Enumerating Pattern Matches in Texts and Trees

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[3]CNRS CRIL

[4]Universität Bayreuth

Problem: Finding Patterns in Text

- We have a long text T:

```
Antoine Amarilli Description Name Antoine Amarilli.  Handle:  a3nm.  Identity Born 1990-02-07.
French national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP
a3nm@a3nm.net Affiliation Associate professor of computer science (office C201-4) in the DIG team
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→ How to find the pattern P efficiently in the text T?

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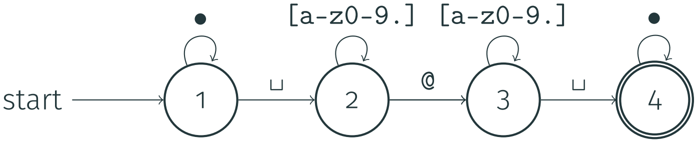

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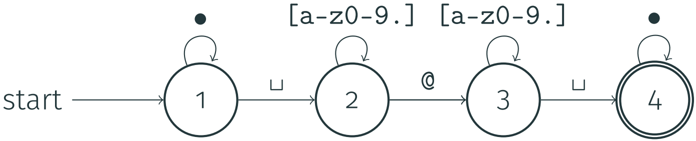

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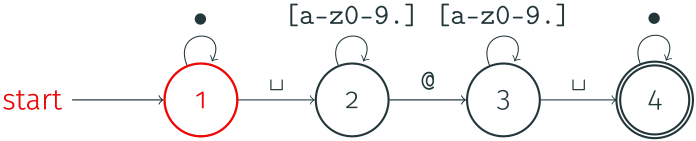

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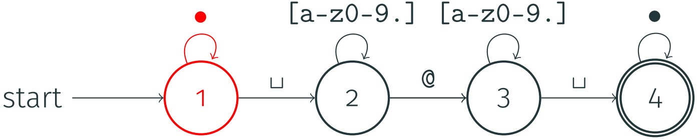

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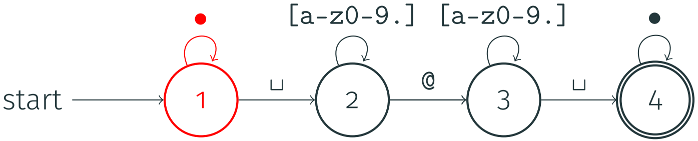

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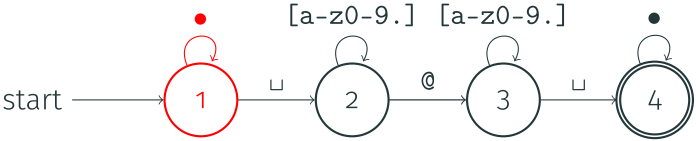

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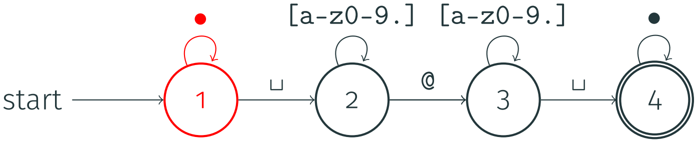

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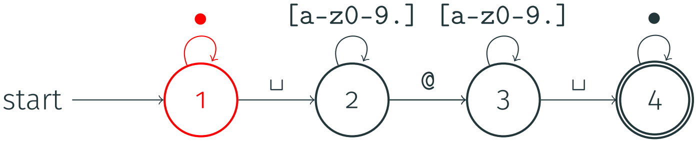

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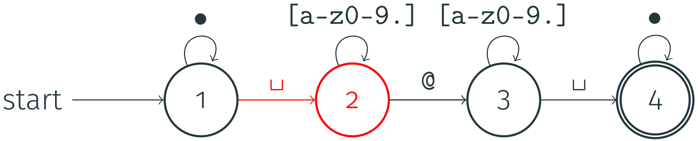

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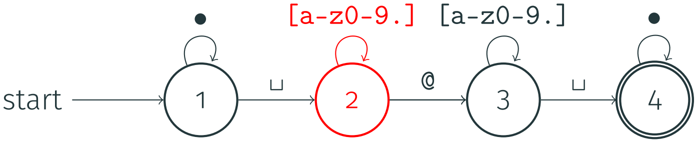

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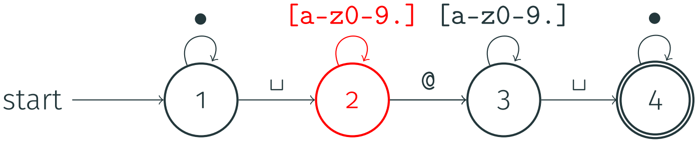

