Probabilistic Databases: Introduction

EDBT-Intended Summer School

Antoine Amarilli

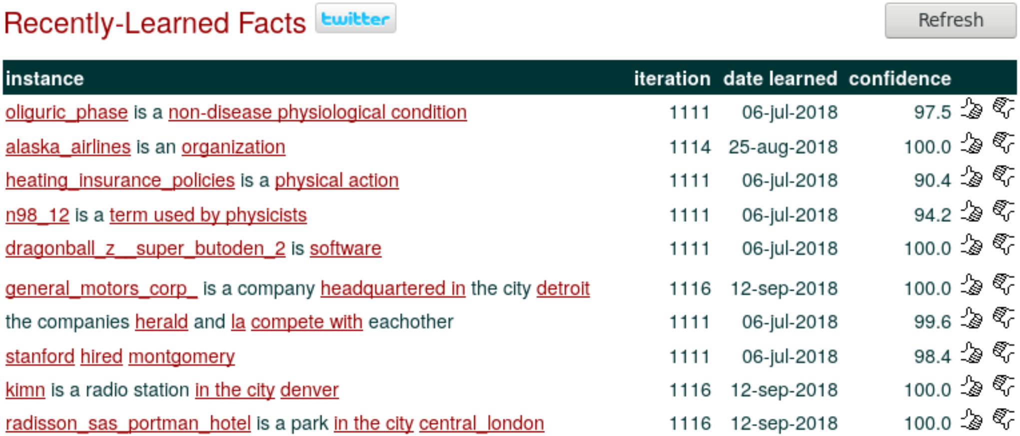
TELECOM Paris

Uncertain data: Practical motivations

Numerous sources of uncertain data:

- Measurement errors
- Data integration from contradicting sources
- Imprecise mappings between heterogeneous schemata
- Imprecise automated processes (information extraction, NLP, etc.)
- Imperfect human judgment
- Lies, opinions, rumors

Use case: Web information extraction



Recently-Learned Facts

instance	iteration	date learned	confidence
oliguric_phase is a non-disease physiological condition	1111	06-jul-2018	97.5
alaska_airlines is an organization	1114	25-aug-2018	100.0
heating_insurance_policies is a physical action	1111	06-jul-2018	90.4
n98_12 is a term used by physicists	1111	06-jul-2018	94.2
dragonball_z__super_butoden_2 is software	1111	06-jul-2018	100.0
general_motors_corp_ is a company headquartered in the city detroit	1116	12-sep-2018	100.0
the companies herald and la compete with eachother	1111	06-jul-2018	99.6
stanford hired montgomery	1111	06-jul-2018	98.4
kimn is a radio station in the city denver	1116	12-sep-2018	100.0
radisson_sas_portman_hotel is a park in the city central_london	1116	12-sep-2018	100.0

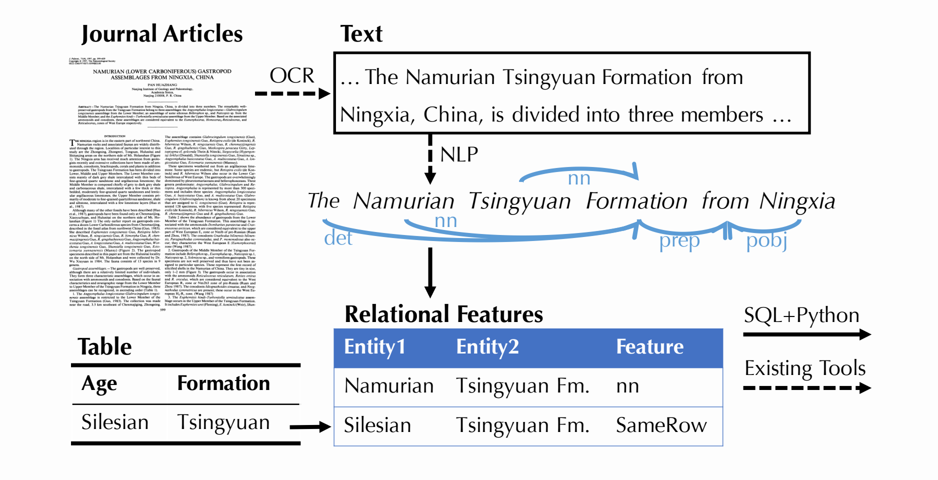
Never-ending Language Learning (NELL, CMU), `http://rtw.ml.cmu.edu/rtw/kbbrowser/`

Use case: Web information extraction

Subject	Predicate	Object	Confidence
Elvis Presley	diedOnDate	1977-08-16	97.91%
Elvis Presley	isMarriedTo	Priscilla Presley	97.29%
Elvis Presley	influences	Carlo Wolff	96.25%

YAGO, `https://www.yago-knowledge.org/`

Other use case: Information extraction from scientific articles



Journal Articles

NAMURIAN (LOWER CARBONIFEROUS) GASTROPOD ASSEMBLAGES FROM NINGXIA, CHINA

PAN HUAZHANG

OCR

Text

… The Namurian Tsingyuan Formation from Ningxia, China, is divided into three members …

NLP

The Namurian Tsingyuan Formation from Ningxia

Table

Age	Formation
Silesian	Tsingyuan

Relational Features

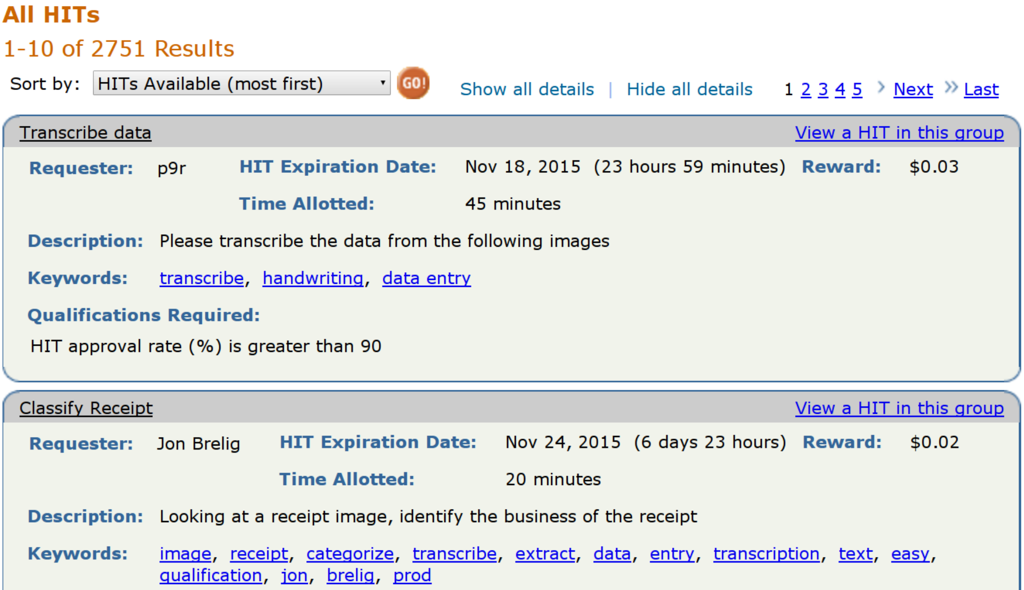
Entity1	Entity2	Feature
Namurian	Tsingyuan Fm.	nn
Silesian	Tsingyuan Fm.	SameRow

SQL+Python

Existing Tools

From GeoDeepDive / xDD

Other use case: Crowdsourcing



All HITs

1-10 of 2751 Results

Sort by: HITs Available (most first) GO!   Show all details | Hide all details   1 2 3 4 5 › Next » Last

Transcribe data   View a HIT in this group

Requester: p9r   **HIT Expiration Date:** Nov 18, 2015 (23 hours 59 minutes)   **Reward:** $0.03

Time Allotted: 45 minutes

Description: Please transcribe the data from the following images

Keywords: transcribe, handwriting, data entry

Qualifications Required:

HIT approval rate (%) is greater than 90

Classify Receipt   View a HIT in this group

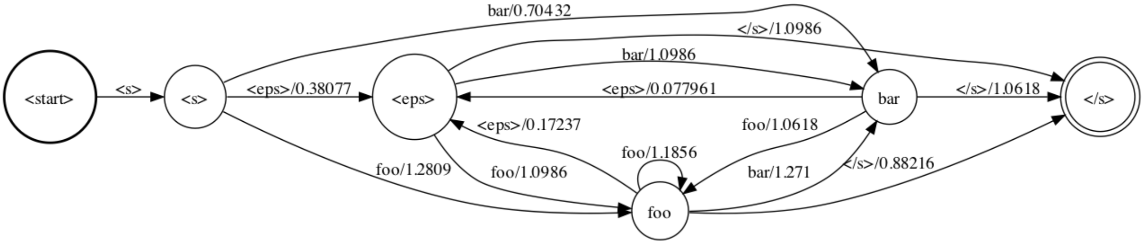
Requester: Jon Brelig   **HIT Expiration Date:** Nov 24, 2015 (6 days 23 hours)   **Reward:** $0.02

Time Allotted: 20 minutes

Description: Looking at a receipt image, identify the business of the receipt

Keywords: image, receipt, categorize, transcribe, extract, data, entry, transcription, text, easy, qualification, jon, brelig, prod

Other use case: Speech recognition and OCR



Different types of uncertainty

- The uncertainty can be **qualitative** (e.g., NULL)…
- … or **quantitative** (e.g., 95%)

Further, there are different types:

- **Unknown** value: NULL in an RDBMS
- **Alternative** between several possibilities: either A or B or C
- **Imprecision on a numeric value**: a sensor gives a value that is an approximation of the actual value
- **Confidence in a fact as a whole**: cf. information extraction
- **Structural uncertainty**: the schema of the data itself is uncertain
- **Missing data**: we know that some data is missing (open-world semantics)

What happens to this uncertainty?

Naive solution

Forget about uncertainty, or apply a threshold after each computation step

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Instead of neglecting uncertainty, let’s manage it rigorously throughout the whole process of answering a query

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Instead of neglecting uncertainty, let's manage it rigorously throughout the whole process of answering a query

Also: it leads to interesting theoretical questions! :)

Possible worlds semantics

Idea: use a representation system

Possible world: A regular (deterministic) relational database

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- **finite:** a set of possible worlds, each with their probability
- **continuous:** more complicated

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date	**teacher**	
08	Diego	0.9
09	Paolo	0.8
09	Floris	0.7

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- Present the most common **models** of probabilistic data
  - → Focus on the **simplest one**, tuple-independent databases (TID)

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- Present **treewidth-based approaches** to efficient PQE
- Give an overview of **other topics** on probabilistic databases

Probabilistic Databases: Models and PQE

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Relational model by example

`Guest`

`id`	`name`	`email`
1	John Smith	john.smith@gmail.com
2	Alice Black	alice@black.name
3	John Smith	john.smith@ens.fr

`Reservation`

`id`	`guest`	`room`	`arrival`	`nights`
1	1	504	2022-01-01	5
2	2	107	2022-01-10	3
3	3	302	2022-01-15	6
4	2	504	2022-01-15	2
5	2	107	2022-01-30	1

Relations and databases

Formally:

- A **database schema** $\mathcal{D}$ maps each **relation name** to an **arity** (we add **attribute names** in our examples)

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We can write tuples as **table rows** or as **ground facts**:

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`id`	`name`	`email`
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Guest(1, John Smith, john.smith@gmail.com),
Guest(2, Alice Black, alice@black.name),
Guest(3, John Smith, john.smith@ens.fr)

Queries

- A **query** is an arbitrary **function** over database instances over a fixed schema $\mathcal{D}$
- We only study **Boolean queries**, i.e., queries returning only **true** or **false**

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  - **Conjunctive queries** (CQ)
    - $\exists \bigwedge \cdots$: existentially quantified conjunctions of atoms
    - $Q : \exists x\, y\, z\, x'\, y'\; Guest(x, y, z) \wedge Guest(x', y', z)$

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  - **Unions of conjunctive queries** (UCQ)
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    - **Unions of conjunctive queries** (UCQ)
        - $\bigcup \exists \bigwedge \cdots$: unions of **CQs**
    - First-Order logic (FO)
    - **Monadic-Second Order** logic (MSO)

TID

Tuple-independent databases (TID)

- The **simplest** model: tuple-independent databases
- Annotate each **instance fact** with a **probability**

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→ Assume independence between facts

Semantics of TID

- Each fact is **kept** or **discarded** with the indicated probability
- Probabilistic choices are **independent** across facts

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Getting a probability distribution

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→ the possible worlds are the subsets of facts of I

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Formally, for a TID I, the probability of $J \subseteq I$ is:

- product of $\Pr(F)$ for each fact F kept in J
- product of $1 - \Pr(F)$ for each fact F not kept in J

Is it a probability distribution?

Do the probabilities of the possible words always sum to 1?

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→ The sum of these probabilities is the result of expanding the expression:
$(\Pr(\mathrm{F}_1) + (1 - \Pr(\mathrm{F}_1))) \times \cdots \times (\Pr(\mathrm{F}_\mathrm{N}) + (1 - \Pr(\mathrm{F}_\mathrm{N})))$

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- All factors are **equal to 1**, so the probabilities **sum to 1**

Expressiveness of TID

Can we represent all probabilistic instances with TID?

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U_1

teacher
Jane

$\pi(U_1) = 80\%$

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U_2

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U_3

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→ We **cannot** forbid that both teach the class!

BID

Block-independent disjoint instances

- A more expressive framework than TID
- Call some attributes the key (underlined)

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- Probabilities must sum up to ≤ 1 in each block

BID semantics

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BID captures TID

- Each TID can be expressed as a BID...

