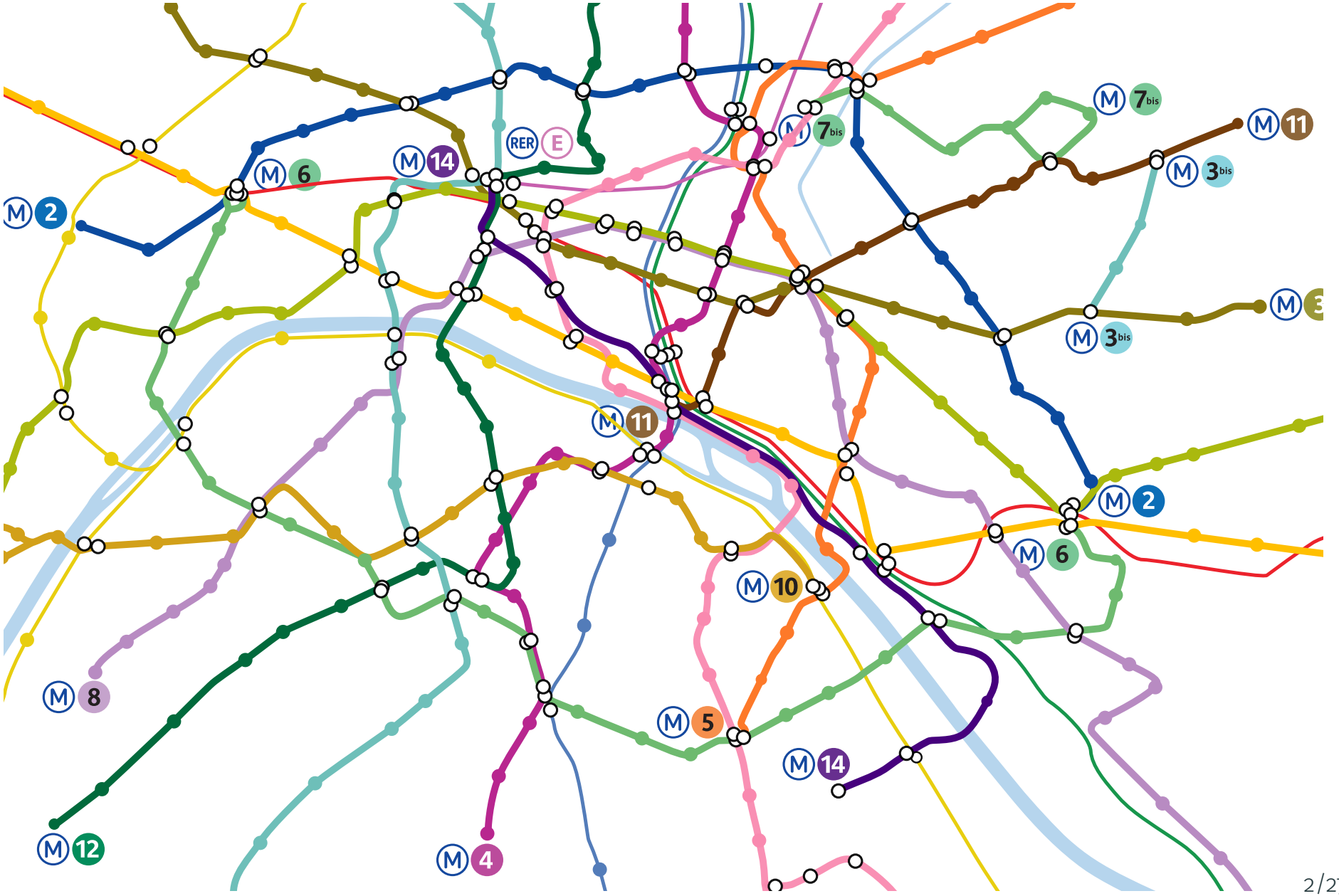
Leveraging the structure of uncertain data

Antoine Amarilli

May 26, 2017

Example application: Subway routing



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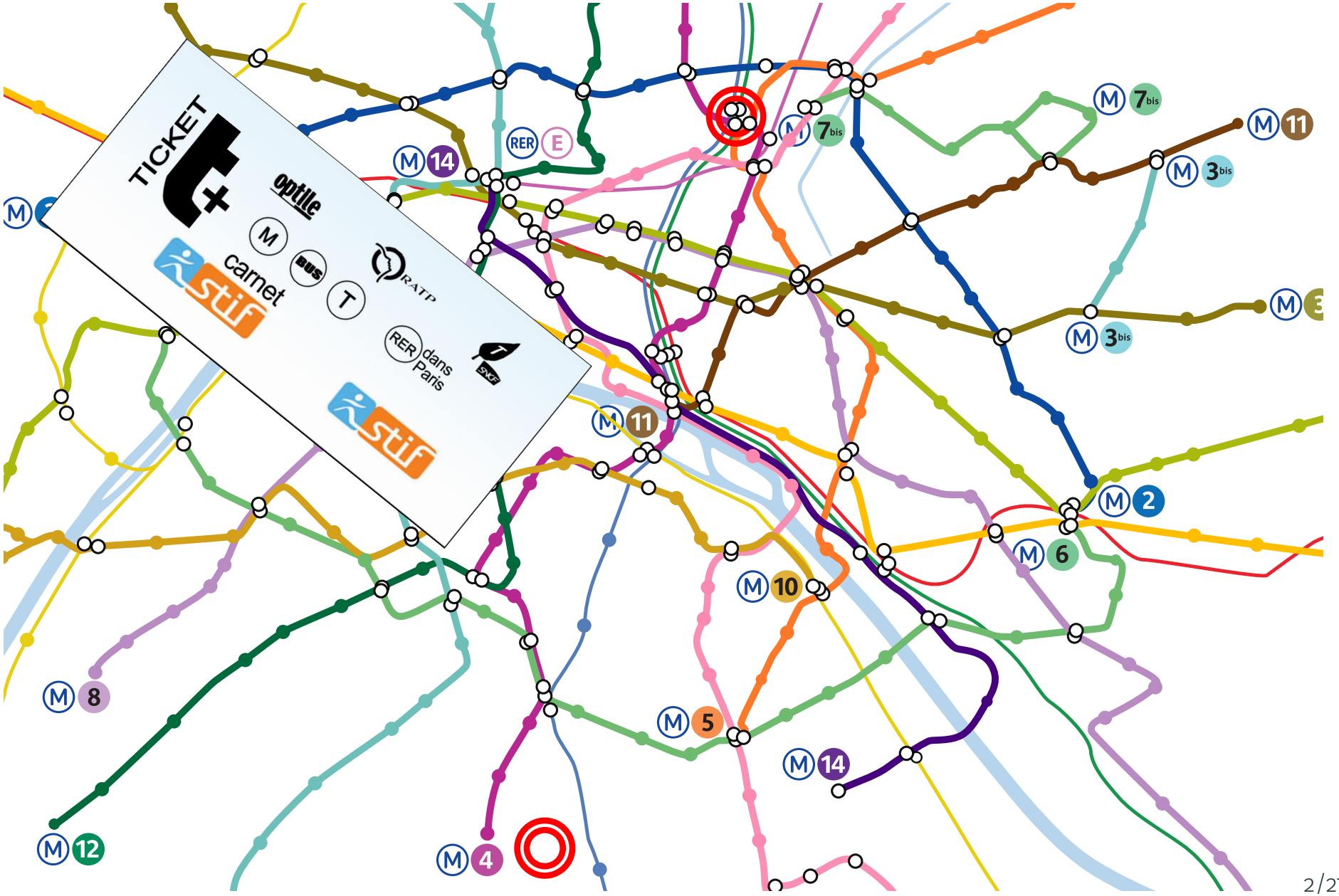

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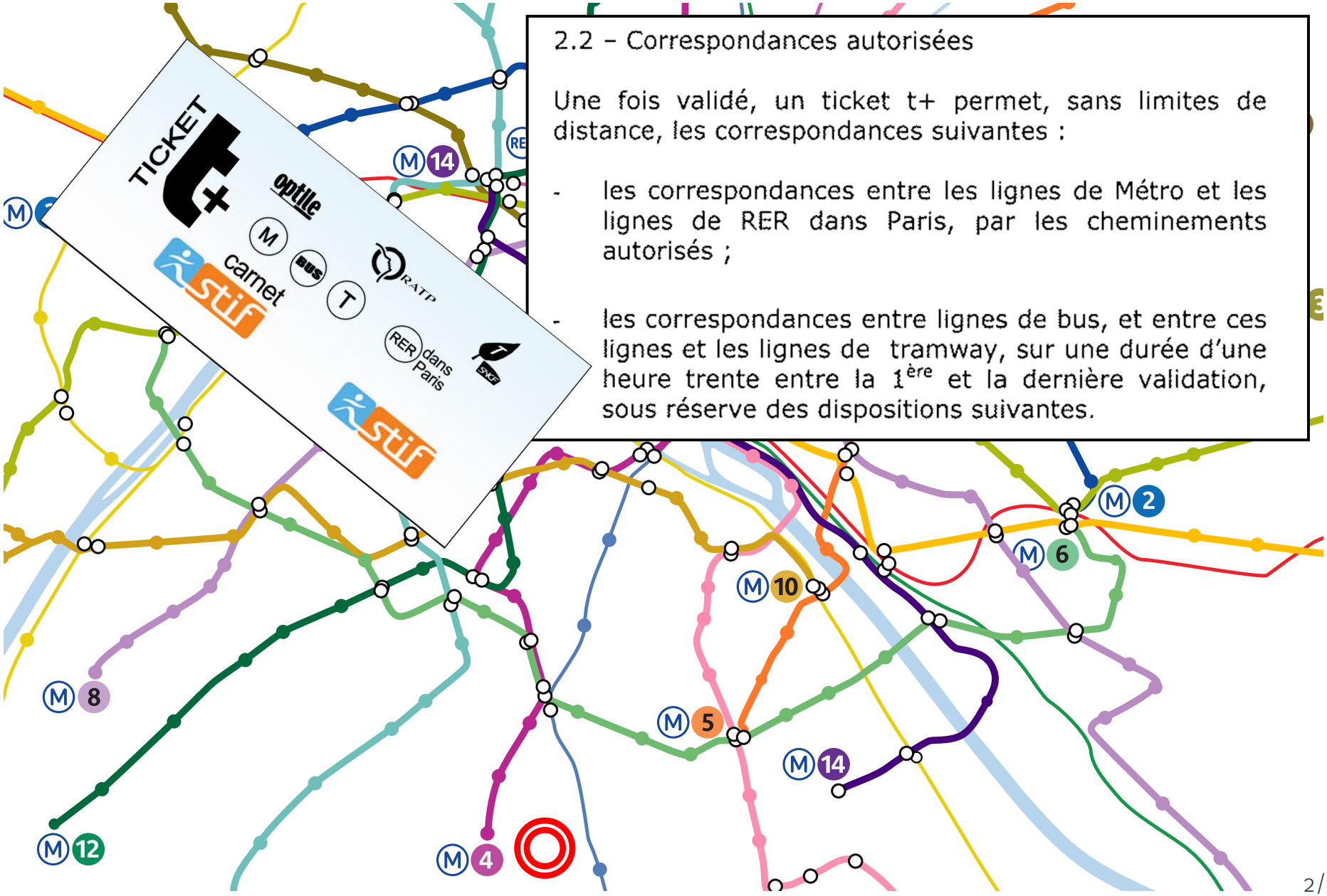

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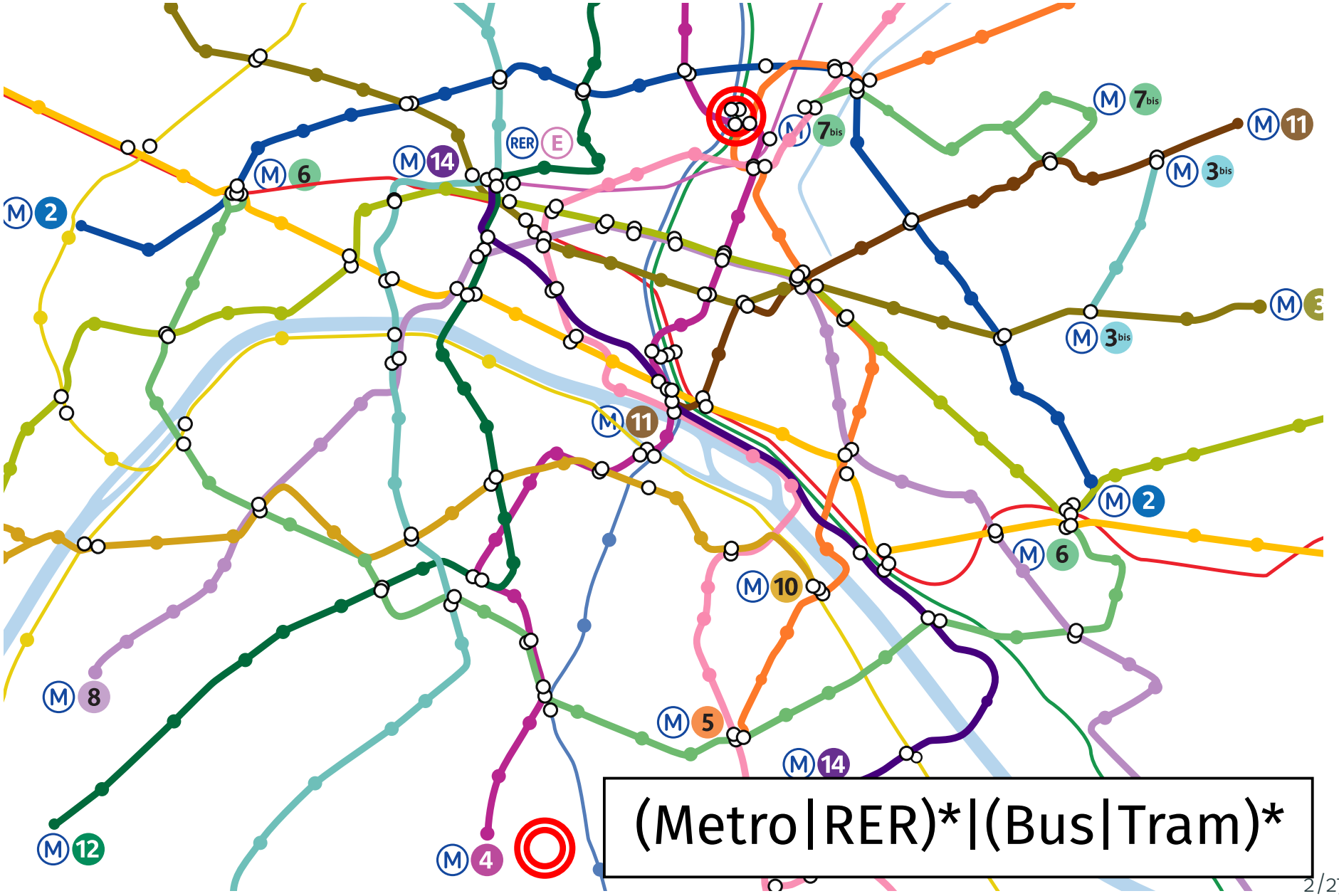
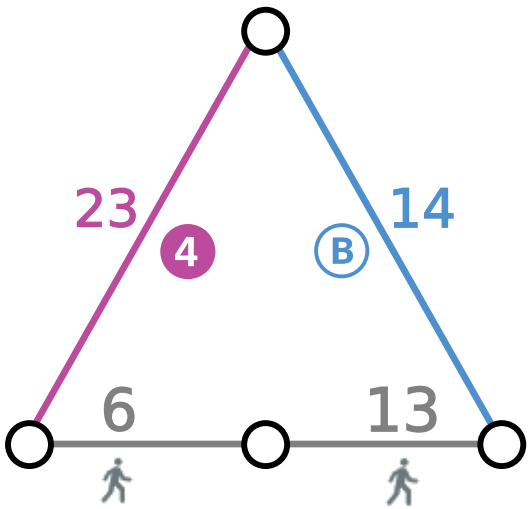


Database

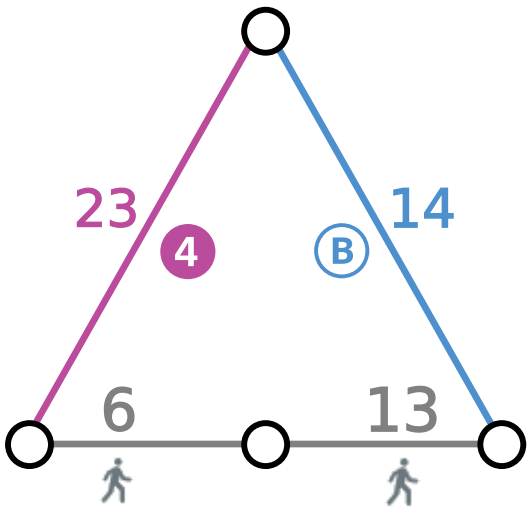


- (Hyper)graph
- Collection of ground facts

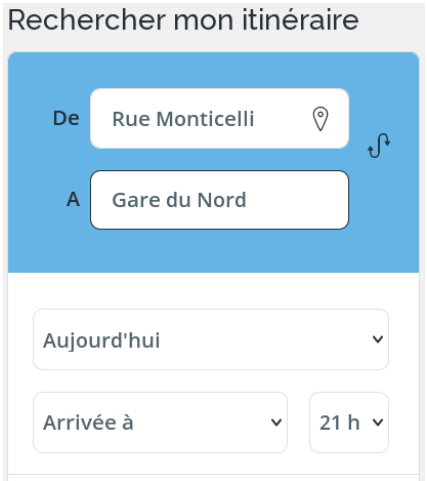
$G(aa_1, ab_2)$, $G(ab_2, ac_3)$,
$S(aa_1, m_4)$, $S(ab_2, r_B)$, ...

Database theory and query evaluation

Database



Query



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  (Metro|RER)*
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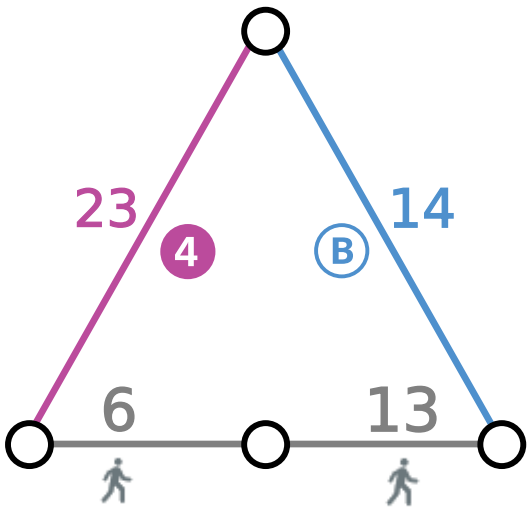

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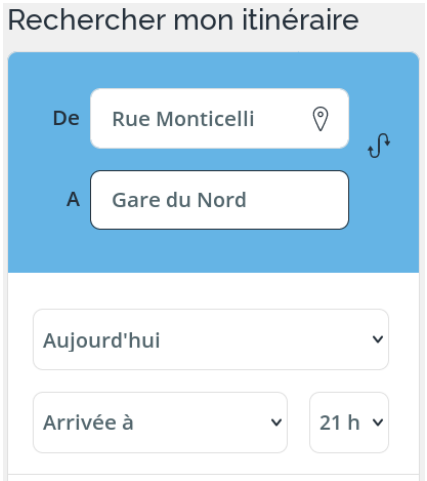

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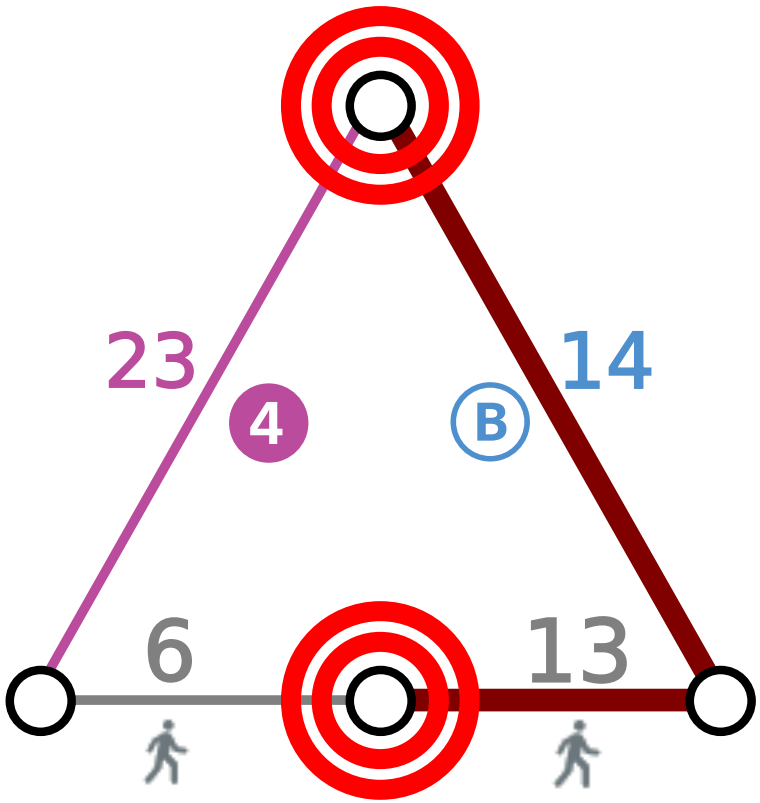
Result

- TRUE/FALSE

↔ Model checking

Probabilistic query evaluation



Probabilistic query evaluation

Panne du RER B : trafic interrompu entre Paris et Roissy, des TGV en renfort

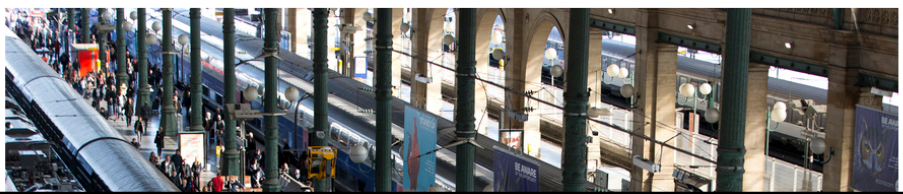
> Transports | 06 décembre 2016, 9h56 | MAJ : 06 décembre 2016, 17h03 |

Probabilistic query evaluation

Panne du RER B : trafic interrompu entre

Paris : pourquoi il y a autant de perturbations sur le RER B et à Gare du Nord

La circulation de l'ensemble des trains au départ de gare du Nord est totalement interrompue à la suite d'une panne électrique.



Probabilistic query evaluation

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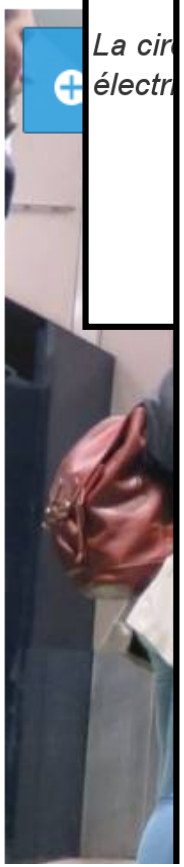


INCIDENT SUR LE RER B : QUE S'EST-IL PASSÉ CE MATIN ?

Malaise voyageur et application des mesures de sécurité : pour quelles raisons le trafic a-t-il été perturbé ce matin sur la ligne B ?

Pour beaucoup, le voyage a été difficile ce matin. Au fil de vos réactions sur Twitter notamment, je constate que les raisons de ces perturbations ne paraissent pas cohérentes. Je tiens donc à vous apporter des premiers

Probabilistic query evaluation

Panne du RER B : trafic interrompu entre

Paris : pourquoi il y a autant de perturbations sur le

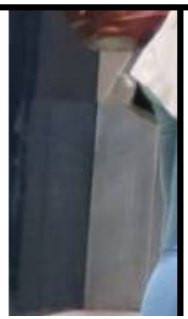
INCIDENT SUR LE RER B · QUE

ACTUALITÉS

Le RER B en panne, les voyageurs n'ont pas eu d'autre choix que de descendre sur les voies

Alors que la circulation alternée a augmenté le nombre de voyageurs dans les transports en commun, le RER B s'est retrouvé à l'arrêt.