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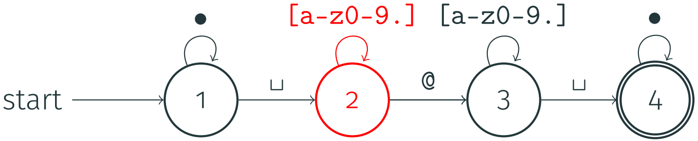

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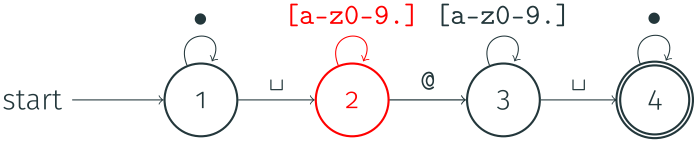

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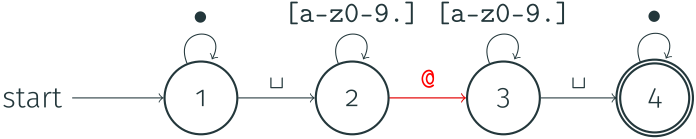

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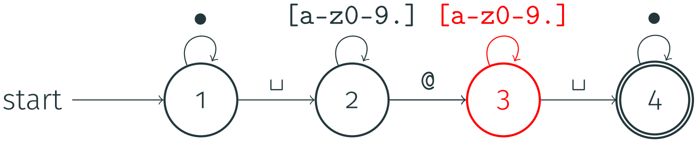

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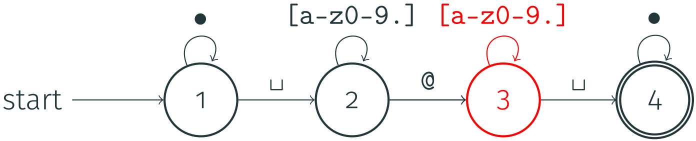

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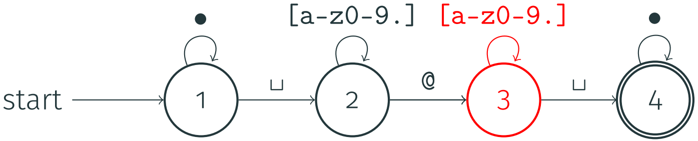

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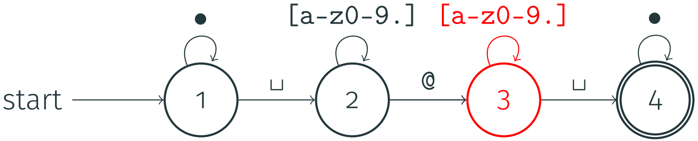

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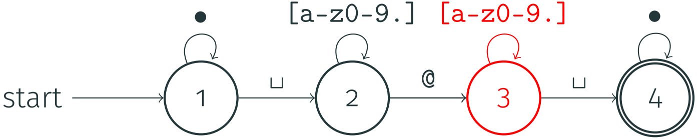

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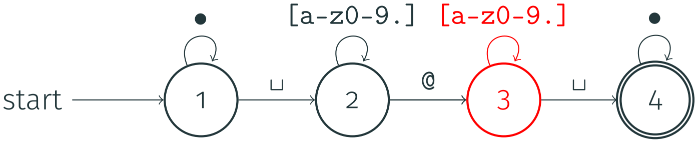

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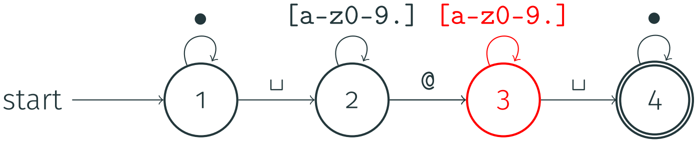

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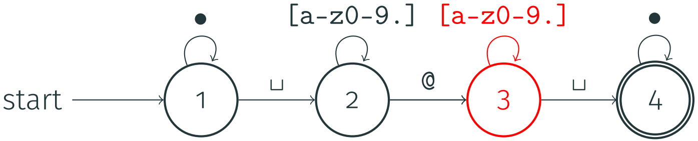

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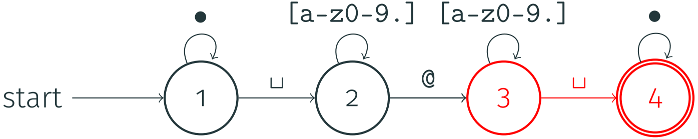

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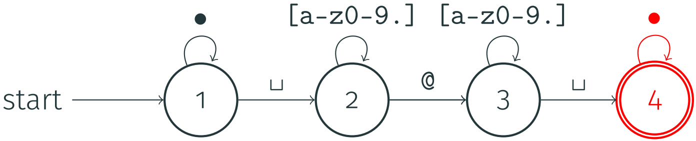

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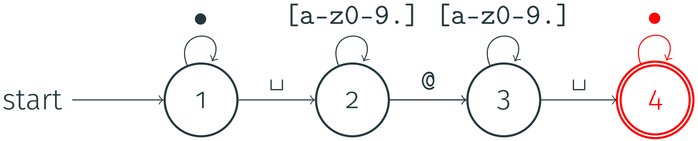

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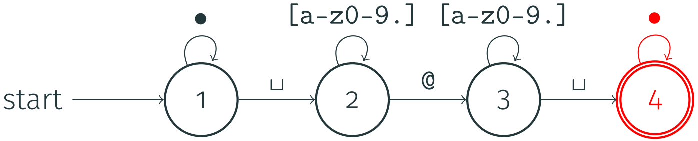

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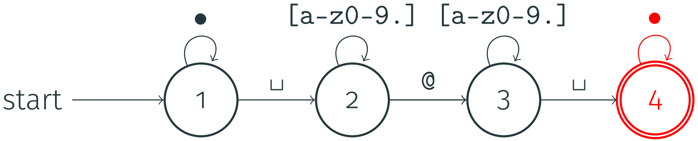

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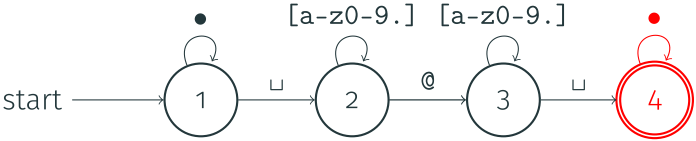