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  - → Take all attributes as key
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U

date	**teacher**	
09	Diego	90%
09	Paolo	80%
09	Floris	70%

Expressiveness of BID

Can we represent all probabilistic instances with BID?

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U_1

teacher
Diego
Paolo
$\pi(U_1) = 80\%$

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U_1

teacher
Diego
Paolo
$\pi(U_1) = 80\%$

U_2

teacher
Diego
Floris
$\pi(U_2) = 10\%$

Expressiveness of BID

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teacher
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Paolo
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U_2

teacher
Diego
Floris
$\pi(U_2) = 10\%$

U_3

teacher
Paolo
Floris
$\pi(U_3) = 10\%$

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→ If **teacher** is not a key, then **only one fact**

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Diego
Floris

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U_3

teacher
Paolo
Floris

$\pi(U_3) = 10\%$

→ If **teacher** is a key **teacher**, then **TID**

→ If **teacher** is not a key, then **only one fact**

→ We **cannot represent** this probabilistic instance as a BID

pc-tables

Boolean c-tables

- Set of Boolean variables $x_1, x_2, \ldots$
- Each fact has a condition: Variables, Boolean operators

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11	Joe	Amphi B	$x_2 \land x_1$
11	Jane	Amphi C	$\neg x_2 \land \neg x_1$
11	Joe	Amphi C	$\neg x_2 \land x_1$

x_1	Jane is sick
x_2	Amphi B is available

pc-tables

A (Boolean) pc-table is:

- a database I where each tuple is annotated by a Boolean function on variables x_i
- a probability p_i that each variable x_i is true (assuming independence)

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  → This is like TIDs
- The **probability** of a possible world $J \subseteq I$ is the total probability of the valuations ν such that $I_\nu = J$

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11	Joe	Amphi C	$\neg x_2 \wedge x_1$

x_1 Jane is sick

→ Probability 10%

x_2 Amphi B is available

→ Probability 20%

pc-table semantics example

date	**teacher**	**room**	$x_1 : 10\%, x_2 : 20\%$
04	Jane	Amphi A	$\neg x_1$
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- Take ν mapping x_1 to 0 and x_2 to 1
- Probability of ν: $(100\% - 10\%) \times 20\% = 18\%$

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- **Probability** of ν: $(100\% - 10\%) \times 20\% = 18\%$
- Evaluate the **conditions**

→ Probability of possible world: **sum** over the valuations

  → Here: **only** this valuation, 18%

Expressiveness of pc-tables

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  - → Simply give each fact its own probability value

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- Further, they are a **strong representation system**: the union, product, etc., of two pc-tables, can be easily represented as a pc-table

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Yet, in the rest of the talk, we focus on **TIDs** → easier to characterize tractable queries

PQE

Query evaluation on probabilistic databases (PQE)

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How can we evaluate a query *Q* over a probabilistic database *D*?

- Probability that *Q* holds over *D*:

$$\Pr(D \models Q) = \sum_{\substack{D' \subseteq D \\ D' \models Q}} \Pr(D')$$

- **Intuitively:** the probability that Q holds is the probability of drawing a possible world $D' \subseteq D$ which satisfies Q

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Probabilistic query evaluation (PQE) problem for a query Q over TIDs: given a TID, compute the probability that Q holds

Example of PQE on TID

name	position	city	classification	**prob**
John	Director	New York	unclassified	0.5
Paul	Janitor	New York	restricted	0.7
Dave	Analyst	Paris	confidential	0.3
Ellen	Field agent	Berlin	secret	0.2
Magdalen	Double agent	Paris	top secret	1.0
Nancy	HR director	Paris	restricted	0.8
Susan	Analyst	Berlin	secret	0.2

What is the probability to have a tuple with value **New York**?

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- It is one minus the probability of not having such a tuple

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What is the probability to have a tuple with value **New York**?

- It is **one minus** the probability of not having such a tuple
- Not having such a tuple is the **independent AND** of not having each tuple
- So the result is $1 - (1 - 0.5) \times (1 - 0.7) = 0.85$

Complexity of PQE

Formal question:

- We fix a Boolean query, e.g., $\exists xy\, R(x), S(x,y), T(y)$

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- Note that we study data complexity, i.e., Q is fixed and the input is D

Naive probabilistic query evaluation

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- Consider all possible worlds of the input
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- Sum the probabilities of all worlds that satisfy the query

Naive probabilistic query evaluation example

TID D

in	out	
A	B	0.8
B	C	0.2

Query Q

$$R(x, y) \land R(y, z)$$

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Possible worlds and probabilities:

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in	out
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$(1 - 0.8) \times 0.2$

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in	out
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Total probability that Q holds: $0.8 \times 0.2 = 0.16$.

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  - → Even if the query output has **few possible worlds**!
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- Probabilistic query evaluation is **computationally intractable** so it is unlikely that we can beat naive evaluation **in general**
  - → But **some queries** admit an efficient algorithm!

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- What is the probability of the query: $\exists x\, R(x)$?
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→ It is: $1 - \prod_{R(a)} (1 - \mathrm{Pr}(R(a)))$

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- What is the probability of the query: $\exists xy\ R(x), S(x, y)$?
  - It asks: "is there an R-fact which also has an S-fact?"
  - Idea: case disjunction based on the value of x

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Research question: can we **characterize the easy cases** and **prove hardness otherwise**?

Probabilistic Databases: The Dichotomy of PQE

EDBT-Intended Summer School

Antoine Amarilli

Research goal: Understanding the complexity of PQE

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- For $Q : R(x)$ the problem is **easy** (PTIME)
- For $Q : R(x), S(x, y), T(y)$ the problem is **hard** (#P-hard)

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Result of the form:

if Q *has a certain form* then $\mathrm{PQE}(Q)$ is in PTIME, otherwise it is #P-hard

The “small” Dalvi and Suciu dichotomy

- **Conjunctive query (CQ)**: existentially quantified conjunction of atoms

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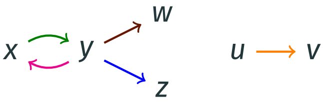
Theorem ([Dalvi and Suciu, 2007])

Let Q be an arity-two self-join-free CQ:

- *If Q is a **conjunction of stars**, then $\mathrm{PQE}(Q)$ is in **PTIME***
- *Otherwise, $\mathrm{PQE}(Q)$ is **#P-hard***

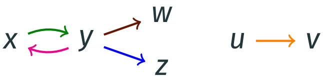
Conjunction of stars

- A **star** is a CQ with a **separator variable** that occurs in all edges
- A **conjunction of stars** is a conjunction of one or several stars



The following is **not a star**: $x \longrightarrow y \longrightarrow z \longrightarrow w$

Proving the small dichotomy (upper bound, 1)



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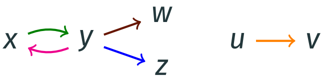

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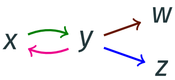

- We consider each connected component separately

→ Independent conjunction over the connected components

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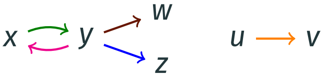

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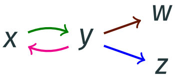


- We consider each connected component separately

→ Independent conjunction over the connected components

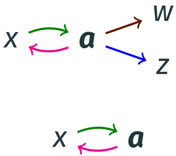
- We can test all possible values of the separator variable

→ Independent disjunction over the values of the separator

Proving the small dichotomy (upper bound, 2)

$$\begin{tikzcd} & & w \\ x \arrow[r, bend left] & a \arrow[l, bend left] \arrow[ur] \arrow[dr] & \\ & & z \end{tikzcd}$$

Proving the small dichotomy (upper bound, 2)



- For every match, we consider every **other variable** separately

→ **Independent conjunction** over the variables

Proving the small dichotomy (upper bound, 2)

$$\begin{tikzcd} & & w \\ x \arrow[r, bend left] & \boldsymbol{a} \arrow[l, bend left] \arrow[ru] \arrow[rd] & \\ & & z \end{tikzcd}$$

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$$\begin{tikzcd} \boldsymbol{b_1} \arrow[r, bend left] & \boldsymbol{a} \arrow[l, bend left] \\ \boldsymbol{b_2} \arrow[r, bend left] & \boldsymbol{a} \arrow[l, bend left] \\ \boldsymbol{b_3} \arrow[r, bend left] & \boldsymbol{a} \arrow[l, bend left] \\ \vdots & \end{tikzcd}$$

- We consider every value for the other variable

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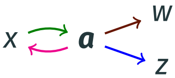

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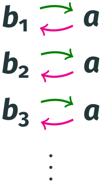

- We consider every value for the **other variable**

→ **Independent disjunction** over the possible assignments

- We consider every fact

→ **Independent conjunction** over the facts

→ Just **read the probability** of the ground fact $R(b, a)$.

Proving the small dichotomy (lower bound, 1)

Every arity-two self-join-free CQ which is not a conjunction of stars contains a pattern essentially like:

$$x \longrightarrow y \longrightarrow z \longrightarrow w$$

Proving the small dichotomy (lower bound, 1)

Every arity-two self-join-free CQ which is **not a conjunction of stars** contains a pattern essentially like:

$$x \longrightarrow y \longrightarrow z \longrightarrow w$$

We can **add facts with probability 1** to instances so the other facts are always satisfied, and focus on **only these three facts**

→ **Let us show #P-hardness of this query**

Proving the small dichotomy (lower bound, 2)

Let us show that $\mathrm{PQE}(Q)$ is **#P-hard** for the CQ Q : $x \longrightarrow y \longrightarrow z \longrightarrow w$

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    - e.g., given $(x \lor y) \land z$, compute that it has 3 satisfying valuations

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- Example: $\phi : (X_1 \land Y_1) \lor (X_1 \land Y_2) \lor (X_2 \land Y_2) \lor (X_3 \land Y_1) \lor (X_3 \land Y_2)$

Proving the small dichotomy (lower bound, 3)

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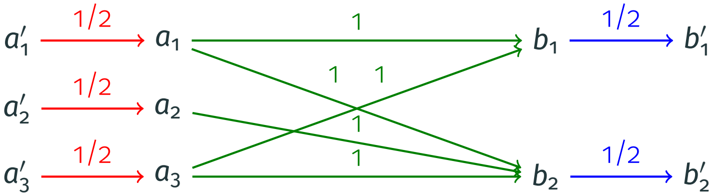

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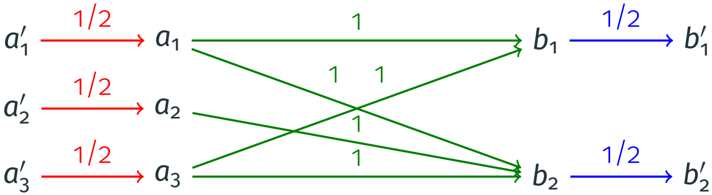

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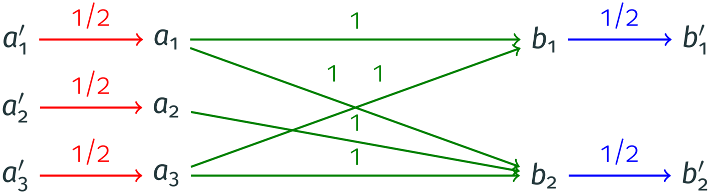

Idea:

- **Valuations** of ϕ correspond to **possible worlds** of I_ϕ
- A valuation **satisfies** ϕ iff the corresponding possible world **satisfies** Q

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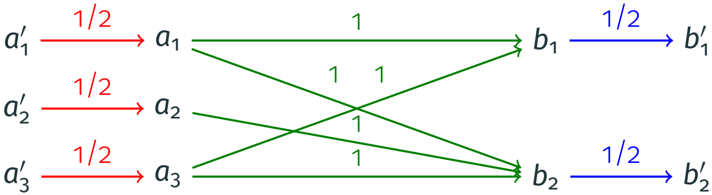
Example: $\phi : (X_1 \land Y_1) \lor (X_1 \land Y_2) \lor (X_2 \land Y_2) \lor (X_3 \land Y_1) \lor (X_3 \land Y_2)$

Build an **instance** I_ϕ from ϕ:



Idea:

- **Valuations** of ϕ correspond to **possible worlds** of I_ϕ
- A valuation **satisfies** ϕ iff the corresponding possible world **satisfies** Q

→ The **probability** of Q on I_ϕ is the **number of accepting valuations of** ϕ, divided by the number of valuations ($2^{-|\mathrm{Vars}|}$)

Extending beyond arity-two (1)

How can we extend beyond arity-two queries?

Theorem ([Dalvi and Suciu, 2007])

Let Q be a ~~arity-two~~ self-join-free CQ:

- *If Q is ~~a conjunction of stars~~ hierarchical, then $\mathrm{PQE}(Q)$ is in PTIME*
- *Otherwise, $\mathrm{PQE}(Q)$ is #P-hard*

Extending beyond arity-two (2)

Class of Hierarchical CQs defined inductively:

- A query with no variables is hierarchical

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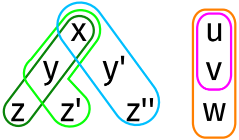
$$\exists x\,(\exists y\,(\exists z\; R_1(x,y,z)) \land (\exists z'\; R_2(x,y,z'))) \land (\exists y' \exists z''\; R_3(x,y',z''))$$
$$\land\,(\exists u\,(\exists v\; R_4(u,v)) \land (\exists w\; R_5(u,v,w)))$$



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Via **equivalent characterization**: a non-hierarchical query has two variables x and y and:

- One atom containing **x and y**
- One atom containing **x but not y**
- One atom containing **y but not x**

The “big” Dalvi and Suciu dichotomy

Full dichotomy on the **unions of conjunctive queries** (UCQs):

Theorem ([Dalvi and Suciu, 2012])

Let Q *be a UCQ:*

- *If* Q *is handled by a complicated algorithm then* $\mathrm{PQE}(Q)$ *is in* ***PTIME***
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This result is **far more challenging**:

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This result is far more challenging:

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  - many unpleasant details (e.g., a ranking transformation)
- Lower bound: hardness proof on minimal cases where the algorithm does not work (very challenging)

References i

Dalvi, N. and Suciu, D. (2007).
The dichotomy of conjunctive queries on probabilistic structures.
In *Proc. PODS*.

