Efficient Enumeration of Query Answers via Circuits

Antoine Amarilli

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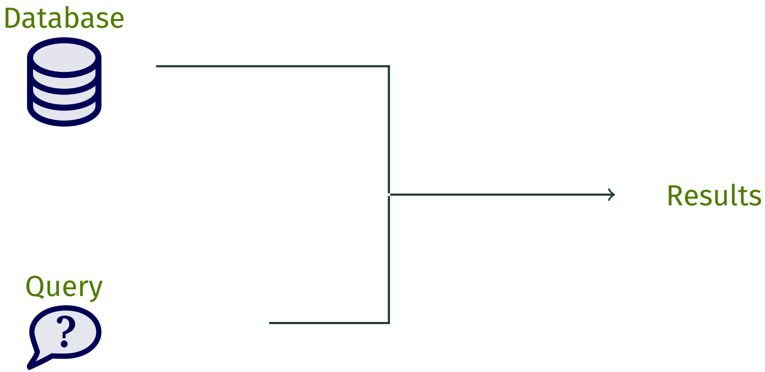
Télécom Paris

Query evaluation

Central problem in database theory and practice: query evaluation



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Database

Follows

from	to
Alice	Bob
Bob	Carol
Bob	Dave
Carol	Eve

Query

→ Results

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Combined complexity and data complexity

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- Data complexity: the query is fixed, the input is only the data

  $\rightarrow$ Motivation: the data is usually much larger than the query

Data complexity for large output size

- Consider the query Q: *"Find all users* x*,* y*, and* z *such that* x *follows* y *and* y *follows* z*"*

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→ We need a **better measure of complexity**

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Step 1:
Preprocessing
in $O(|D|)$

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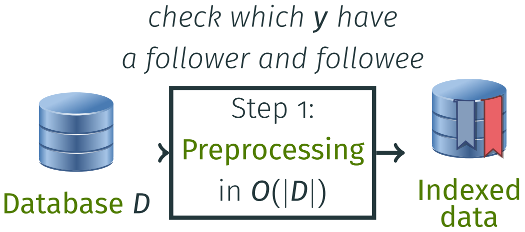

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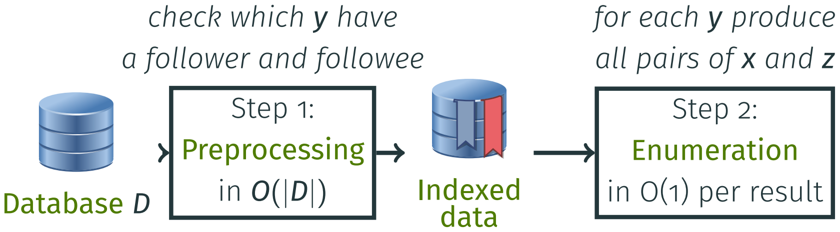

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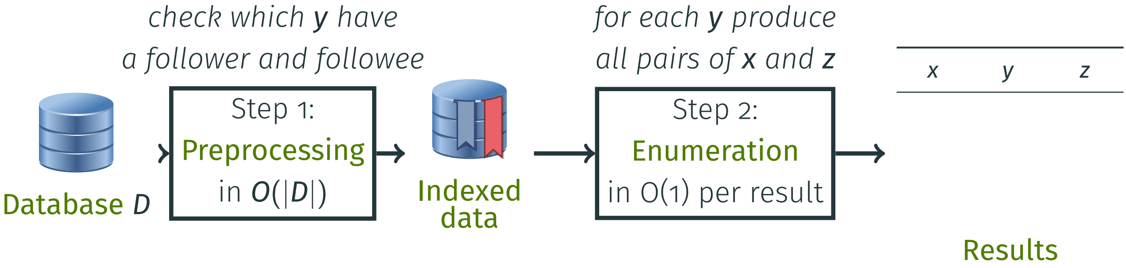

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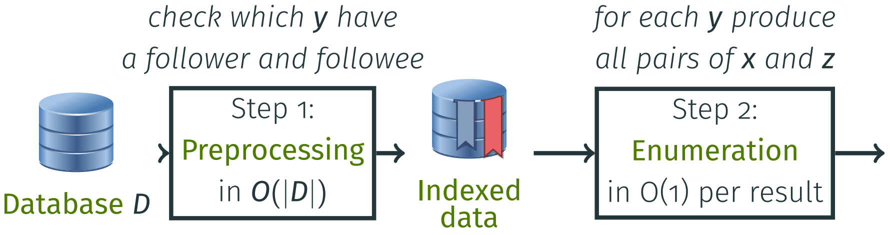