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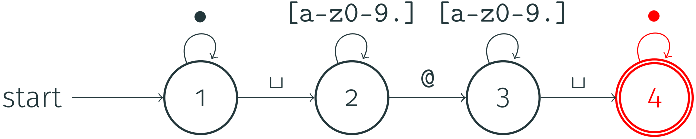

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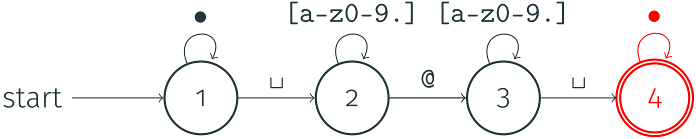

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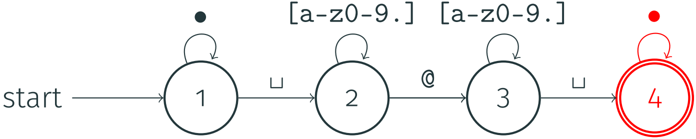

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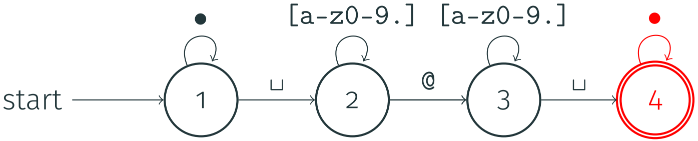

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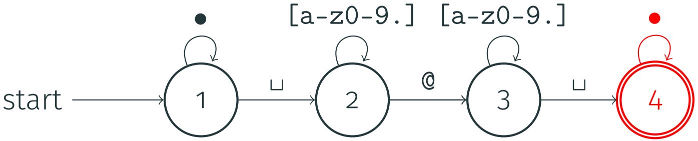

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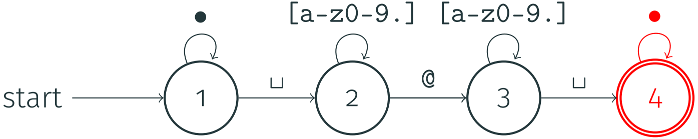

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`Email␣a3nm@a3nm.net␣Affiliation`

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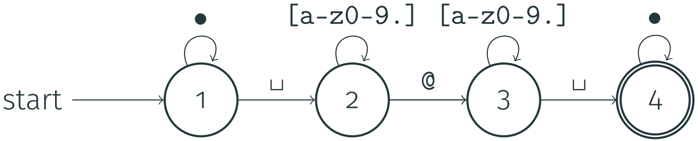

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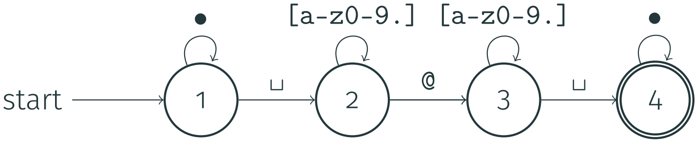

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E	m	a	i	l	␣	a	3	n	m	@	a	3	n	m	.	n	e	t	␣	A	f	f	i	l	i	a	t	i	o	n

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- Naive algorithm: Run the automaton A on each substring of T

`l o l`

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[⟩ l o l

Measuring the Complexity

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[l ⟩ o   l

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```
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    ```
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    ```

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    ```
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  - The **number of matches** is $\Omega(|T|^2)$

$\rightarrow$ We need a **different way** to measure complexity