Dalvi, N. and Suciu, D. (2012).
The dichotomy of probabilistic inference for unions of conjunctive queries.
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Probabilistic Databases: Provenance Circuits and Knowledge Compilation

EDBT-Intended Summer School

Antoine Amarilli

Recall: Boolean Provenance

- Relational database instance I: set of facts
- Boolean query Q: take an instance and answer yes/no

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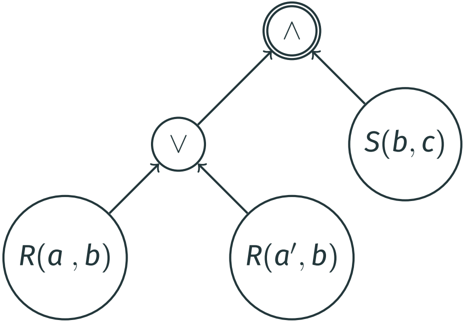

Related work: Semiring provenance

Semiring provenance ([Green et al., 2007], on Tuesday): annotate results of a **relational algebra** query with a **semiring expression**

A B C	
a b c	p
d b e	r
f g e	s

(a)

A C	
a c	$\{p\}$
a e	$\{p, r\}$
d c	$\{p, r\}$
d e	$\{r, s\}$
f e	$\{r, s\}$

(b)

A C	
a c	$2p^2$
a e	pr
d c	pr
d e	$2r^2 + rs$
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Figure 5: Why-prov. and provenance polynomials

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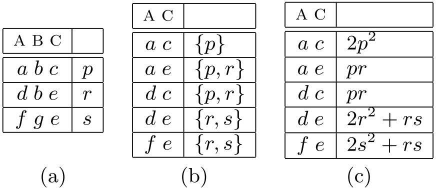

A	B	C	
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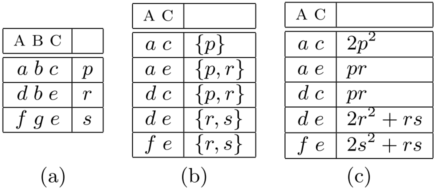

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- We only care about **Boolean provenance**
  → No **multiplicity** of facts or derivations
- **Circuit representation:** more concise

The intensional approach to PQE

- **Previously**, for a tractable query Q: we can solve $\mathrm{PQE}(Q)$
- Now, let's see the **intensional approach**
  - Compute a **circuit** representing the **Boolean provenance** of Q
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$O(n^2)$ algorithms

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Setting A → Task 2

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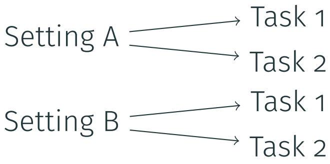
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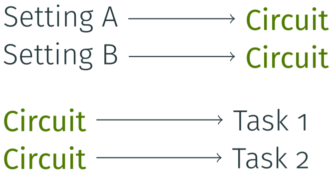
Setting A → **Circuit**
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Circuit → Task 1
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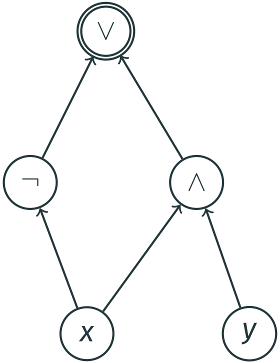
Boolean circuit representations

Circuits are just a way to represent **Boolean formulas**
while factoring common subexpressions (more concise)



- Directed acyclic graph of **gates**
- **Output** gate: ◎
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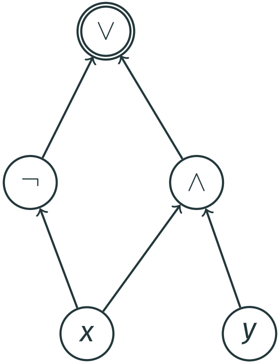

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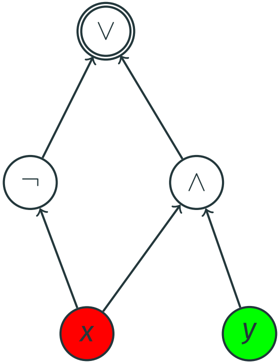

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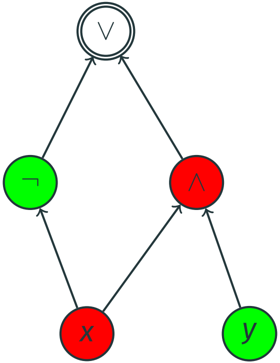

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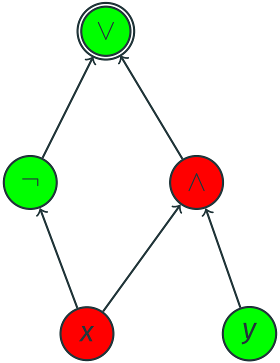

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- Unions of Conjunctive Queries (UCQ)

Theorem

For any UCQ, given an instance, we can construct its Boolean provenance in PTIME

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- Regular path queries (RPQ), Datalog, etc.

Theorem [Deutch et al., 2014]

For any Datalog query, given an instance, we can get its Boolean provenance in PTIME

Computing Boolean provenance: practice

- **ProvSQL:** PostgreSQL extension to compute provenance
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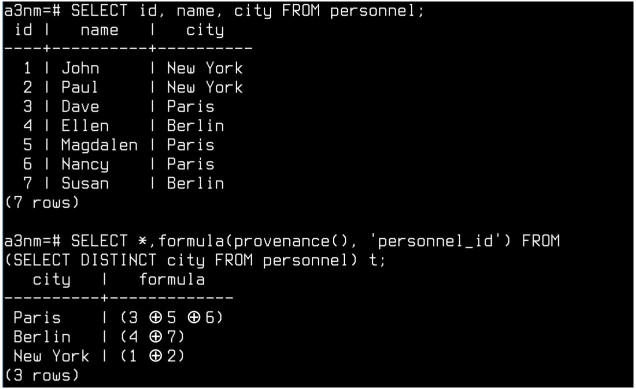

```
a3nm=# SELECT id, name, city FROM personnel;
 id |   name   |   city
----+----------+----------
  1 | John     | New York
  2 | Paul     | New York
  3 | Dave     | Paris
  4 | Ellen    | Berlin
  5 | Magdalen | Paris
  6 | Nancy    | Paris
  7 | Susan    | Berlin
(7 rows)

a3nm=# SELECT *,formula(provenance(), 'personnel_id') FROM
(SELECT DISTINCT city FROM personnel) t;
   city   |   formula
----------+-------------
 Paris    | (3 ⊕ 5 ⊕ 6)
 Berlin   | (4 ⊕ 7)
 New York | (1 ⊕ 2)
(3 rows)
```

You can run it! `https://github.com/PierreSenellart/provsql`

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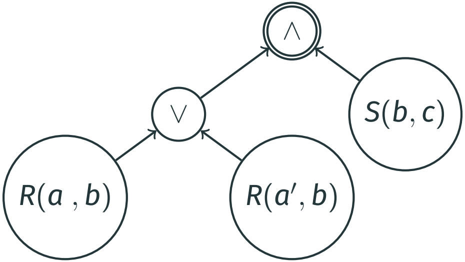

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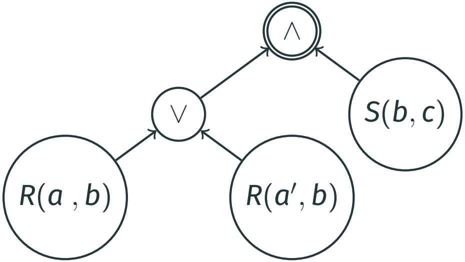

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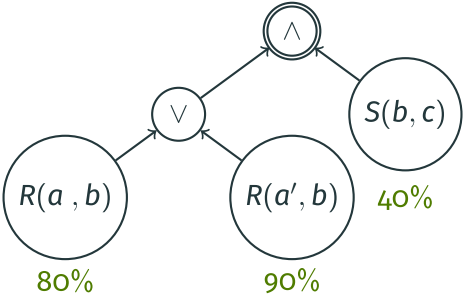

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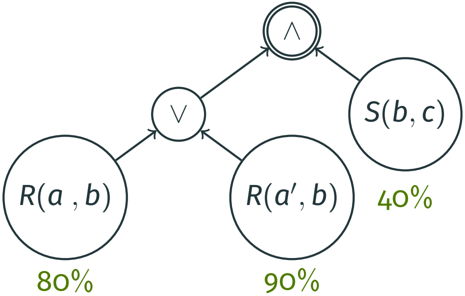

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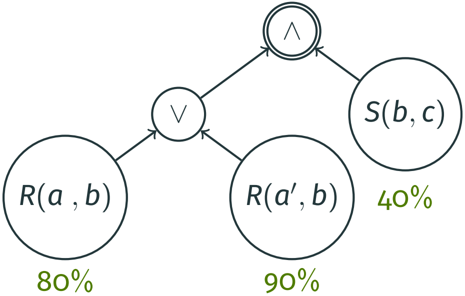
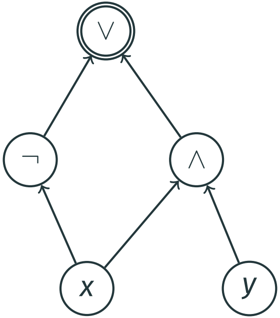


Computing the probability of the circuit

- We now have a Boolean provenance circuit over the database facts



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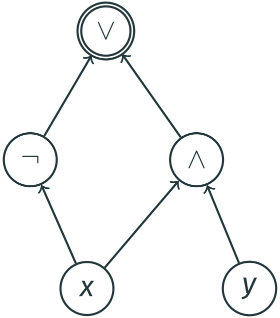

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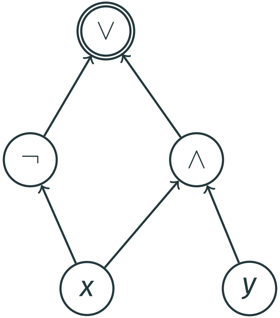

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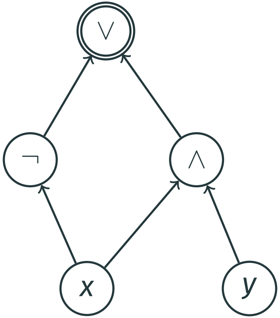

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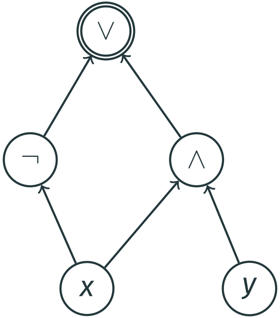

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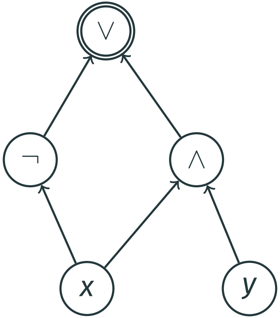

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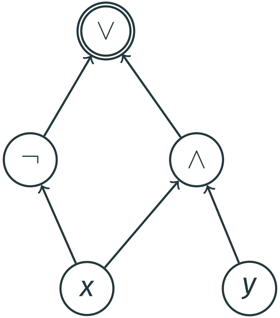

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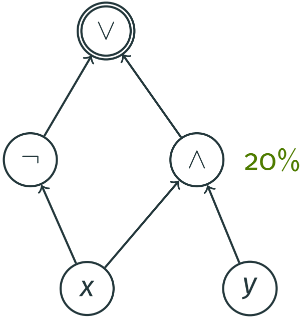

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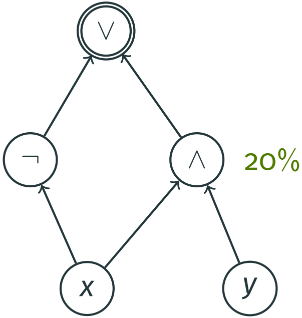

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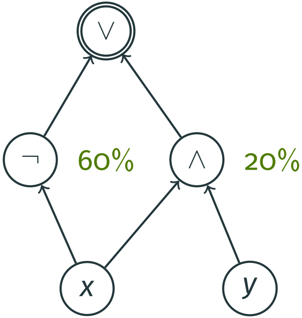

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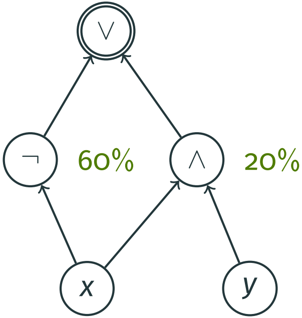

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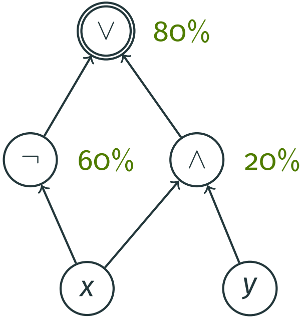

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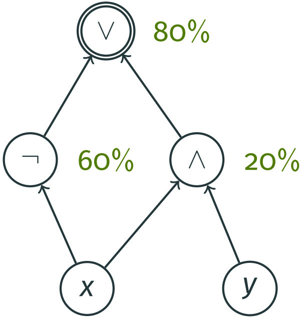

