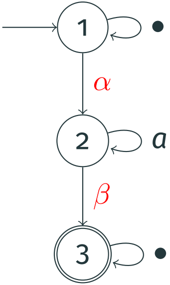
a	a	a	b	a



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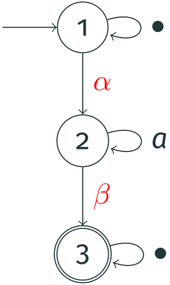

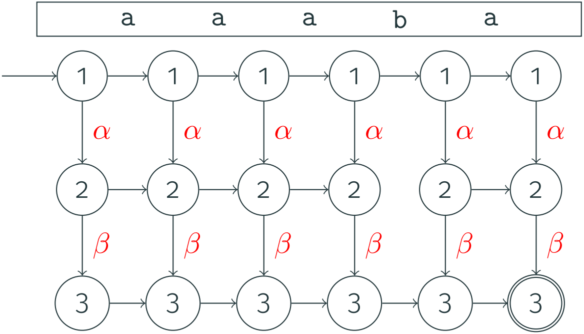

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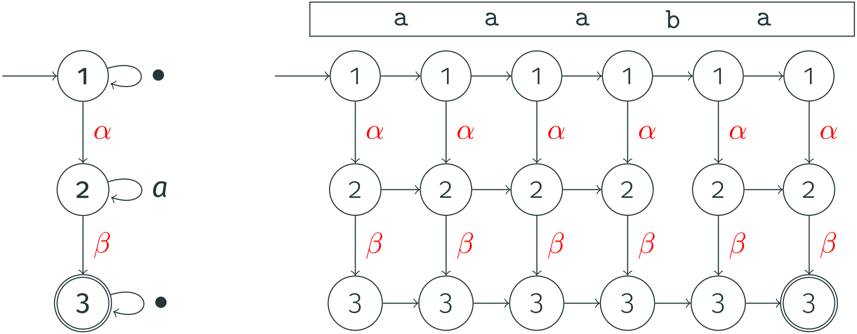

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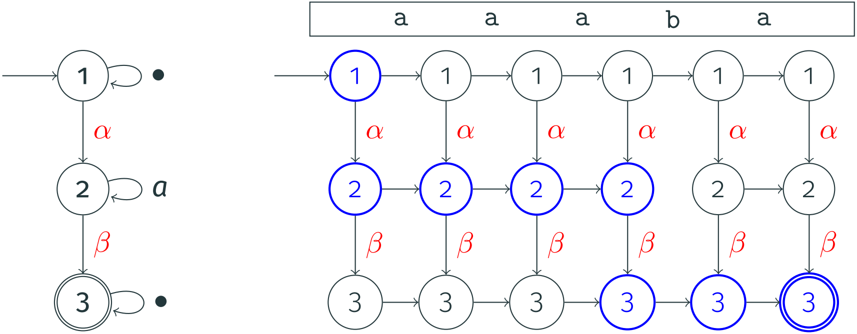
Example: Text $T :=$ `aaaba` and $P := \bullet^* \ \alpha \ a^* \ \beta \ \bullet^*$, match $\langle \alpha : 0, \beta : 3 \rangle$



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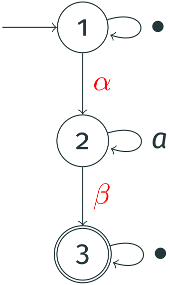

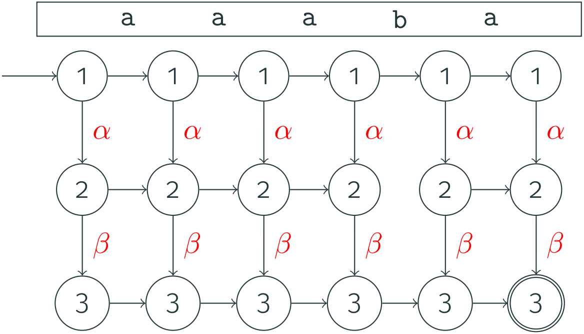

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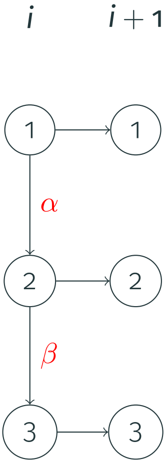
$\rightarrow$ **Challenge:** Enumerate paths but avoid **duplicate matches** and do not **waste time** to ensure constant delay

Proof idea: on-the-fly computation to avoid duplicates



- We are at a position i and set of states in blue

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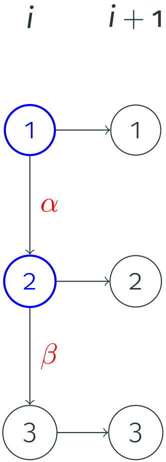

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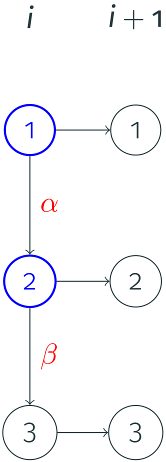

- We are at a position i and set of states in blue
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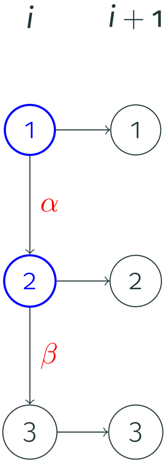

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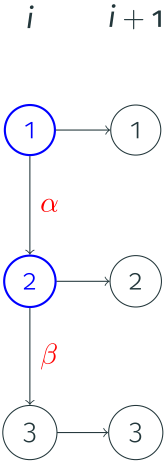

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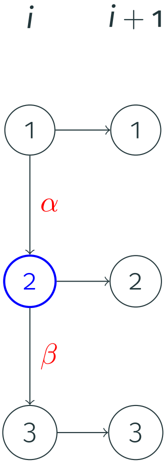

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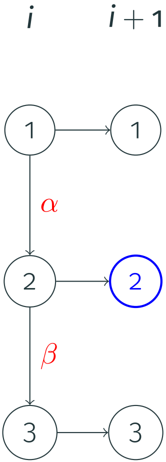

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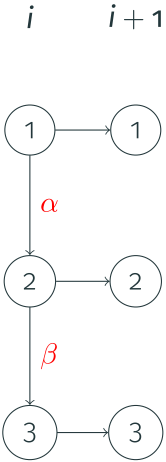