Enumeration Algorithms

Idea: In real life, we do not want to compute all the matches
we just need to be able to enumerate matches quickly

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→ Formalization: enumeration algorithms

Formalizing Enumeration Algorithms

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Text T

␣ [a-z0-9.]*@
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Pattern P

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Phase 1:
Preprocessing

Formalizing Enumeration Algorithms

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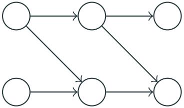
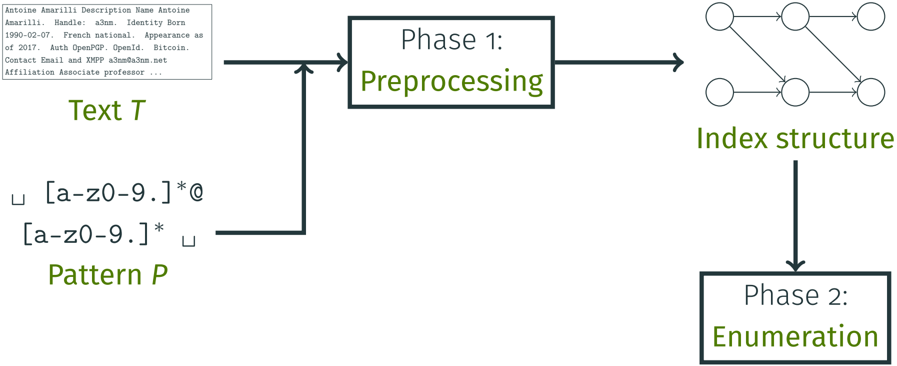


Index structure

Formalizing Enumeration Algorithms



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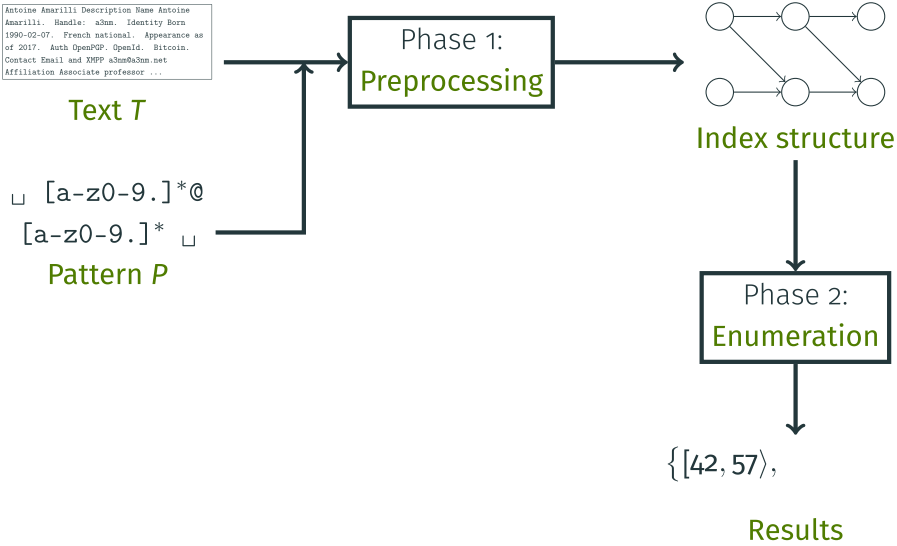

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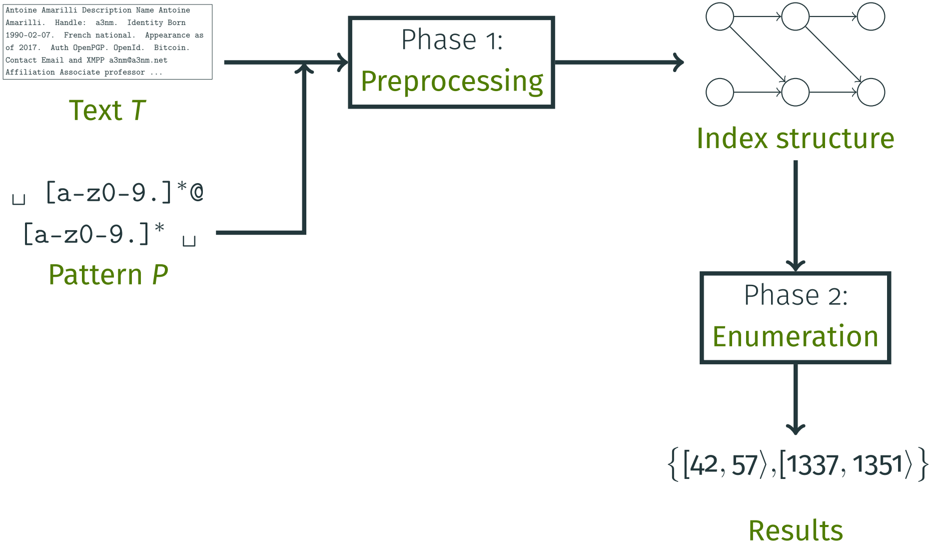

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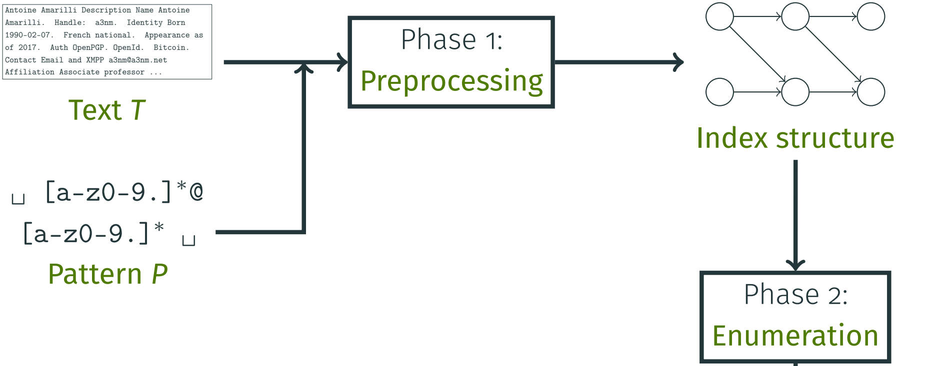

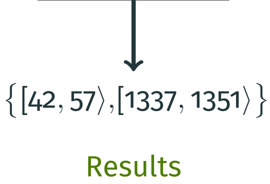

Two ways to measure performance:

- Total time for phase 1
- Delay between two results in phase 2

... as a function of the text and pattern

Complexity of Enumeration Algorithms

- Recall the inputs to our problem:
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→ Can we do **better**?

Results for Enumerating Pattern Matches

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Theorem

We can enumerate all matches of a pattern P on a text T with:

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- *Delay polynomial in P and independent from T*