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→ The circuit that we constructed falls in a **restricted class** satisfying such conditions

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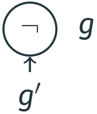

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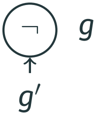

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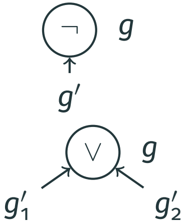

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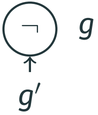

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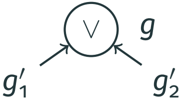

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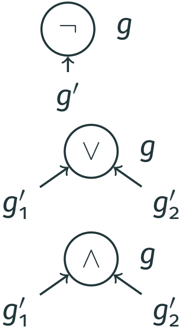

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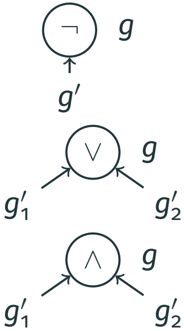

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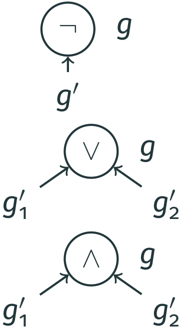

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→ d-DNNFs are one of many **tractable circuit classes** in **knowledge compilation**

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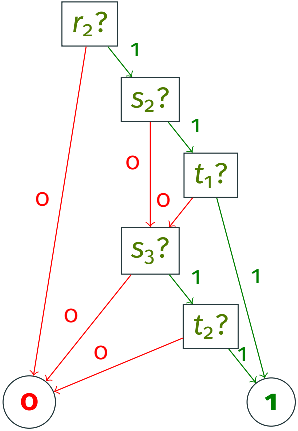

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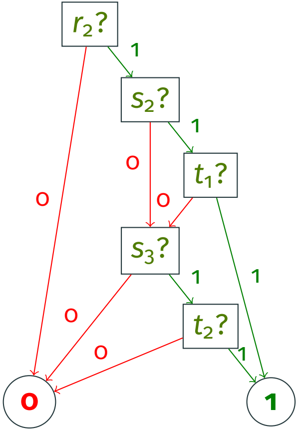

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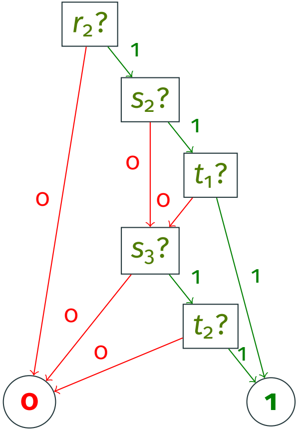

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Corollary

For any hierarchical self-join-free CQ Q, the problem $\mathrm{PQE}(Q)$ is in linear time up to the cost of arithmetic operations

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- **Crux of the problem:** capture arithmetic operations on probabilities with a d-D circuit, specifically **inclusion-exclusion**; see [Monet, 2020]

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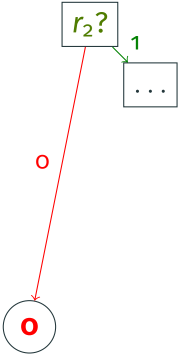

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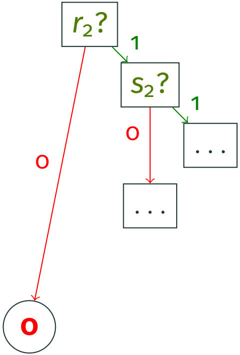

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b	v	s_2
b	w	s_3

T	
v	t_1
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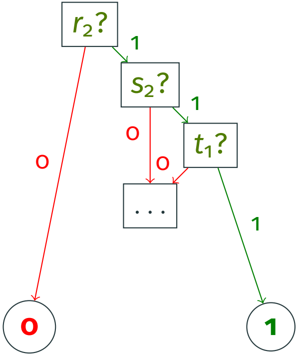

Ordered Binary Decision Diagram (OBDD)

OBDD for a Boolean query Q on database I:
ordered decision diagram on the facts of I to decide whether Q holds

$$Q : \pi_{\emptyset}(R \bowtie S \bowtie T)$$

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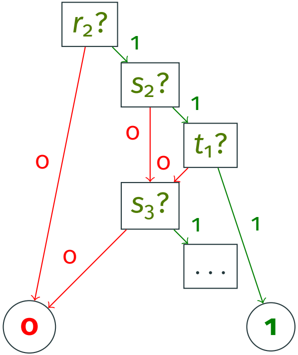

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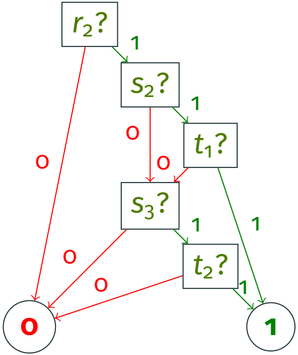

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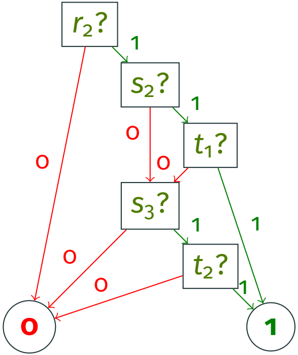

→ We can compute the probability of an OBDD **bottom-up**

Probabilistic Databases: Width-Based Approaches

EDBT-Intended Summer School

Antoine Amarilli

Going back to more restricted instances

OK, PQE *is* ***intractable*** *for essentially all queries. What now?*

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Theorem (A., Bourhis, Senellart, 2015, 2016)

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Conversely, there is a query Q *for which* $\mathrm{PQE}(Q)$ *is intractable on **any** input instance family of unbounded treewidth (under some technical assumptions)*

Non-probabilistic query evaluation on words

Database: a **word** w where nodes have a color from an alphabet ○ 🔴 🔵

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Computational complexity as a function of w
(the query Q is **fixed**)

Monadic second-order logic (MSO)

- $P_{\text{🔵}}(x)$ means “x is blue”; also $P_{\text{🔴}}(x)$, $P_{\bigcirc}(x)$
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- **Monadic second-order logic (MSO):** adds **quantifiers over sets**
  - $\exists S\; \forall x\; S(x)$ means *"there is a set S containing every element x"*
  - Can express **transitive closure** $x \rightarrow^{*} y$, i.e., *"x is before y"*
  - $\forall x\, P_{🔴}(x) \Rightarrow \exists y\, P_{🔵}(y) \wedge x \rightarrow^{*} y$
    means *"There is a blue node after every pink node"*

Word automata

Translate the query Q to a deterministic word automaton

Alphabet: ○ 🔴 🔵

w: ○—🔴—○—🔵—○

Q: $\exists x\, y\ P_{\text{🔴}}(x) \land P_{\text{🔵}}(y)$

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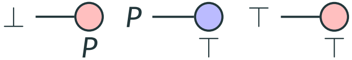

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MSO and word automata have the same expressive power on words

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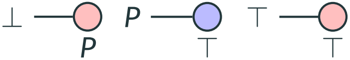

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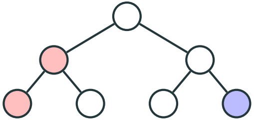
Corollary

Query evaluation of MSO on words is in linear time (in data complexity)

Non-probabilistic query evaluation on trees

Database: a **tree** T where nodes have a color from an alphabet ○ 🟥 🟦



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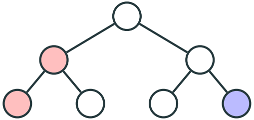

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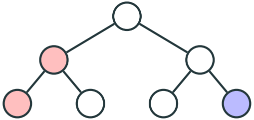

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Result: YES/NO indicating if the tree T satisfies the query Q

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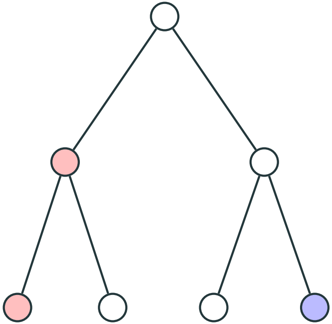
Tree alphabet: ○ 🔴 🔵



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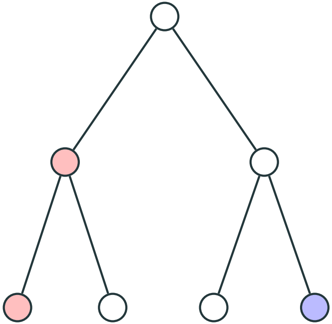

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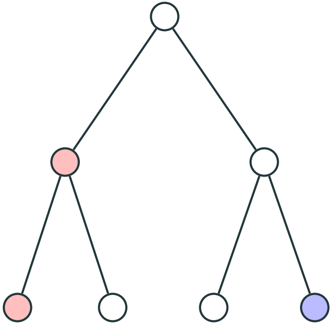

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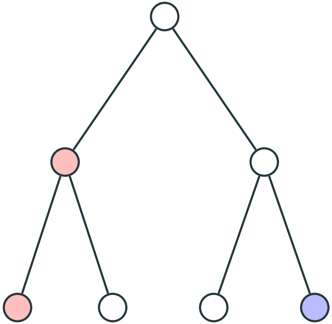

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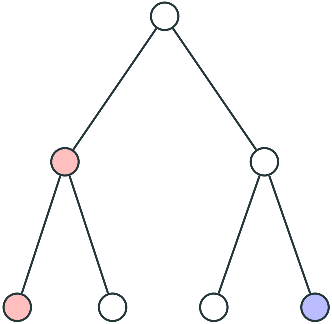

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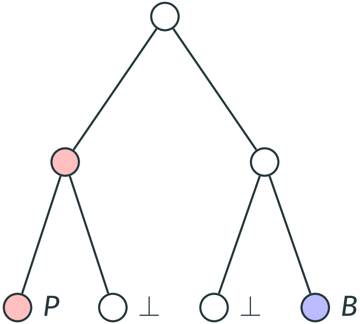

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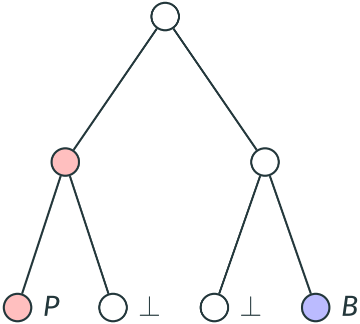

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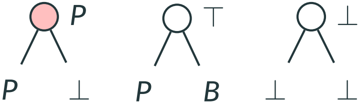

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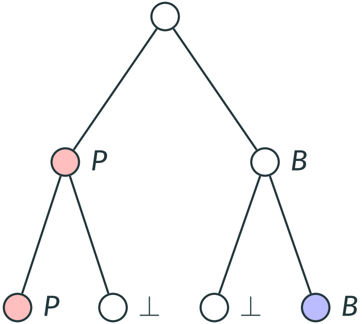

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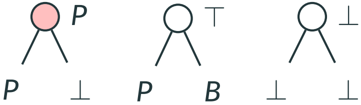

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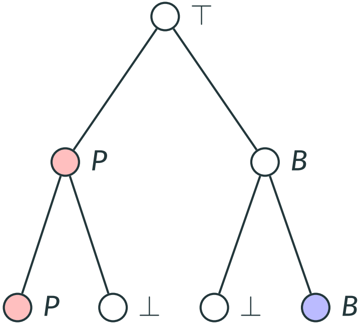

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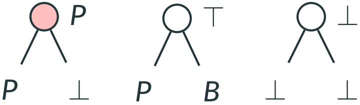

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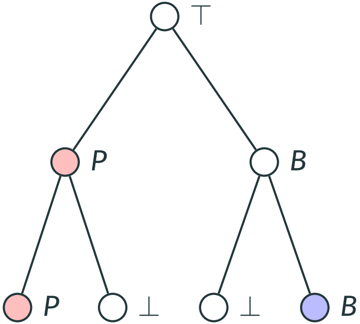

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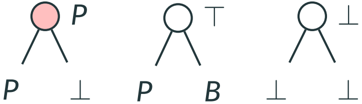

Theorem ([Thatcher and Wright, 1968])

MSO and tree automata have the same expressive power on trees

Probabilistic query evaluation on trees

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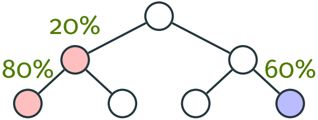

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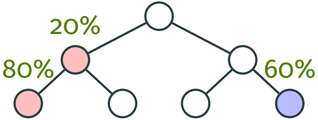

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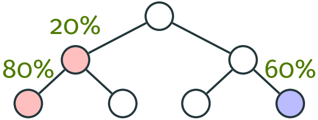

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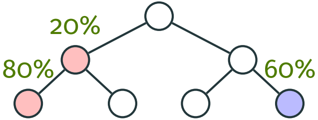

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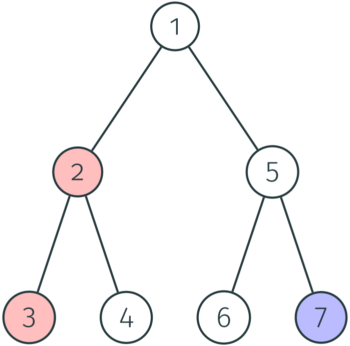
Theorem

*For any fixed **MSO query** Q, the problem $\mathrm{PQE}(Q)$ on trees is in **linear time** assuming constant-time arithmetics*