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$\rightarrow$ We must have **preprocessed** the DAG to make sure that we can always finish the run

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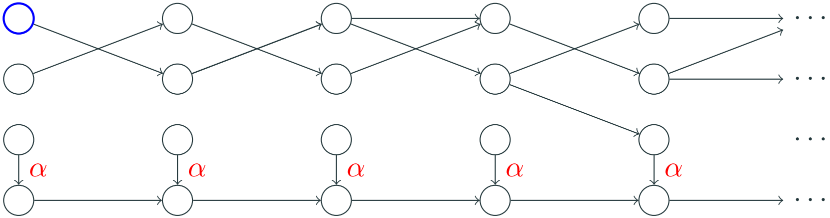

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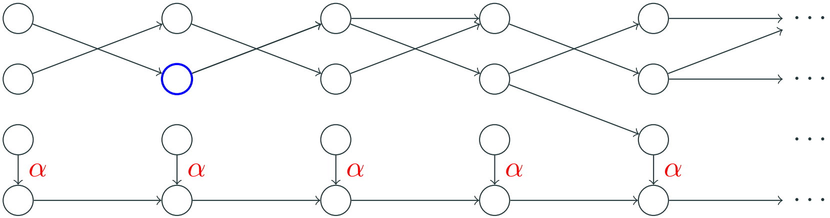

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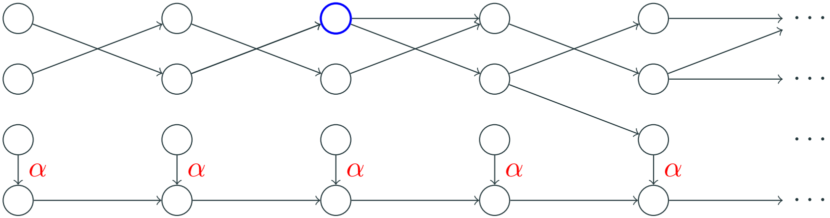

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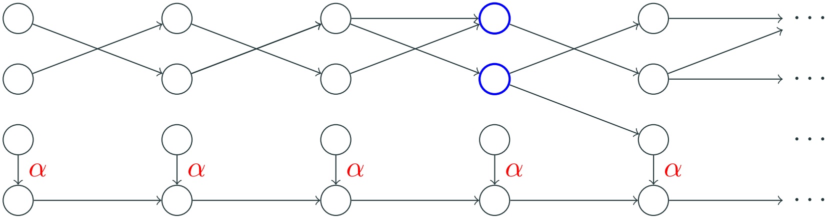

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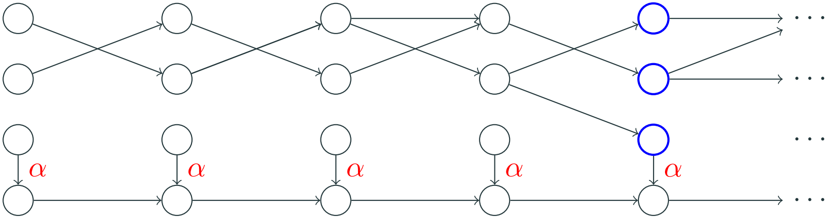

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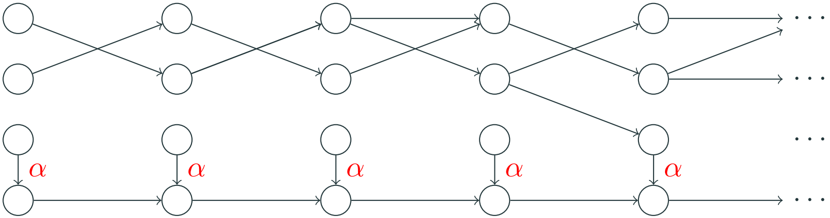

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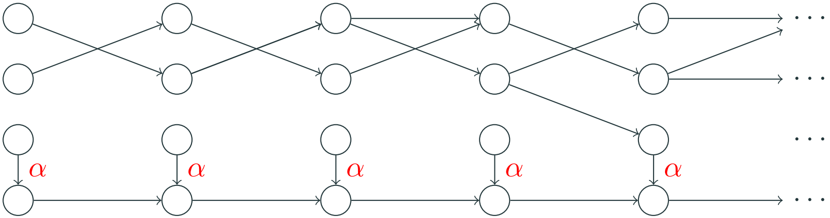

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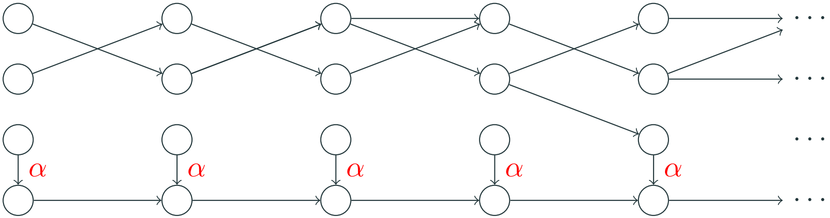

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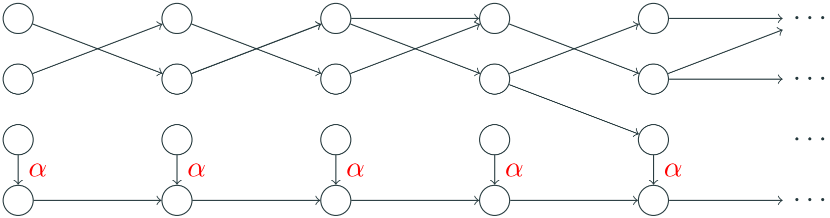

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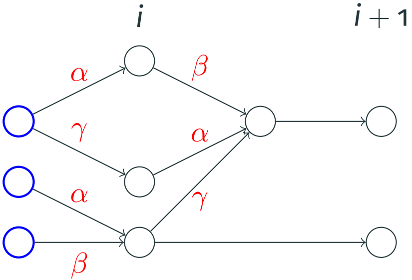
  - $\rightarrow$ Compute at each position i the transitive closure to all positions j such that j is the next position of some state at i (there are $\leq |A|$)

Proof idea: flashlight search

- **Issue:** Finding which **variable sets** we can assign at position i?