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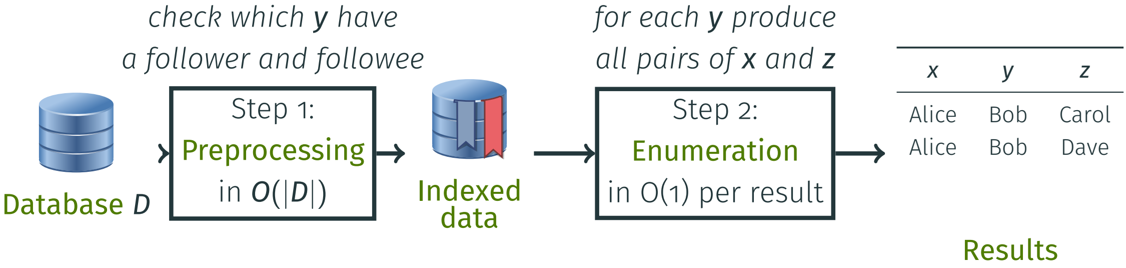

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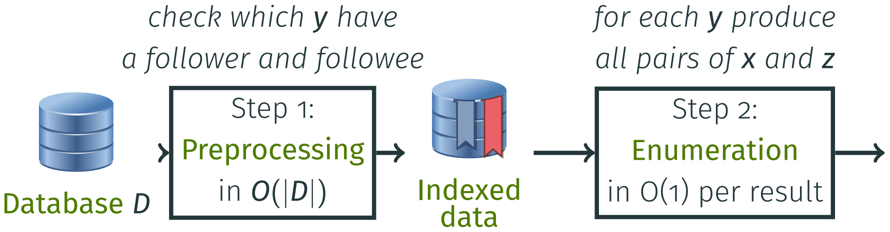

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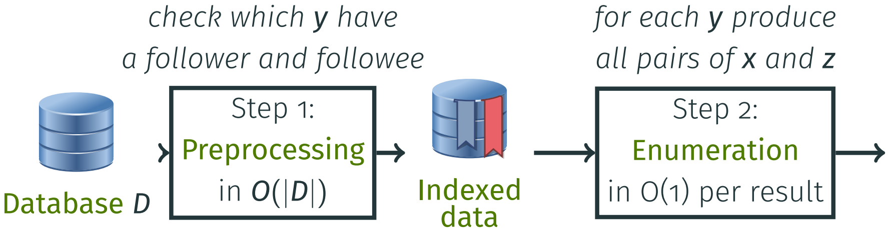

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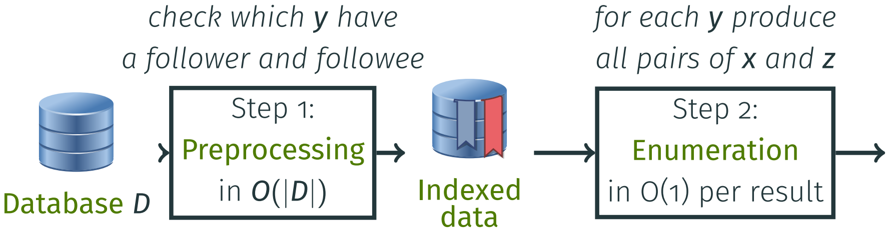

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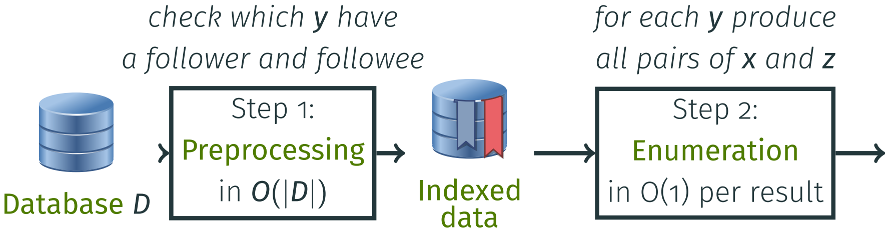

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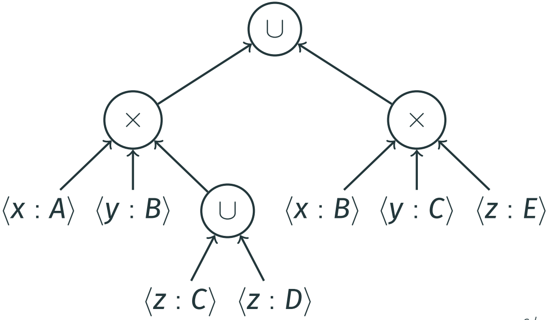
Factorized representation of $Q(D)$