Uncertain trees: capturing how the query result depends on the choices



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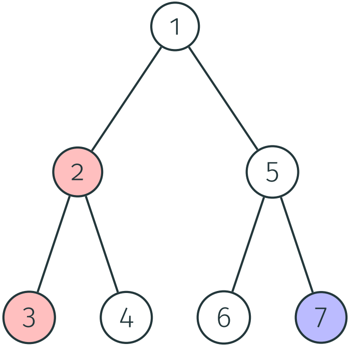

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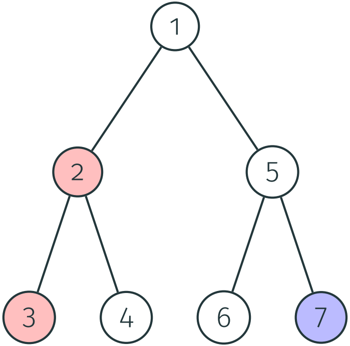

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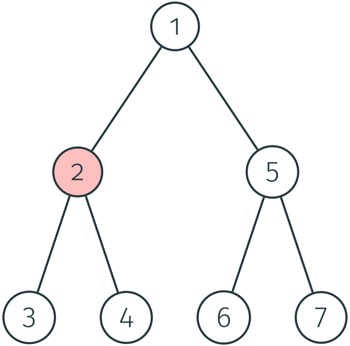

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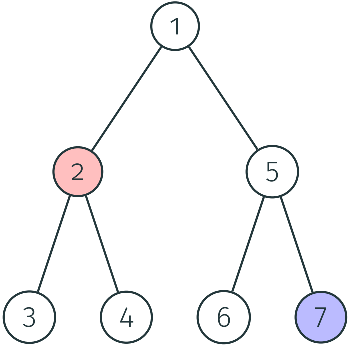

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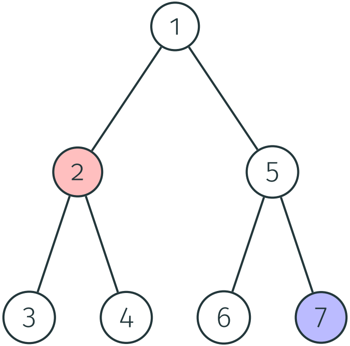

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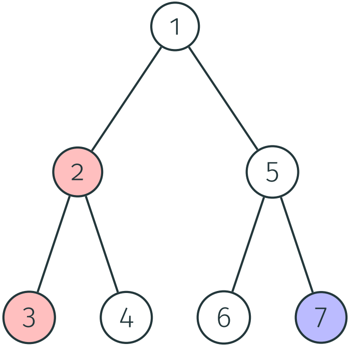

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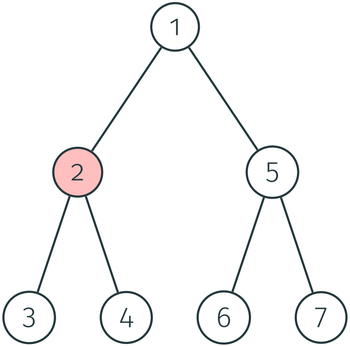

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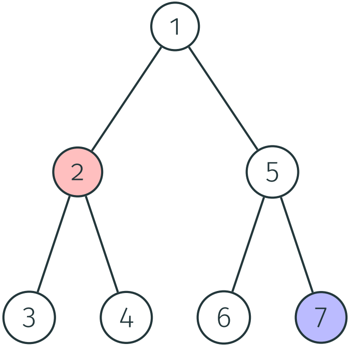

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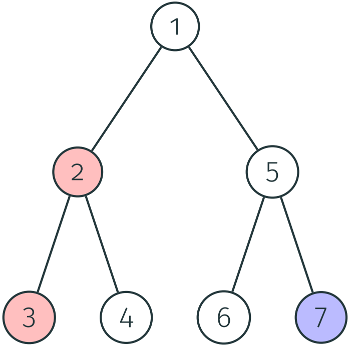

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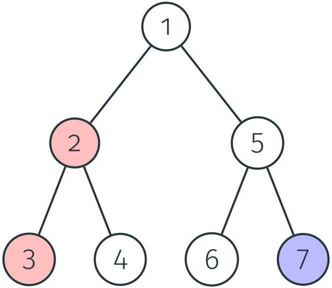
→ This is just a **Boolean provenance circuit** on the “color facts” of the tree nodes!

Example: Provenance circuit



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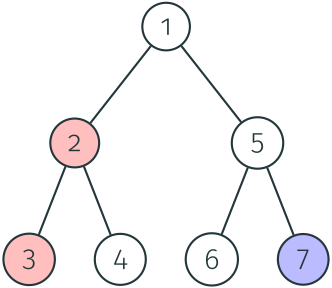

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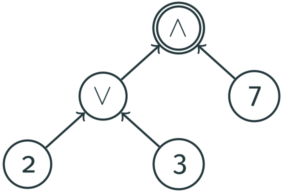

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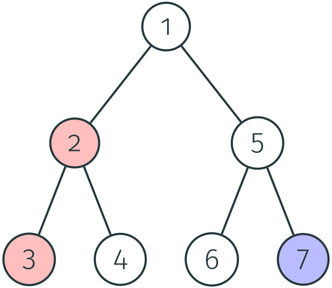

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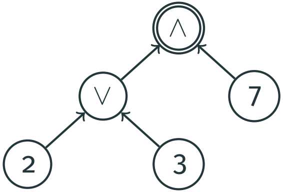

Formal definition of provenance circuits:

- Boolean query Q, uncertain tree T, circuit C
- **Variable gates** of C: nodes of T
- **Condition:** Let ν be a valuation of T, then $\nu(C)$ iff $\nu(T)$ satisfies Q

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For any bottom-up tree automaton A and input tree T, we can build a Boolean SDNNF provenance circuit of A on T in $O(|A| \times |T|)$

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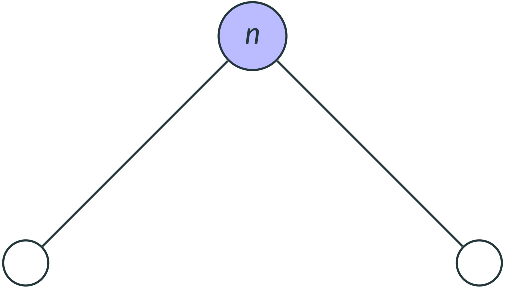

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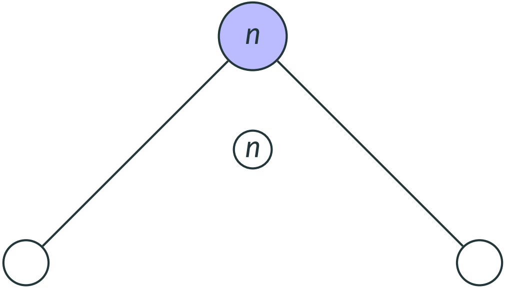

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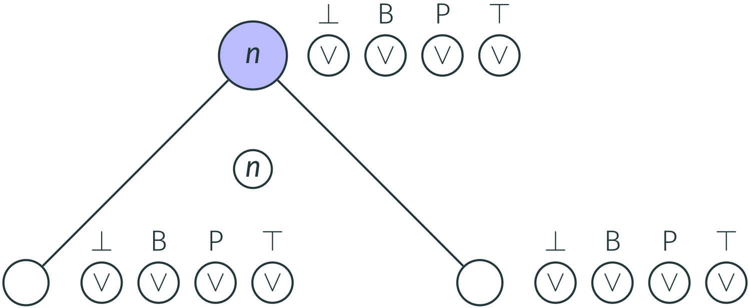

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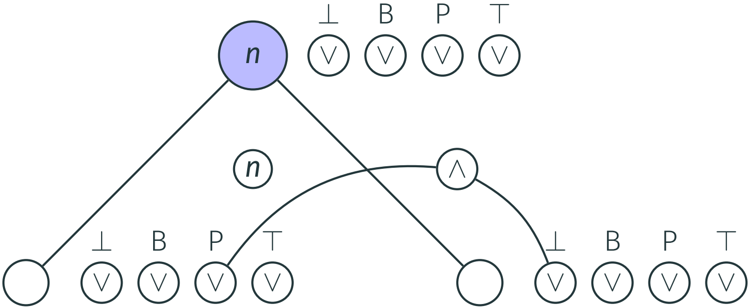

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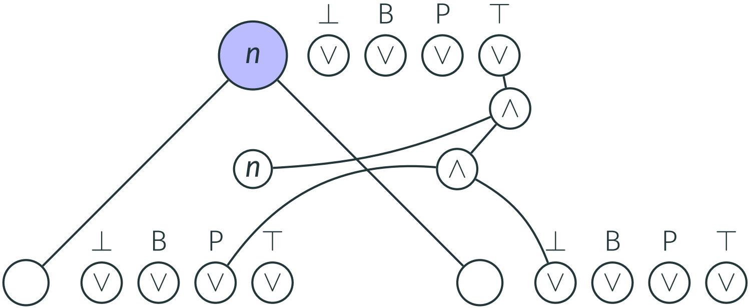

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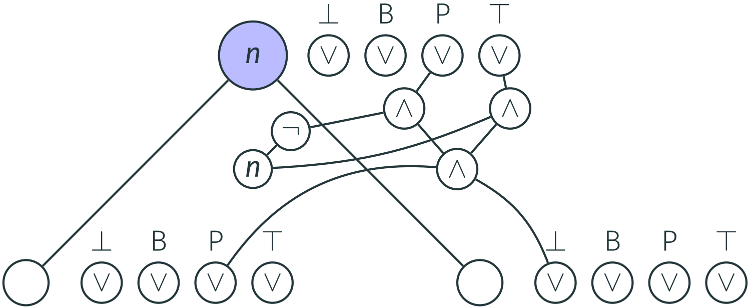

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For any bottom-up ***unambiguous tree automaton*** A *and input* ***tree*** T,
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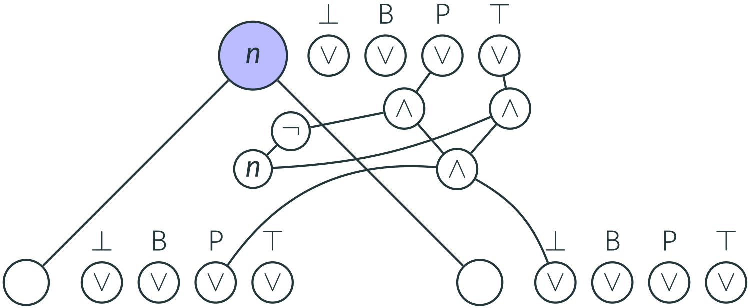

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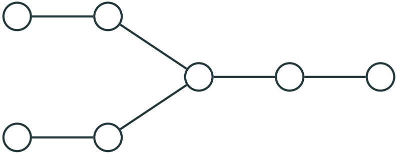
We have shown tractability of PQE on **trees**; let us extend to **bounded treewidth**

Treewidth by example:



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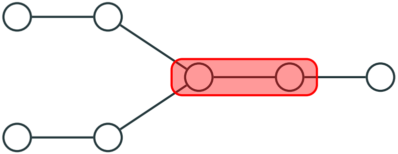

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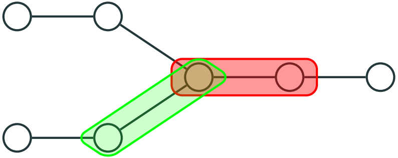

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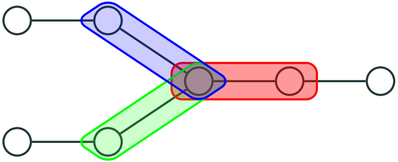

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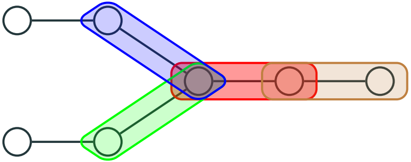

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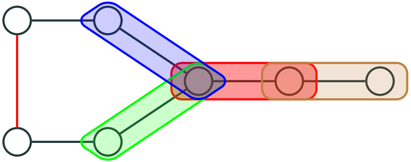

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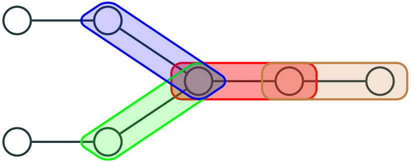

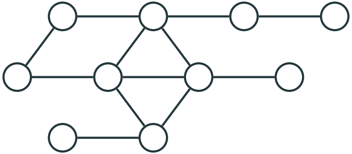

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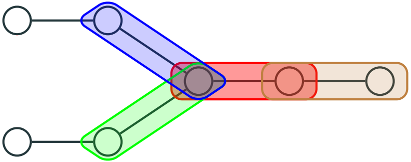

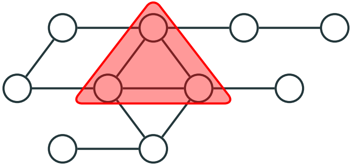

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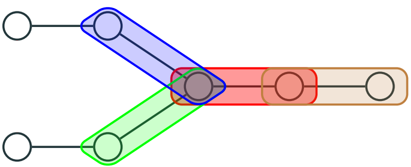

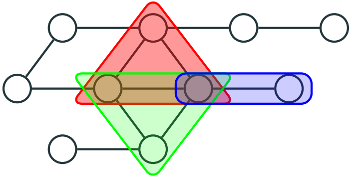

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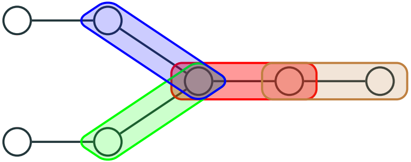