Automaton Formalism

- We use automata that read letters and capture variables

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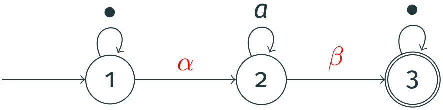

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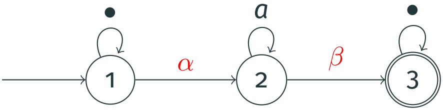

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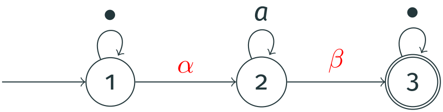

- Semantics of the automaton A:
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  - **Guesses** variables at positions in the text
  - → **Output:** tuples $\langle \alpha : i, \beta : j \rangle$ such that A has an accepting run reading α at position i and β at j

Automaton Formalism

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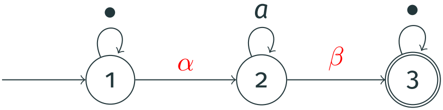

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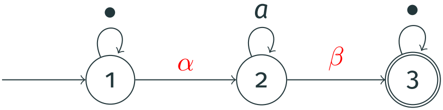

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Compute a product DAG of the text T and of the automaton A

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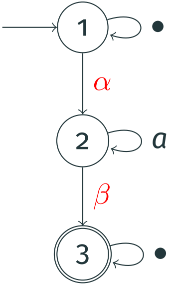

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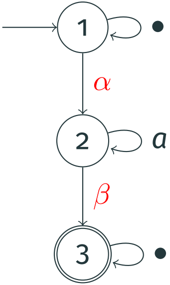

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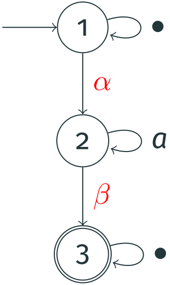

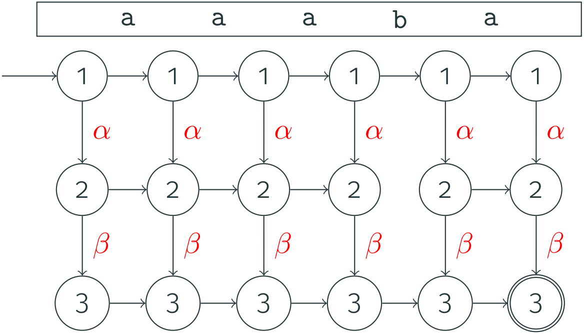

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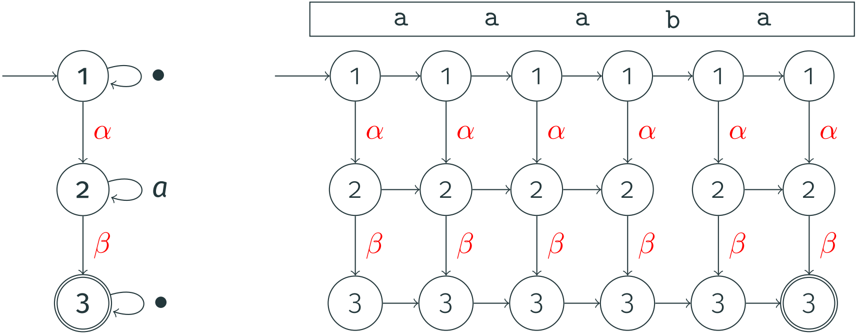

$\rightarrow$ Each **path** in the **product DAG** corresponds to a **match**

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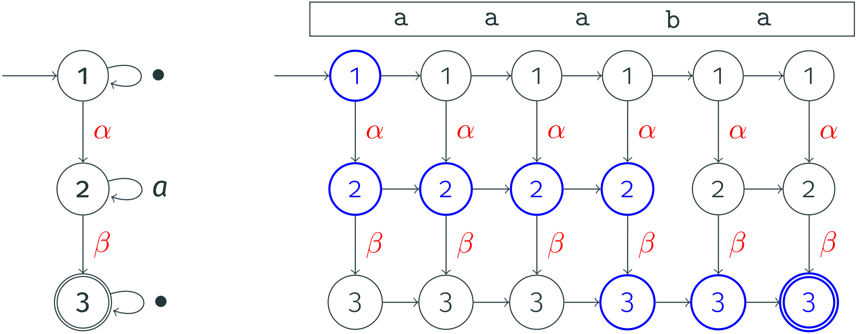
Example: Text $T :=$ `aaaba` and $P := \bullet^* \ \alpha \ a^* \ \beta \ \bullet^*$, match $\langle \alpha : 0, \beta : 3 \rangle$