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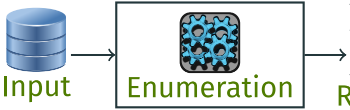

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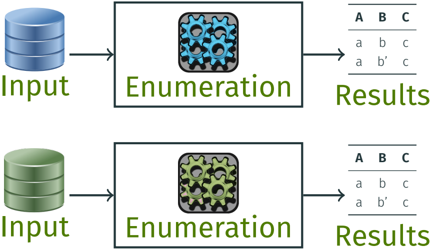

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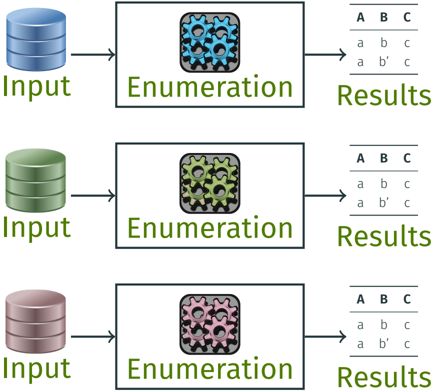

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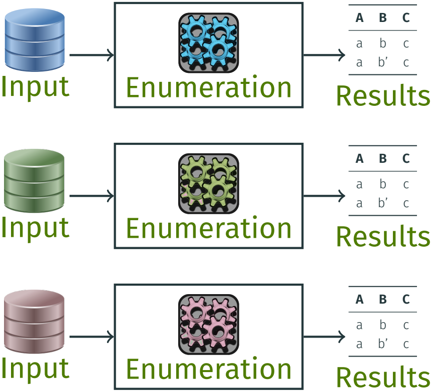

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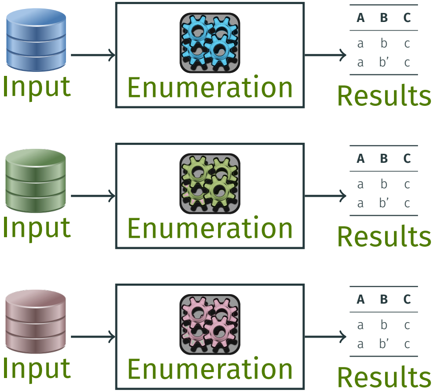

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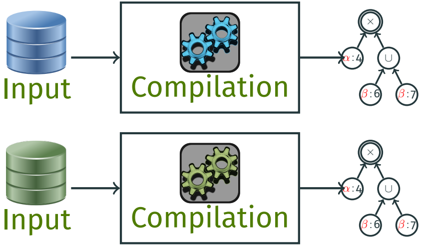

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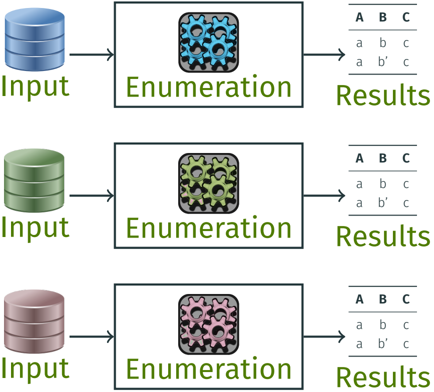

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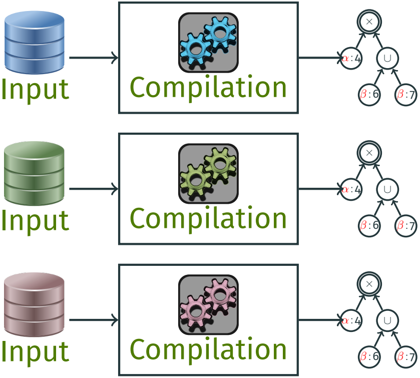

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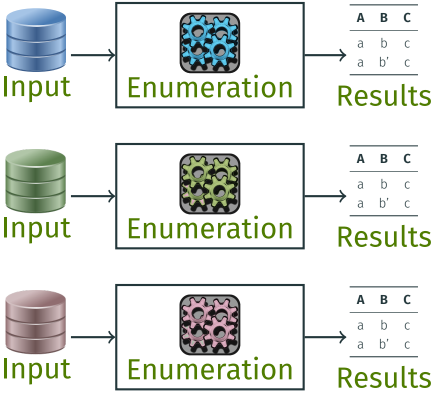