06/12/2016 11:57 CET | **Actualisé** 06/12/2016 20:14 CET



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Probabilistic query evaluation

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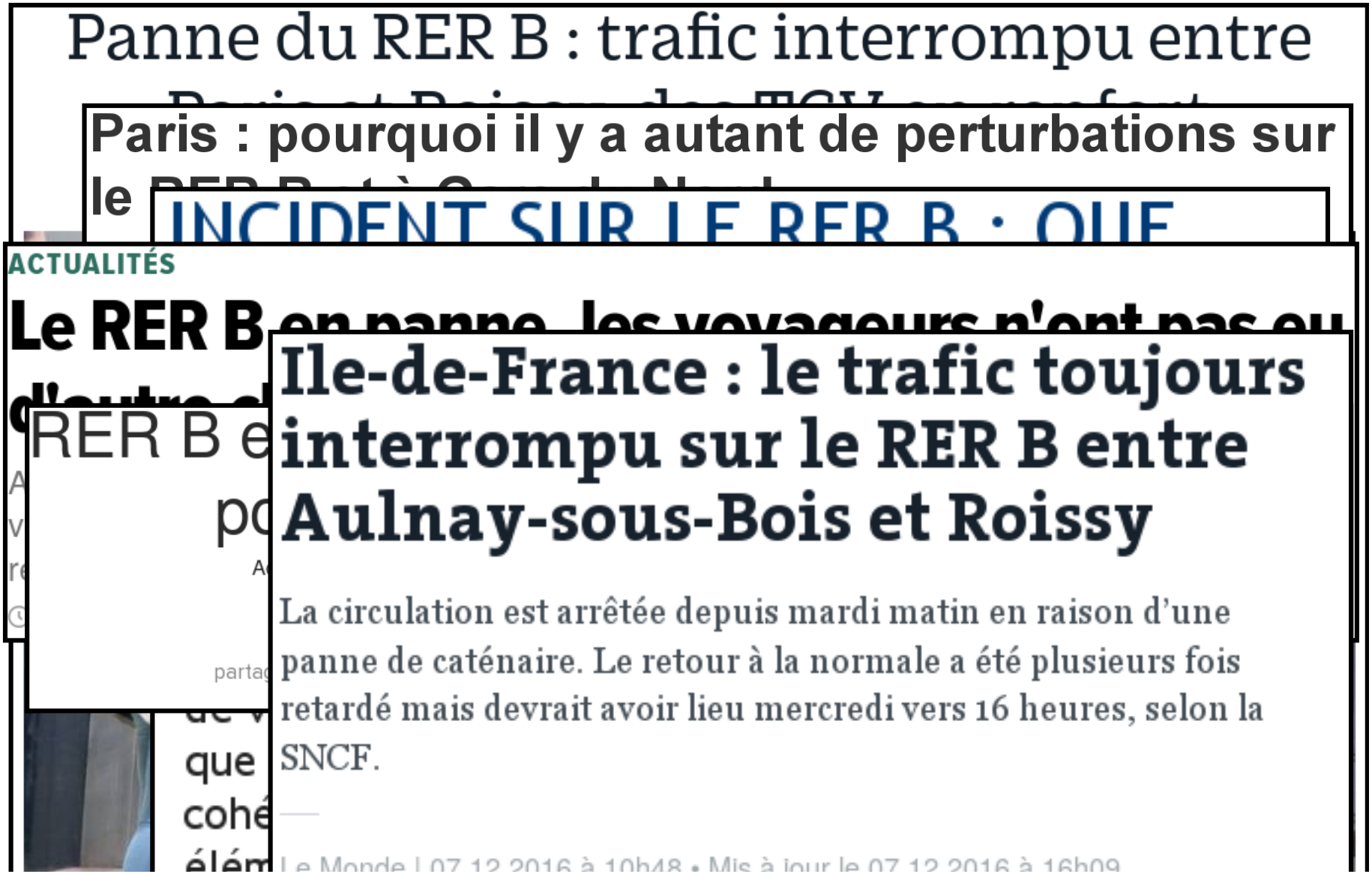

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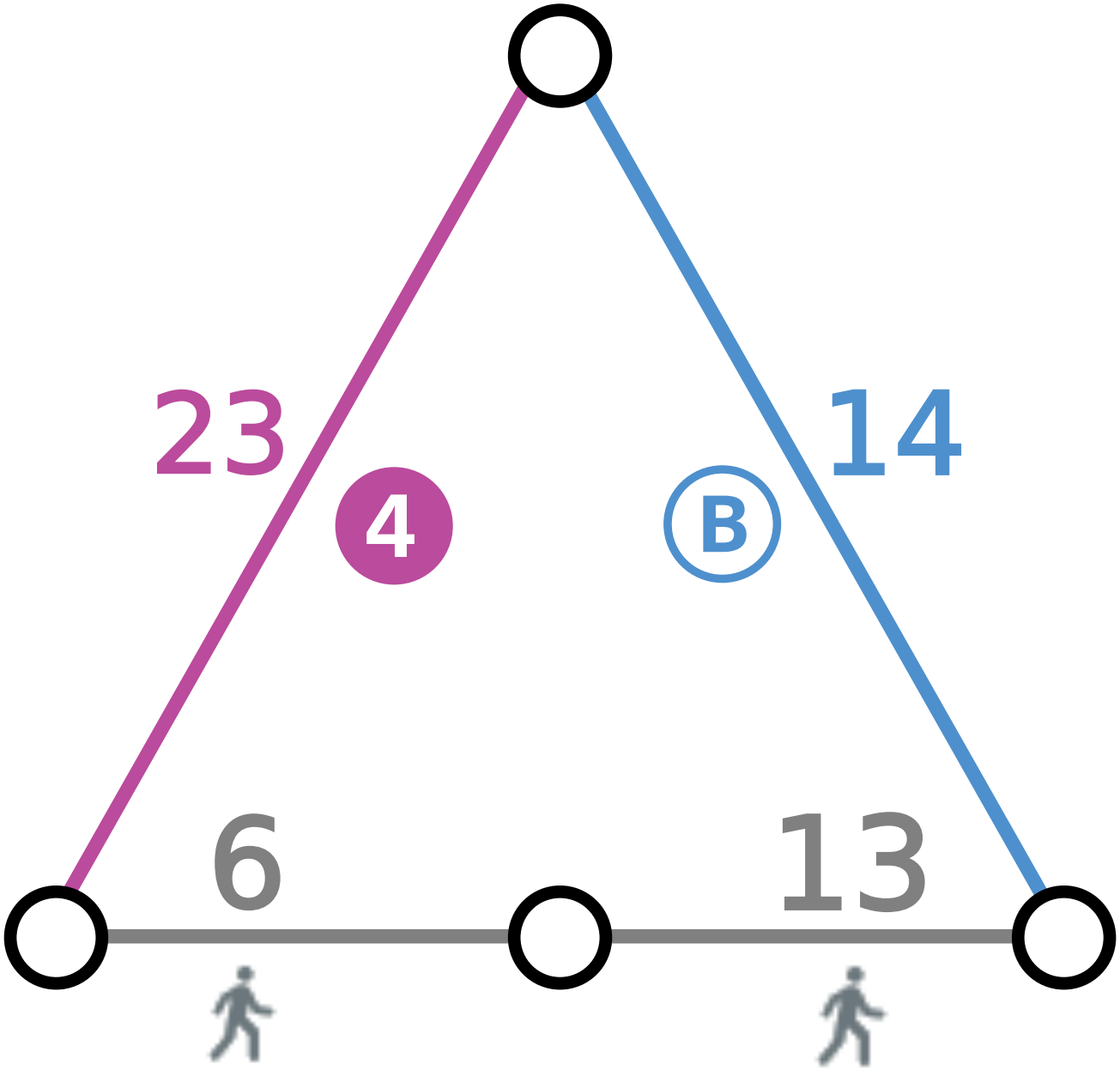

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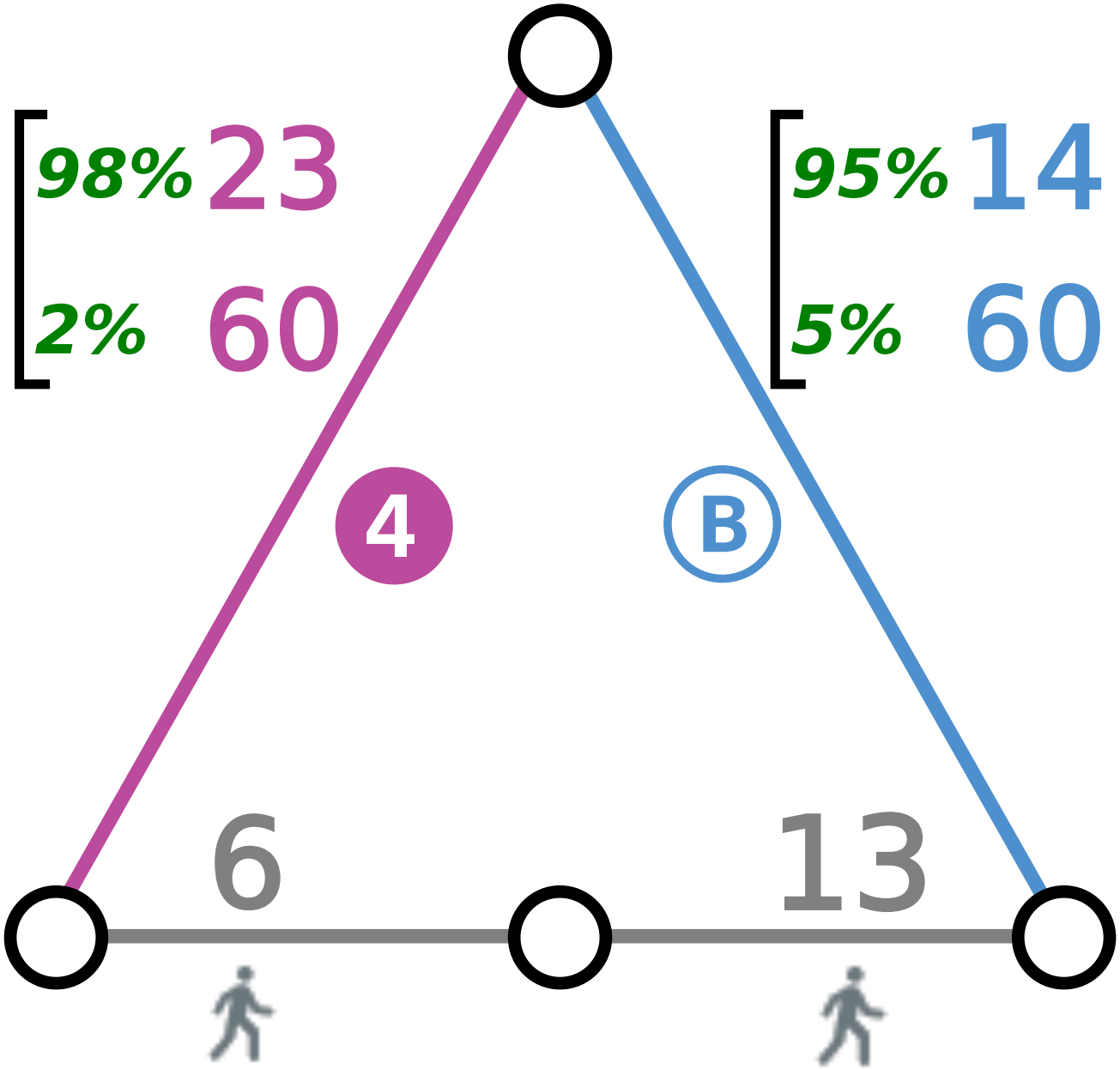
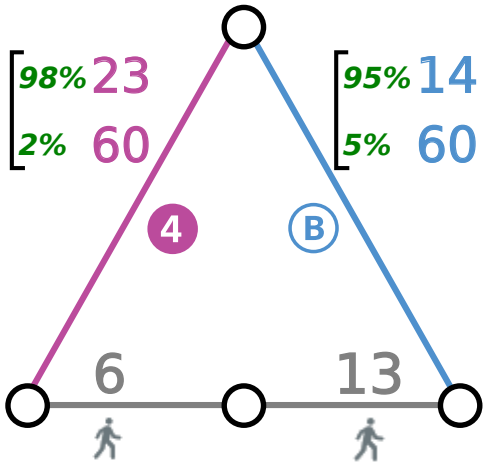


Probabilistic query evaluation

Probabilistic database



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- Collection of ground facts

+ independent probabilities

Probabilistic query evaluation

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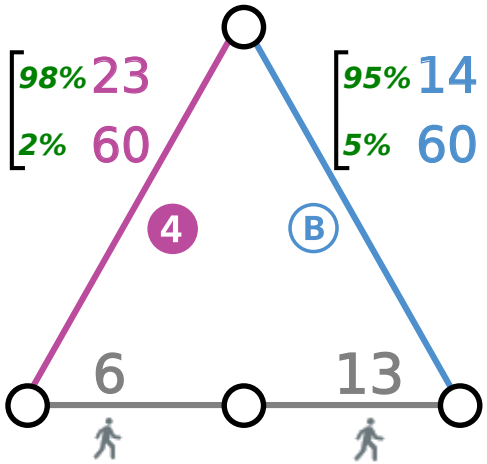
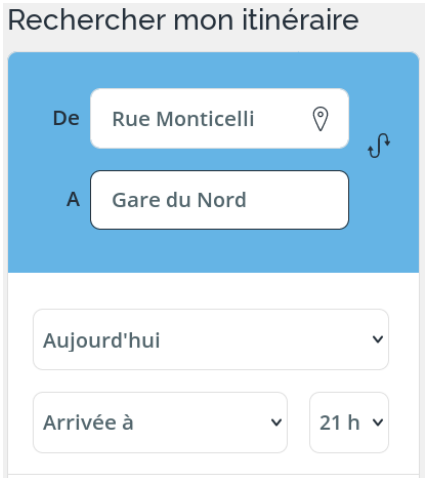


+

Query



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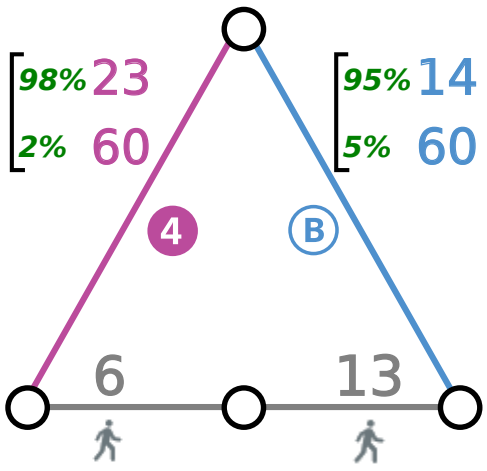

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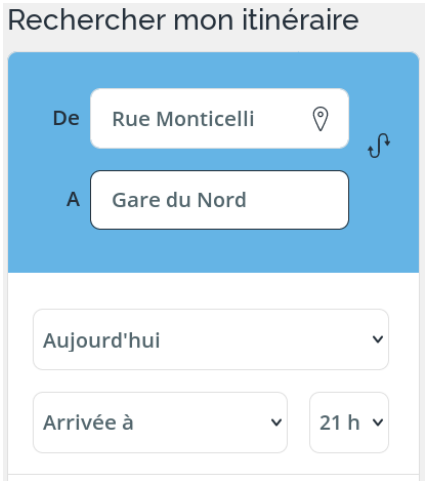

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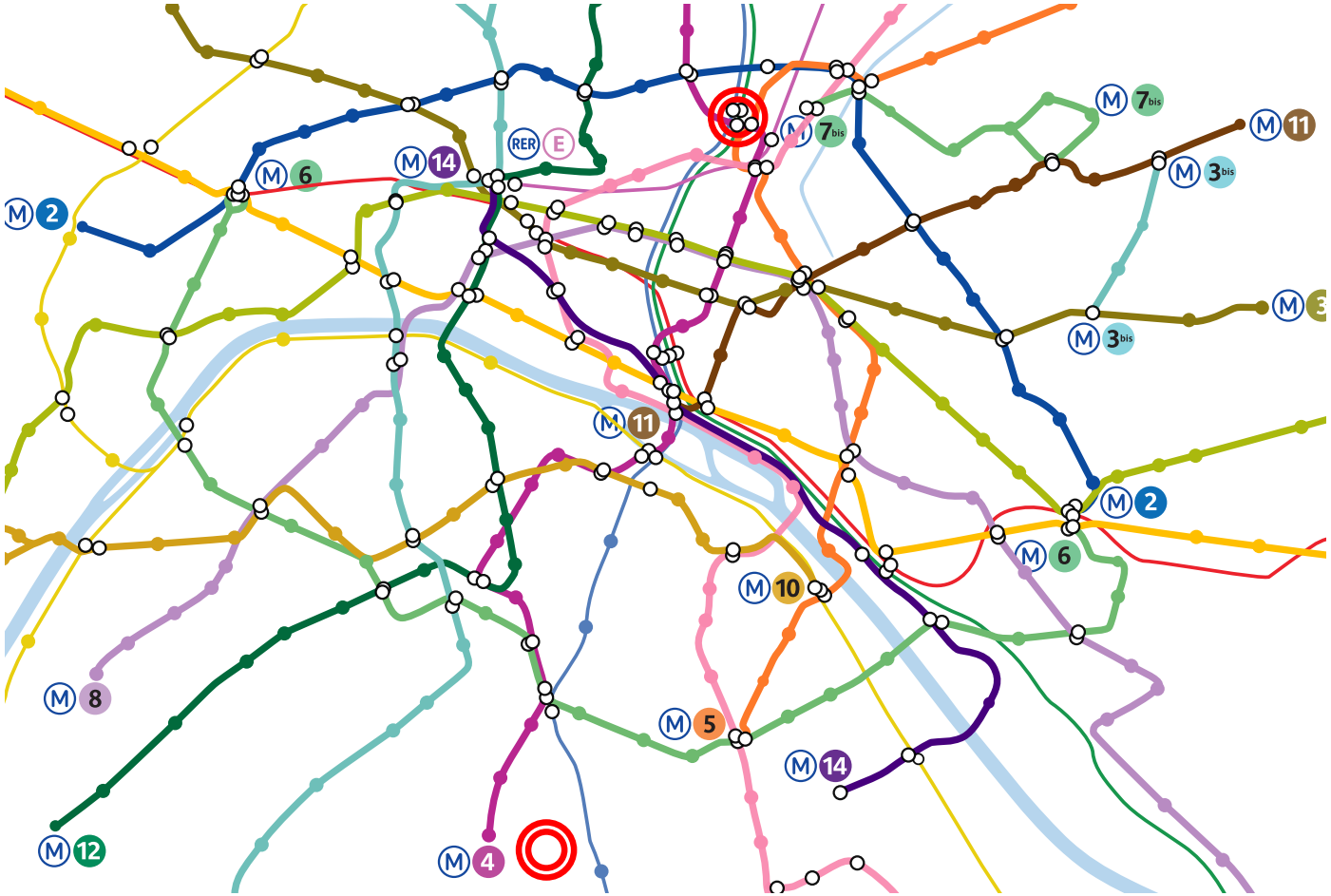
Probabilistic Result

proba to be on time: **98%**

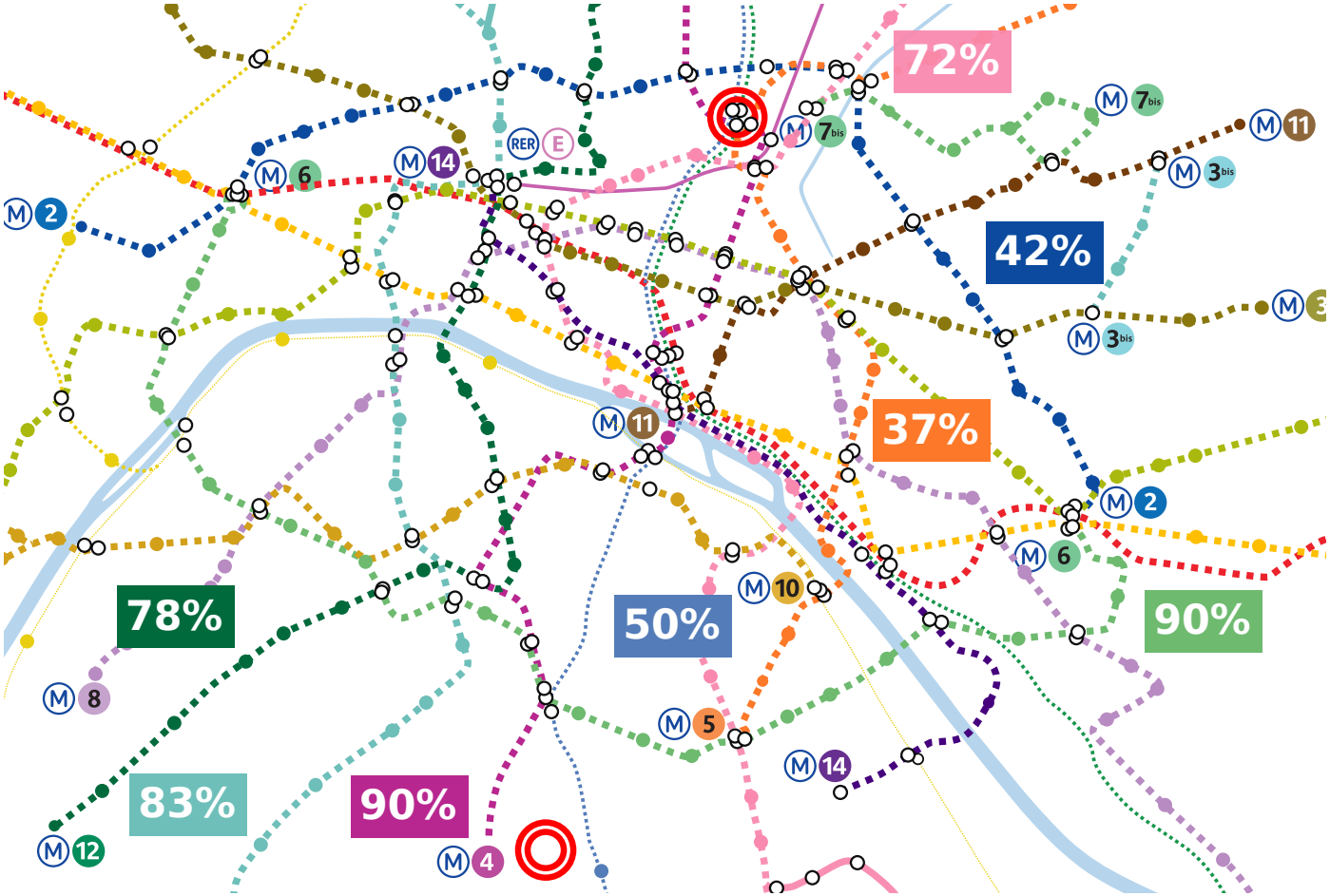
- Probability according to the input distribution

Computational complexity



- Computing paths on a large graph:
  - → Well-studied problem, efficient algorithms

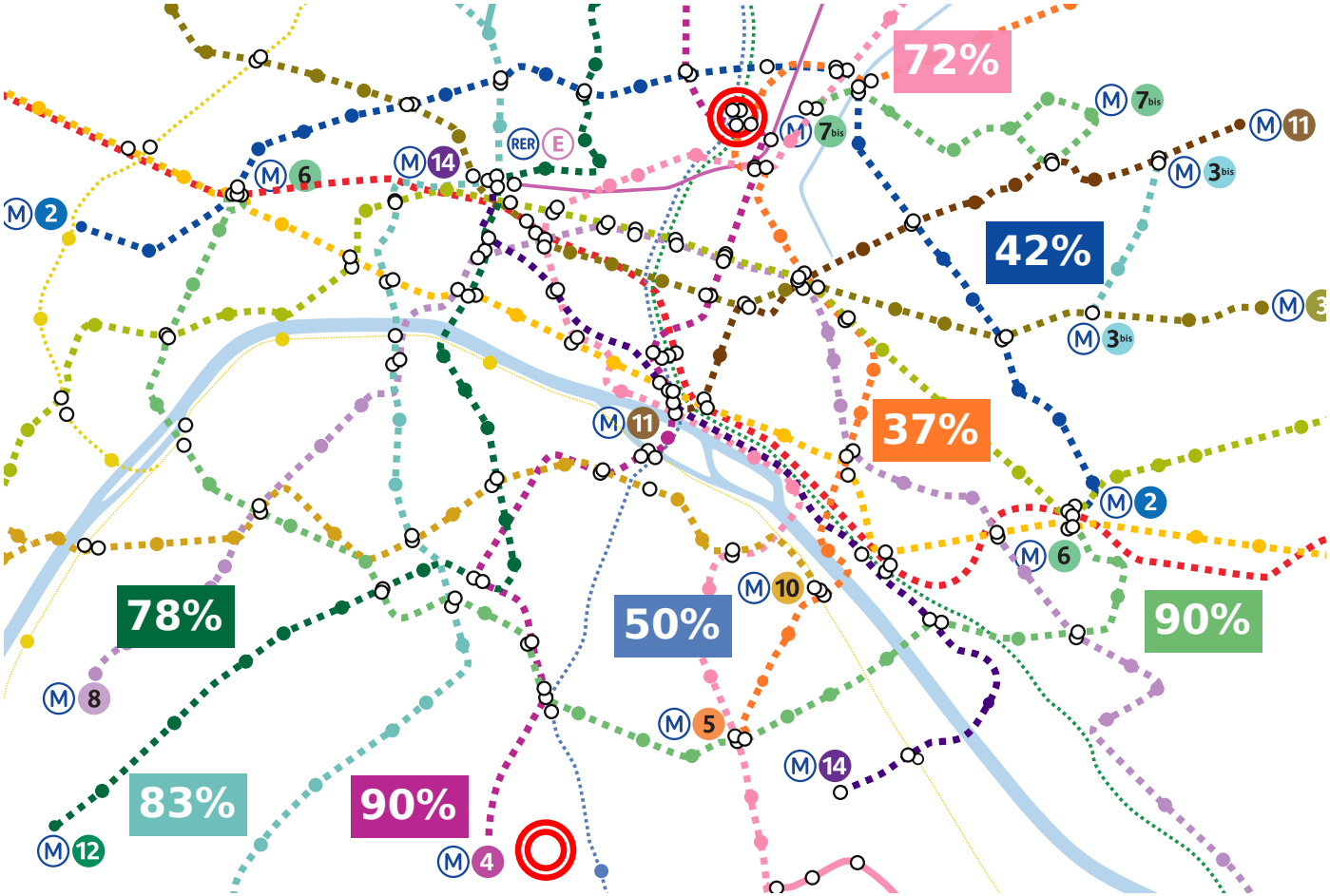
Computational complexity



- Computing paths on a large probabilistic graph:

  → ???