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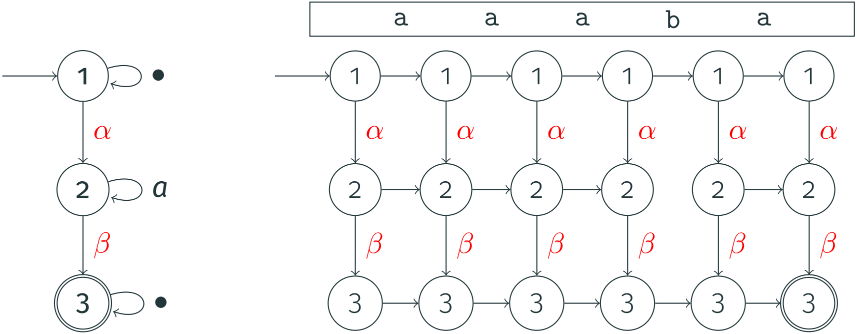


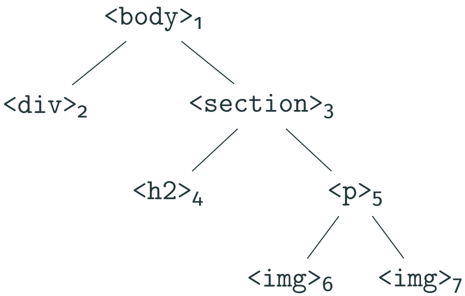
→ Each **path** in the **product DAG** corresponds to a **match**

→ **Challenge:** Enumerate paths but avoid **duplicate matches** and do not **waste time** to ensure constant delay

Extension: From Text to Trees

Pattern Matching on Trees

- The data *T* is no longer text but is now a tree:



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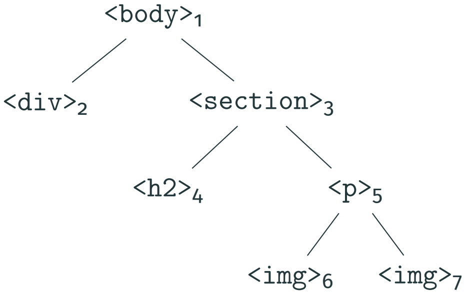

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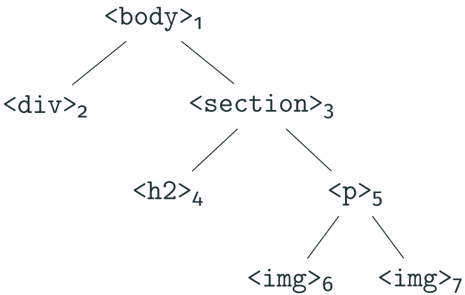

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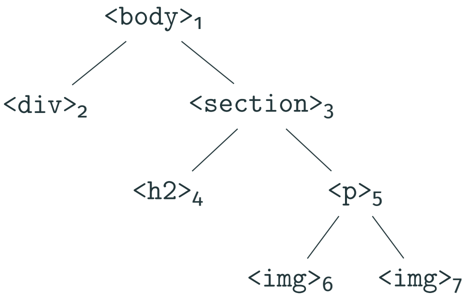

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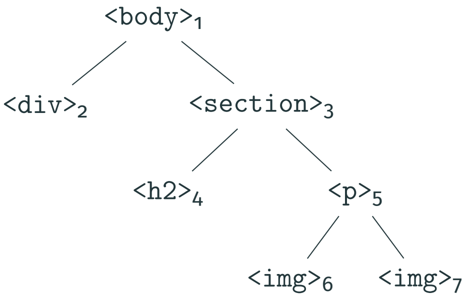

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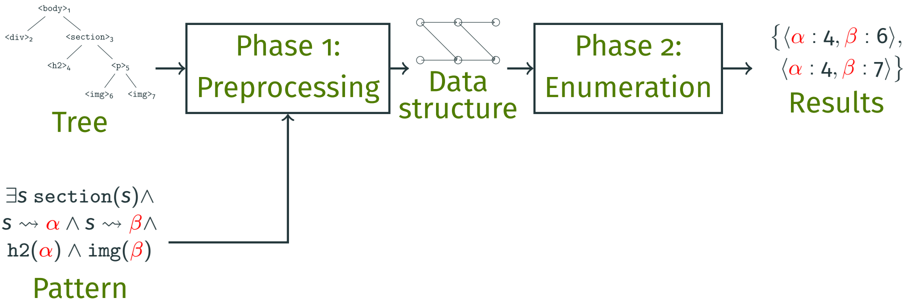
Theorem [Amarilli et al., 2019]

- *Preprocessing in* $O(|T| \times Poly(P))$
- *Delay polynomial in* P *and independent from* T

Proof Idea for Trees: Structure

Similar structure to the previous proof, but with a circuit:



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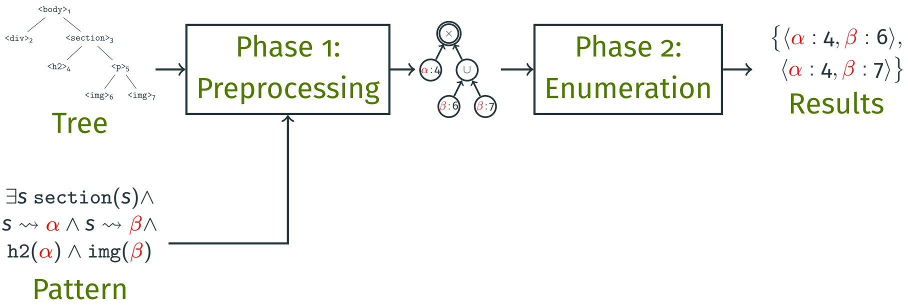
Similar structure to the previous proof, but with a circuit:

- **Preprocessing:** Compute a circuit representation of the answers
- **Enumeration:** Apply a generic algorithm on the circuit



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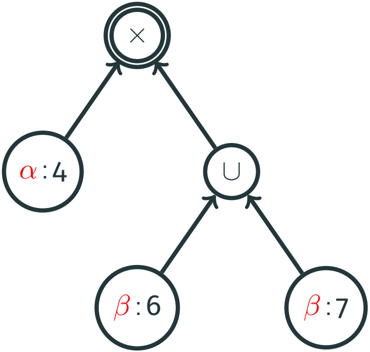

Three kinds of set-valued gates:

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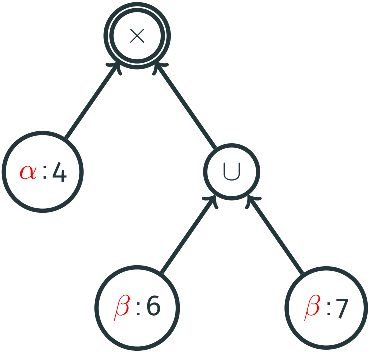

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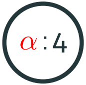
- **Variable gate** 

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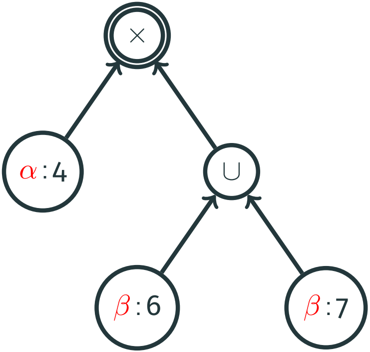

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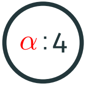
- **Variable gate**  :

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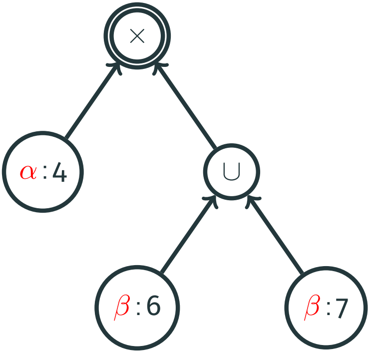

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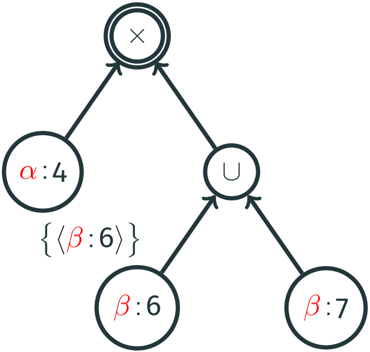

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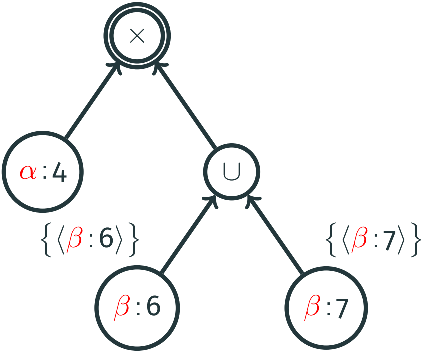

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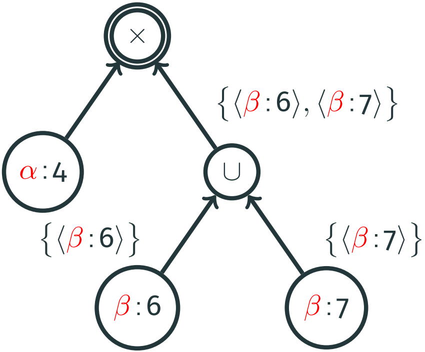

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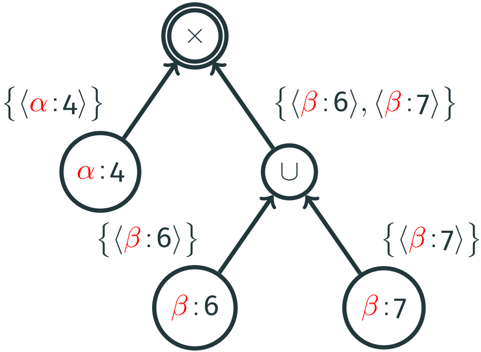

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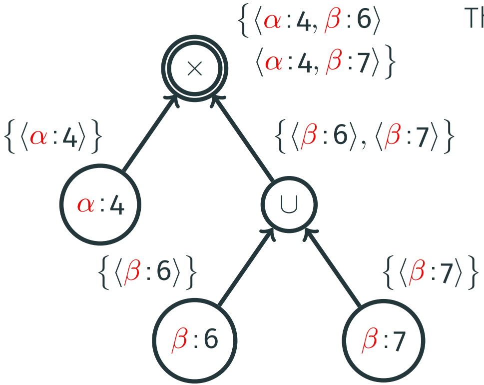

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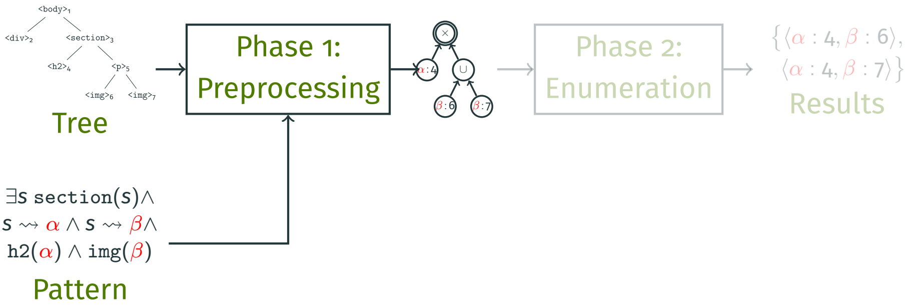

Theorem

For any ***tree automaton*** A *with capture variables* $\alpha_1, \ldots, \alpha_k$*, given a* ***tree*** T*, we can build in* $O(|T| \times |A|)$ *a* ***set circuit*** *capturing exactly the set of tuples* $\{\langle \alpha_1 : n_1, \ldots, \alpha_k : n_k \rangle$ *in the output of* A *on* T

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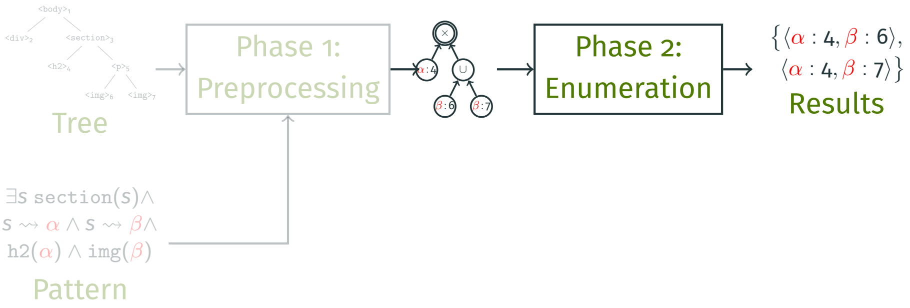


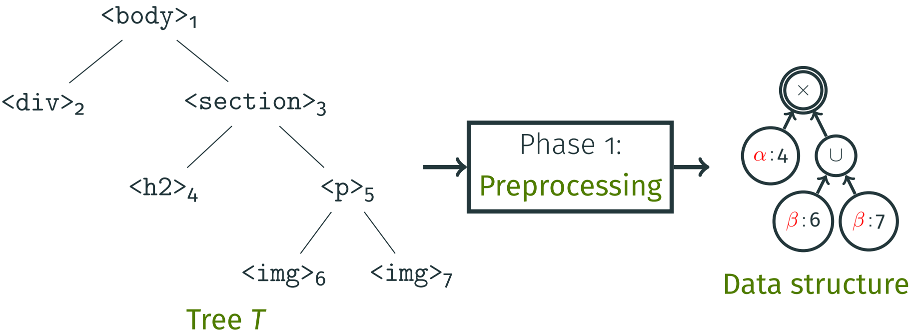
Theorem

Given a set circuit satisfying some conditions, we can enumerate all tuples that it captures with linear preprocessing and constant delay