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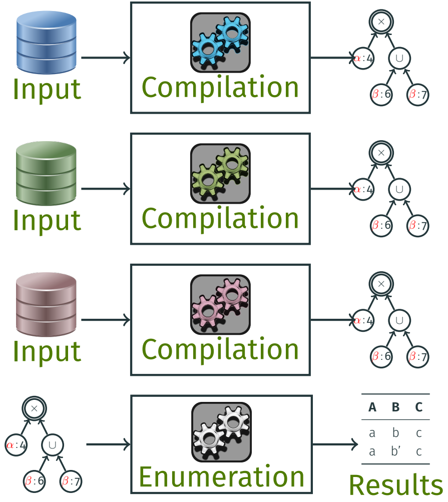

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Results on enumeration for query evaluation, especially via factorized representations

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- **Other tasks**: ranked enumeration, direct access, incremental maintenance, etc.

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a''	b''

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- There are **four answers**:

  $(a, b, c), (a, b, c'), (a, b', c'), (a', b', c')$

Cyclic vs acyclic CQs

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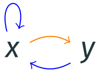

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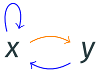

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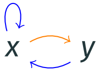

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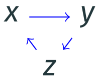

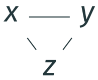

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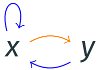

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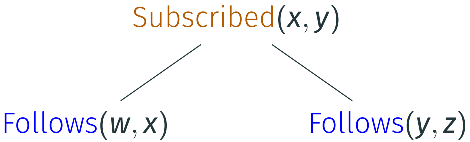
We can generalize **acyclic CQs** to arbitrary arity (= α-acyclic Gaifman hypergraph)

Join trees for acyclic CQs

Fact: a CQ is **acyclic** iff it has a **join tree**:

- The vertices are the **atoms** of the query
- For each variable, its occurrences form a **connected subtree**
- (For experts: width-1 generalized hypertree decomposition of the Gaifman hypergraph)

Take the query: $Q(w, x, y, z) : \text{Follows}(w, x) \wedge \text{Subscribed}(x, y) \wedge \text{Follows}(y, z)$



Yannakakis's algorithm for acyclic CQs

Theorem ([Yannakakis, 1981])

Given an acyclic CQ Q and database D, we can compute $Q(D)$ in time $O(|Q| \times (|D| + m))$, where m is the output size

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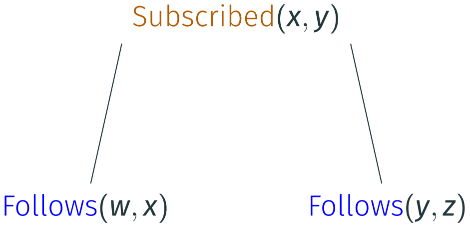

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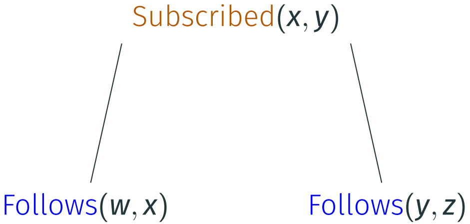

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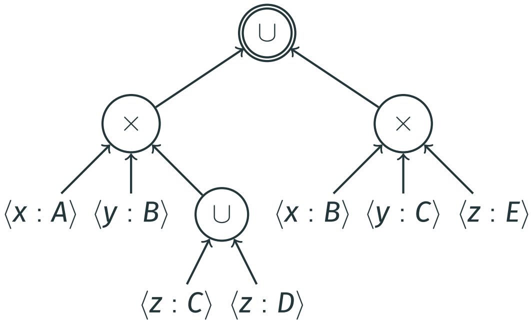
- Join together all relations to get the full result

Factorized representations [Olteanu and Závodný, 2015]



- Directed acyclic graph of **gates**
- **Output** gate: 

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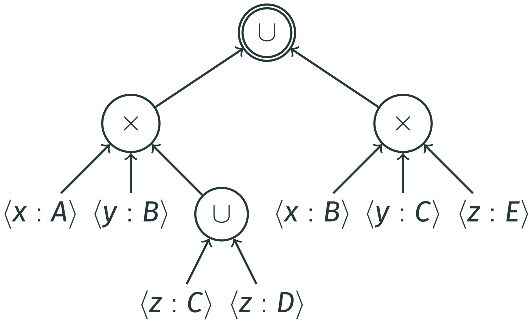