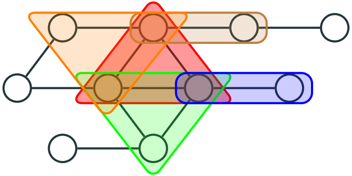

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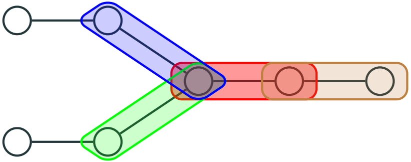

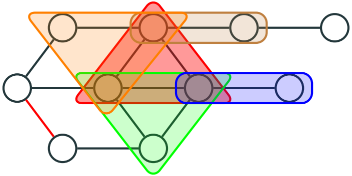

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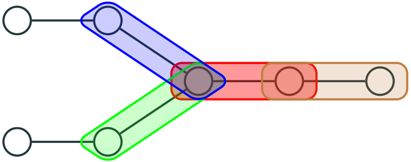

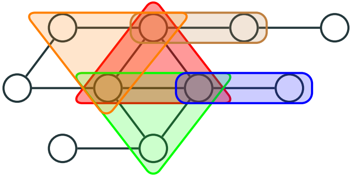

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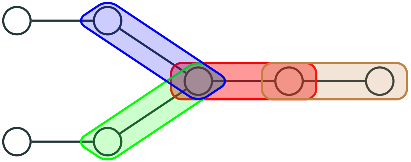

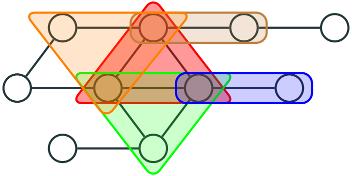

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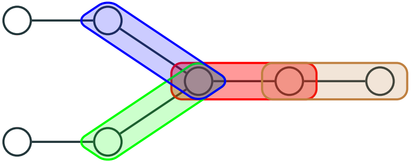

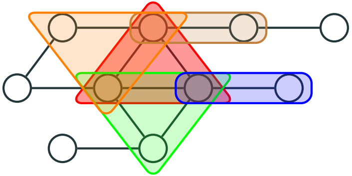

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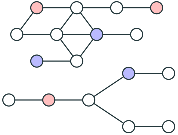
→ **Treelike**: the **treewidth** is bounded by a **constant**

Courcelle's theorem and extension to PQE

Treelike **data**



MSO query

$$\exists x\, y$$
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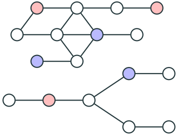
Courcelle's theorem and extension to PQE

Treelike **data**



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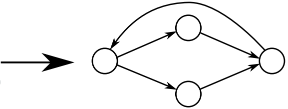

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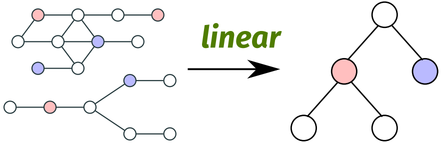
Treelike **data** — Tree **encoding**



MSO query — Tree **automaton**

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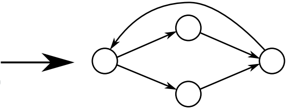
P🔴$(x) \wedge P$🔵(y)



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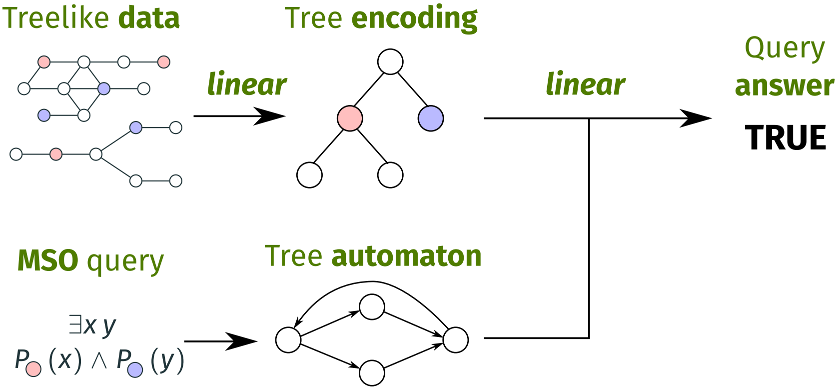

Treelike **data**

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Tree **encoding**

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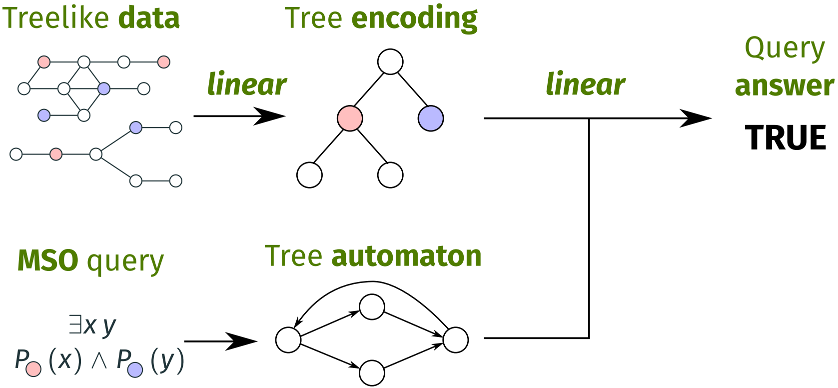

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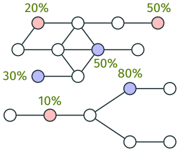
Theorem ([Courcelle, 1990])

For any fixed Boolean MSO query $\boldsymbol{Q}$ *and* $\boldsymbol{k} \in \mathbb{N}$, *given a database* $\boldsymbol{D}$ *of treewidth* $\leq \boldsymbol{k}$, *we can compute in linear time in* $\boldsymbol{D}$ *whether* $\boldsymbol{D}$ *satisfies* $\boldsymbol{Q}$

Courcelle's theorem and extension to PQE

Probabilistic treelike **data**



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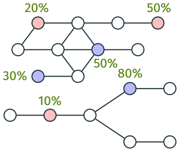

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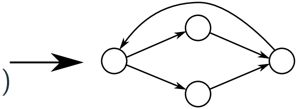

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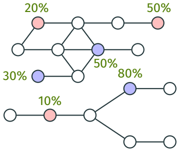

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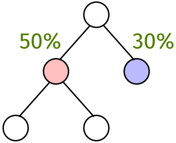

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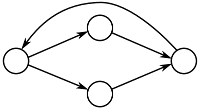

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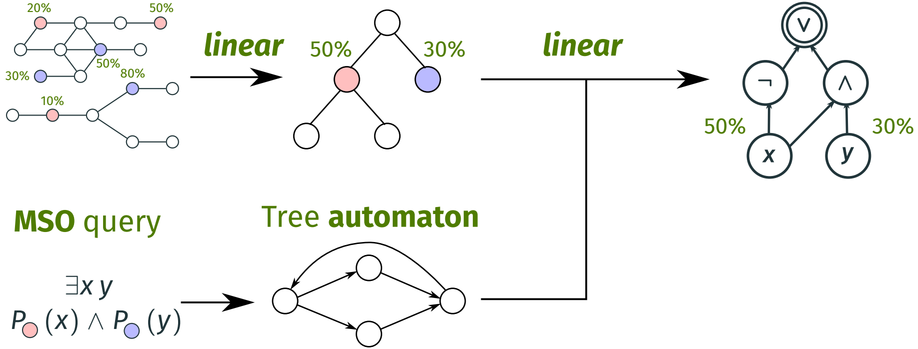

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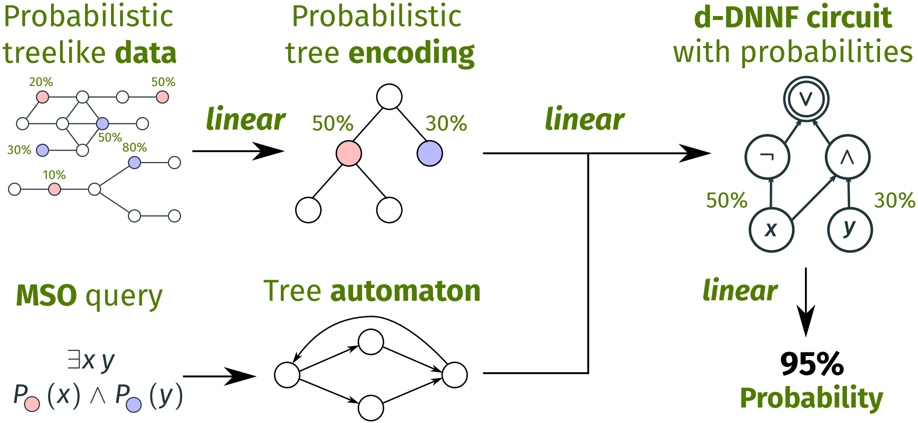

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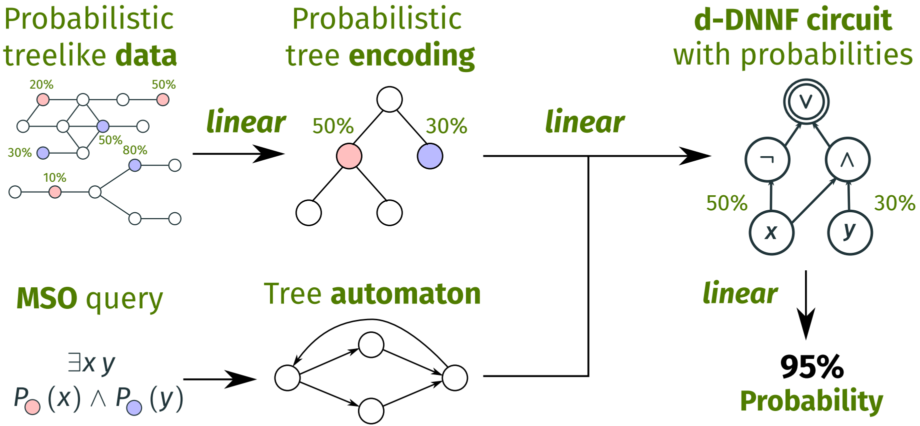

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→ Proof idea: **extract wall graphs as topological minors** ([Chekuri and Chuzhoy, 2014]) and use them for a lower bound

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Probabilistic Databases: Other Topics and Conclusion

EDBT-Intended Summer School

Antoine Amarilli

Table of contents

Recursive and homomorphism-closed queries

Going to more general queries

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Going to more general queries

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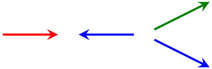
We study the case of **queries closed under homomorphisms**

→ We restrict to **arity-two signatures** (work in progress…)

Homomorphism-closed queries

- A **homomorphism** from a graph G to a graph G' maps the vertices of G to those of G' while preserving the edges



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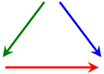

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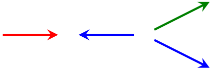

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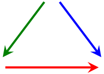

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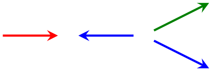

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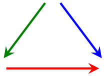

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- Homomorphism-closed queries can equivalently be seen as **infinite unions of CQs** (corresponding to their models)

Our result

We show:

Theorem (A., Ceylan, 2020)

For any query Q closed under homomorphisms on an arity-two signature:

- *Either* Q *is equivalent to a tractable UCQ and* $\mathrm{PQE}(Q)$ *is in PTIME*
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  - Hence, $\mathrm{PQE}(Q)$ is #P-hard

Uniform probabilities

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What if we restricted probabilities on input instances to always be $1/2$*?*

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We limit to **self-join-free CQs** and extend the "small" Dalvi and Suciu dichotomy to UR:

Theorem (A., Kimelfeld, 2022)

Let Q be a self-join-free CQ:

- *If Q is hierarchical, then $\mathrm{PQE}(Q)$ is in PTIME*
- *Otherwise, even $\mathrm{UR}(Q)$ is #P-hard*

Approximate evaluation

Approximation

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- One possibility is to compute a lower bound and upper bound:
  - $\max(\Pr(\phi), \Pr(\psi)) \leq \Pr(\phi \vee \psi) \leq \min(\Pr(\phi) + \Pr(\psi), 1)$
  - $\max(0, \Pr(\phi) + \Pr(\psi) - 1) \leq \Pr(\phi \wedge \psi) \leq \min(\Pr(\phi), \Pr(\psi))$ (by duality)
  - $\Pr(\neg\phi) = 1 - \Pr(\phi)$ (reminder)

Approximation by sampling

Another possibility is to approximate via Monte-Carlo sampling:

- Pick a random possible world according to the fact probabilities:
  - → Keep F with probability $\Pr(F)$ and discard it otherwise
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- Approximate the probability of the formula ϕ as the **proportion of times** when it was true
- **Theoretical guarantees:** on how many samples suffice so that, with high probability, the estimated probability is almost correct

Other method for a **multiplicative approximation**: Karp-Luby algorithm

Using external tools

- Specialized software to compute the probability of a formula: **weighted model counters**
- Examples (ongoing research):
    - **c2d:** `http://reasoning.cs.ucla.edu/c2d/download.php`
    - **d4:** `https://www.cril.univ-artois.fr/KC/d4.html`
    - **dsharp:** `https://bitbucket.org/haz/dsharp`

Repairs

Repairs

- Another kind of uncertainty: we know that the database must satisfy some **constraints** (e.g., functionality)
- The data that we have does **not** satisfy it
- Reason about the ways to **repair** the data, e.g., removing a minimal subset of tuples
- Can we **evaluate queries** on this representation? E.g., is a query true on **every maximal repair**? See, e.g., [Koutris and Wijsen, 2015].