Computational complexity



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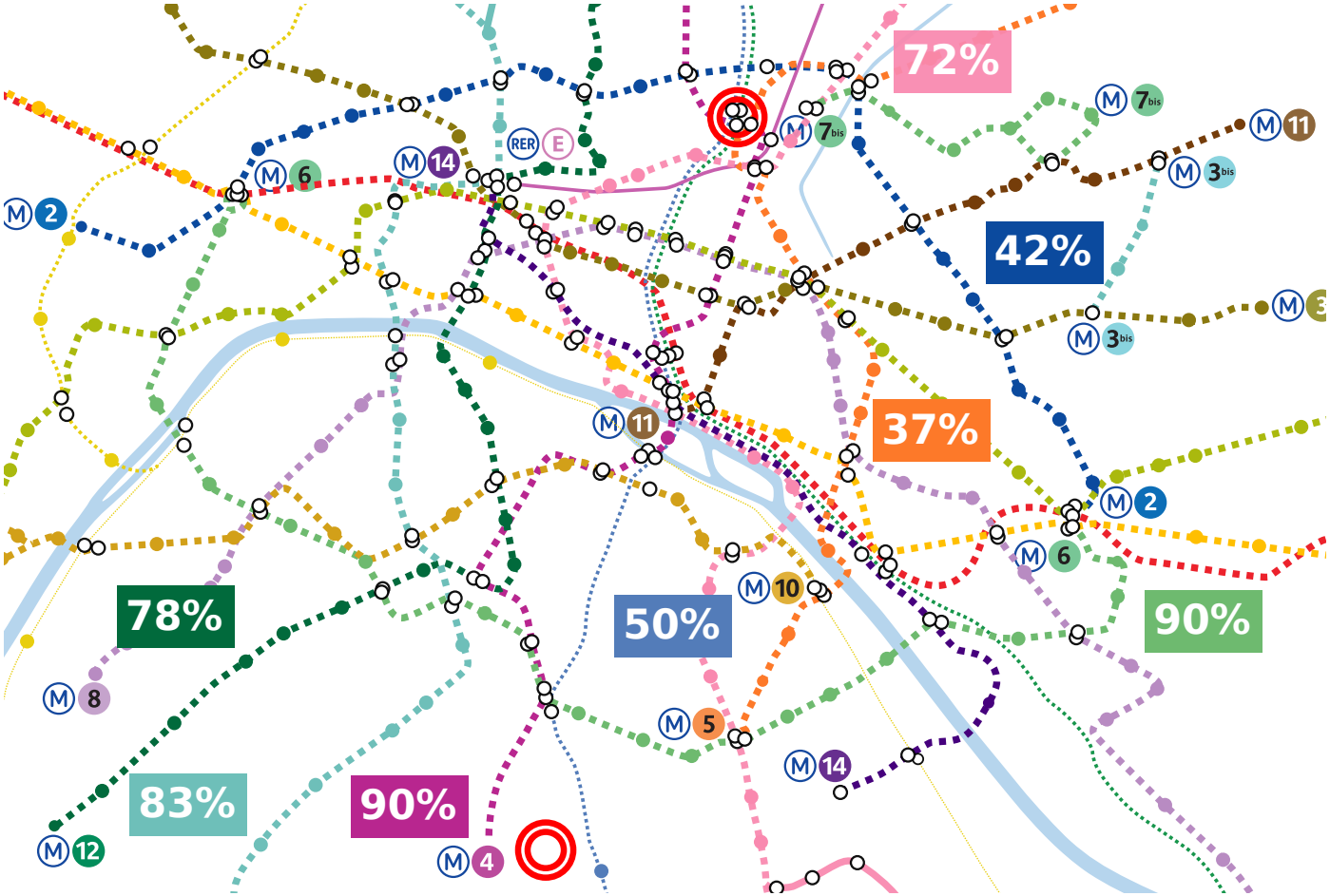
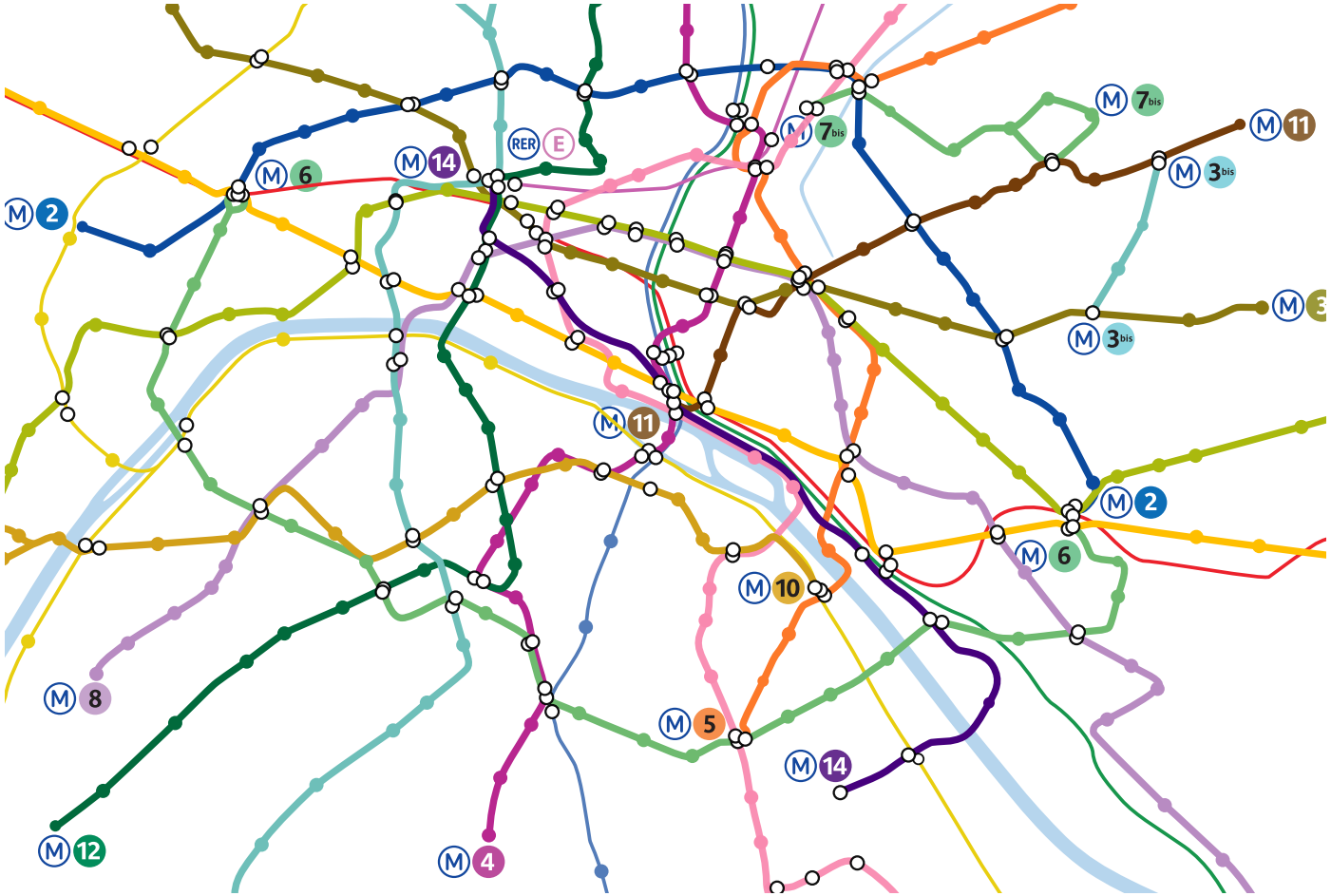


- Computing paths on a large probabilistic graph:
  - → Exponential number of possibilities
  - → #P-hard computational complexity in the database

Idea: use the structure of data



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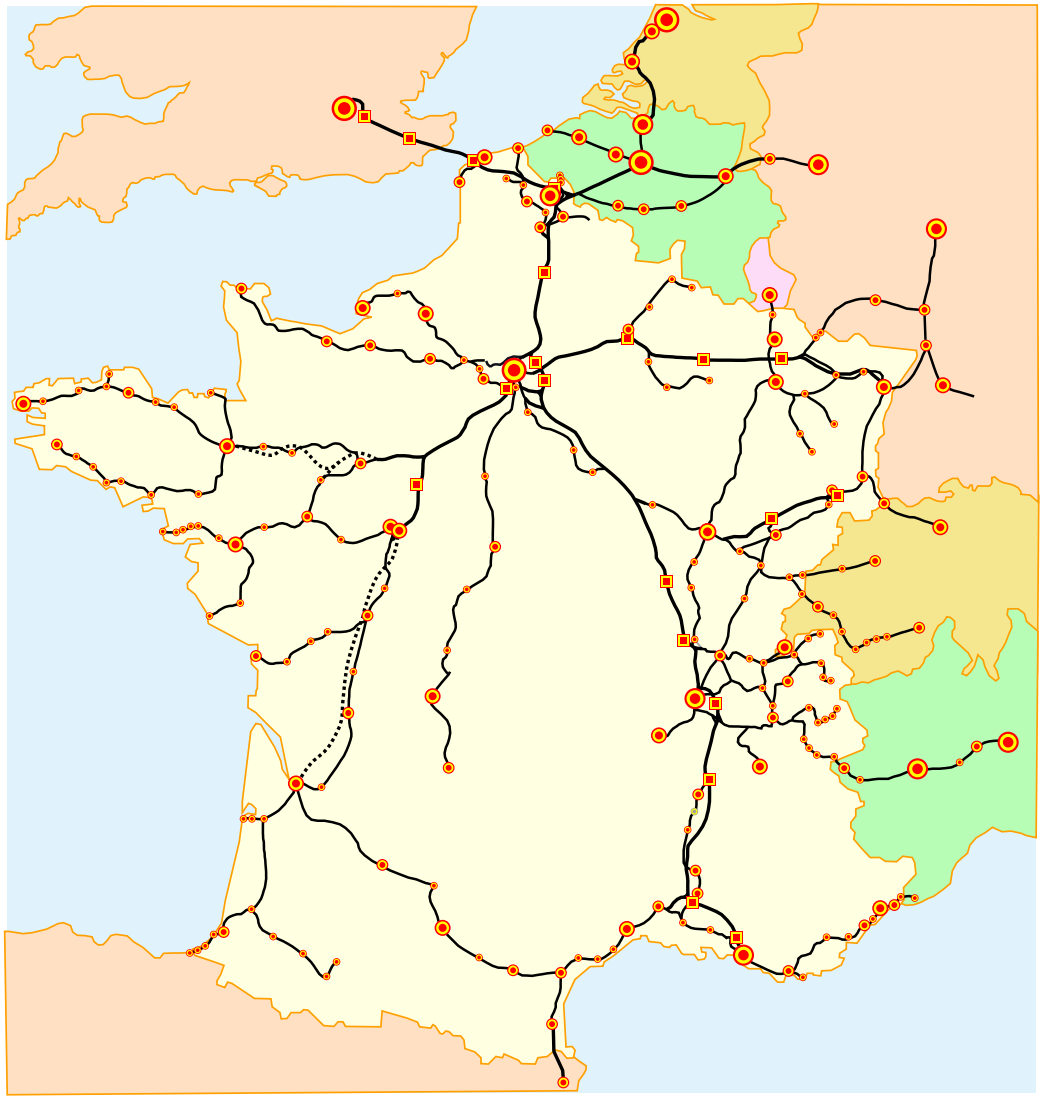

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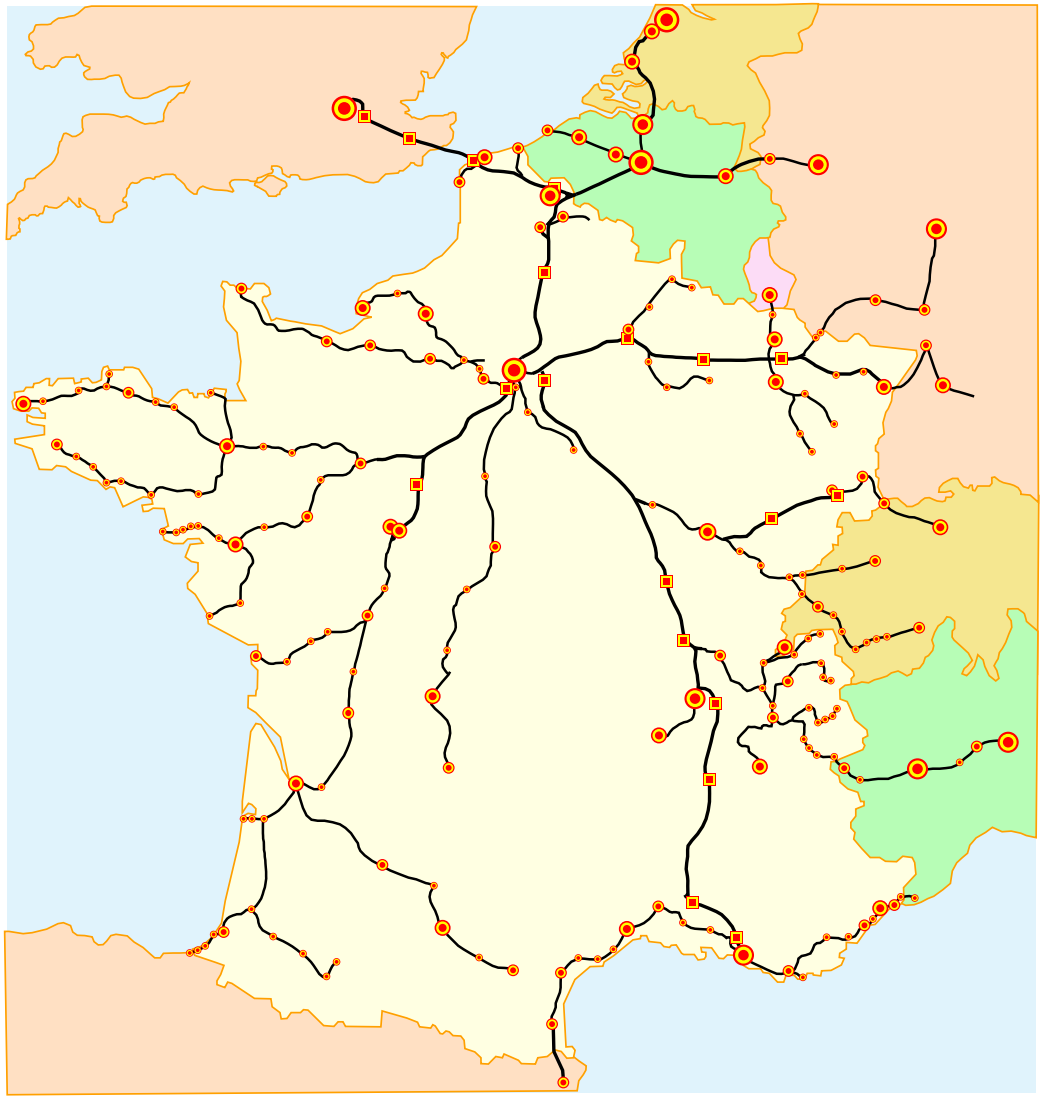

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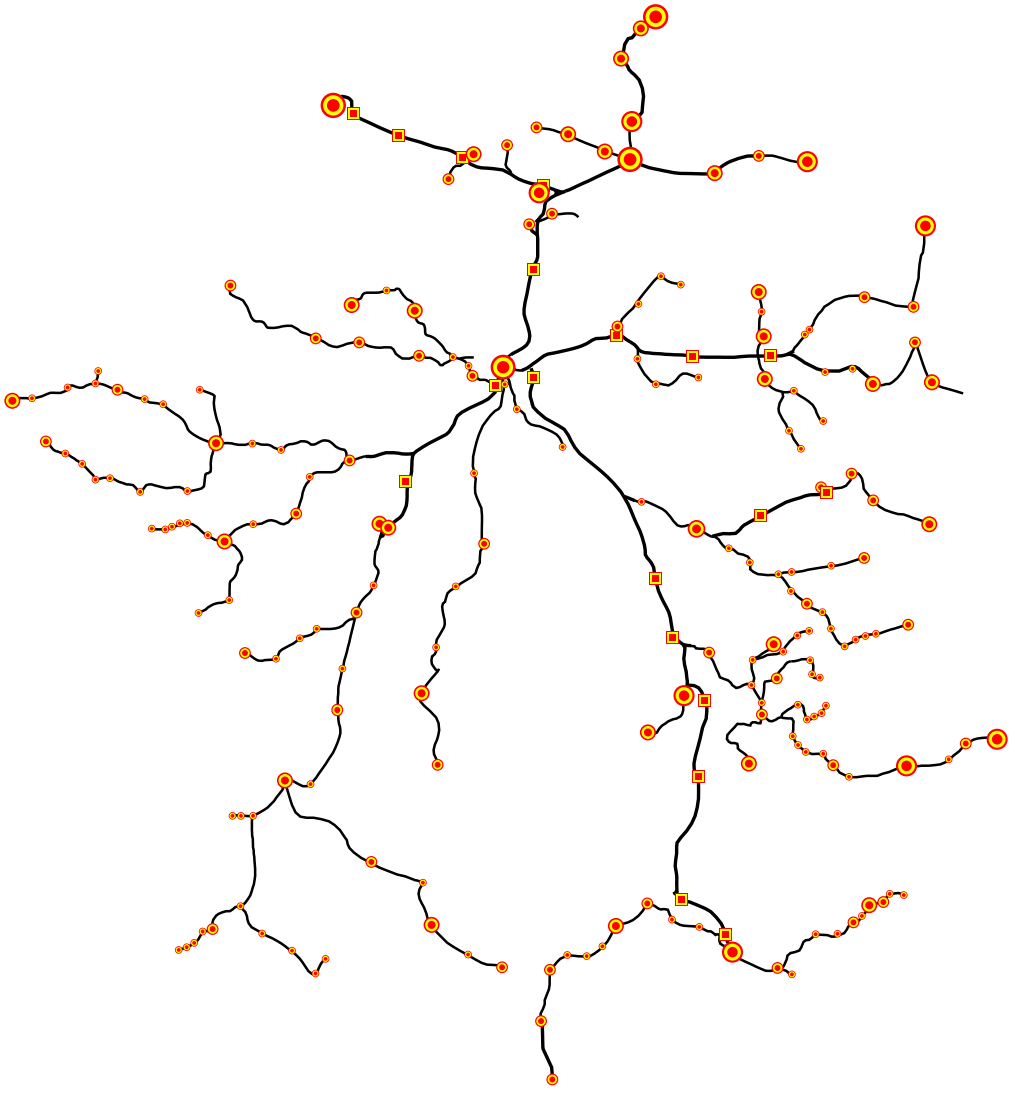

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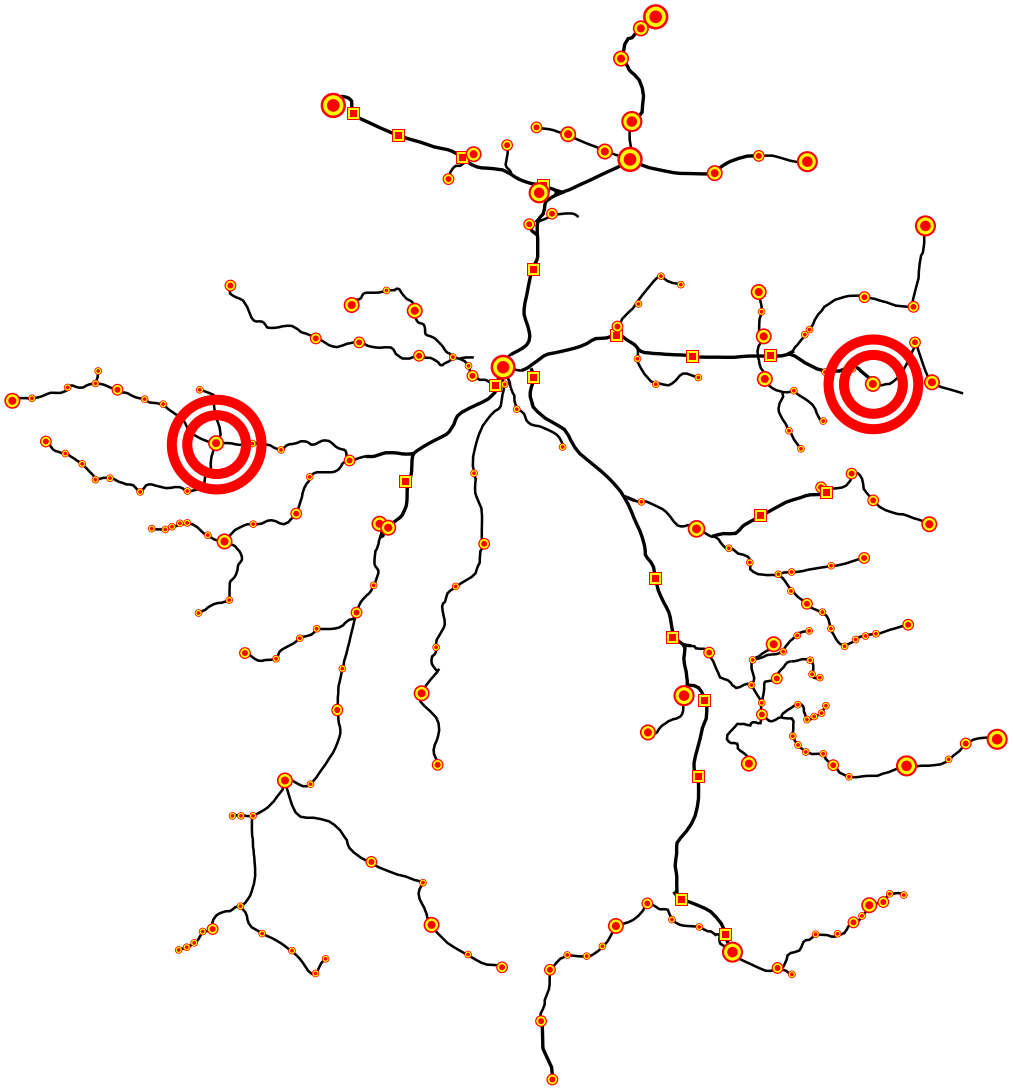

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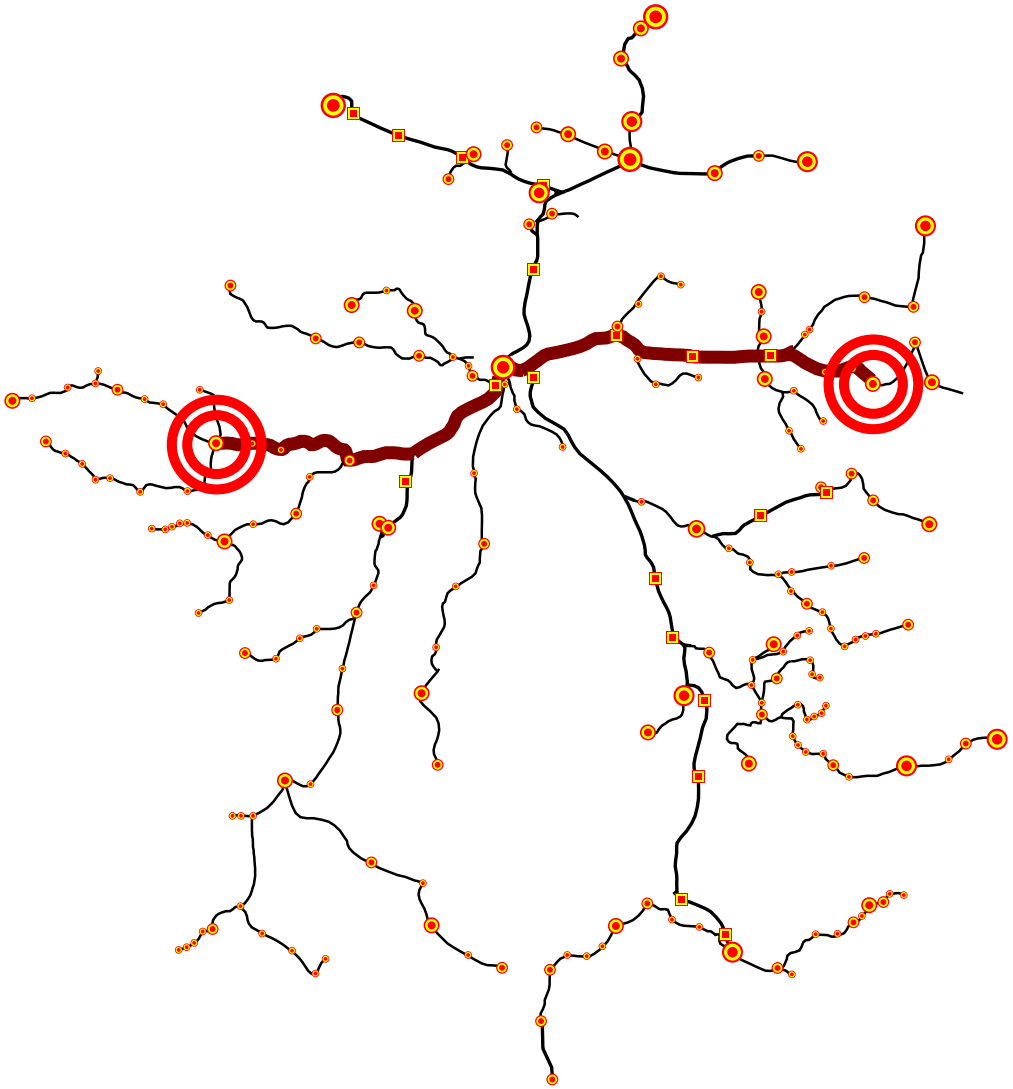

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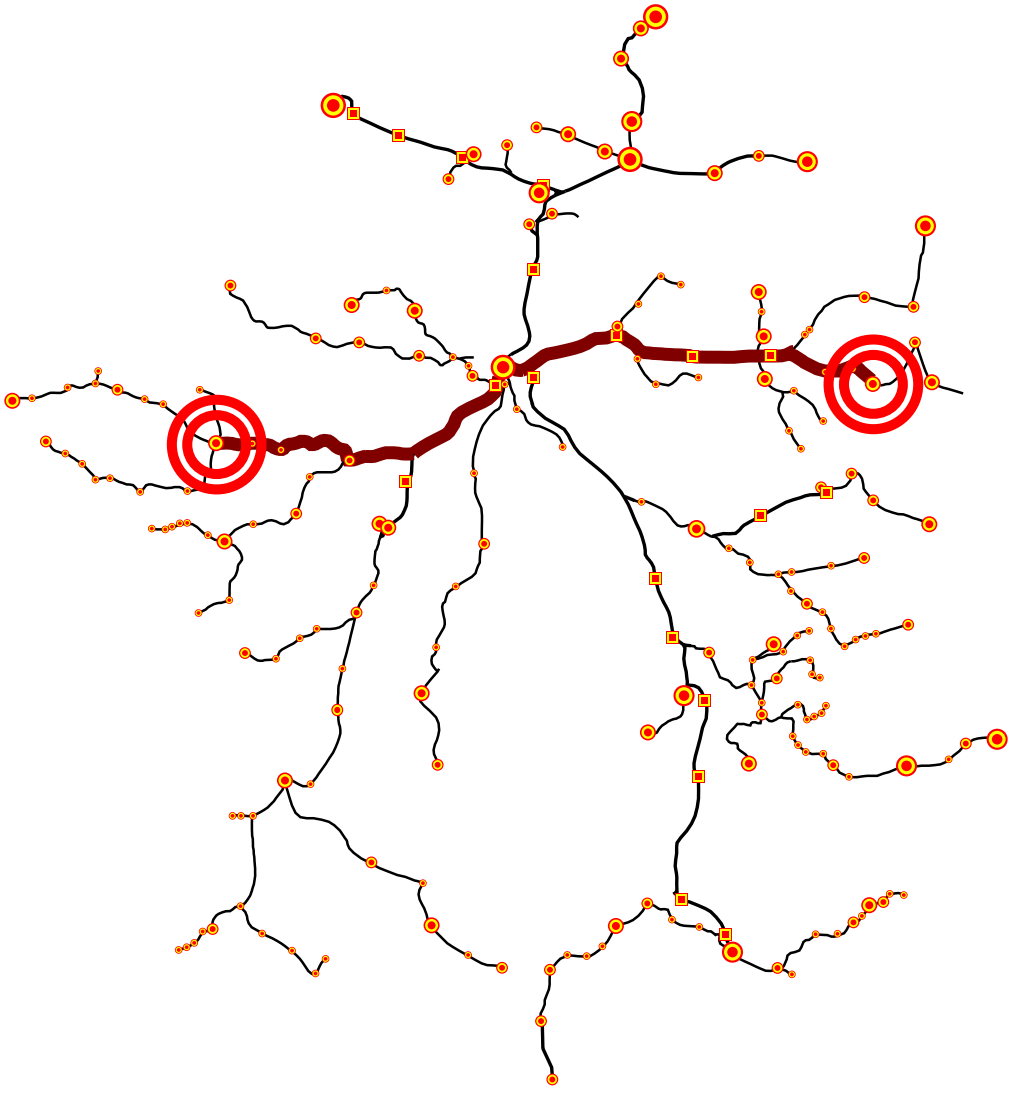

→ Shortest path: very easy on a large tree

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Does query evaluation on probabilistic data have lower complexity when the structure of the data is close to a tree?