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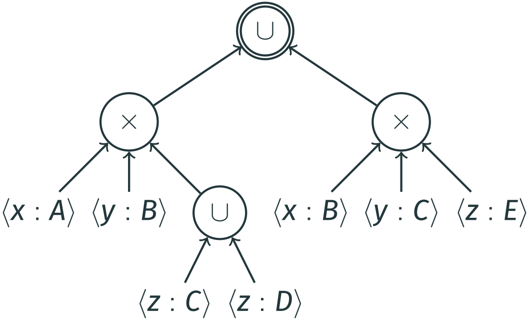

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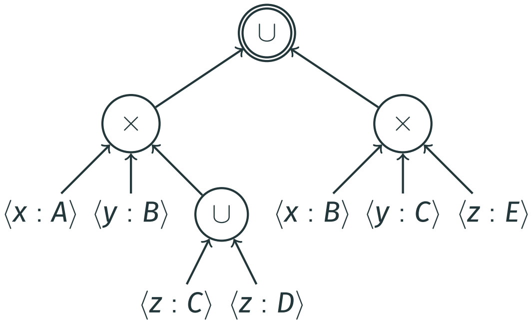

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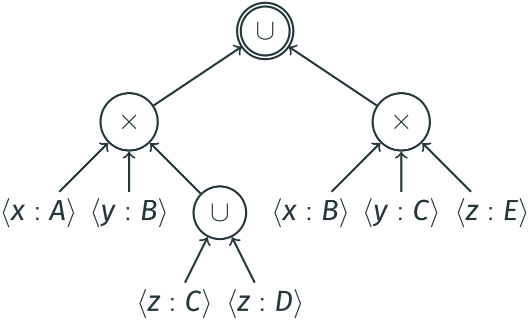

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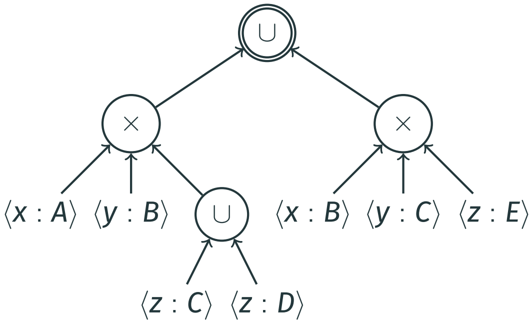

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Conditions on d-representations:

- **Deterministic**: all unions are disjoint
- **Normal**: no union is an input to a union

Enumerating tuples for normal deterministic d-representations

Task: Enumerate the tuples of the relation $R(g)$ captured by a gate g

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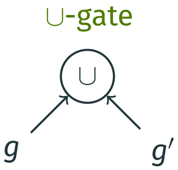

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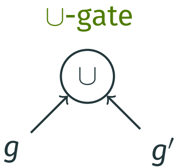

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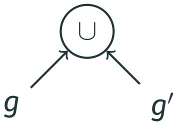

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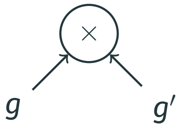

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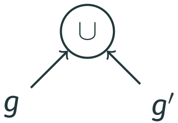

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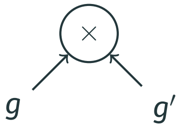

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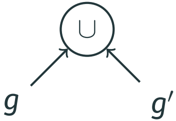

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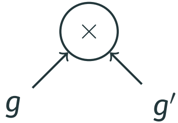

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Product: enumerate $R(g)$ and for each result t
  enumerate $R(g')$ and for each result t'
    concatenate t and t'

Enumerating tuples for normal deterministic d-representations (2)

Theorem ([Olteanu and Závodný, 2015], Theorem 4.11)

For any fixed schema $S = (x_1, \ldots, x_k)$, the tuples of a normal deterministic d-representation with schema S can be enumerated in constant delay

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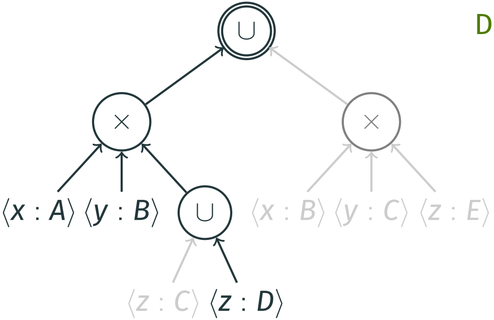