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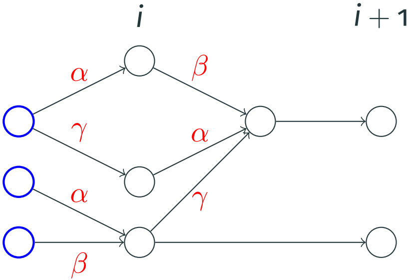

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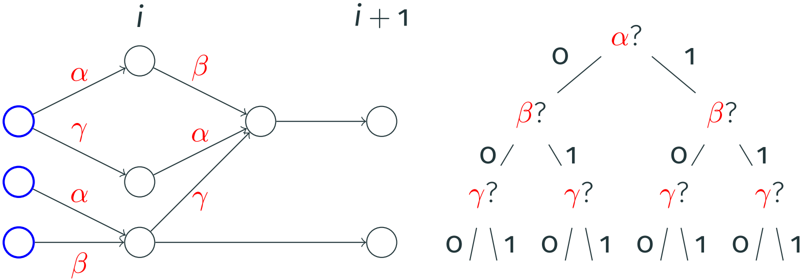

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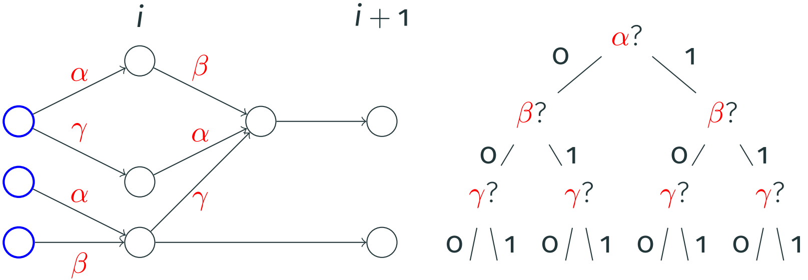

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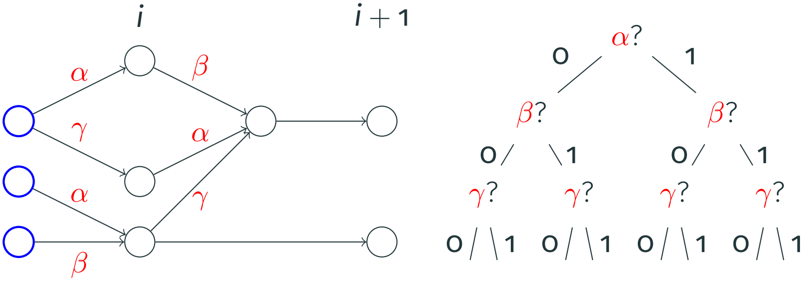

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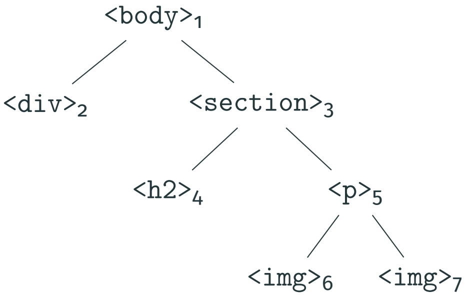
Extension: From Text to Trees

Pattern matching on trees

- The data T is no longer text but is now a tree:



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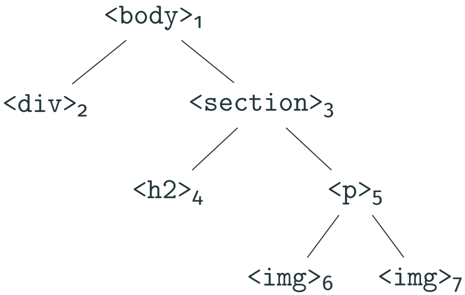

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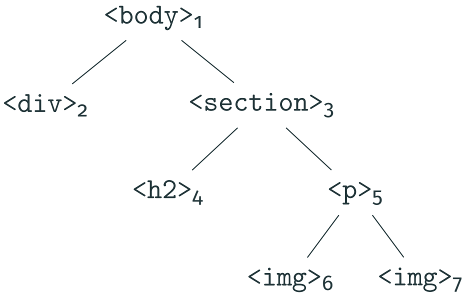

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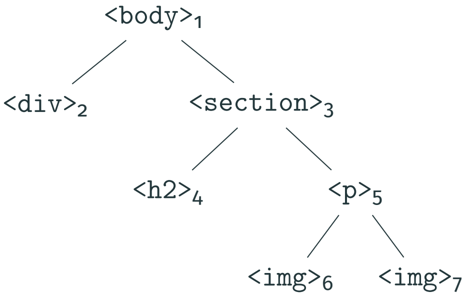

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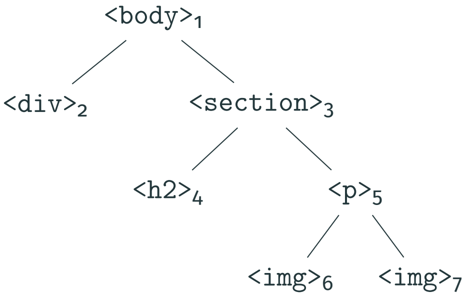

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- We **conjecture** the following bounds for this task (ongoing work):

Conjecture

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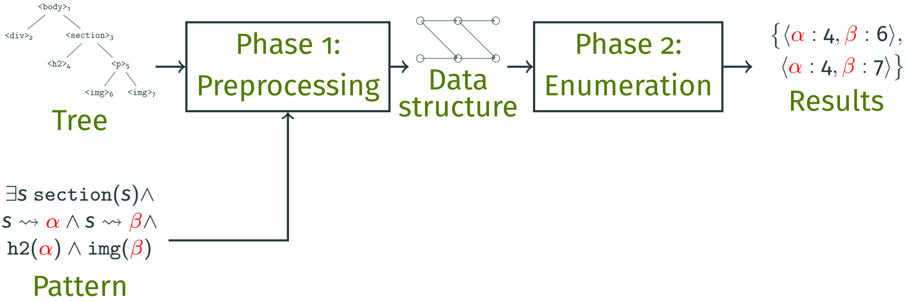
Proof idea for trees: structure