E.g., for $\{\langle \alpha : 4, \beta : 6\rangle, \langle \alpha : 4, \beta : 7\rangle\}$: enumerate $\langle \alpha : 4, \beta : 6\rangle$ then $\langle \alpha : 4, \beta : 7\rangle$

Extension: Supporting Updates

Updates



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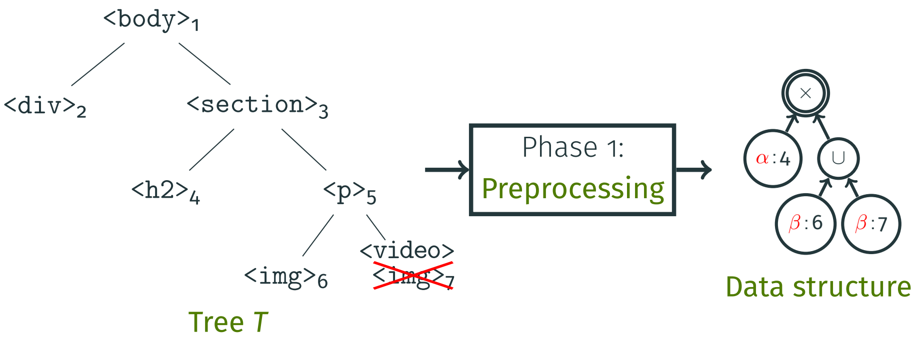

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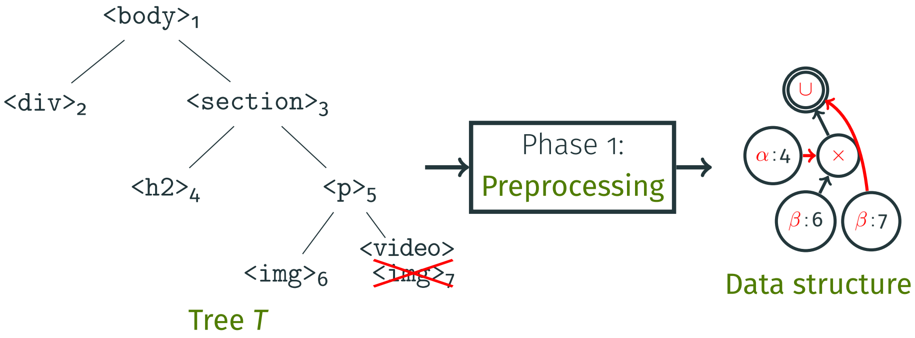

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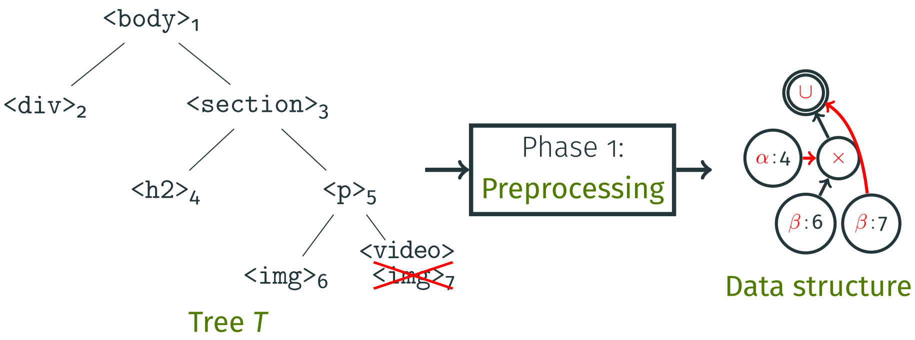

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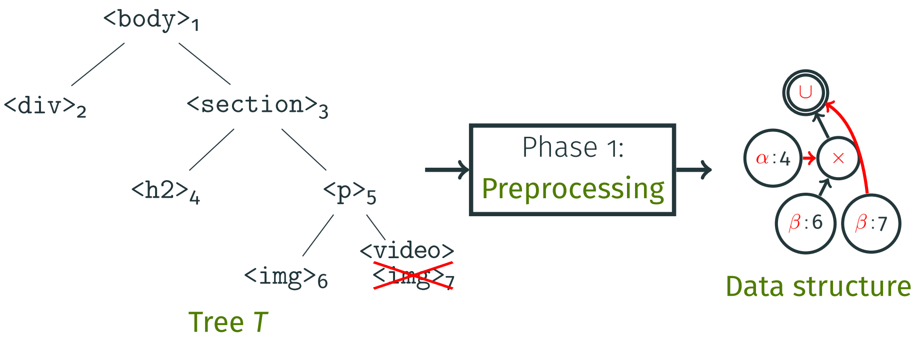

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→ Can we **do better**?

Results on dynamic trees

All these results are on data complexity in T (for a fixed pattern):

Work	Data	Preproc.	Delay	Updates
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[Losemann and Martens, 2014]	text	$O(T)$	$O(\log T)$	$O(\log T)$
[Niewerth and Segoufin, 2018]	text	$O(T)$	$O(1)$	$O(\log T)$

Results on dynamic trees

All these results are on data complexity in T (for a fixed pattern):

Work	Data	Preproc.	Delay	Updates
[Bagan, 2006], [Kazana and Segoufin, 2013]	trees	$O(T)$	$O(1)$	$O(T)$
[Losemann and Martens, 2014]	trees	$O(T)$	$O(\log^2 T)$	$O(\log^2 T)$
[Losemann and Martens, 2014]	text	$O(T)$	$O(\log T)$	$O(\log T)$
[Niewerth and Segoufin, 2018]	text	$O(T)$	$O(1)$	$O(\log T)$
[Amarilli et al., 2019]	trees	$O(T)$	$O(1)$	$O(\log T)$

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Thanks for your attention!

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