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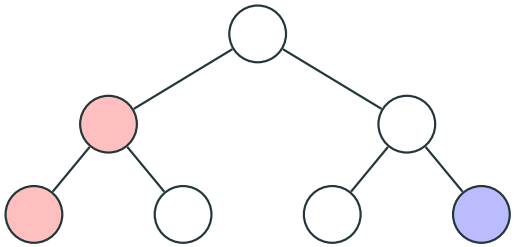
  - Applications to probabilistic query evaluation
- Other applications: Counting, enumeration, provenance...

Table of contents

Non-probabilistic query evaluation on trees

Database: a tree T where nodes have a color from an alphabet ○ ● ●



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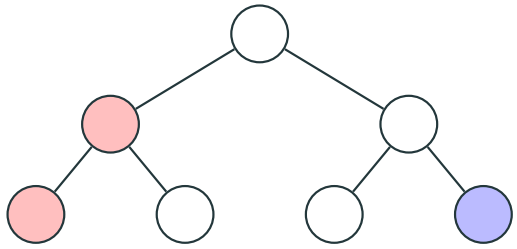

Query Q: a sentence in monadic second-order logic (MSO)

- $P_{\bullet}(x)$ means "x is blue"
- $x \to y$ means "x is the parent of y"

"Is there both a pink and a blue node?"

$\exists x\, y\; P_{\bullet}(x) \wedge P_{\bullet}(y)$

Non-probabilistic query evaluation on trees

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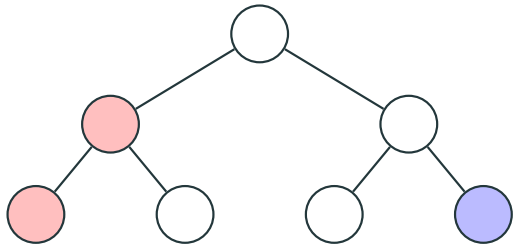

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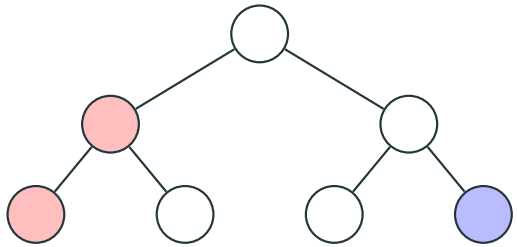

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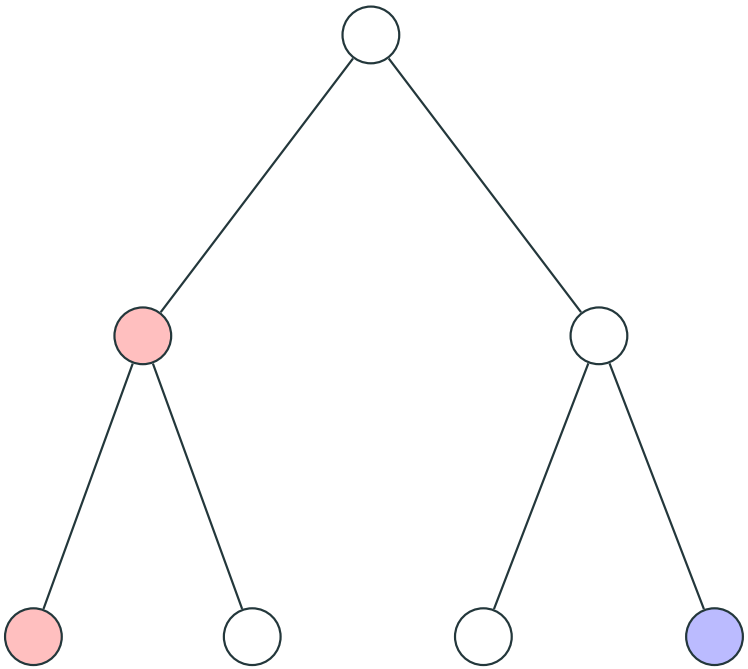
Computational complexity as a function of T
(the query Q is fixed)

Tree automata

Tree alphabet: 



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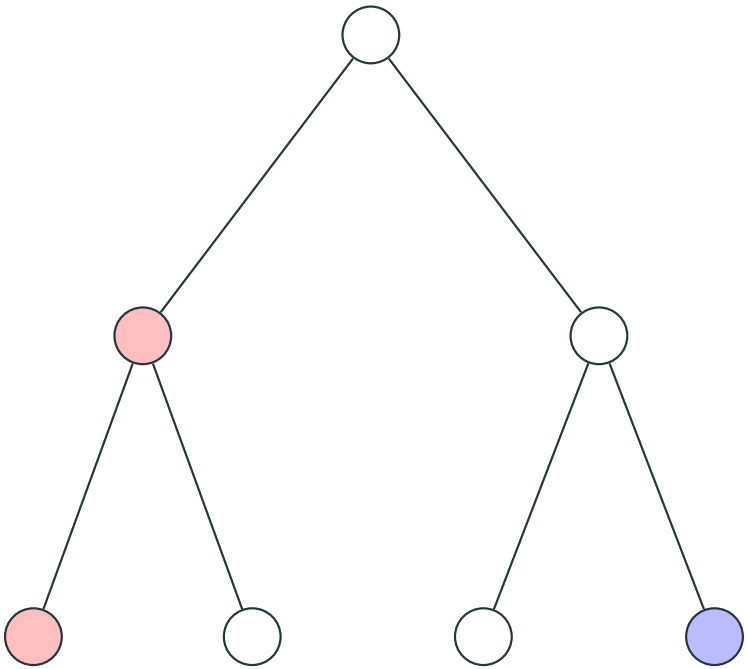

- Bottom-up deterministic tree automaton
- *“Is there both a pink and a blue node?”*

Tree automata

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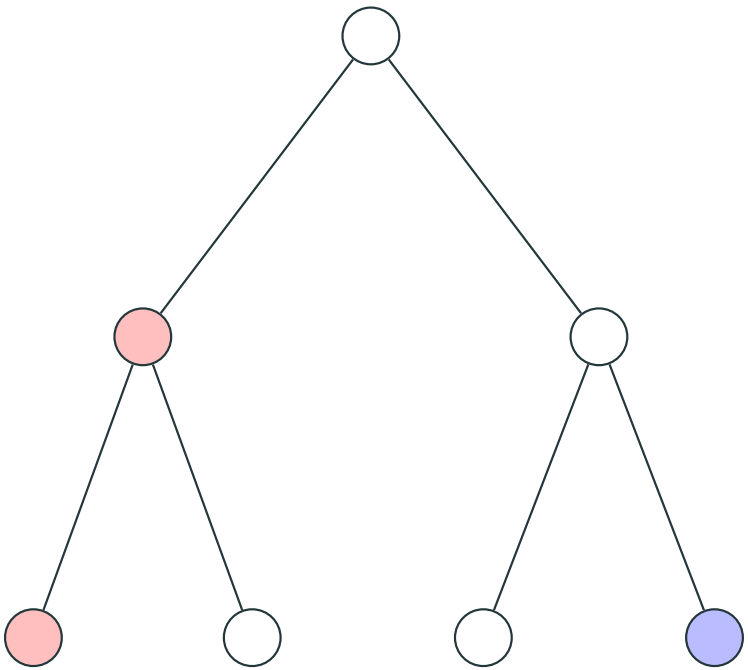

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- *"Is there both a pink and a blue node?"*
- States: $\{\bot, B, P, \top\}$

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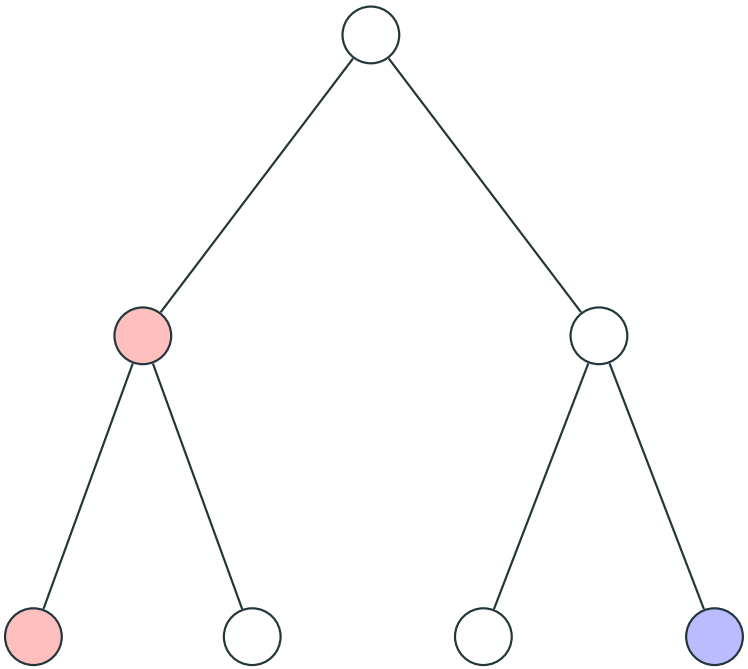

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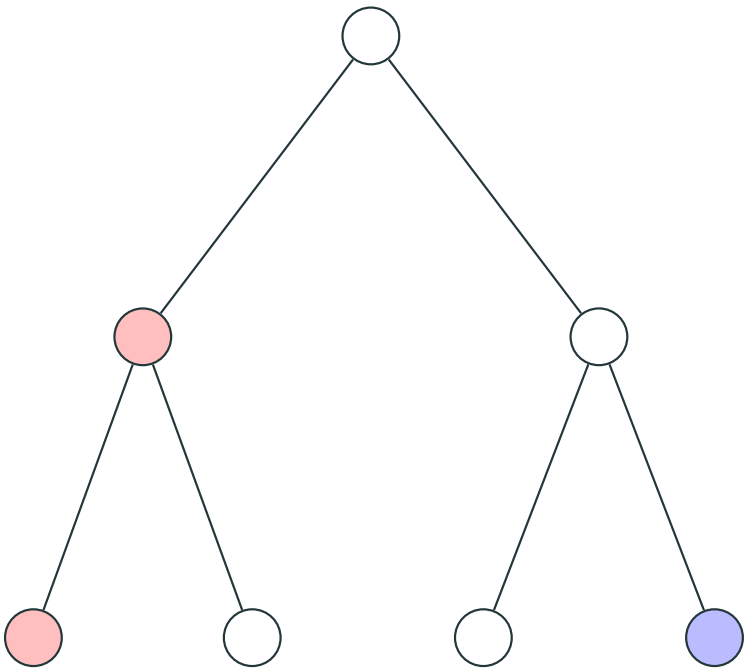
- States: $\{\bot, B, P, \top\}$
- Final states: $\{\top\}$

Tree automata

Tree alphabet: ○ ● ●



- Bottom-up deterministic tree automaton
- *"Is there both a pink and a blue node?"*
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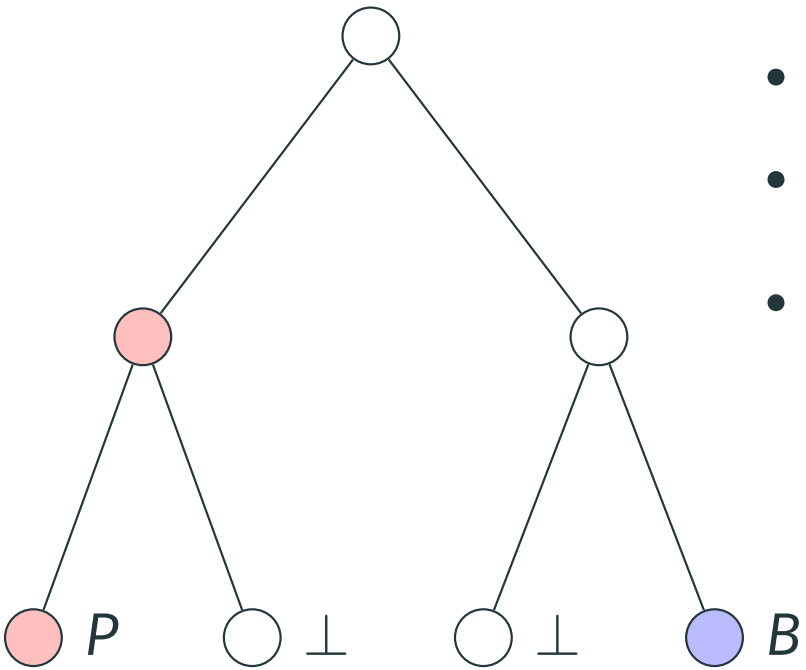

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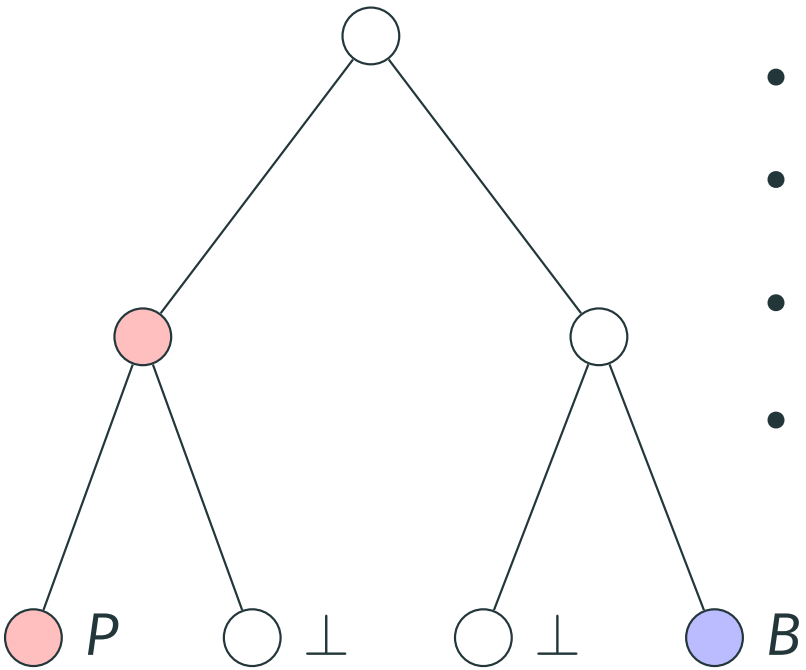

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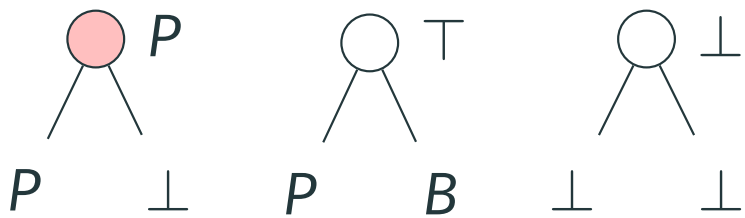

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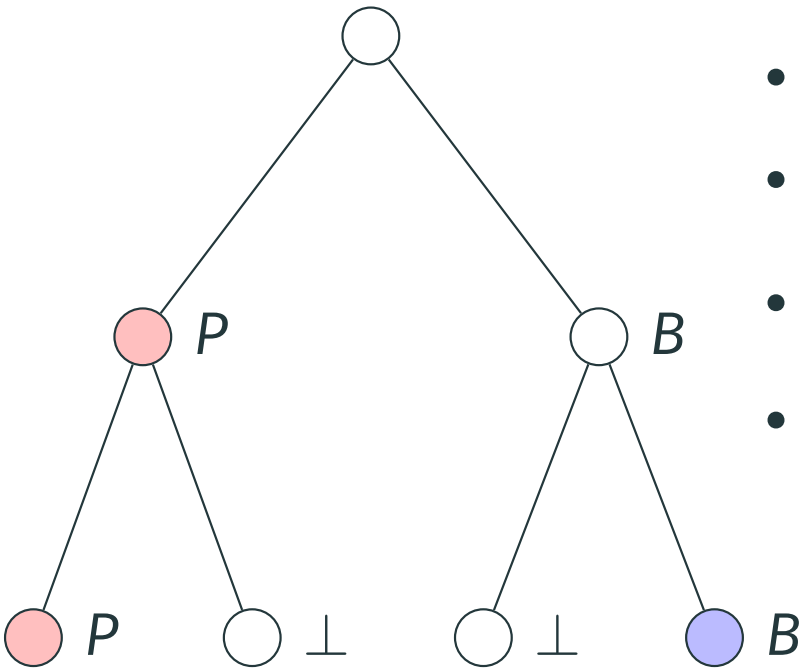

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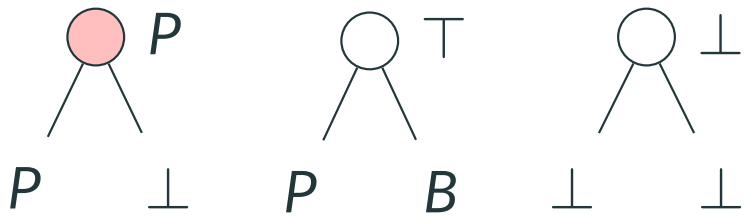

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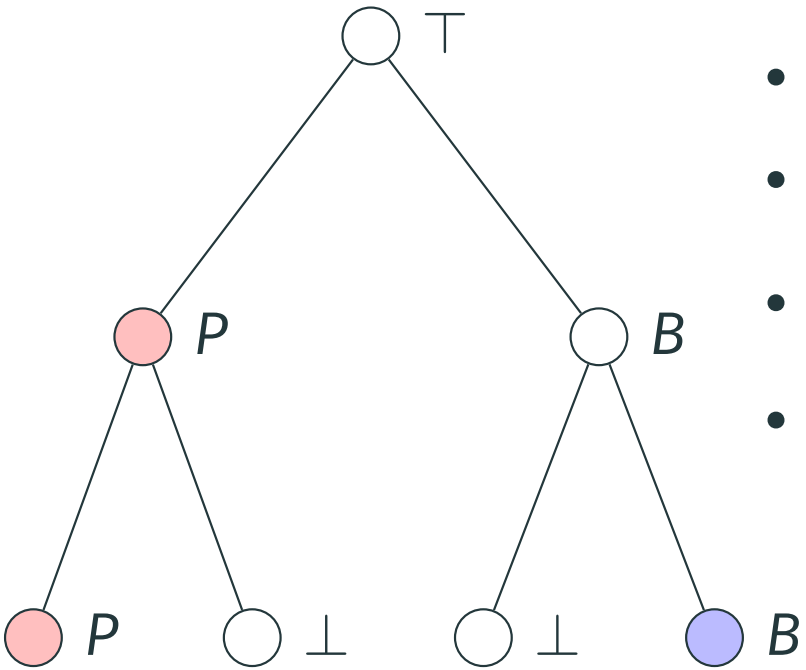

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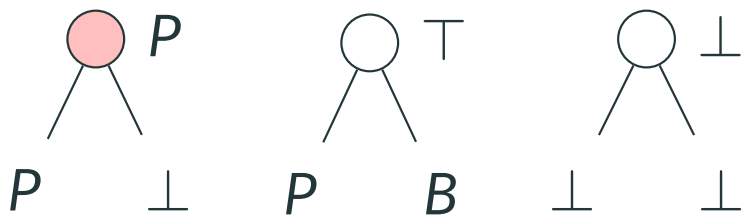

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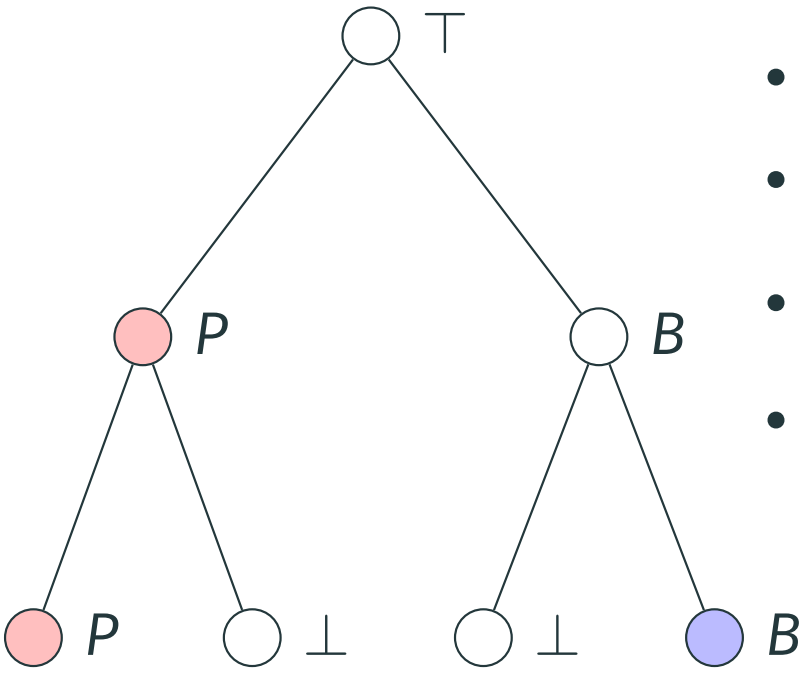

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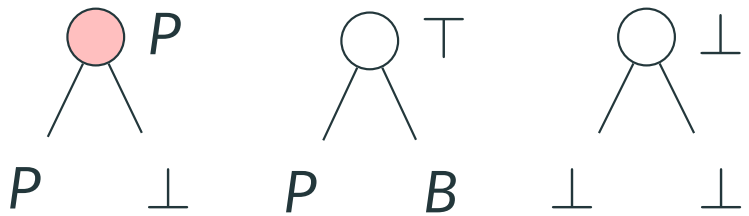

Theorem [Thatcher and Wright, 1968]

MSO and tree automata have the same expressive power on trees

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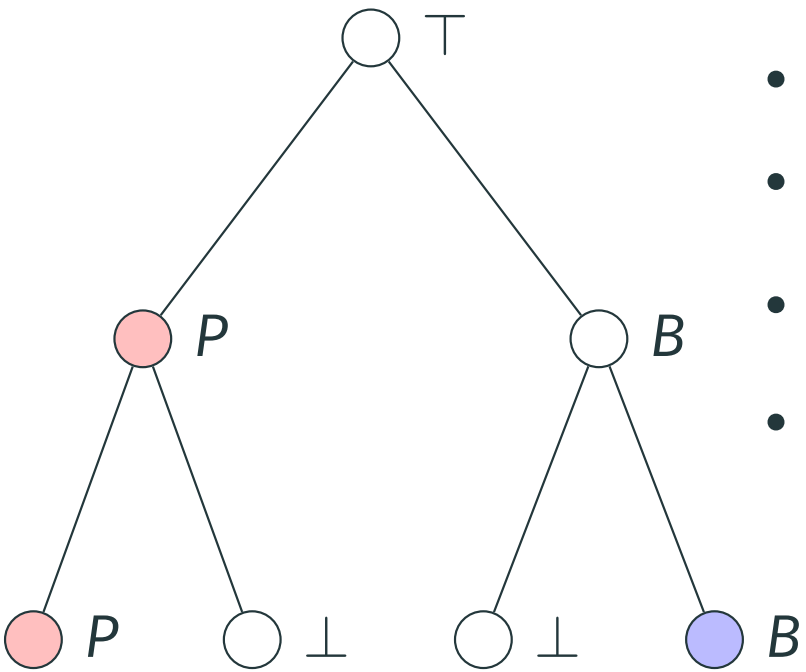

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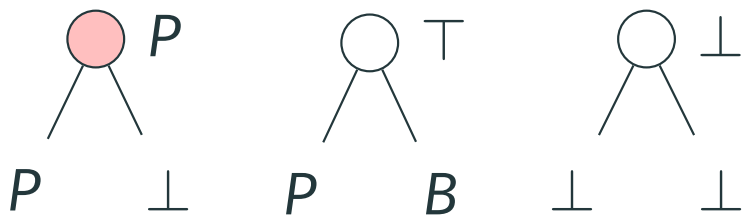


Theorem [Thatcher and Wright, 1968]

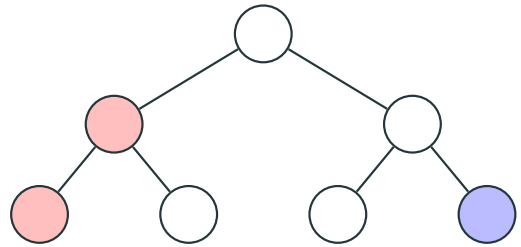
MSO and tree automata have the same expressive power on trees

Corollary

Non-probabilistic query evaluation of MSO on trees is in linear time.

Non-probabilistic query evaluation on trees

Database: a tree T where nodes have a color from an alphabet ○ ● ●



Query Q: a sentence in monadic second-order logic (MSO)

- $P_{\bullet}(x)$ means "x is blue"
- $x \to y$ means "x is the parent of y"

"Is there both a pink and a blue node?"

$\exists x\, y\; P_{\bullet}(x) \wedge P_{\bullet}(y)$

Result: TRUE/FALSE indicating if T satisfies the query Q

Computational complexity as a function of the tree T
(the query Q is fixed)

Non-probabilistic query evaluation on treelike data

Database: a treelike database T

???