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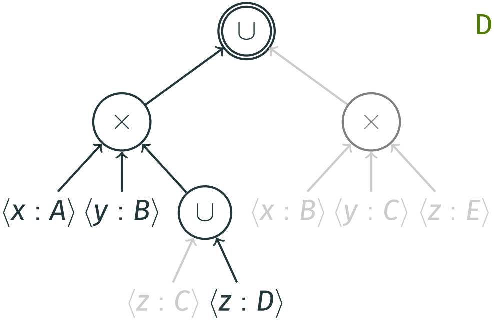

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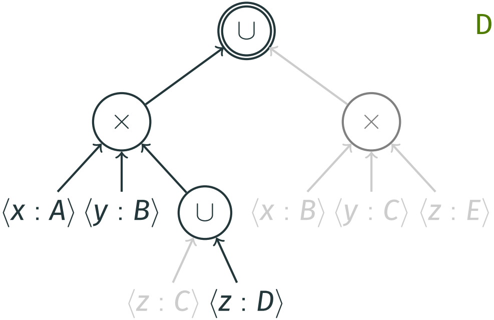

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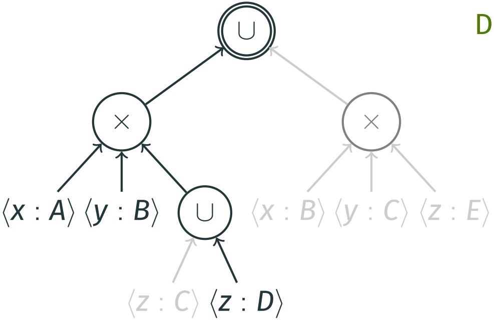

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- The inputs to union gates are not union gates (the representation is normal)
- Hence, the trace (gates visited to get a tuple) has size linear in the tuple arity, hence constant

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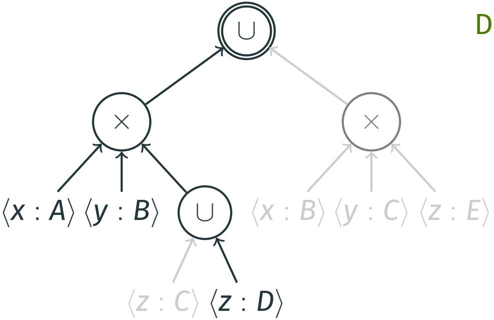

Delay analysis:

- Every product gate **nontrivially splits** the assignment to produce
- The inputs to union gates are **not union gates** (the representation is **normal**)
- Hence, the **trace** (gates visited to get a tuple) has size **linear in the tuple arity**, hence **constant**

Note: normal deterministic d-representations also allow us to:

- **Count** the number of solutions in linear time
- **Access** the i-th solution, given i, in logarithmic time

Factorized representations for full acyclic CQs

Theorem

*Given an acyclic CQ Q and database D, we can compute a **deterministic normal d-representation of $Q(D)$** in time $O(|Q| \times |D|)$ and hence enumerate $Q(D)$ with **linear preprocessing and constant delay***

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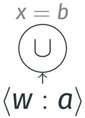

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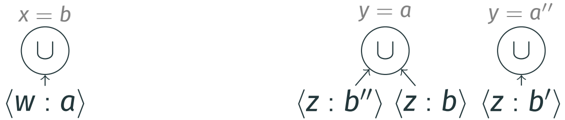

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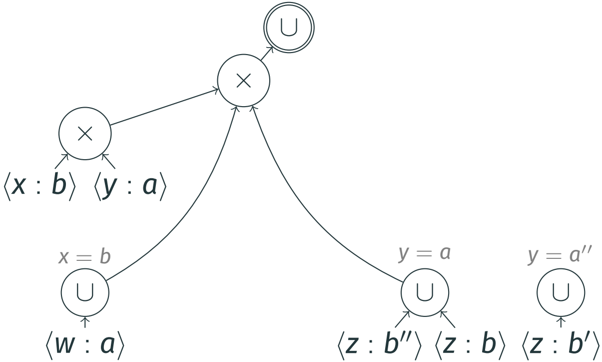