Similar structure to the previous proof, but with a circuit:



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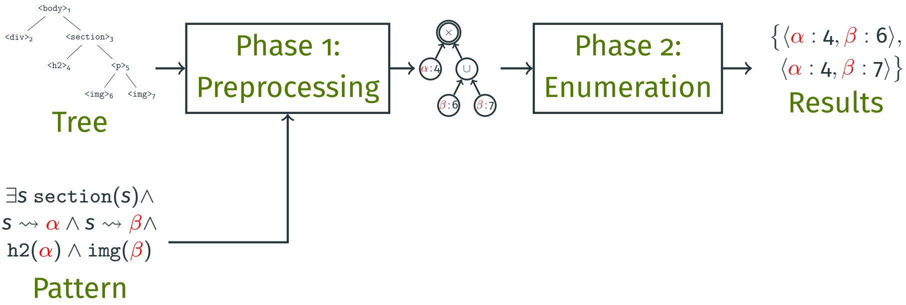
- **Enumeration:** Apply a **generic algorithm** on the circuit



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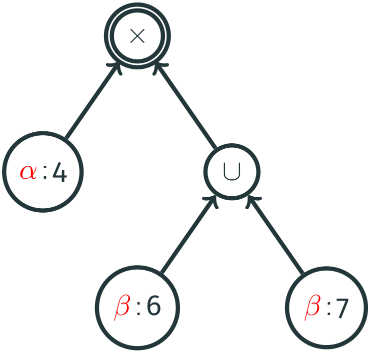

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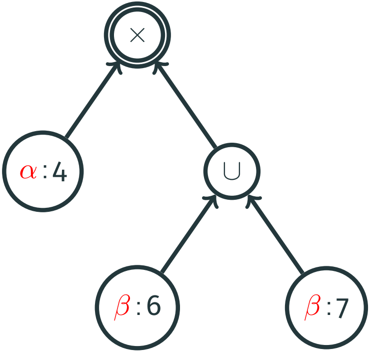

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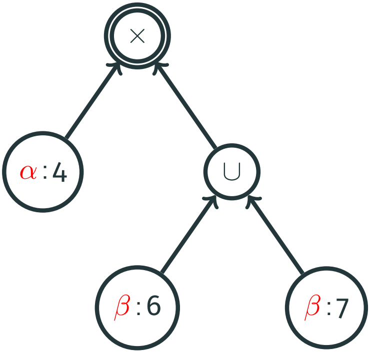

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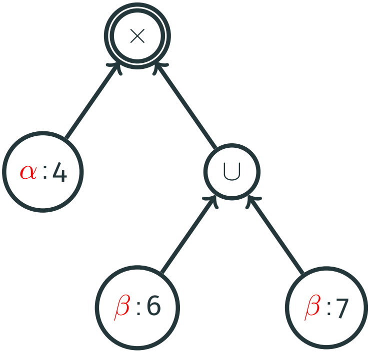

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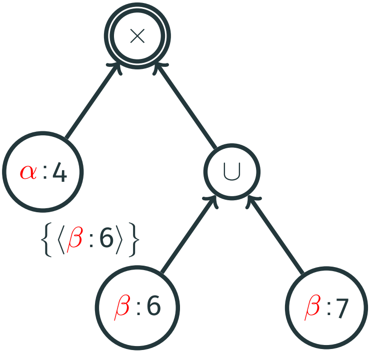

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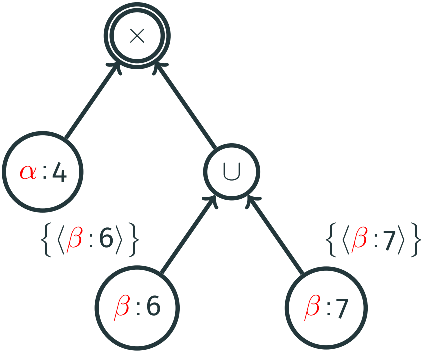

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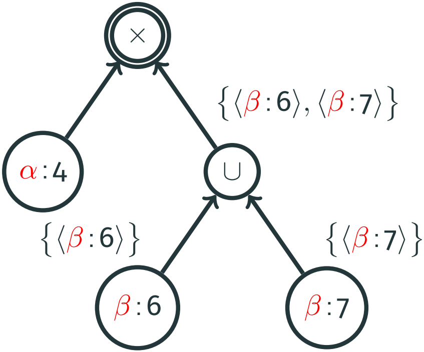

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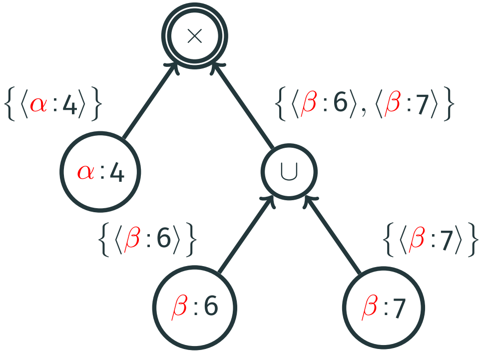

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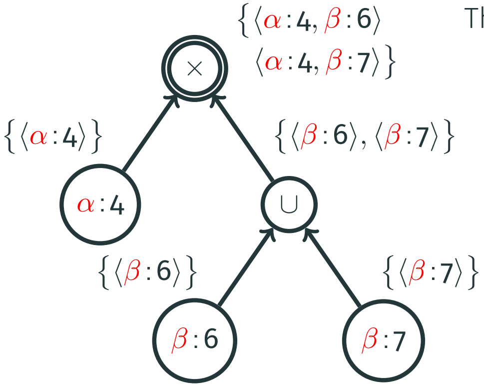

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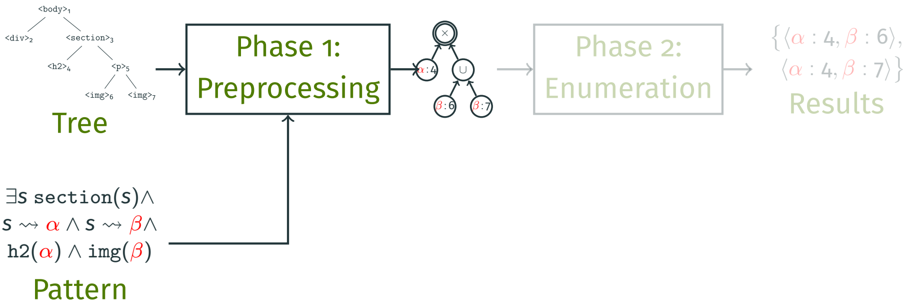