Query Q: a sentence in monadic second-order logic (MSO)

(Metro|RER)*
| (Bus|Tram)*

Result: TRUE/FALSE indicating if T satisfies the query Q

Computational complexity as a function of the tree T
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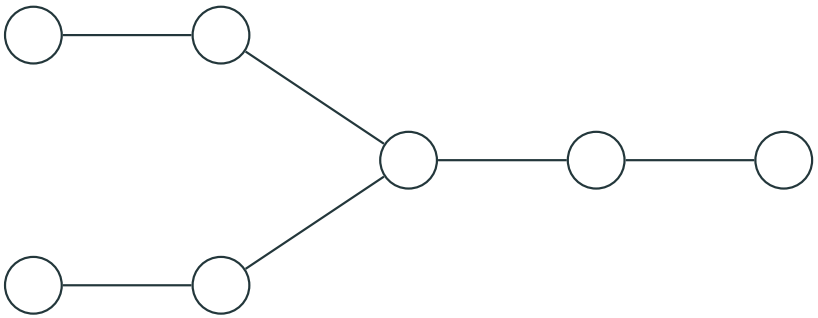
Treewidth by example:



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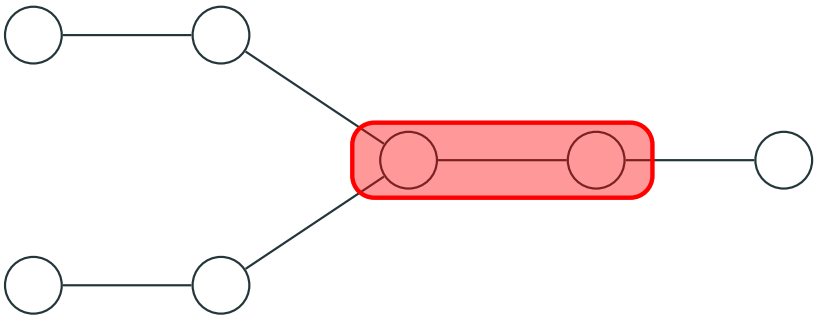

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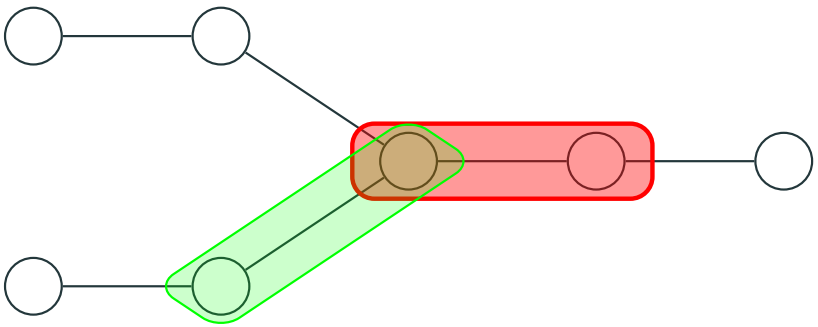

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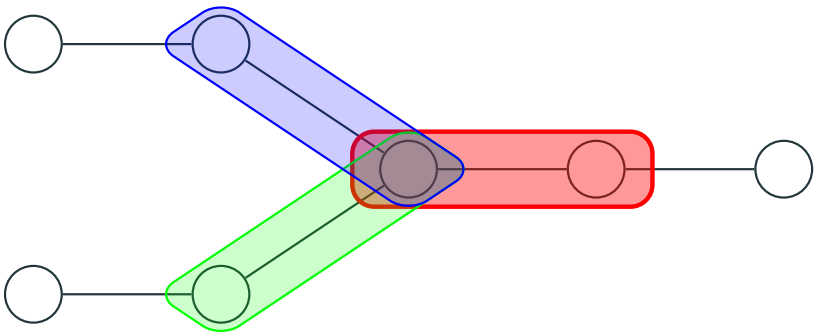

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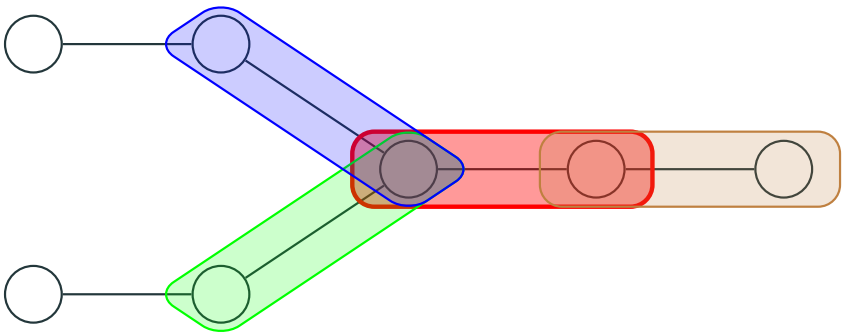

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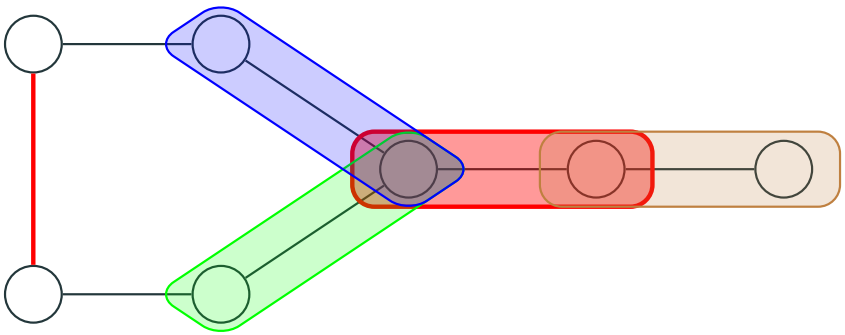

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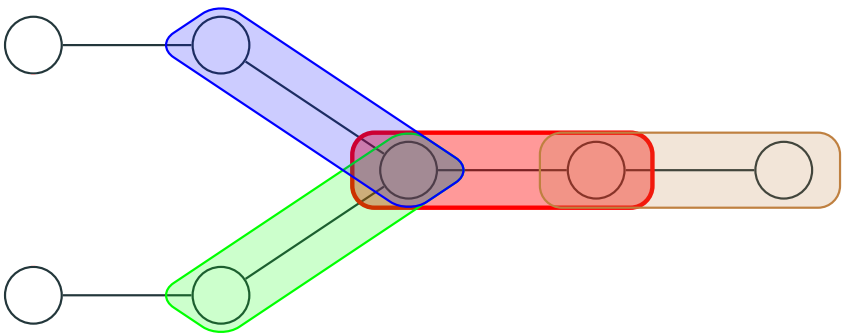

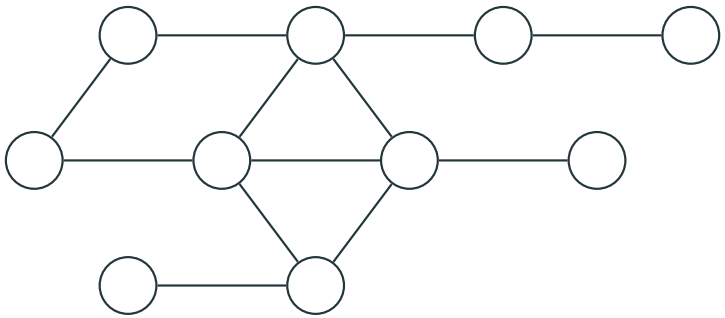

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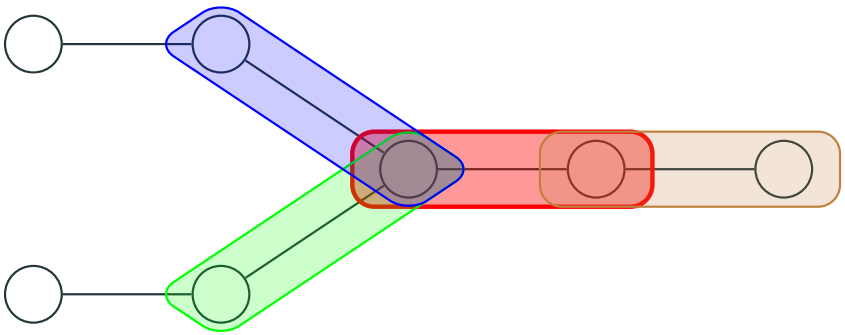

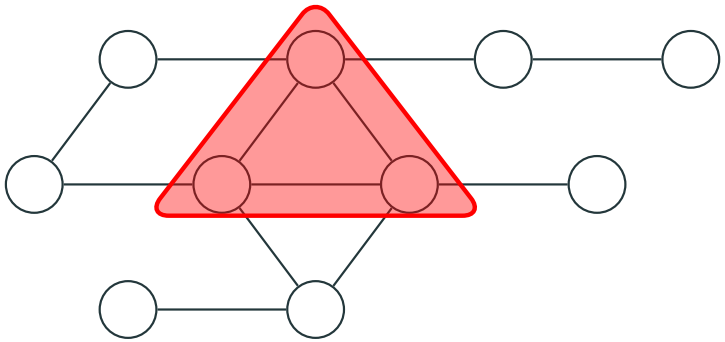

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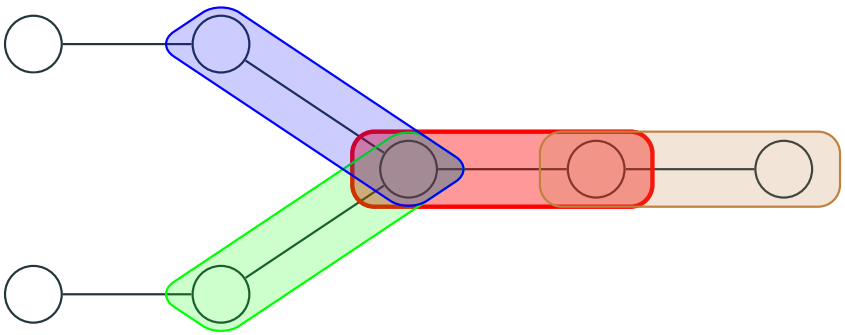

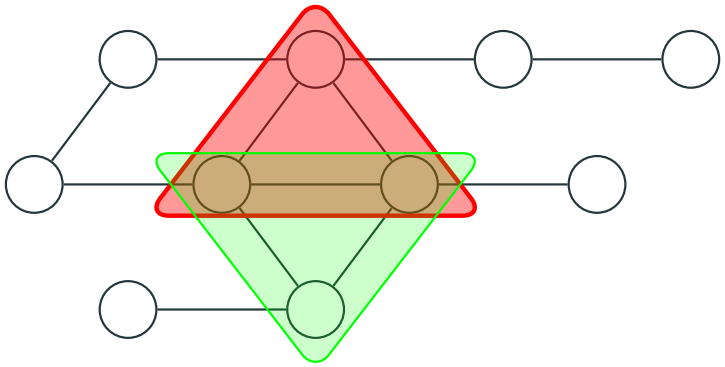

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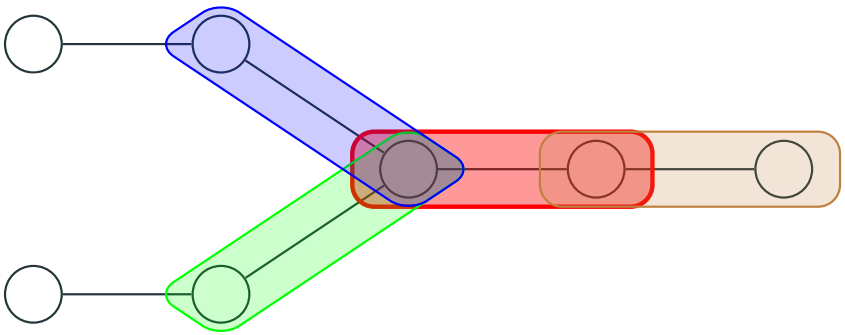

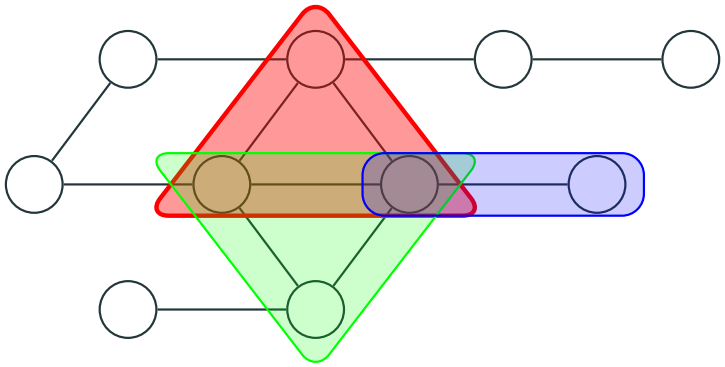

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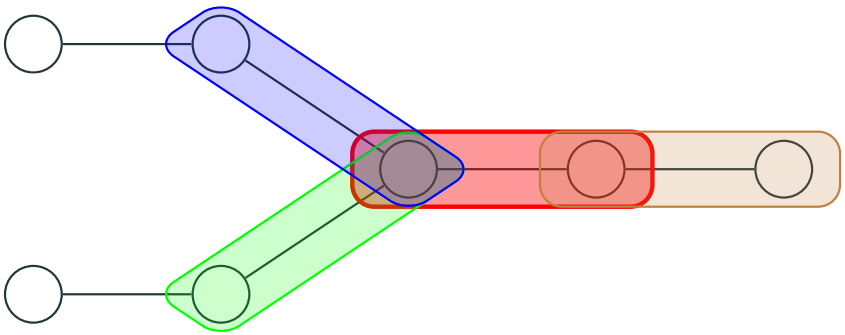

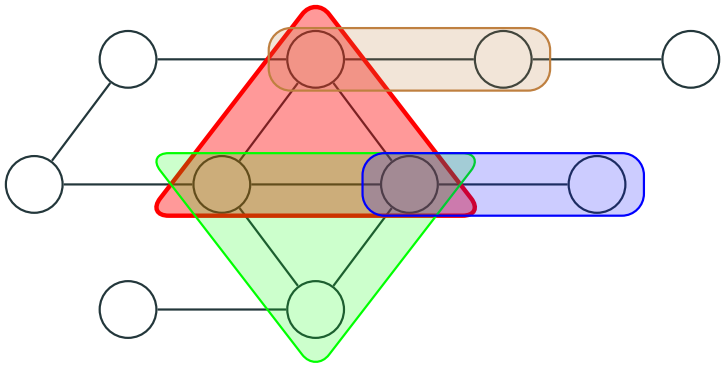

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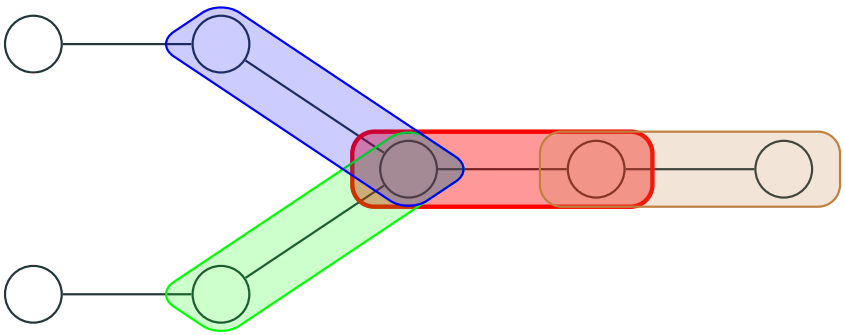

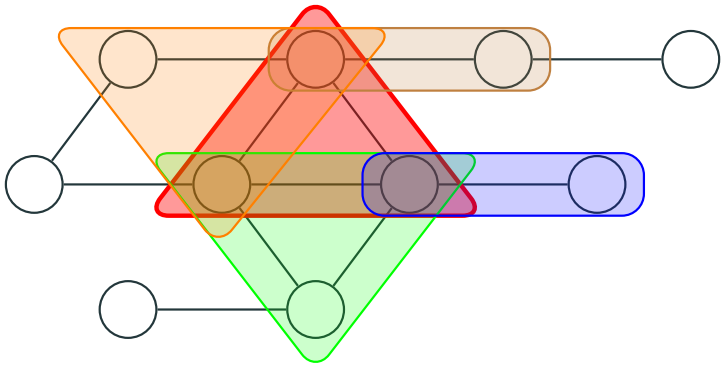

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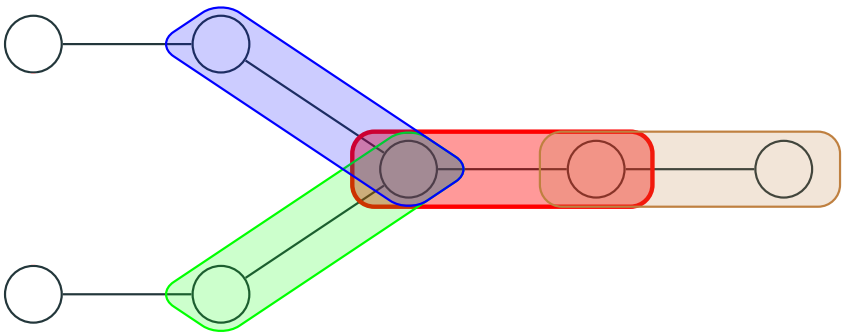

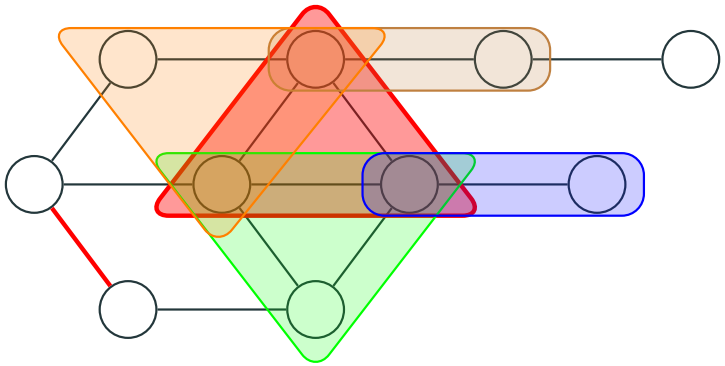

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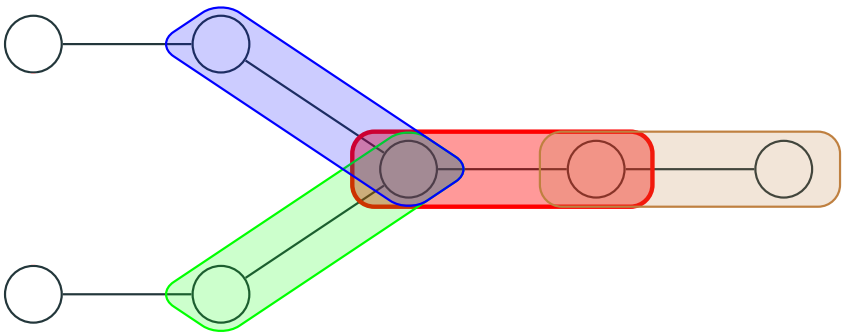

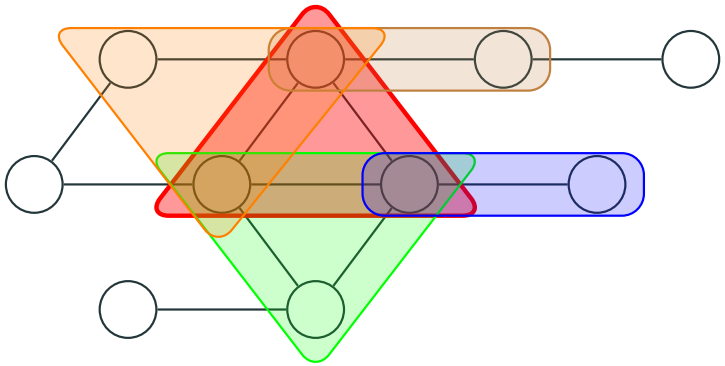

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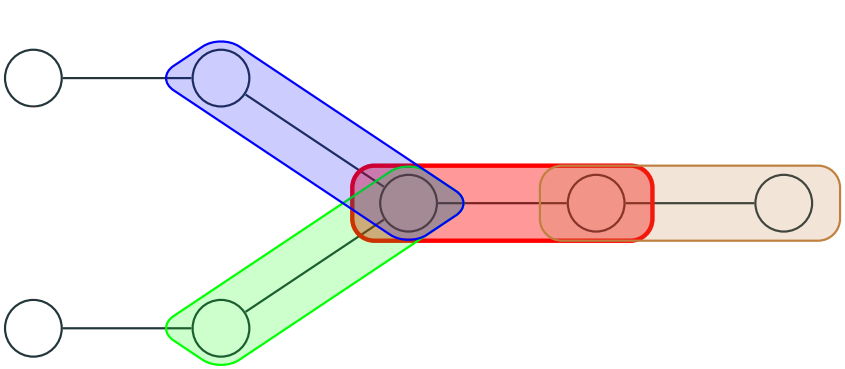

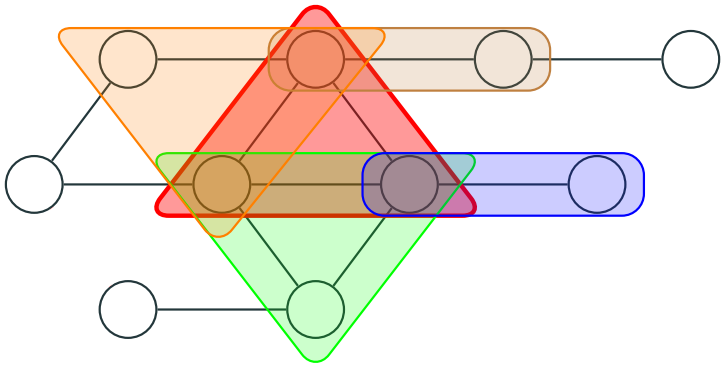

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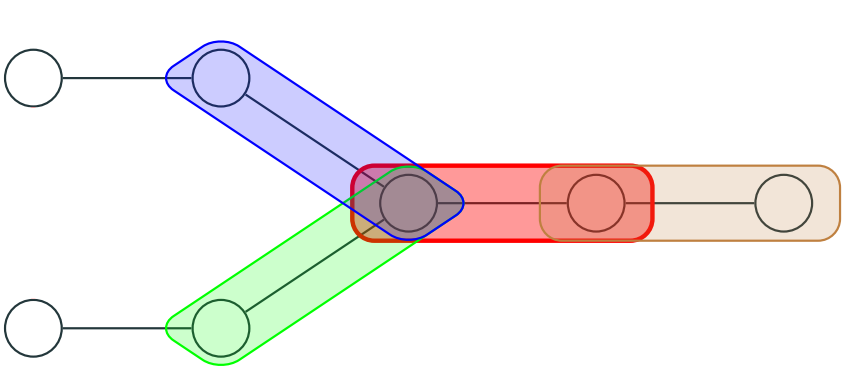

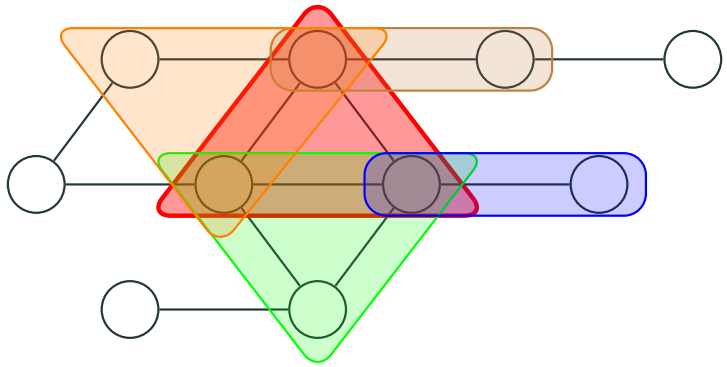

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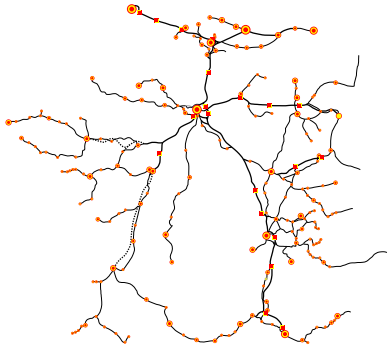
$\rightarrow$ Treelike: the treewidth is bounded by a constant

Courcelle's theorem

Treelike **data**



MSO query

(RER|metro)*
|(bus|tram)*

Courcelle's theorem

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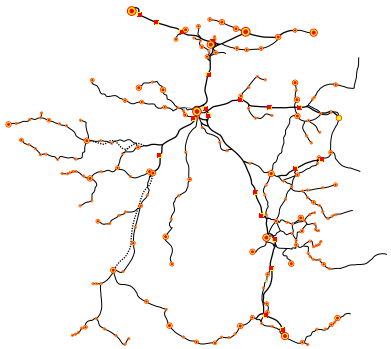

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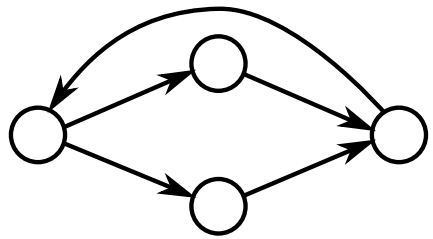

Courcelle's theorem

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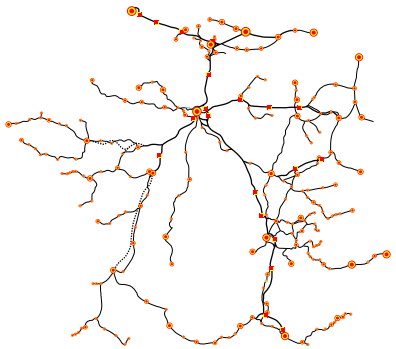
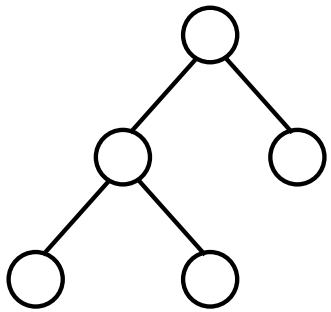


linear

Tree **encoding**



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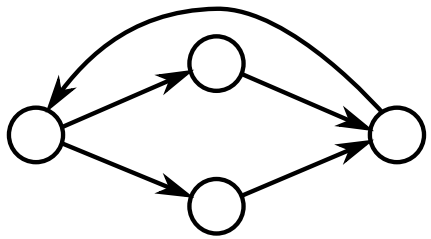