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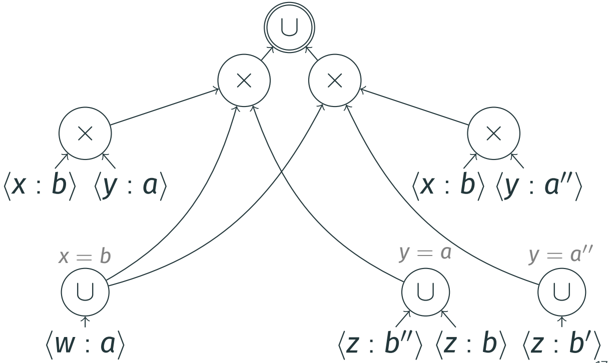

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Lower bounds for CQ enumeration

What about enumeration for non-free-connex CQs? Let us assume:

- The query is **minimized**: can always be done without loss of generality
- The query is **without self joins**: uses only each relation name once
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- *If Q is **acyclic but not free-connex**, then we can multiply n-by-n matrices in $O(n^2)$*
  - → *we can even do it on sparse matrices*

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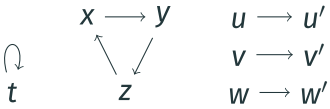

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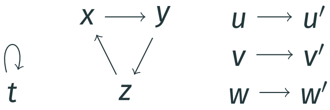

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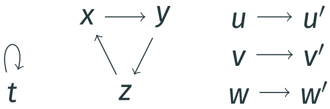

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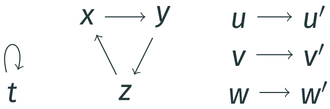

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Open problem: dichotomy on CQs with self-joins? see [Carmeli and Segoufin, 2023]

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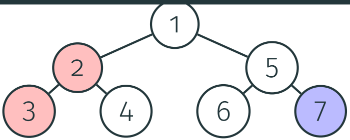
  - For monadic second-order logic (MSO) on trees [Bagan, 2006, Kazana and Segoufin, 2013]
- Now: review enumeration results for MSO, in terms of factorized representations (not necessarily normal or deterministic)

MSO query evaluation on trees

Data: a **tree** *T* where nodes have a color from an alphabet



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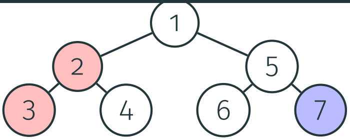

Query Q in monadic second-order logic (MSO)

- $P_{\bigcirc}(x)$ means "x is blue"
- $x \rightarrow y$ means "x is the parent of y"

Equivalent formalism: **tree automata**

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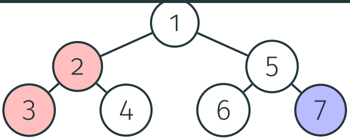

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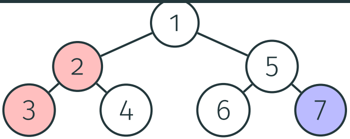

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Data complexity: Measure efficiency as a function of T (the query Q is **fixed**)

Results for MSO on trees

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Note that the d-representation is no longer normal, but we show with some effort:

Theorem (A., Bourhis, Jachiet, Mengel, ICALP'17)

For any fixed schema $S = (x_1, \ldots, x_k)$, the tuples of a deterministic d-representation with schema S can be enumerated with linear preprocessing and constant delay

Enumerating matches of nondeterministic document spanners

Data: a text *T*

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.  French national.  Appearance as
of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP a3nm@a3nm.net Affiliation Associate professor of computer
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test@example.com More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
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Enumerating matches of nondeterministic document spanners

Data: a text T

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Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.  French national.  Appearance as
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Query: a pattern P given as a regular expression

$$P := \texttt{␣ [a-z0-9.]}^{*}\ \texttt{@ [a-z0-9.]}^{*}\ \texttt{␣}$$

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Query: a **pattern** P given as a regular expression

$$P := \textvisiblespace\ \texttt{[a-z0-9.]}^*\ \texttt{@}\ \texttt{[a-z0-9.]}^*\ \textvisiblespace$$

Output: the list of **substrings** of T that match P:

$$[186, 200\rangle, \quad [483, 500\rangle, \quad \ldots$$

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Query: a **pattern** P given as a regular expression

$$P := \sqcup\ \texttt{[a-z0-9.]}^*\ \texttt{@}\ \texttt{[a-z0-9.]}^*\ \sqcup$$

Output: the list of **substrings** of T that match P:

$$[186, 200\rangle,\quad [483, 500\rangle,\quad \dots$$

Goal:

- be **very efficient** in T (constant-delay)
- be **reasonably efficient** in P (polynomial-time)

Results for nondeterministic document spanners

Theorem (A., Bourhis, Mengel, Niewerth, ICDT'19; see also PODS'19)