Theorem

*For any **tree automaton** A with capture variables $\alpha_1, \ldots, \alpha_k$, given a **tree** T, we can build in $O(|T| \times |A|)$ a **set circuit** capturing exactly the set of tuples $\{\langle \alpha_1 : n_1, \ldots, \alpha_k : n_k \rangle$ in the output of A on T*

Proof idea for trees: set circuit construction (details)

- **Automaton:** *"Select all node pairs* (α, β)*"*
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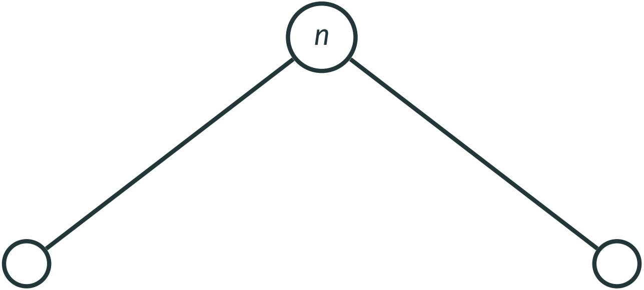

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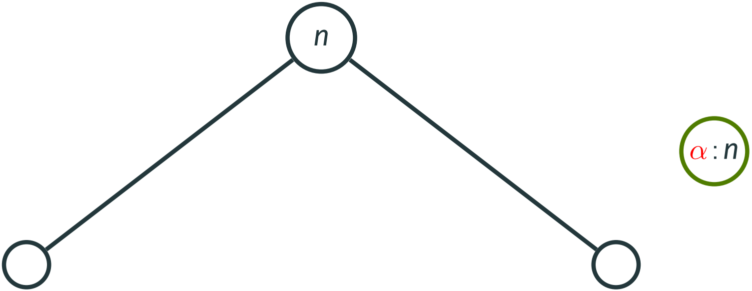

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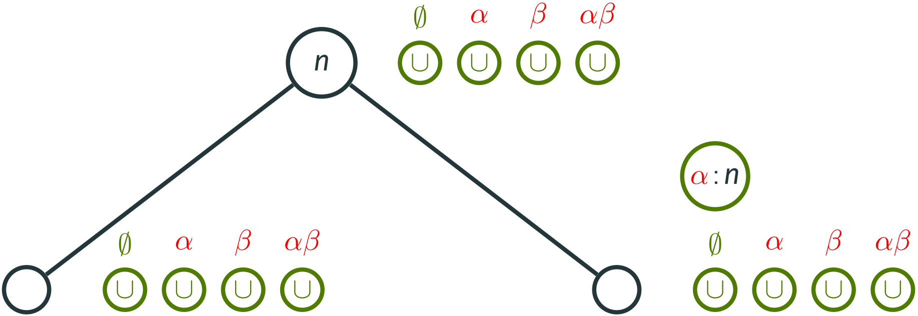

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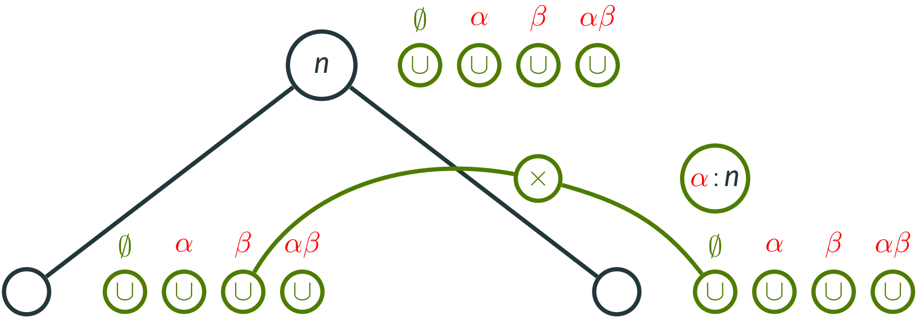

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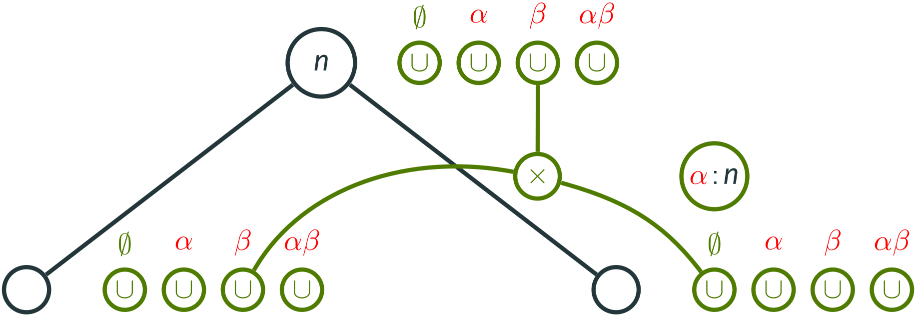

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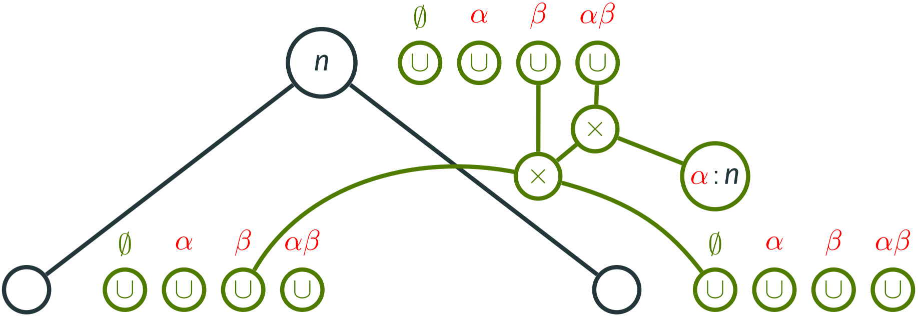