→ Tutorial by Jef Wijsen

Incompleteness: Open-World Query Answering

Open-world query answering

- Most data sources are **incomplete**, e.g., Wikidata
- **Idea:** see an incomplete data source as representing **all possible completions**
- A query result is **certain** if it is true on **every possible completion**
- We also assume **constraints** to restrict the possible completions (e.g., IDs and FDs, see Andreas's talk)

Open-world query answering problem

Definition of the **open-world query answering** problem (OWQA):

- Given:
  - An incomplete **database** D
  - Logical **constraints** Σ on the true state of the world
  - A **query** Q
- Determine if Q is true in **every completion** of D that satisfies Σ

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Note: We assume that the incomplete database D satisfies the constraints.
(Otherwise we need to repair it.)

Results on OWQA

- The OWQA problem can be **undecidable** if we allow **arbitrary first-order logic** for Σ
- It is also undecidable for **common database constraint languages**, e.g., tuple-generating dependencies
- It is **decidable** for **better-behaved** logical fragments, e.g., the **guarded fragment**

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- Two main techniques:
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  - **Backward chaining**, aka "query rewriting": change the query to reflect the constraints

Incompleteness: NULLs

Codd tables, a.k.a. SQL NULLs

Patient	Examin. 1	Examin. 2	Diagnosis
A	23	12	α
B	10	23	$\perp_1$
C	2	4	γ
D	15	15	$\perp_2$
E	$\perp_3$	17	β

- Most **simple** form of incomplete database
- **Widely used** in practice, in DBMS since the mid-1970s!
- All NULLs ($\perp$) are considered **distinct**
- Possible world semantics: all possible completions of the table (infinitely many)
- In SQL, **three-valued logic**, weird semantics:

```
SELECT * FROM Tel WHERE tel_nr = '333' OR tel_nr <> '333'
```

Problem: Codd tables and query evaluation

Appointment

Doctor	Patient
D1	A
D2	A

Illness

Patient	Diagnosis
A	$\perp$

Let's join the two tables...

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- We know that $\perp_1 = \perp_2$, but we cannot **represent it**
- Simple solution: **named nulls** aka v-tables
- More expressive solution: **c-tables**

Summary and directions

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- Extensions: homomorphism-closed queries, uniform reliability...

Other topics of research

- Queries with **negation** [Fink and Olteanu, 2016]
- Queries with **inequalities** [Olteanu and Huang, 2009]
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Thanks for your attention!

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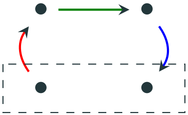

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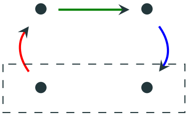

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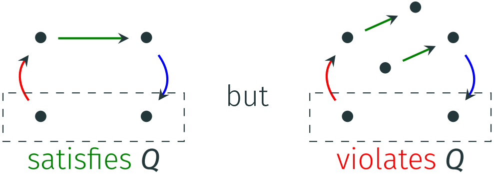

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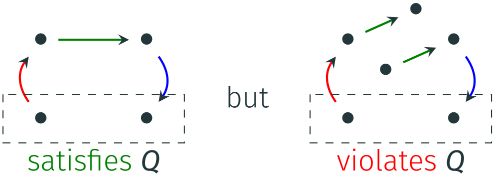

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Using tight patterns to show hardness of PQE

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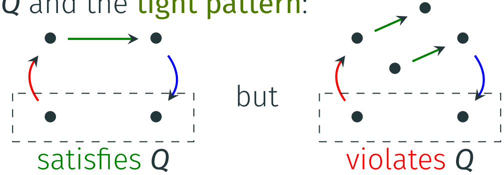

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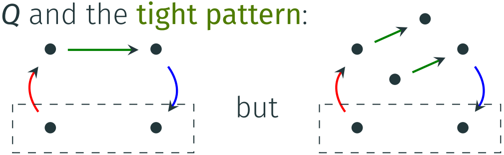

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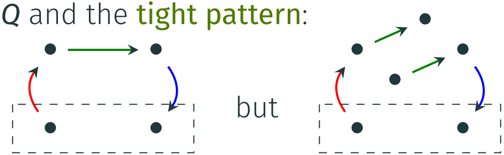

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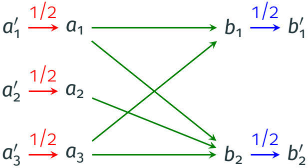

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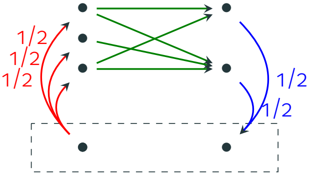

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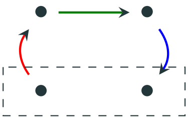

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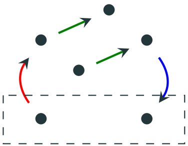

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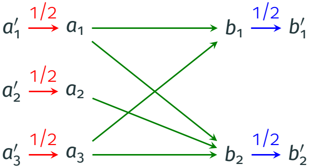

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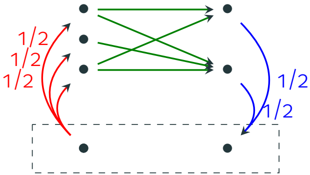

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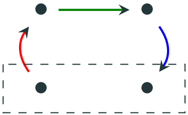

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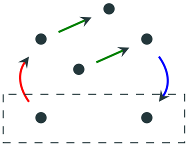

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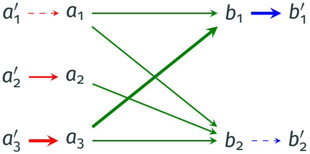

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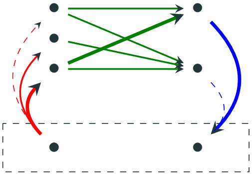

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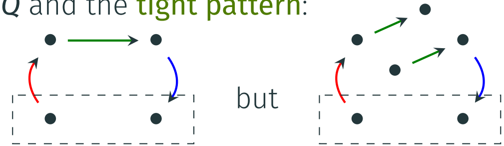

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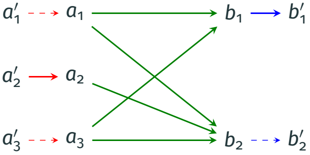

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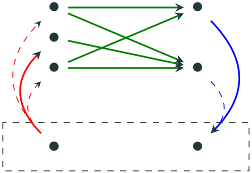

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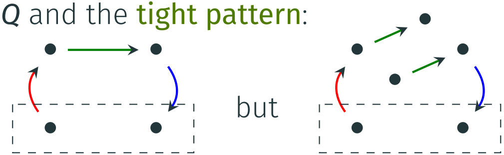

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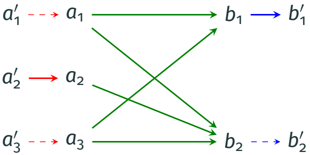

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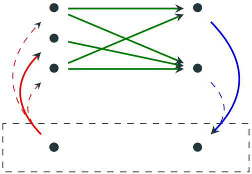

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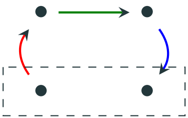

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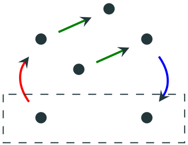

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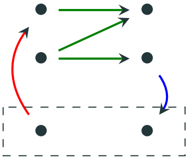
AND



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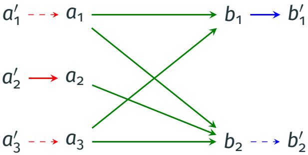
- We reduce from PQE for the **intractable** CQ: $Q_0 : x \longrightarrow y \longrightarrow z \longrightarrow w$



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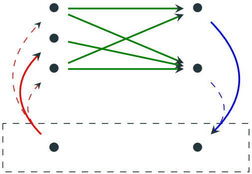


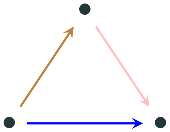
Idea: possible worlds at the **left** have a path that matches Q_0

iff the corresponding possible world of the TID at the **right** satisfies the query *Q*…

… except we need **more** from the tight pattern!

Why can we always find tight patterns?

- Unbounded queries have arbitrarily large minimal models
- Take a large minimal model *D* and disconnect its edges:



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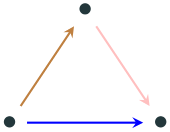
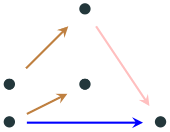


to



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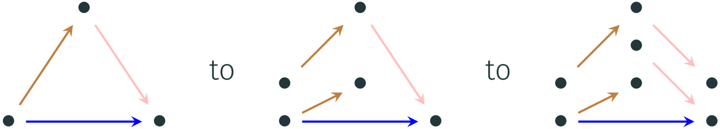

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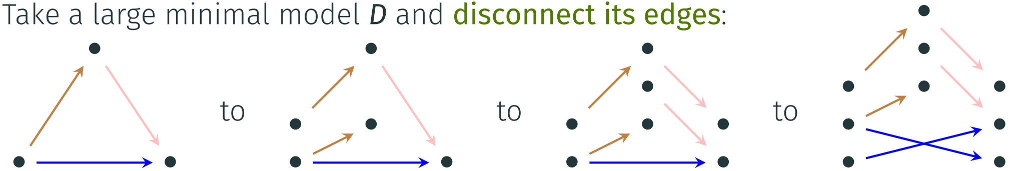

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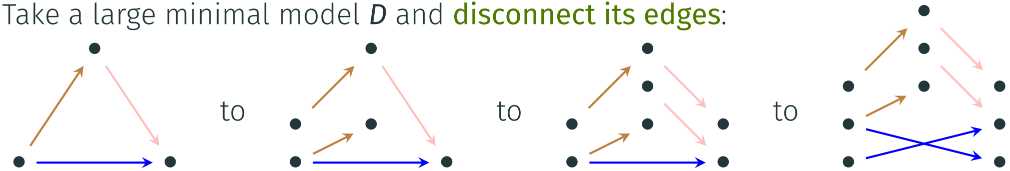

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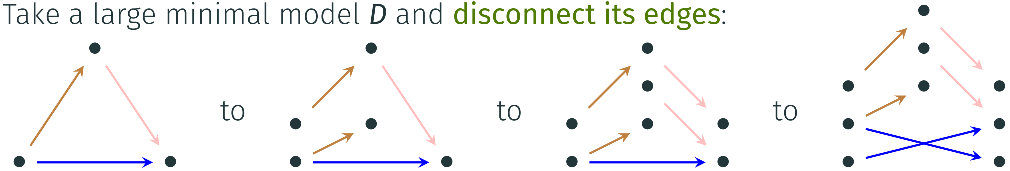

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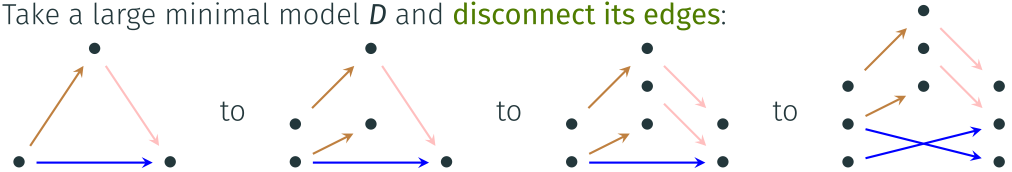

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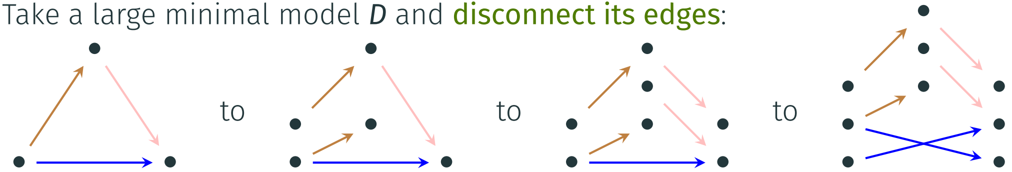

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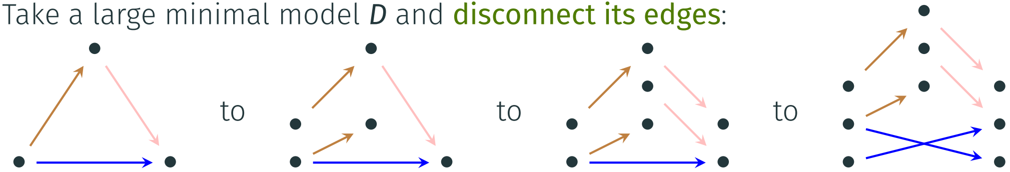

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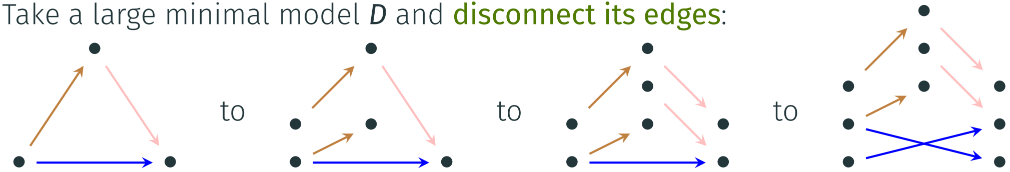

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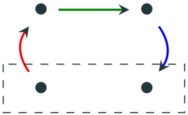
  - D'' has a **homomorphism** back to *D*
  - This contradicts the **minimality** of the large *D*

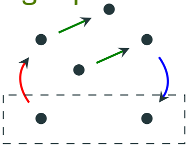
Rescuing the proof

We know that we have a **tight pattern**:



satisfies Q

but



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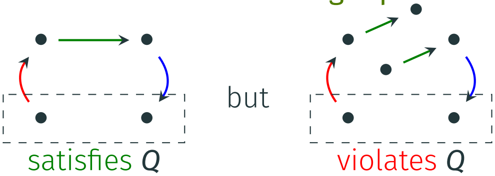