Courcelle's theorem

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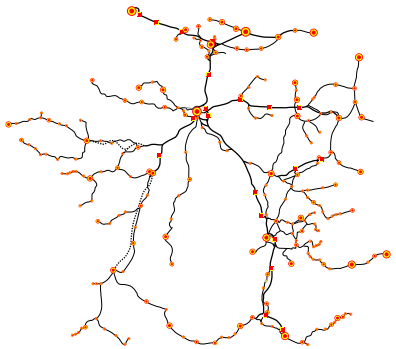

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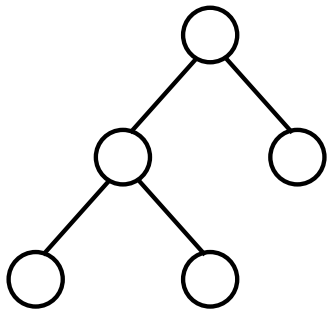


linear

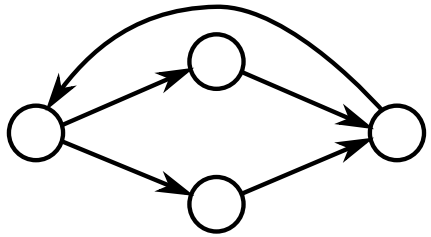
Query **answer**

TRUE

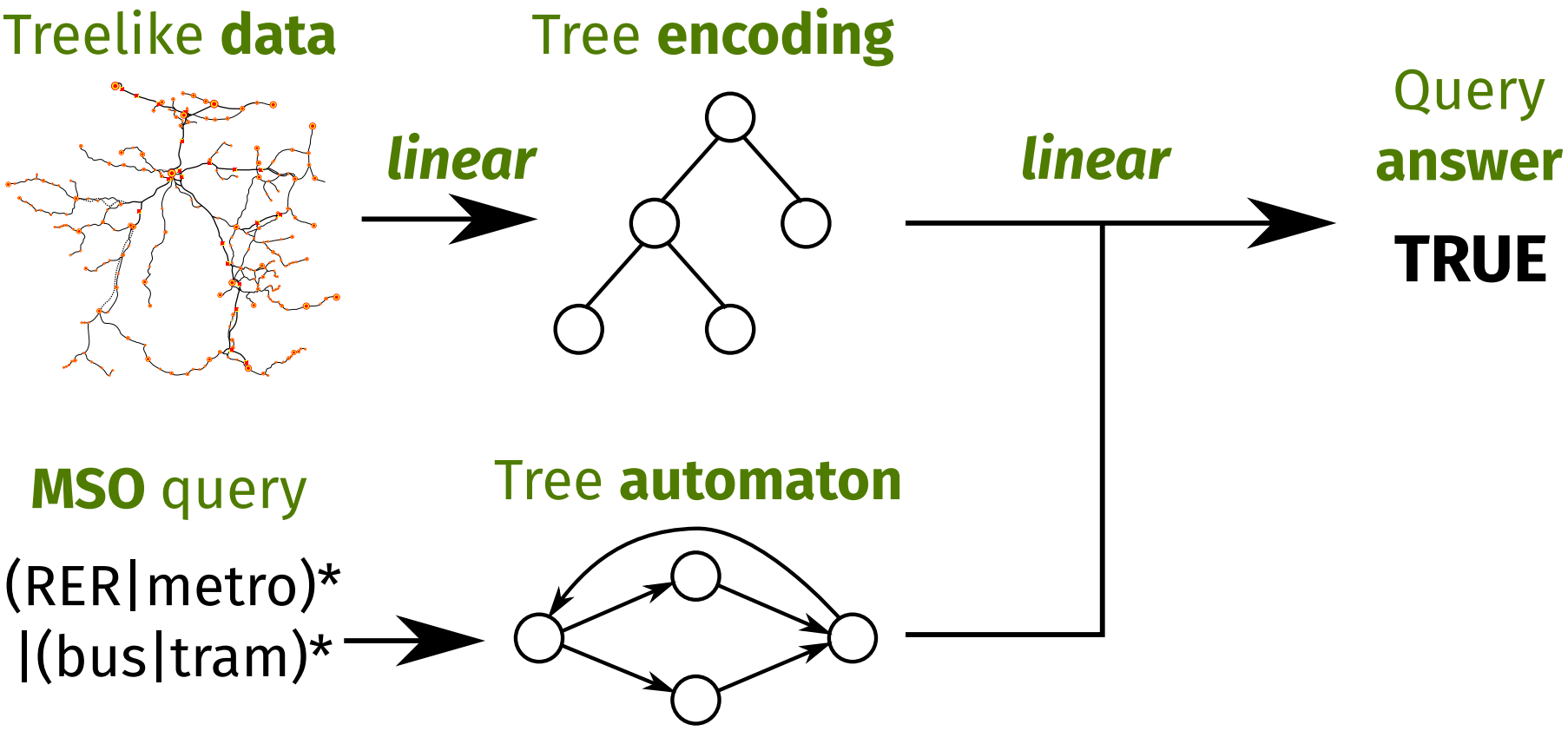
MSO query

(RER|metro)*
|(bus|tram)*

Tree **automaton**



Courcelle's theorem



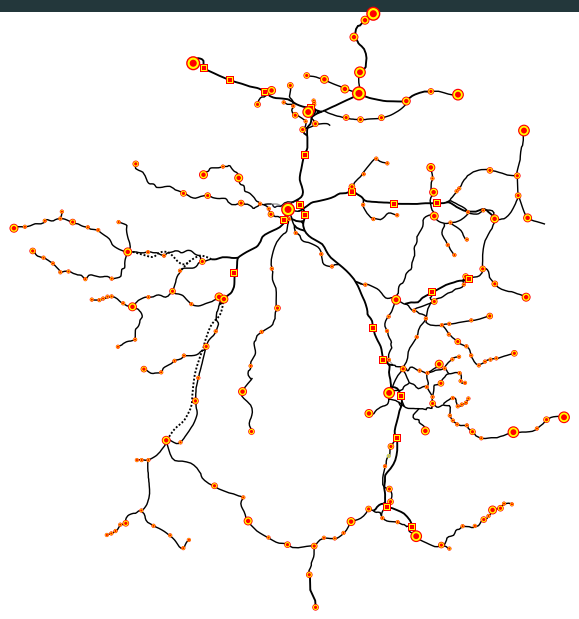
Theorem [Courcelle, 1990]

For any fixed Boolean MSO query Q and $k \in \mathbb{N}$,
given a database D of treewidth $\leq k$,
we can compute in linear time in D whether D satisfies Q

Table of contents

Probabilistic query evaluation on treelike data

- Database D with treewidth $\leq k$ for some constant k
- Probability of each fact of D to be actually present in the data (independently from other facts)



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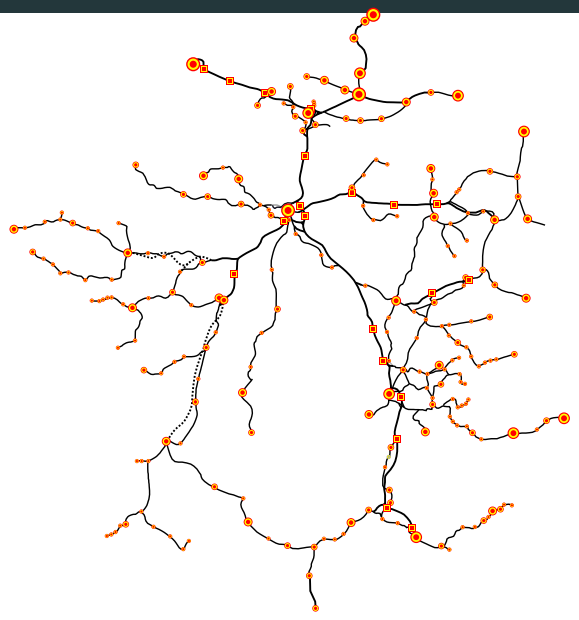

Query Q: a sentence in monadic second-order logic (MSO)

(Metro|RER)*
| (Bus|Tram)*

Probabilistic query evaluation on treelike data

- Database D with treewidth $\leq k$ for some constant k
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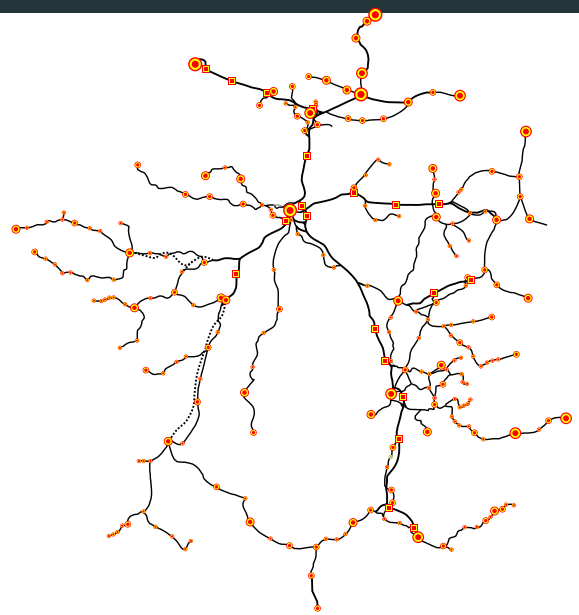

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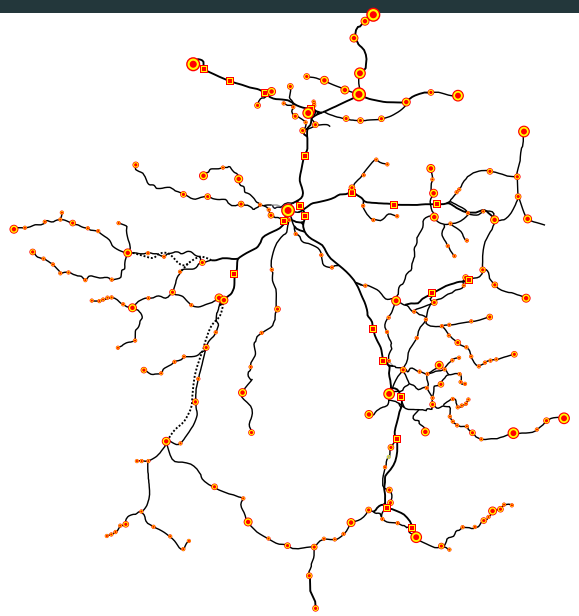

Query Q: a sentence in monadic second-order logic (MSO)

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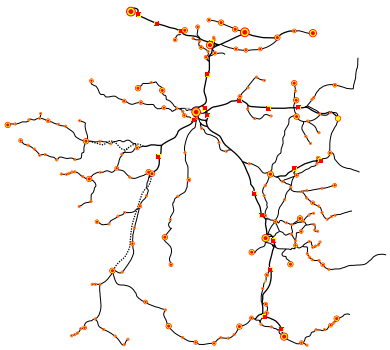
Computational complexity as a function of the database D
(the query Q is fixed)

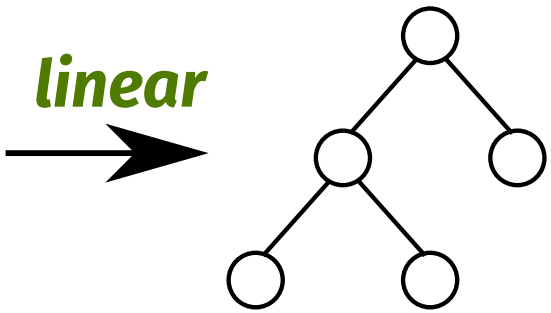
Roadmap

Treelike **data**



Tree **encoding**

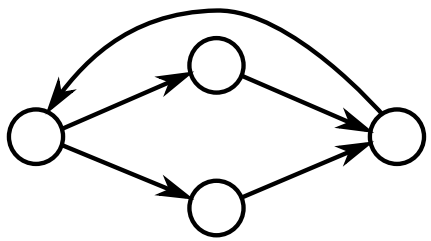
linear



MSO query

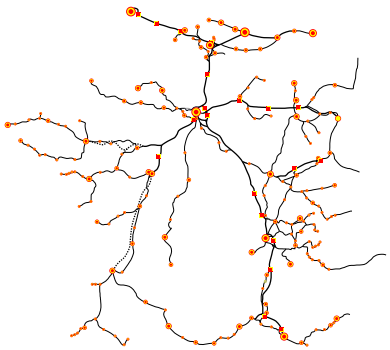
(RER|metro)*
|(bus|tram)*

Tree **automaton**



Roadmap

Treelike **data**



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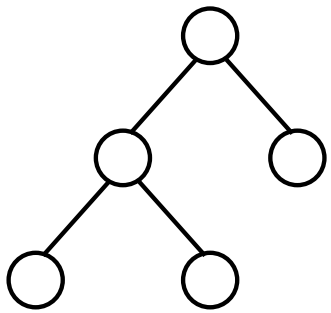
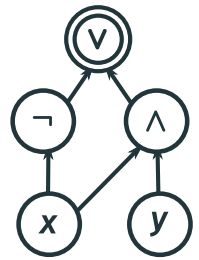


linear

Provenance circuit



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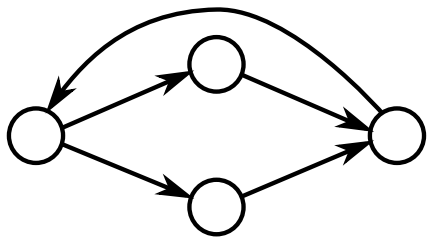
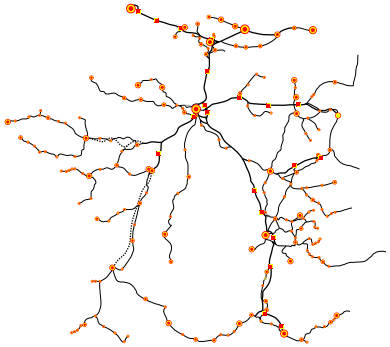


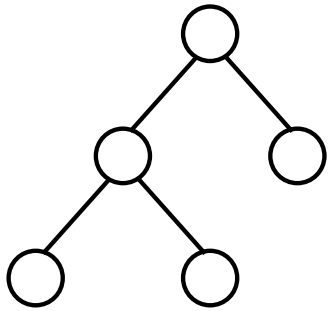
Roadmap

Probabilistic treelike **data**



linear

Uncertain tree **encoding**



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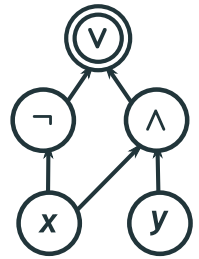
Provenance circuit +probabilities



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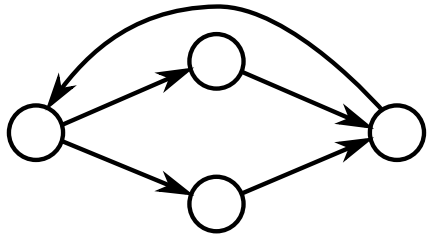

95%
Probability

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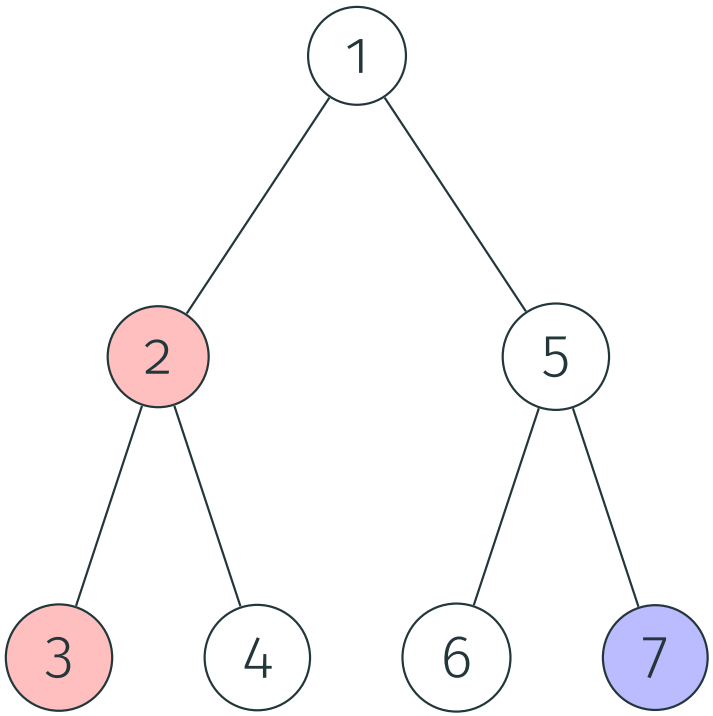

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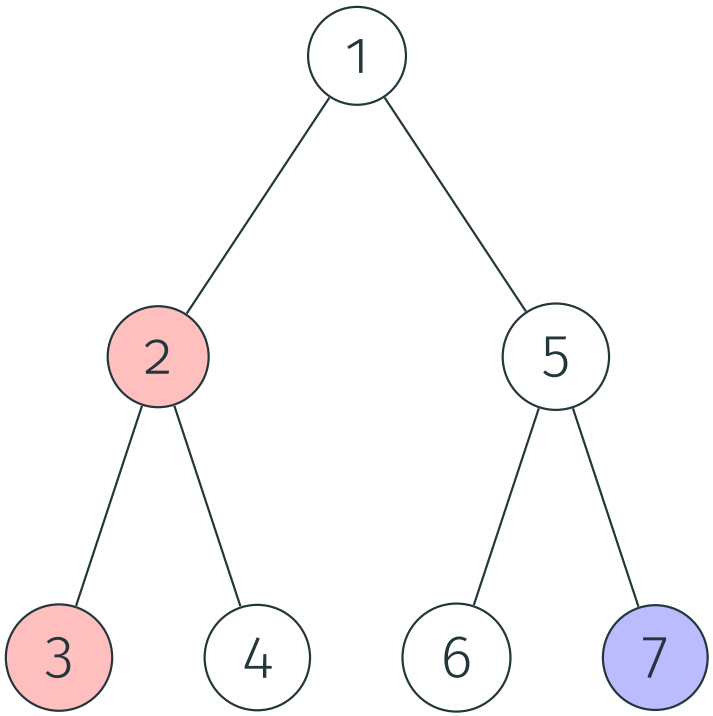

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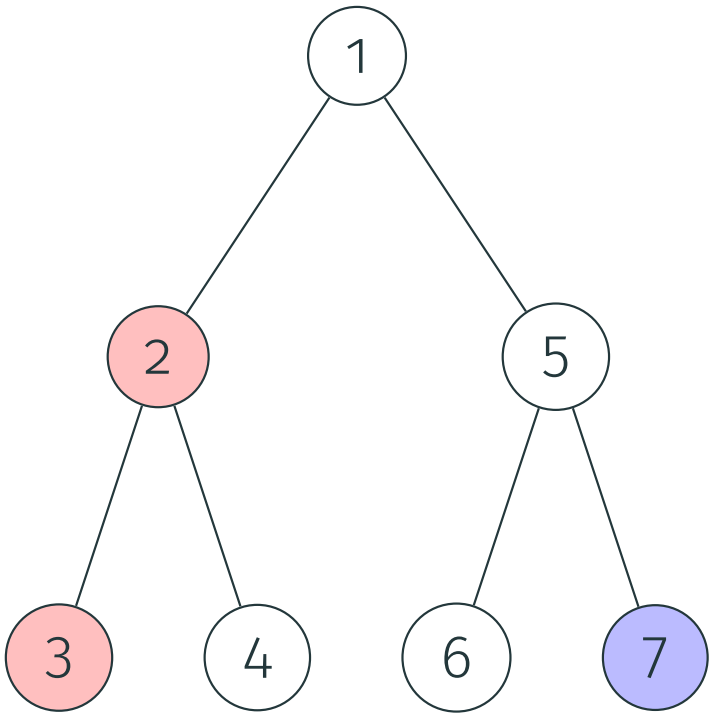

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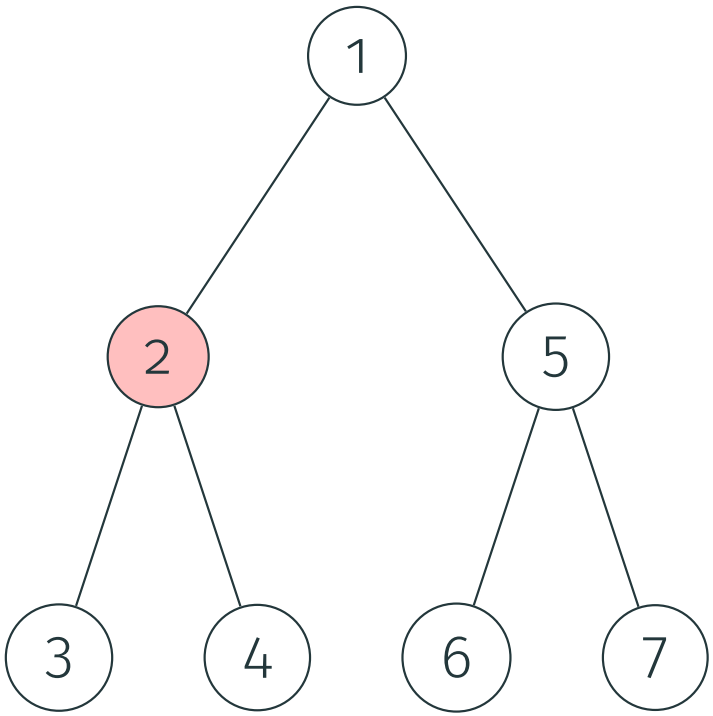

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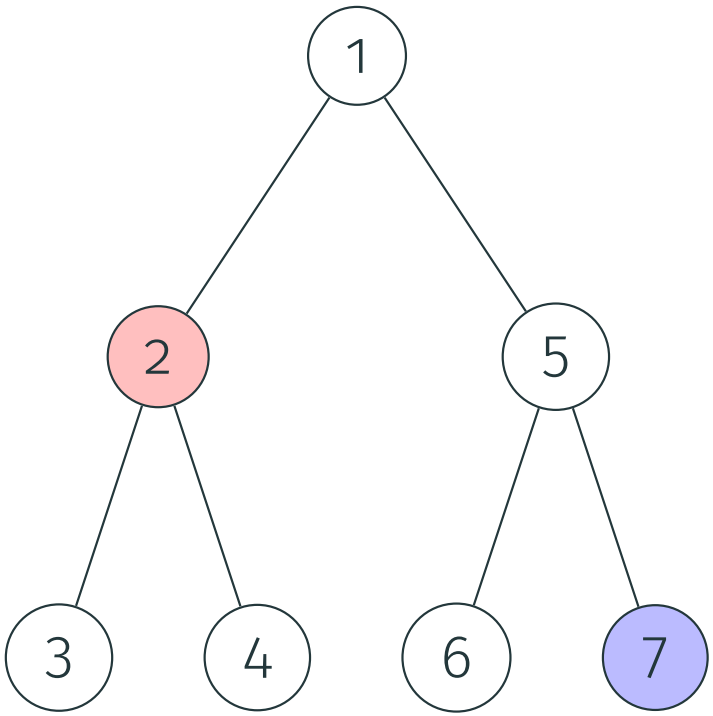

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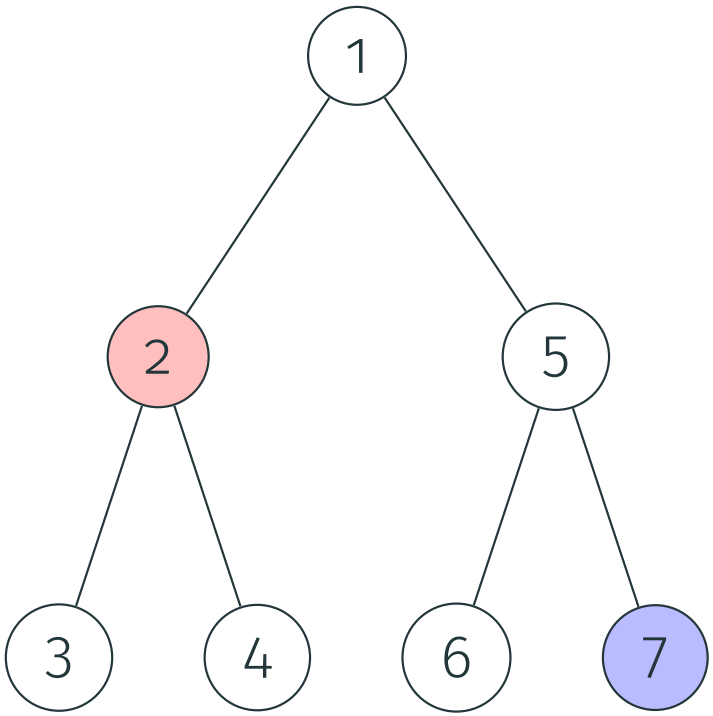

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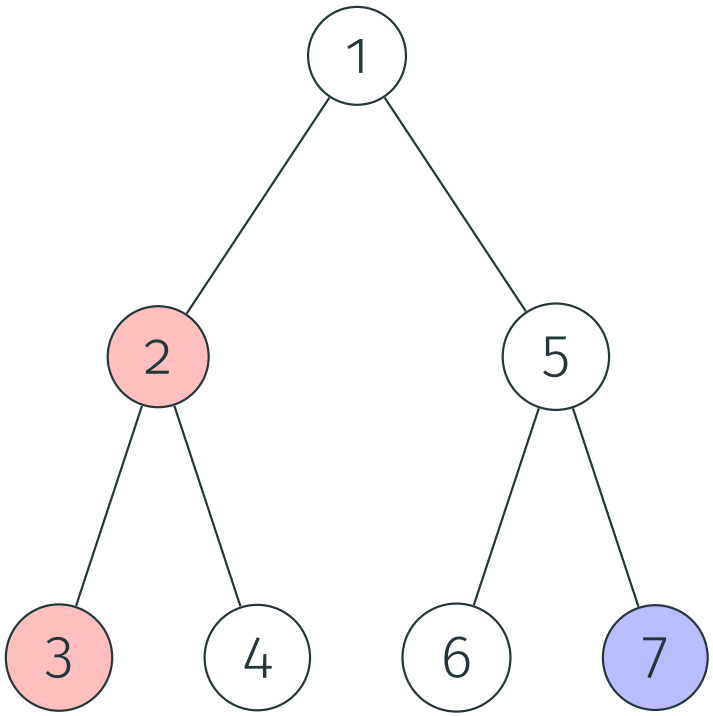