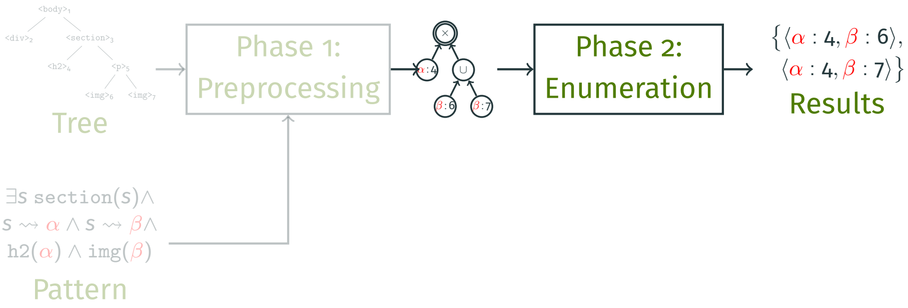

Theorem

Given a set circuit satisfying some conditions, we can enumerate all tuples that it captures with linear preprocessing and constant delay

E.g., for $\{\langle \alpha : 4, \beta : 6 \rangle, \langle \alpha : 4, \beta : 7 \rangle\}$: enumerate $\langle \alpha : 4, \beta : 6 \rangle$ then $\langle \alpha : 4, \beta : 7 \rangle$

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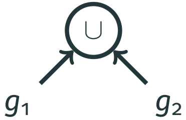

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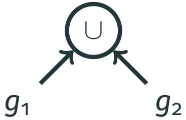

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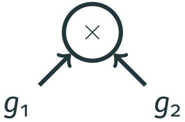

Lexicographic product:

for every t_1 in $T(g_1)$:
  for every t_2 in $T(g_2)$:
    output $t_1 + t_2$

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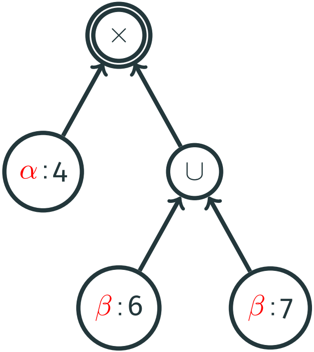
Enumeration relies on some **conditions** on the input circuit (d-DNNF):



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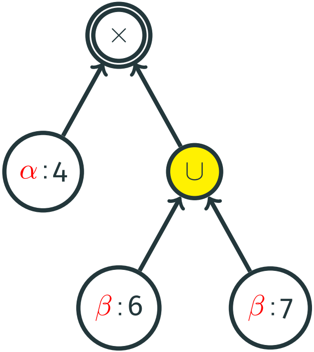

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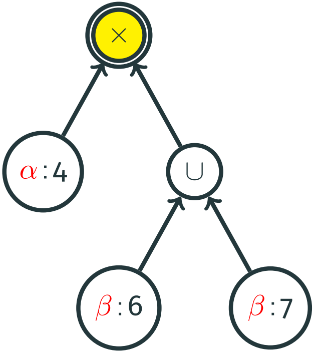

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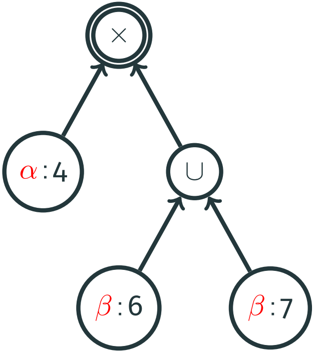

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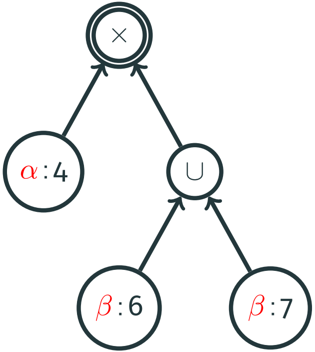

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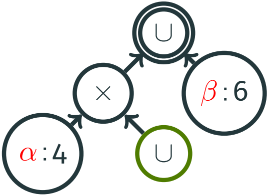

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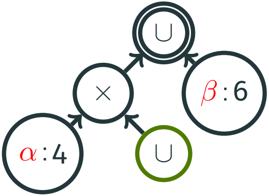

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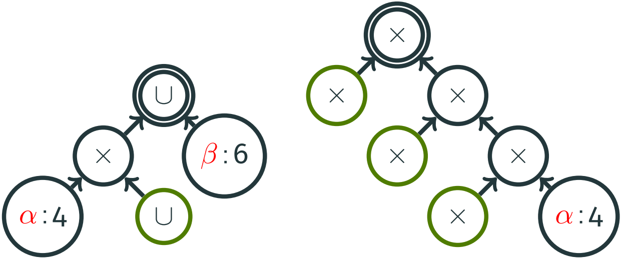

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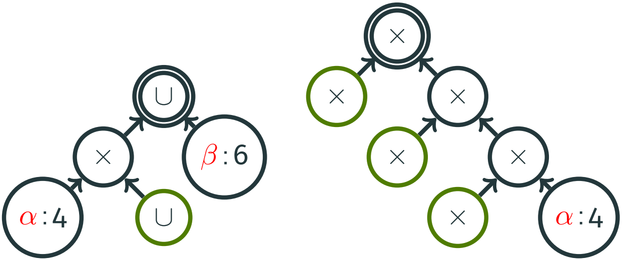

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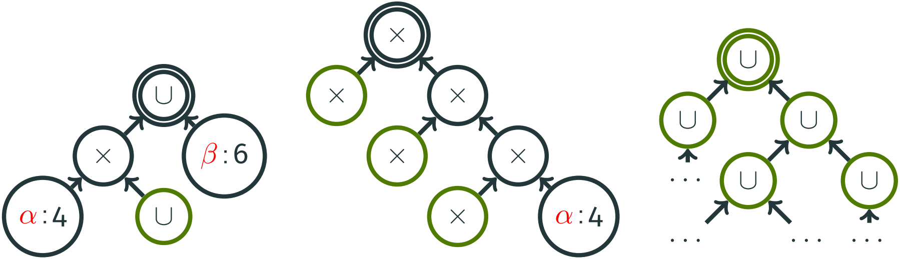