We can enumerate all matches of an input nondeterministic automaton with captures on an input text with

- *Preprocessing linear in the text and polynomial in the automaton*
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- *Preprocessing **linear** in the text and **polynomial** in the automaton*
- *Delay **constant** in the text and **polynomial** in the automaton*

- Generalizes earlier result on **deterministic automata** [Florenzano et al., 2018]
- To make the algorithm polynomial in the **(nondeterministic) automaton**, we need efficient enumeration for a certain kind of **non-deterministic d-representations**

Other enumeration settings

Efficient enumeration is now being studied in **many settings** in data management (sometimes with weaker guarantees than linear preprocessing and constant delay):

- For **regular path queries** [Martens and Trautner, 2018, David et al., 2024]
- For **compressed structures**:
  - Compressed trees [Lohrey and Schmid, 2024]
  - SLP-compressed documents [Schmid and Schweikardt, 2021, Muñoz and Riveros, 2023]
- For **visibly pushdown languages** [Muñoz and Riveros, 2022]
- For **context-free languages** with annotations [Peterfreund, 2021], [A., Jachiet, Muñoz, Riveros, 2023]

There are also **software implementations** [Riveros et al., 2023]

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Introduction: From enumeration to more general tasks

Sometimes, we want **more** than enumerating query results in an unspecified order:

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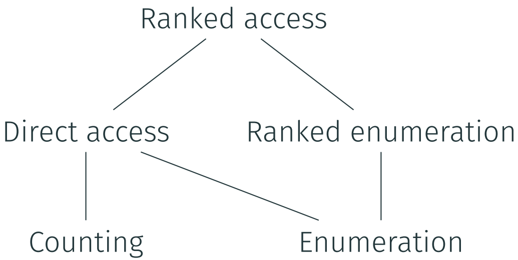

(Adapted from [Carmeli, 2023])

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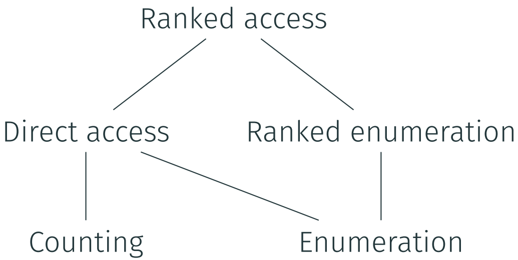

(Adapted from [Carmeli, 2023])

- **Direct access**: getting the i-th answer
- **Counting** the answers
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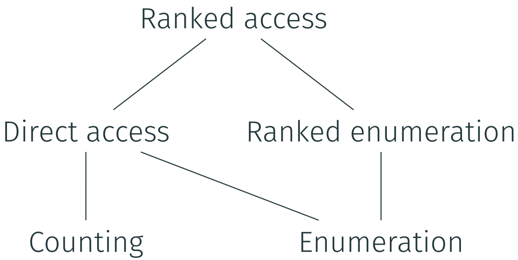

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Another question: **maintain** an enumeration structure under **updates** to the data

Results on ranked enumeration / ranked access

For **CQs and UCQs**:

- Most works study self-join-free CQs under **lexicographic orders** and aim for **logarithmic** access time or delay:
  - Characterization of **tractable orders** for CQs [Carmeli et al., 2023]

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For **MSO** queries on trees:

- **Ranked enumeration** shown with **logarithmic delay** on **words** [Bourhis et al., 2021]
- Recent extension to **trees** [A., Bourhis, Capelli, Monet, 2024]

Incremental maintenance of enumeration structures

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- Notion of **q-hierarchical CQs** that admit linear preprocessing and constant delay enumeration and **constant-time updates**; lower bounds [Berkholz et al., 2017]
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For **MSO** queries on trees, aiming for **logarithmic** update time:

- On **words**, linear preprocessing and constant delay enumeration is possible under **insert/delete updates** [Niewerth and Segoufin, 2018]
- On **trees**, linear preprocessing and constant delay enumeration is possible under **substitution updates** [A., Bourhis, Mengel, 2018] and possibly more

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Summary and future work

- We have seen enumeration algorithms to produce query answers in streaming
  - $\rightarrow$ Ideally, we want linear preprocessing and constant delay
- Modular approach: compute a factorized representation of the results
- Tractable enumeration is possible for free-connex CQs and for MSO queries on trees
- Ongoing research: ranked enumeration, ranked access, incremental maintenance...

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- Enumerating **diverse** / **representative** solutions?
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Thanks for your attention!

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