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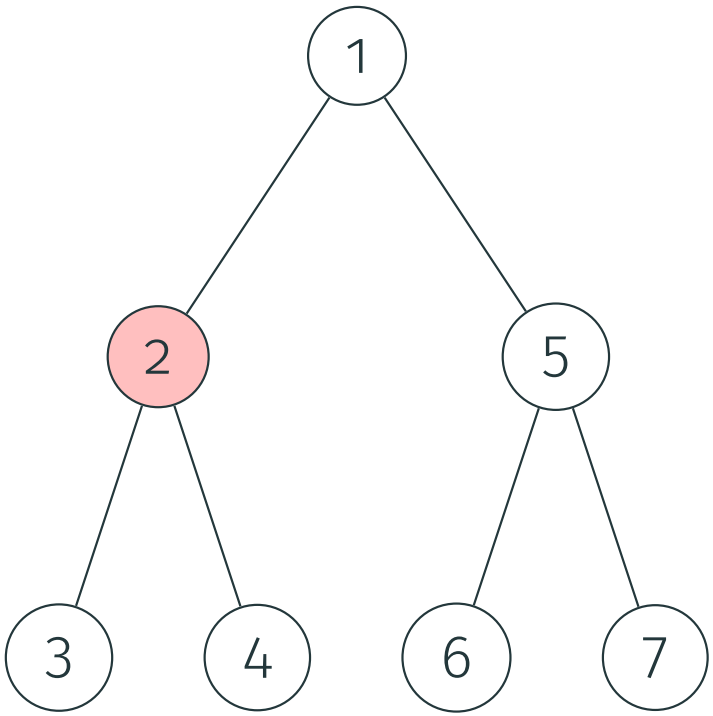

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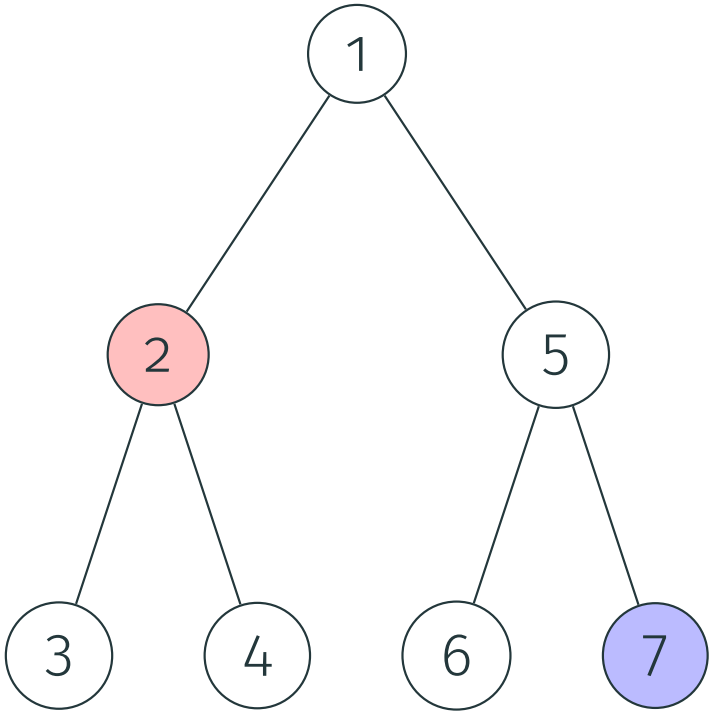

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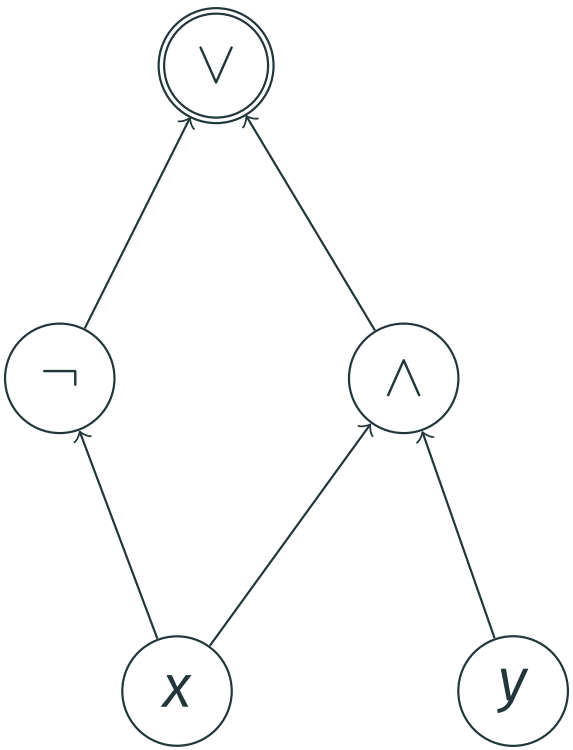
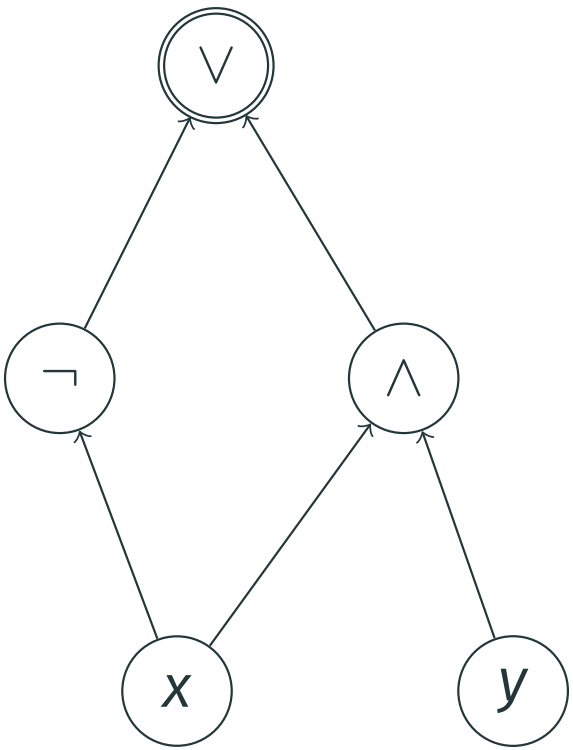


- Directed acyclic graph of gates

Boolean circuit



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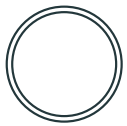
- Output gate: 

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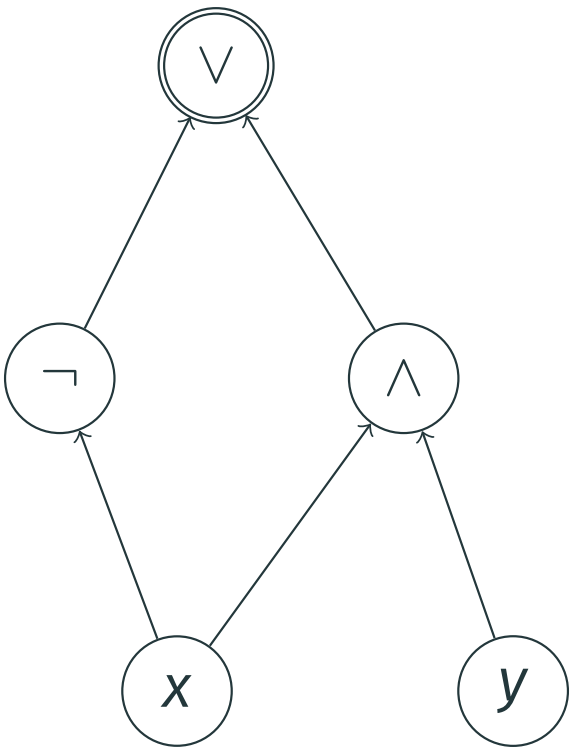

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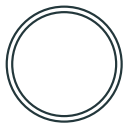
- Output gate: 
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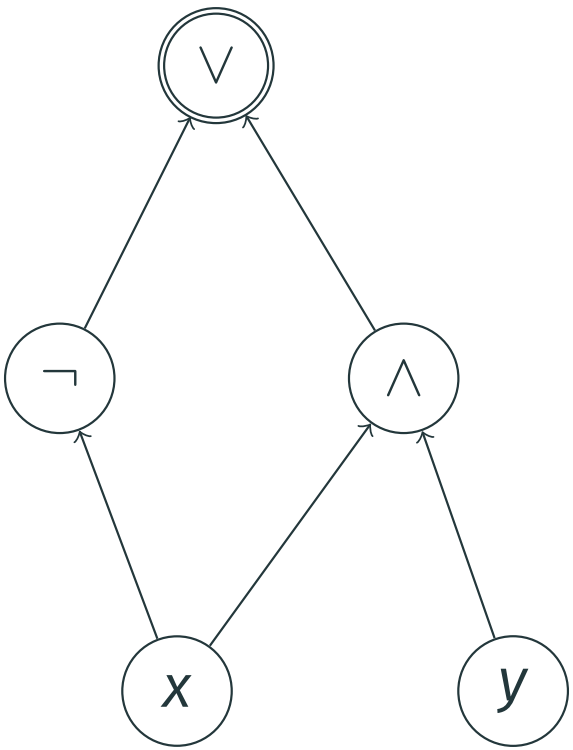
Boolean circuit



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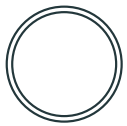
- Output gate: 
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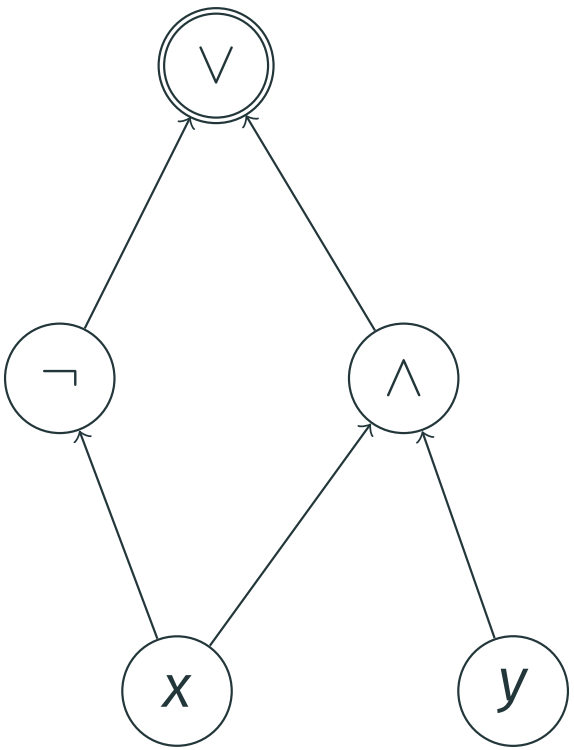
- Internal gates: 

Boolean circuit



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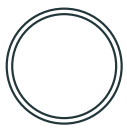
- Output gate: 
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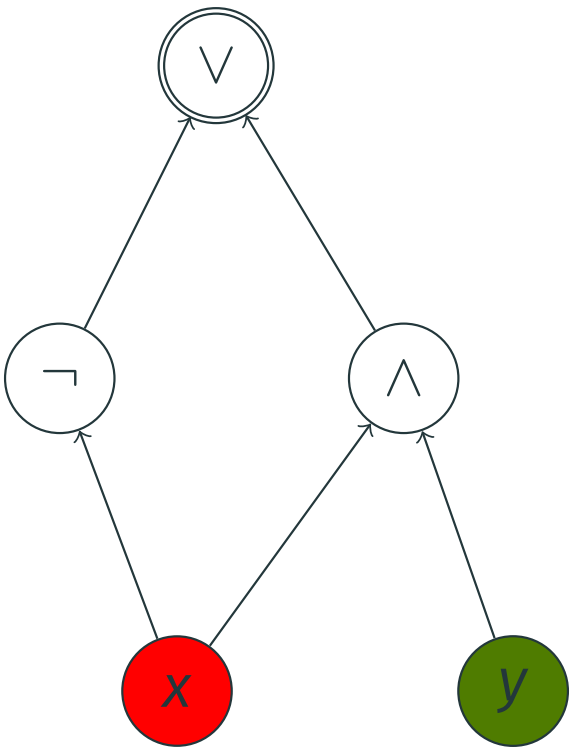
Boolean circuit



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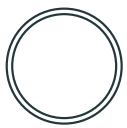
- Output gate: 
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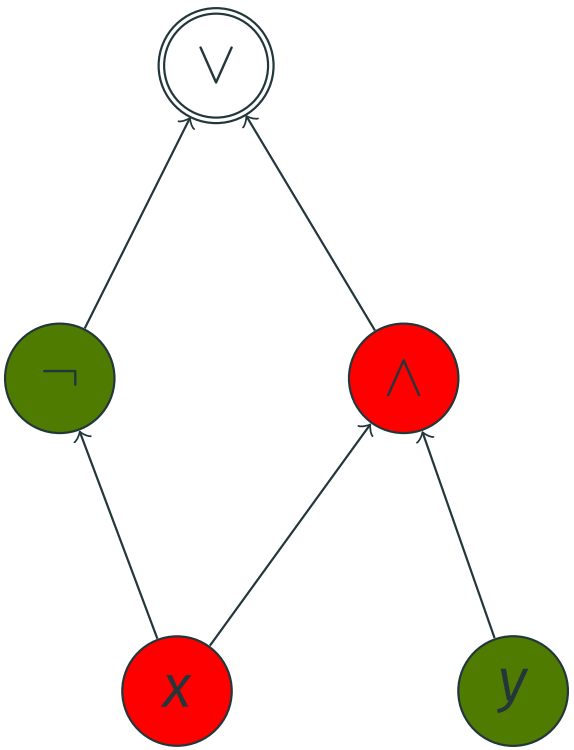

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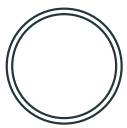
- Output gate: 
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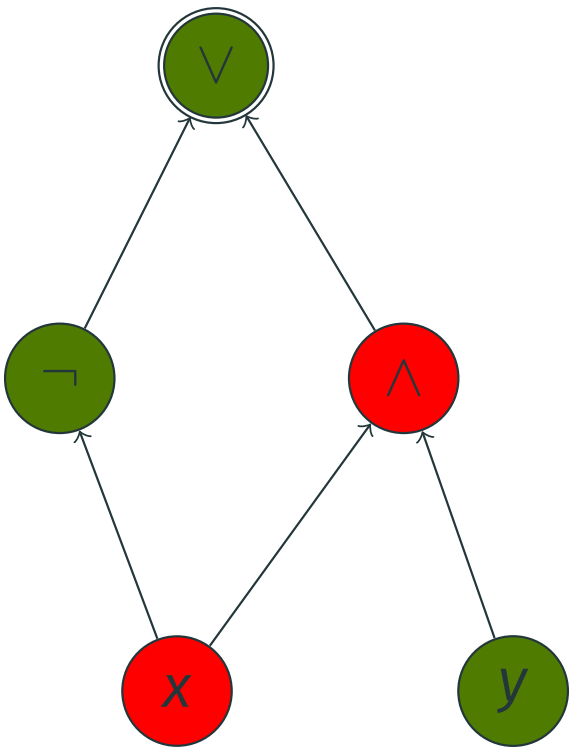

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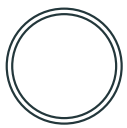
- Output gate: 
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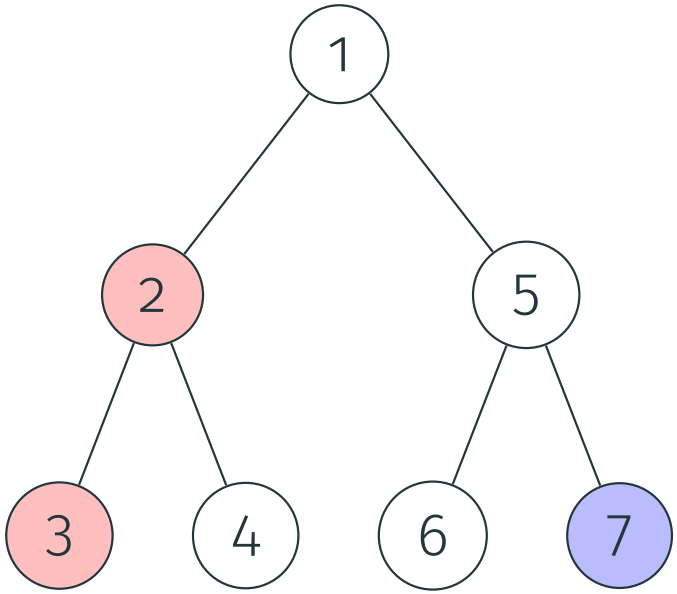
Provenance circuit



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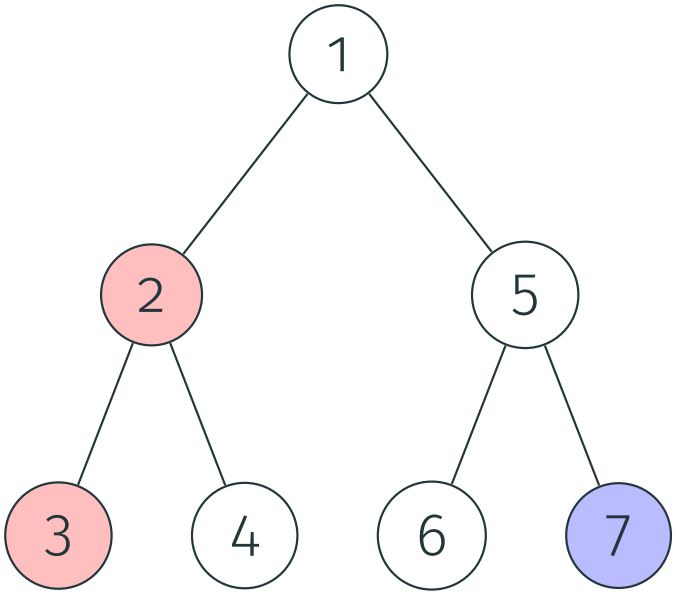

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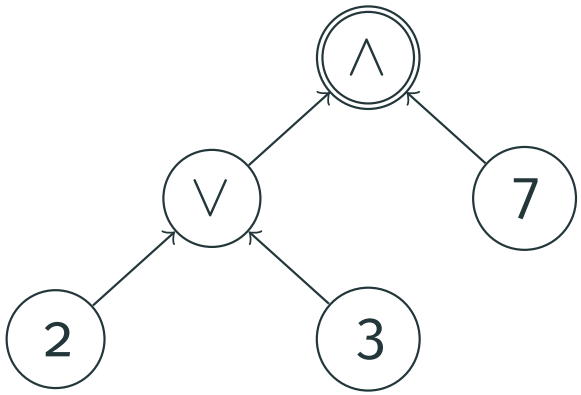

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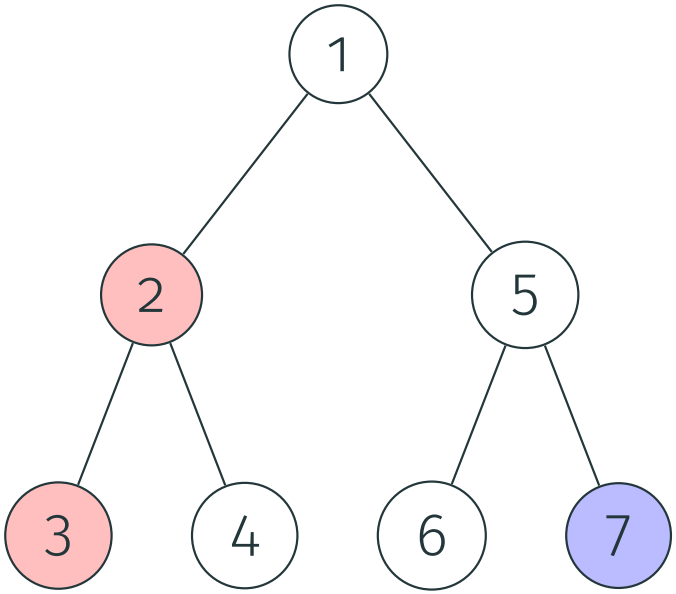

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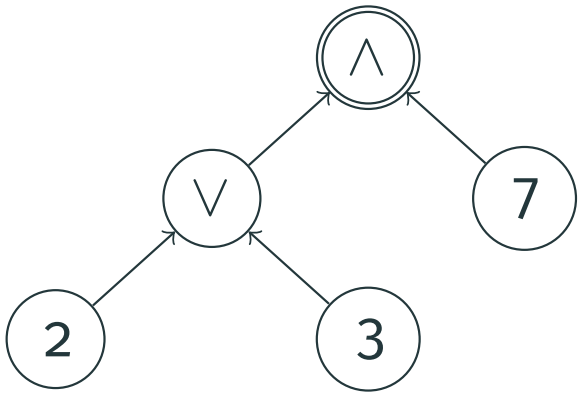

Formally:

- Tree automaton A, uncertain tree T, circuit C
- Variable gates of C: nodes of T
- Condition: Let ν be a valuation of T, then $\nu(C)$ iff A accepts $\nu(T)$

Building provenance circuits on trees

Theorem

For any bottom-up tree automaton A and input tree T,
we can build a provenance circuit of A on T in $O(|A| \times |T|)$

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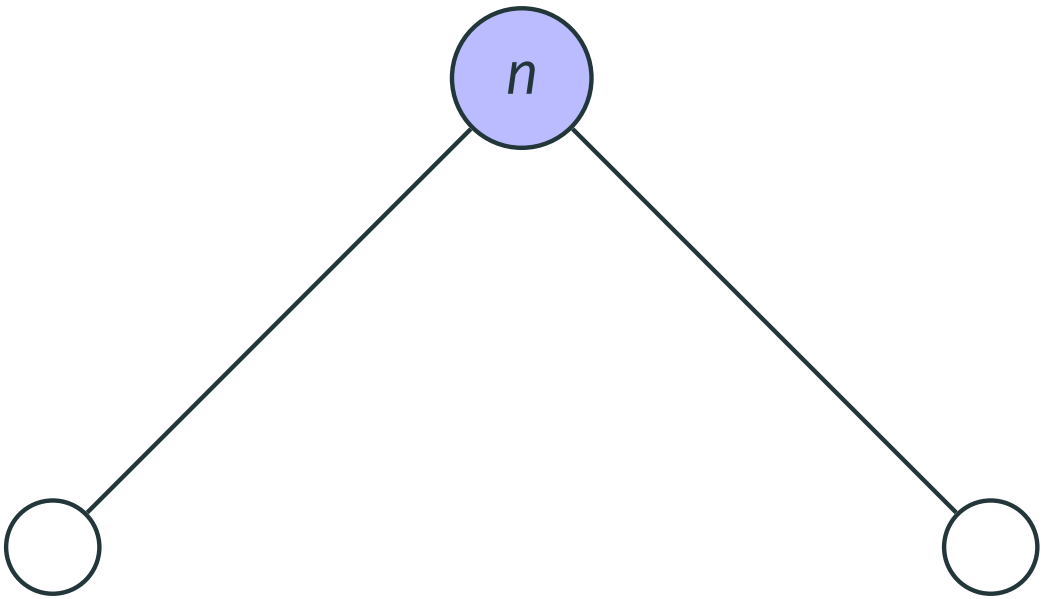

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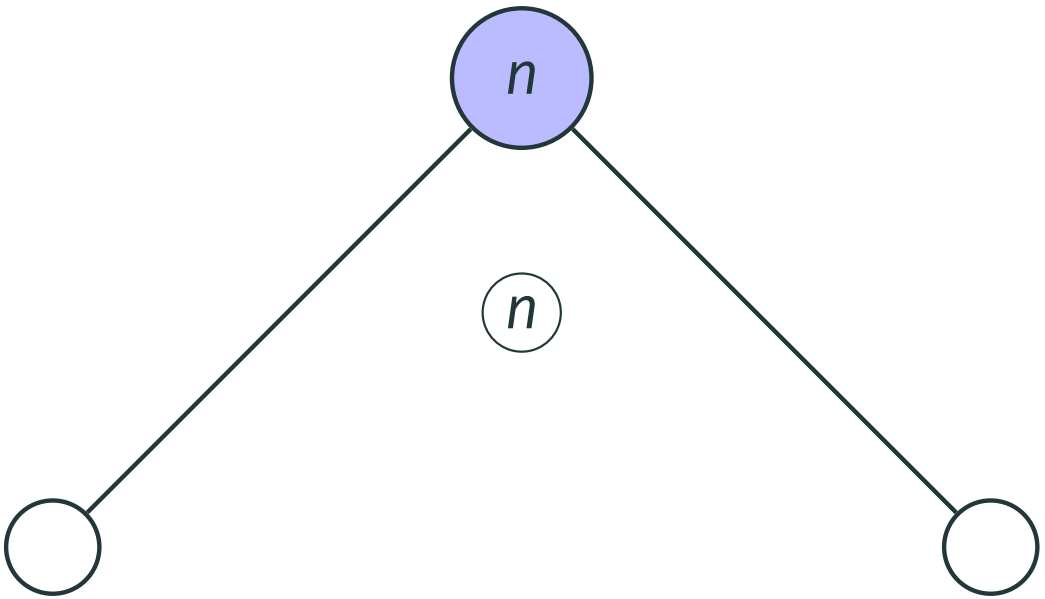