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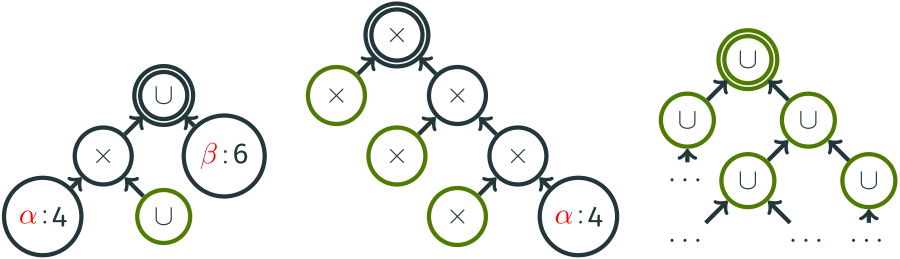

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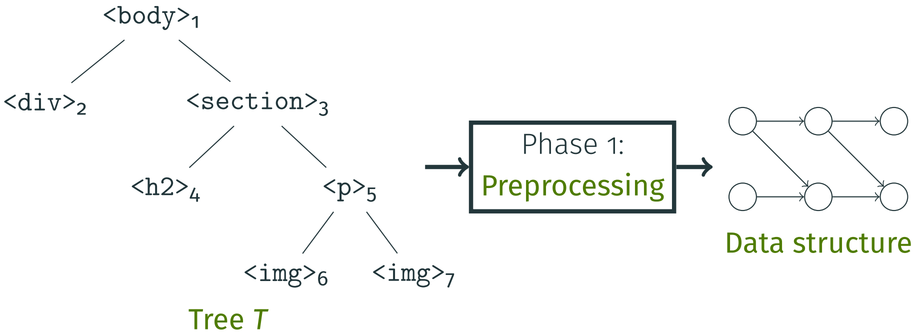
- We must not waste time in **hierarchies of $\cup$-gates**
  - $\rightarrow$ Precompute a **reachability index** (uses **upwards-determinism**)

Extension: Handling Updates

Updates



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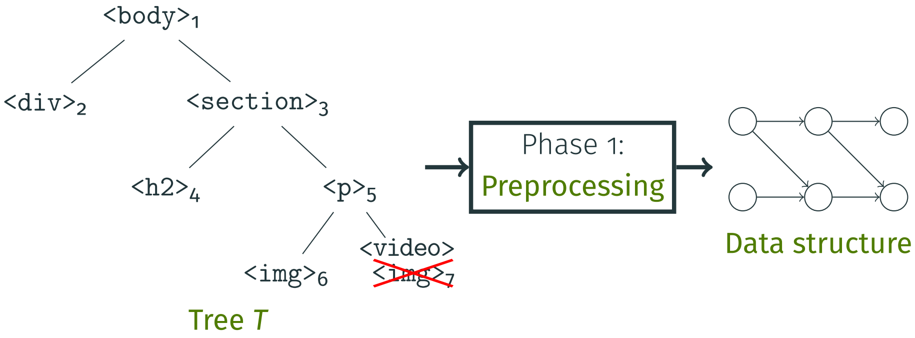

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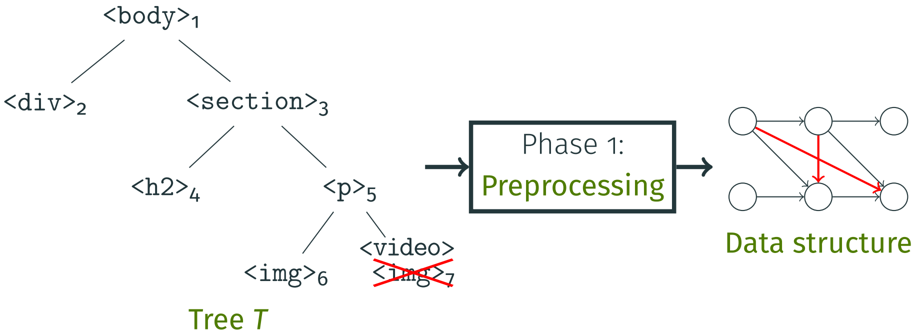

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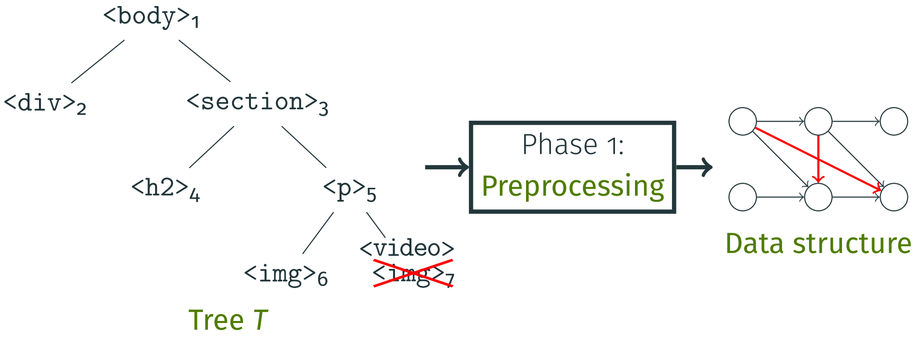

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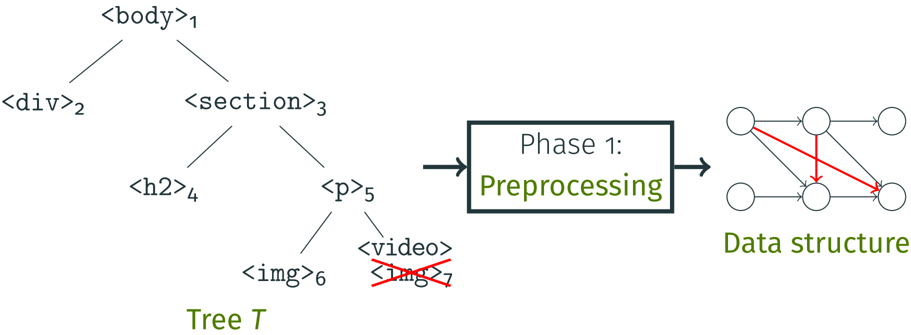

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→ Can we **do better**?