Consider its **iterates**

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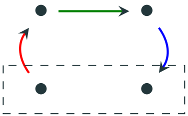

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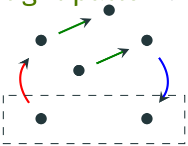
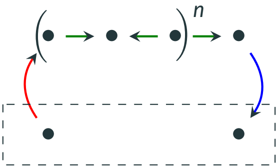


violates Q

Consider its **iterates** for each $n \in \mathbb{N}$:



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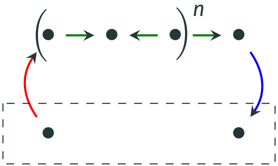

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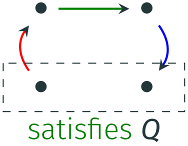
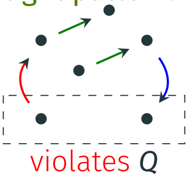


but



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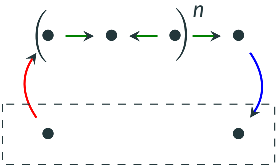
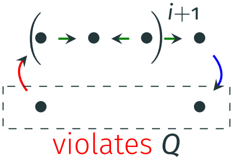


Case 1: some iterate **violates** the query:

but



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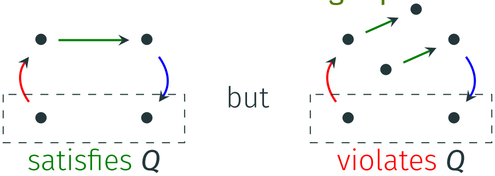

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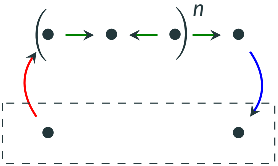

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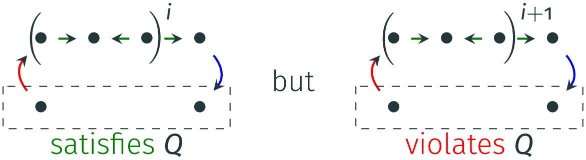

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$\rightarrow$ Reduce from $\mathrm{PQE}(Q_0)$ as we explained

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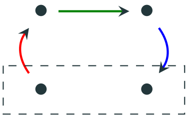

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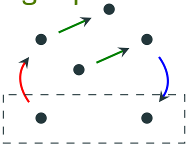

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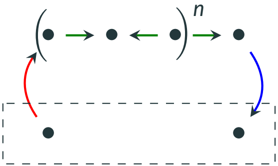

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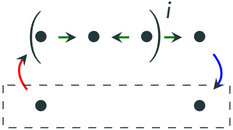

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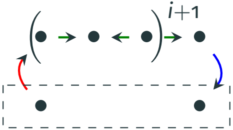

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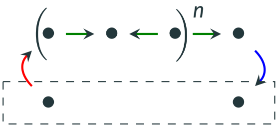
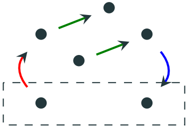


satisfies Q for all $n \in \mathbb{N}$

but



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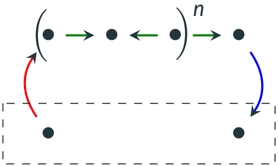
Rescuing the proof

We know that we have a **tight pattern**:

satisfies Q but violates Q

Consider its **iterates** for each $n \in \mathbb{N}$:



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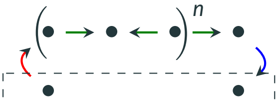
→ Call this an **iterable pattern**

Using iterable patterns to show hardness of PQE

We have an **iterable pattern**:



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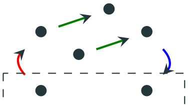

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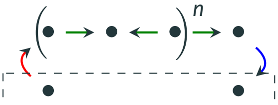

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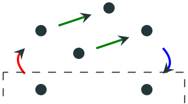

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Idea: reduce from the **#P-hard** problem **source-to-target connectivity**:

- Input: **undirected graph** with a **source** s and **target** t, all edges have probability $1/2$
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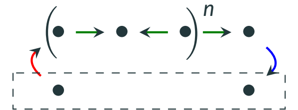

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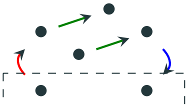

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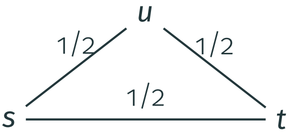

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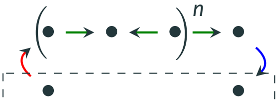

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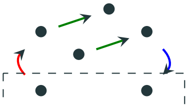

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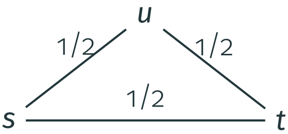

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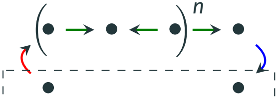

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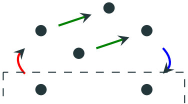

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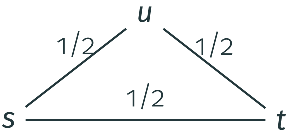

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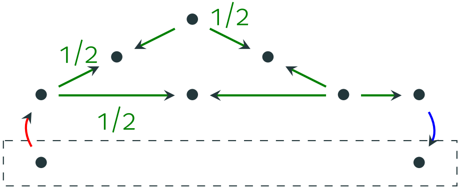

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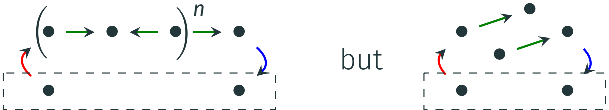

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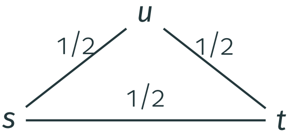

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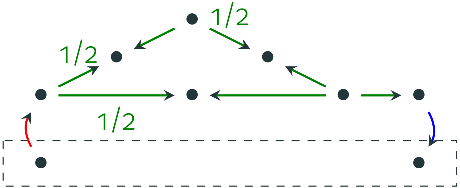

Idea: There is a **path connecting s and t** in a possible world of the graph at the left iff the query Q is **satisfied** in the corresponding possible world of the TID at the right

Using iterable patterns to show hardness of PQE

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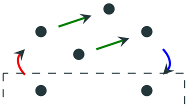

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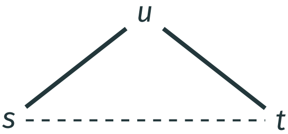
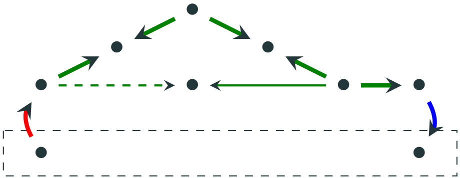


is coded as



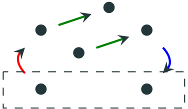
Idea: There is a **path connecting s and t** in a possible world of the graph at the left iff the query Q is **satisfied** in the corresponding possible world of the TID at the right

Using iterable patterns to show hardness of PQE

We have an **iterable pattern**:

satisfies Q for all $n \in \mathbb{N}$

but



violates Q

Idea: reduce from the **#P-hard** problem **source-to-target connectivity**:

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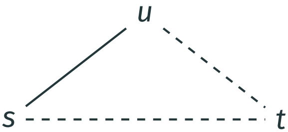

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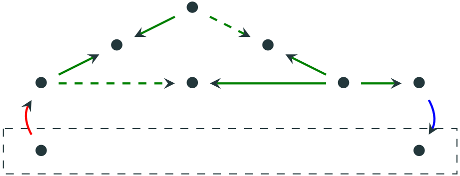

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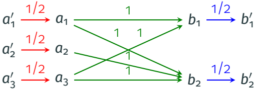
Proof technique

Hard part: show hardness for (variants of) the query Q: $x \longrightarrow y \longrightarrow z \longrightarrow w$

We reduce from $\mathrm{PQE}(Q)$, on **probabilistic graphs** G of the following form:



Task: count the number X of **red-blue edge subsets** that **violate** Q

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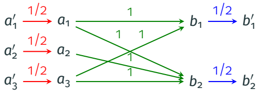

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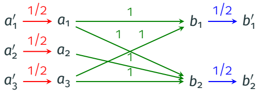

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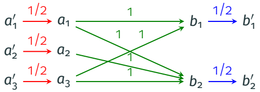

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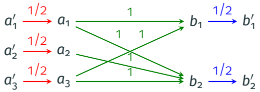

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- Show **invertibility** of this matrix to recover the X_i from the N_i

Using the equation system

We have obtained the system:

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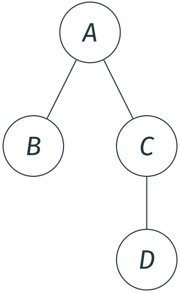
We can choose gadgets and parameters to get a **Vandermonde matrix**, and show invertibility via several **arithmetical tricks**

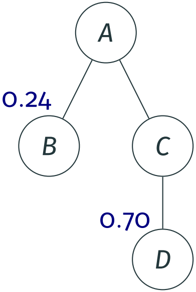
The semistructured model and XML



```
<a>
  <b>...</b>
  <c>
    <d>...</d>
  </c>
</a>
```

- Tree-like structuring of data
- No (or less) schema constraints
- Allow mixing tags (structured data) and text (unstructured content)
- Particularly adapted to tagged or heterogeneous content

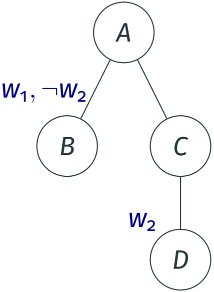
Simple probabilistic annotations



- **Probabilities** associated to tree nodes
- Express parent/child dependencies
- Impossible to express more complex dependencies
- ⇒ some **sets of possible worlds** are not expressible this way!

Annotations with event variables

Event	Prob.
w_1	0.8
w_2	0.7



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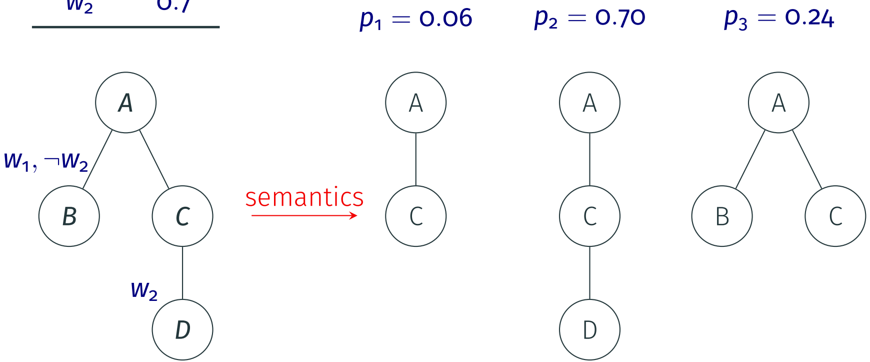

- Expresses **arbitrarily complex** dependencies

Query evaluation on probabilistic XML

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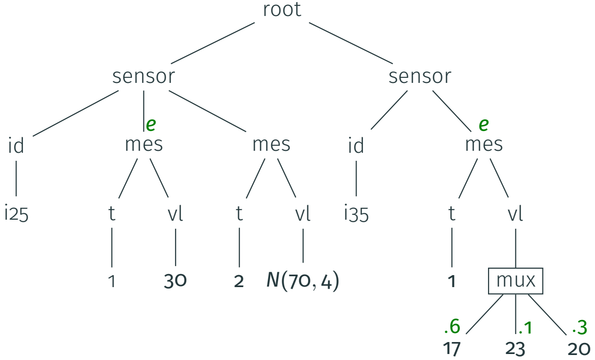
- This generalizes to PQE for **MSO** on **relational databases (TID)** when assuming that the **treewidth** is bounded [Amarilli et al., 2015]
- Bounding the treewidth is **necessary** for tractability in a certain sense [Amarilli et al., 2016]

A general probabilistic XML model [Abiteboul et al., 2009]

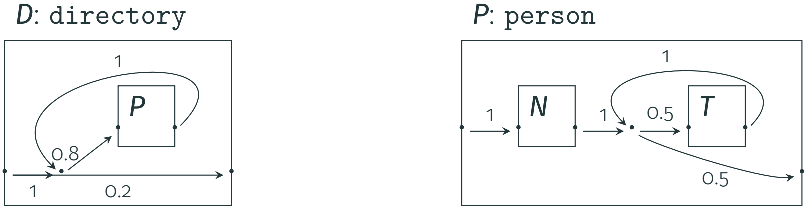


- *e*: event "it did not rain" at time 1
- mux: mutually exclusive options
- $N(70, 4)$: normal distribution

- Compact representation of a **set of possible worlds**
- Two kinds of dependencies: global (*e*) and local (mux)
- Generalizes **all previously proposed models** of the literature

Recursive Markov chains [Benedikt et al., 2010]

```
<!ELEMENT directory (person*)>
<!ELEMENT person (name,phone*)>
```



- Probabilistic model that **extends** PXML with local dependencies
- Generate documents of **unbounded** width or depth

C-tables [Imielinski and Lipski, 1984]

Patient	Examin. 1	Examin. 2	Diagnosis	Condition
A	23	12	α	
B	10	23	$\bot_1$	
C	2	4	γ	
D	$\bot_2$	15	$\bot_1$	
E	$\bot_3$	17	β	$18 < \bot_3 < \bot_2$

- NULLs are labeled, and can be **reused** inside and across tuples
- **Arbitrary correlations** across tuples
- **Closed** under the relational algebra
- Every set of possible worlds can be represented as a database with c-tables