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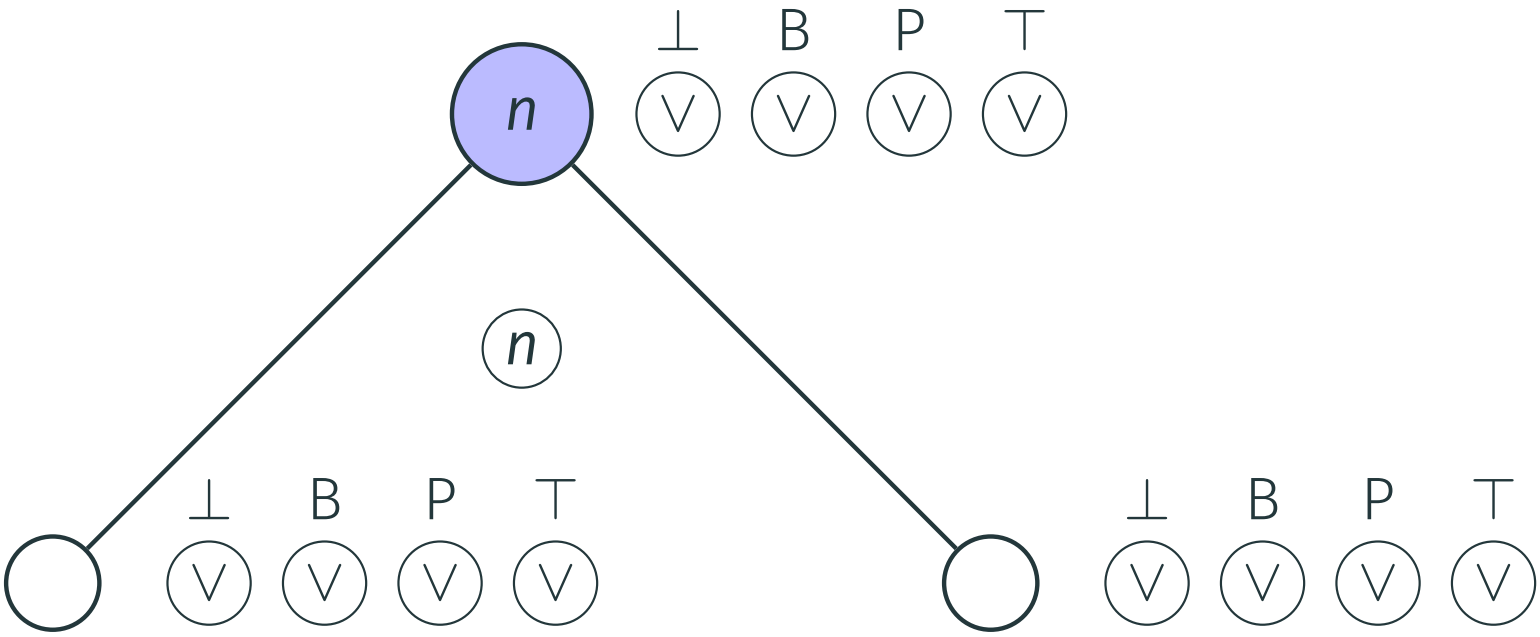

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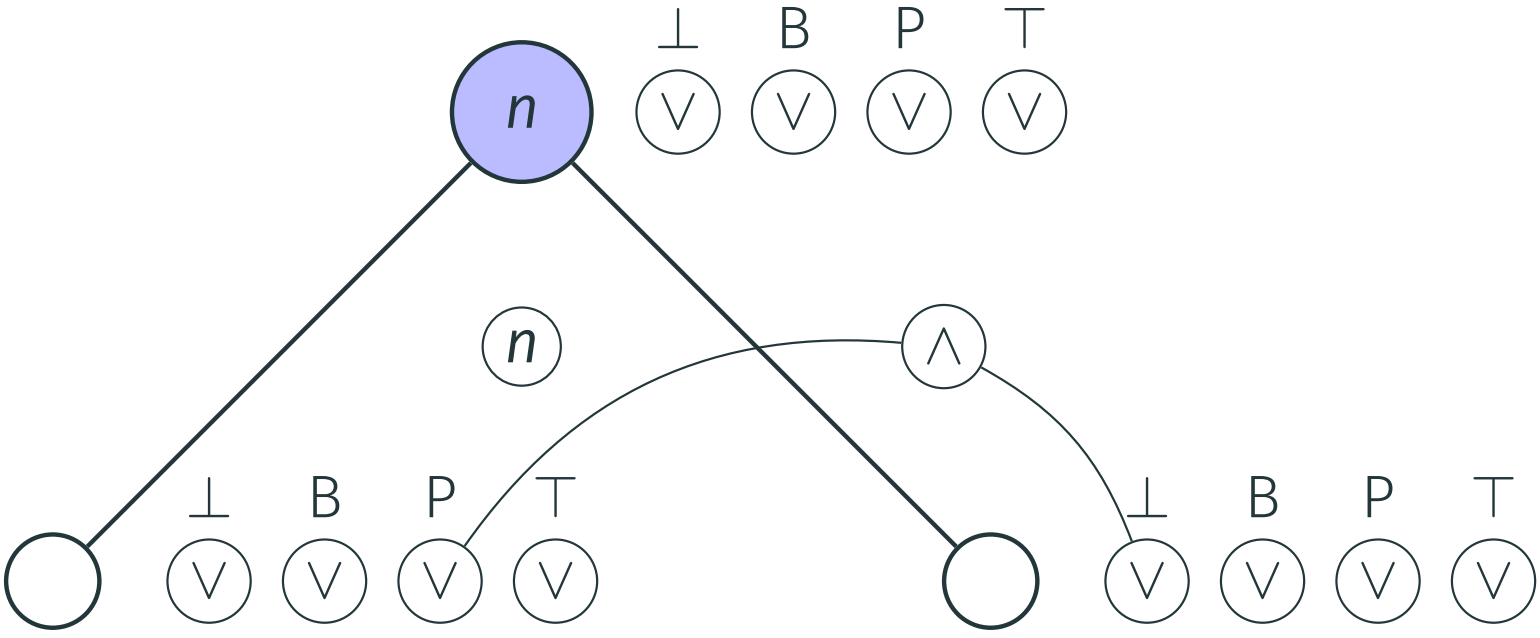

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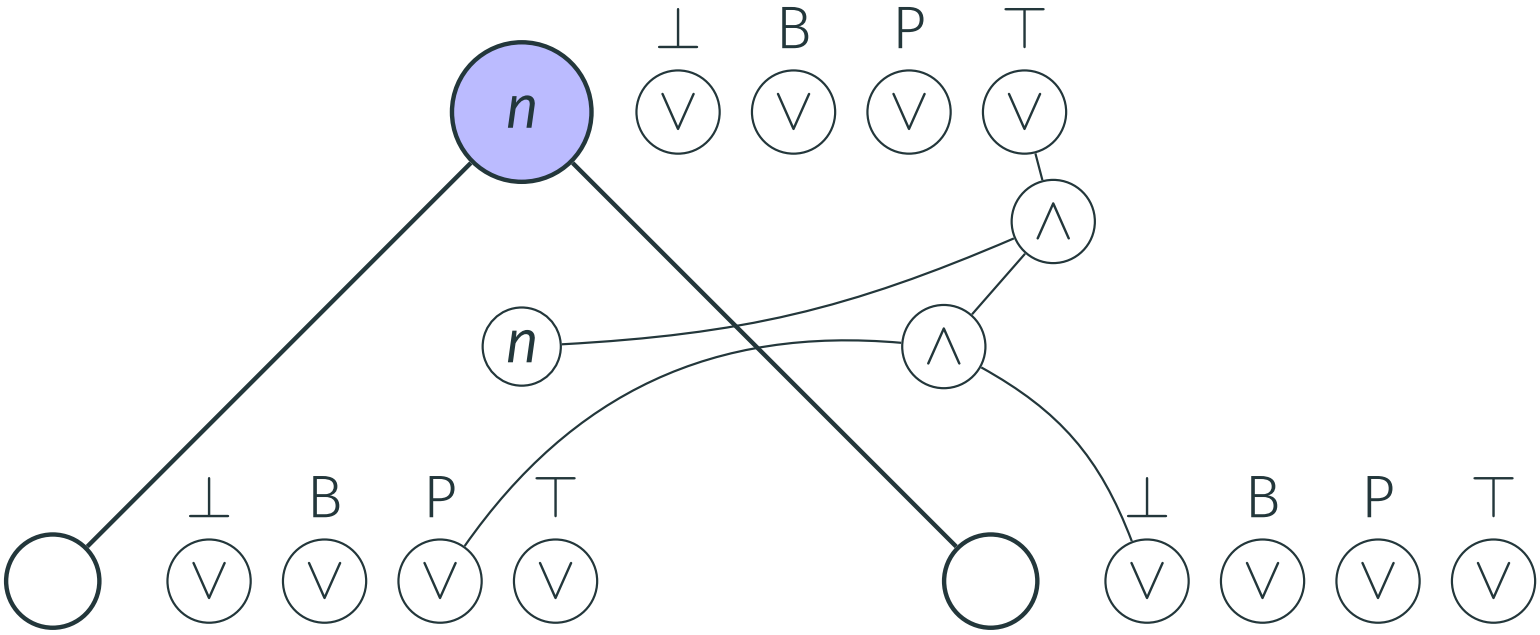
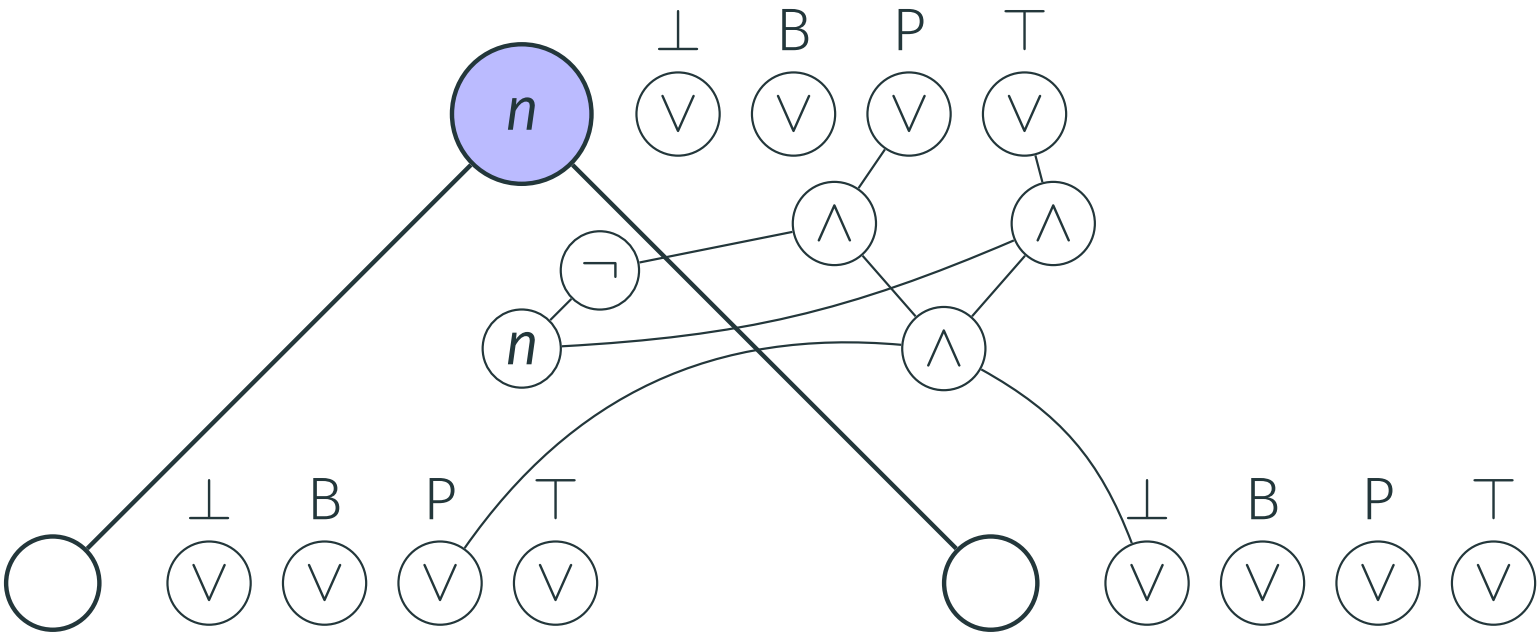


Building provenance circuits on trees

Theorem

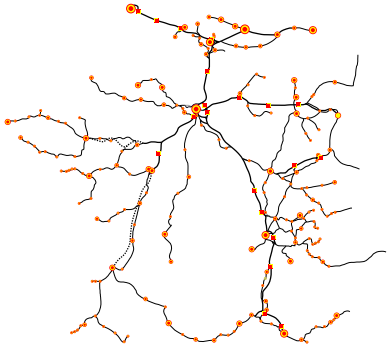
For any bottom-up tree automaton A and input tree T, we can build a provenance circuit of A on T in $O(|A| \times |T|)$

- Alphabet: ○ ● ●
- Automaton: *"Is there both a pink and a blue node?"*
- States: $\{\bot, B, P, \top\}$
- Final: $\{\top\}$
- Transitions:



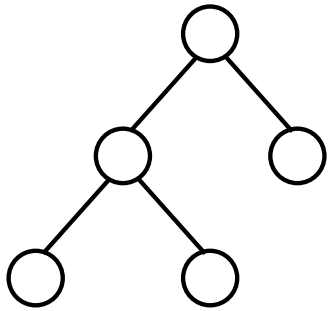
Probabilistic query evaluation

Probabilistic treelike **data**



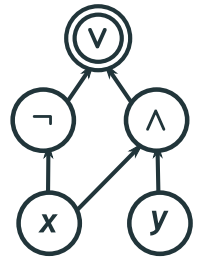
linear

Uncertain tree **encoding**



linear

Provenance circuit +probabilities



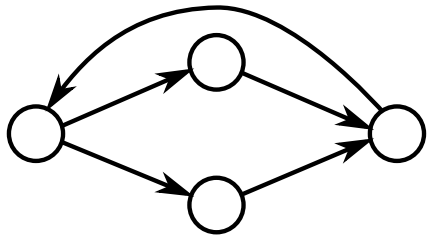
linear

95%
Probability

MSO query

(RER|metro)*
|(bus|tram)*

Tree **automaton**



Details of the approach

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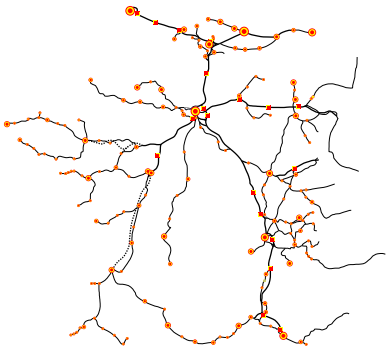

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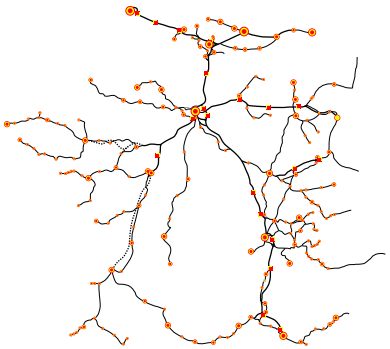

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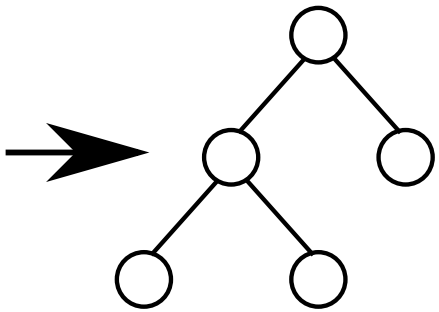

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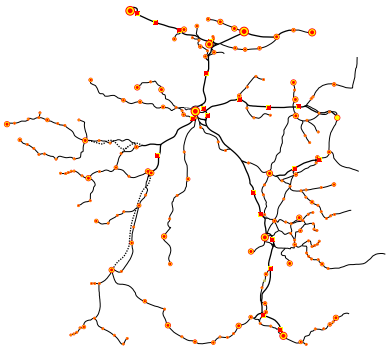

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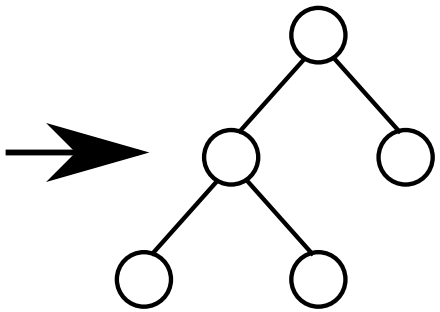

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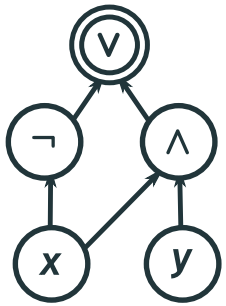

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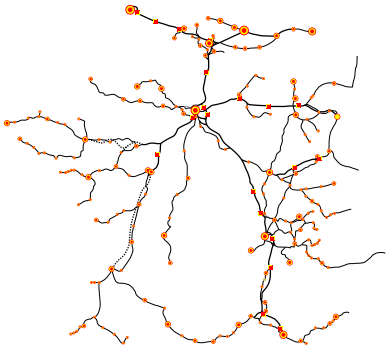

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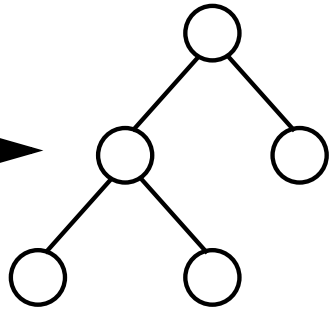

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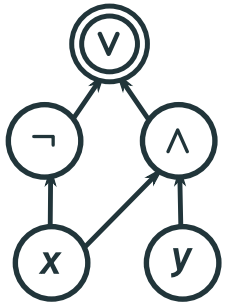

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*Probability that the **circuit** evaluates to **true***

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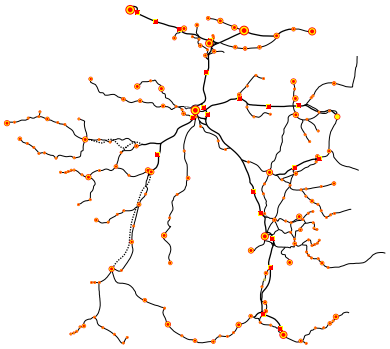

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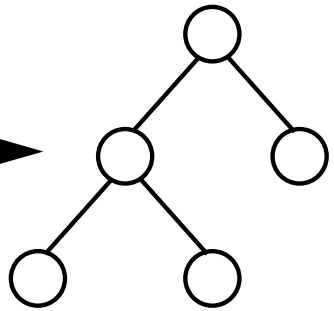

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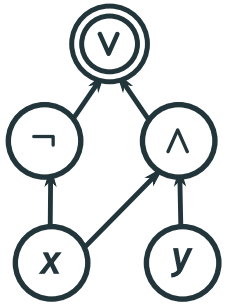

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→ How to compute efficiently the probability of the circuit?

Computing the circuit probability

The circuit is a d-DNNF...

Computing the circuit probability

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-  gates only have variables as inputs

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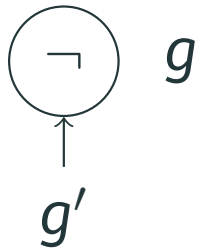

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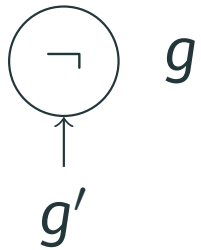

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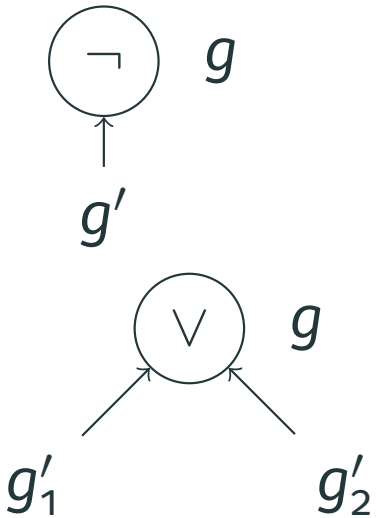

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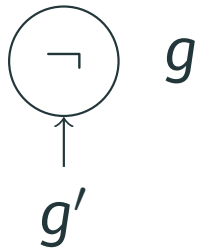

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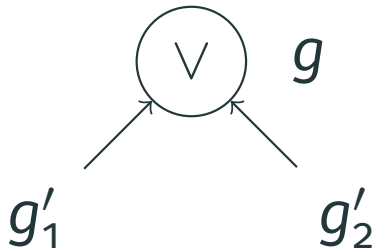

$$P(g) := P(g'_1) + P(g'_2)$$

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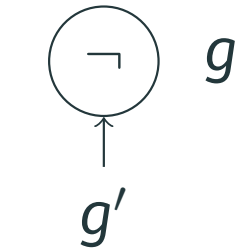

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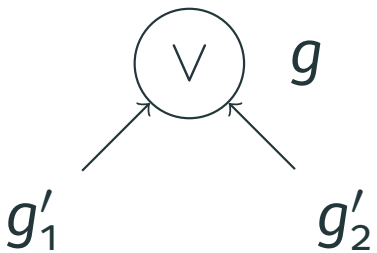

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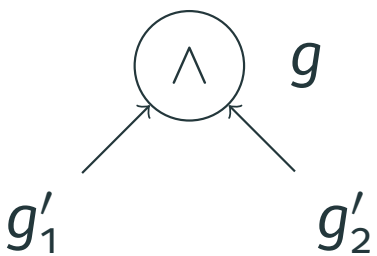

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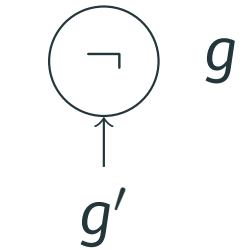

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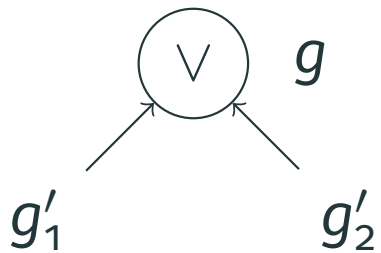

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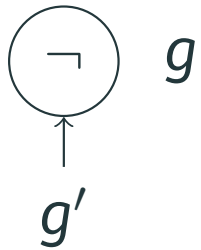

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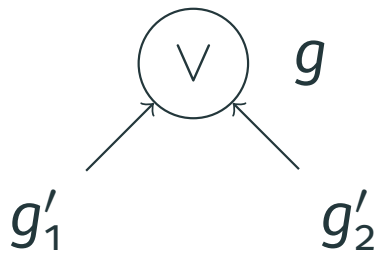

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- $\wedge$ gates are all on independent inputs

$$P(g) := P(g'_1) \times P(g'_2)$$

Theorem [Amarilli et al., 2015]

For any fixed Boolean MSO query Q and $k \in \mathbb{N}$, given a database D of treewidth $\leq k$ with independent probabilities, we can compute in linear time the probability that D satisfies Q

Table of contents

Circuits as factorized representations of query results

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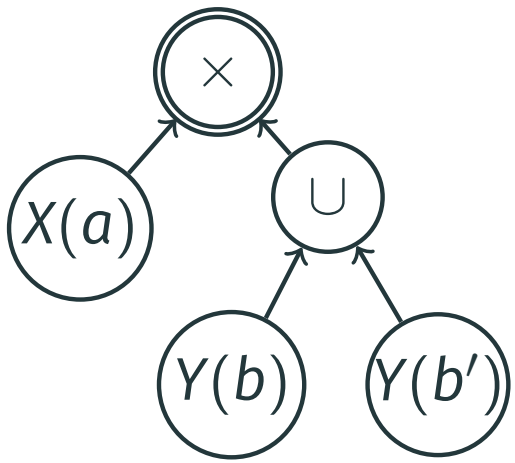

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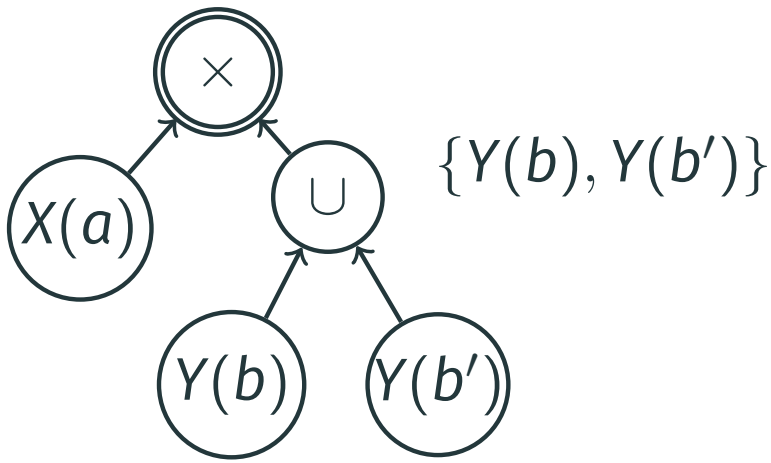

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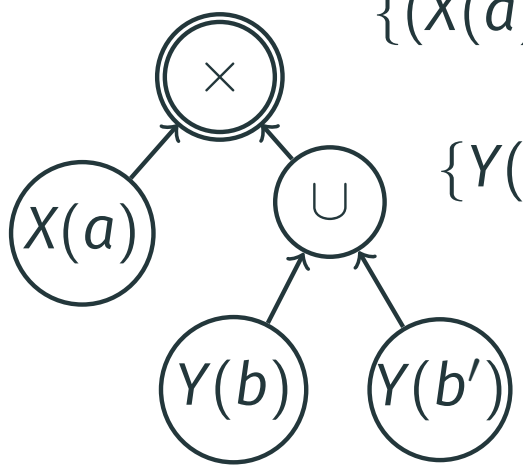

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- Other results:
  - Lower bounds: probabilistic query evaluation is hard unless treewidth is bounded (modulo assumptions) [Amarilli et al., 2016]
  - Complexity in the query: generally nonelementary but can be improved [Amarilli et al., 2017b, Amarilli et al., 2017c]

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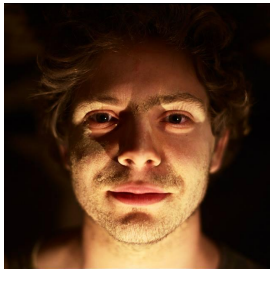
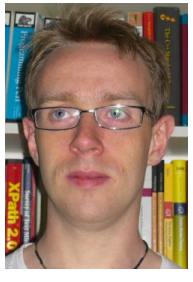
Pierre


Louis


Stefan


Mikaël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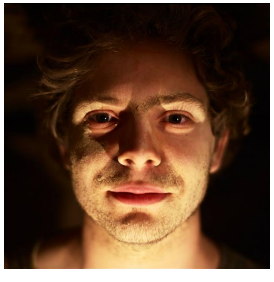
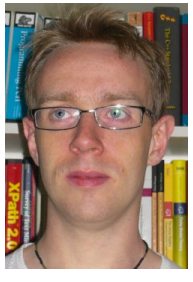
- Ongoing work (with my wonderful co-authors):
  - Efficient enumeration algorithms under updates
  - More lower bounds results, connections to knowledge compilation
  - More expressive provenance: cycluits (circuits with cycles)
  - Combined tractability for probabilistic query evaluation
  - Connections to message passing probabilistic algorithms


Pierre


Louis


Stefan


Mikaël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Thanks for your attention!

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