Known results on dynamic trees

All these results are on data complexity in T (for a fixed pattern):

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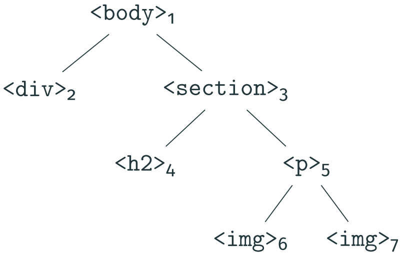

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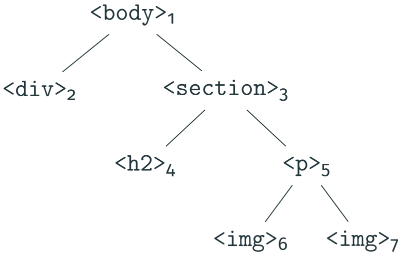

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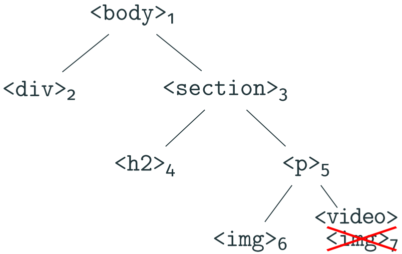

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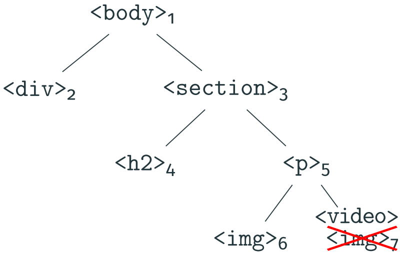

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- Remaining open questions:
  - $\rightarrow$ Does this hold for more general updates (insert/delete, etc.)?
  - $\rightarrow$ Can we also achieve tractable combined complexity?

Extension: Connection to Circuits

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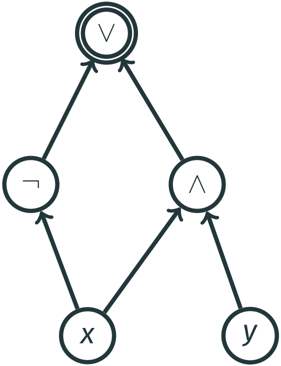
$\rightarrow$ Task: Given a Boolean circuit, how to efficiently enumerate its satisfying valuations?

Boolean circuits



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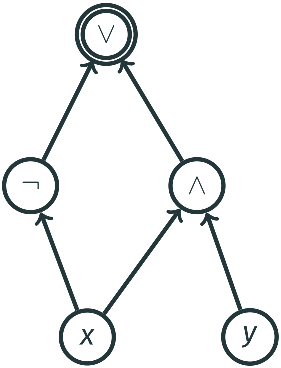

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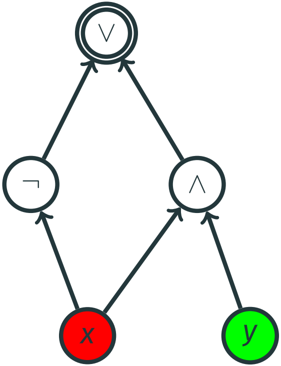

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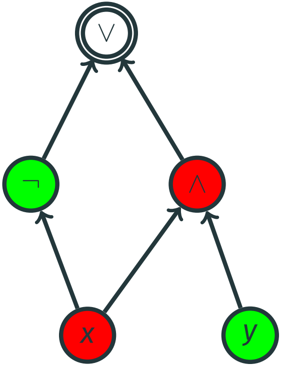

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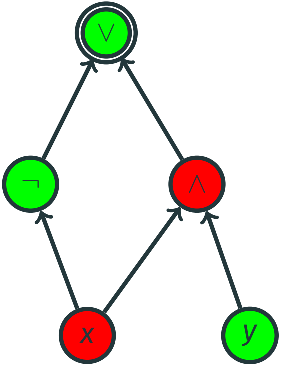

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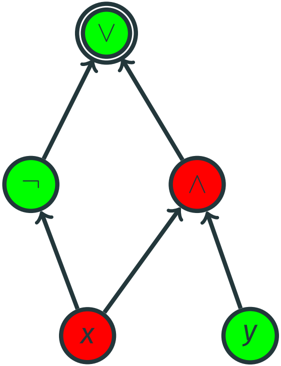

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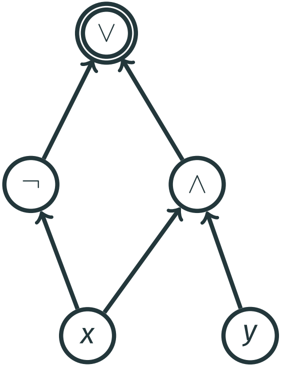

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Our task: Enumerate all **satisfying assignments** of an input circuit

Circuit restrictions

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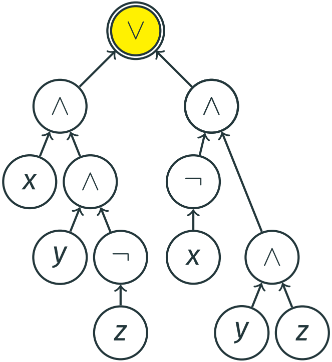

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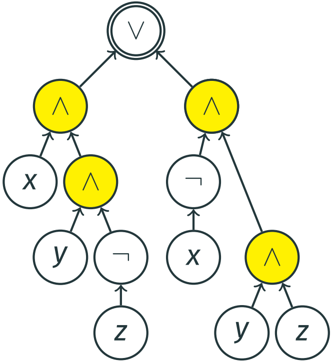

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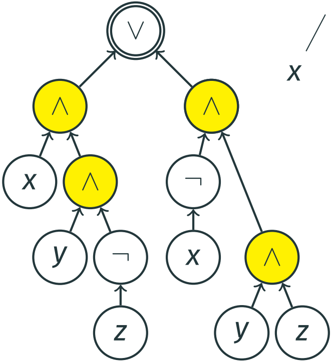
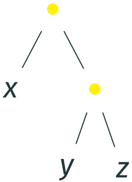
v-tree: $\wedge$-gates follow a **tree** on the variables





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Subtleties: Must **complete** to a set circuit; memory usage problems

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Thanks for your attention!

Amarilli, A., Bourhis, P., and Mengel, S. (2018).
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To appear.