Enumerating Pattern Matches in Words and Trees

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Problem: Finding patterns in text

- We have a long text T:

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    - Write the pattern as a regular expression:

$$P := \texttt{␣}^{+}\ \texttt{[a-z0-9.]}^{*}\ \texttt{@}\ \texttt{[a-z0-9.]}^{*}\ \texttt{␣}^{+}$$

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→ How to find the pattern P efficiently in the text T?

Solution: automata

- Convert the pattern from a regular expression to an automaton

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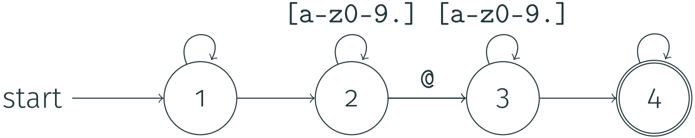

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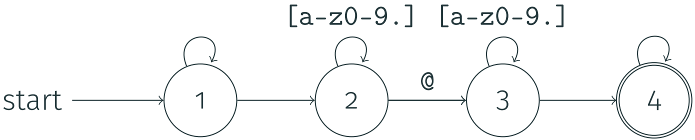

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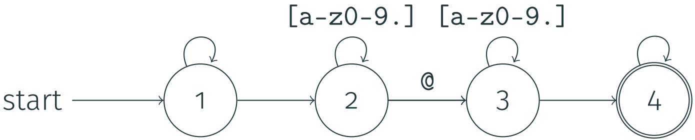

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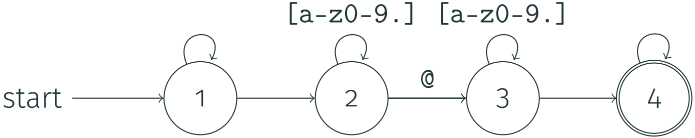

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    - **Data complexity** in the text T: **linear**, i.e., $O(|T|)$

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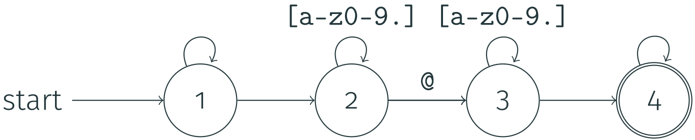

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- How efficient is this?
  - Data complexity in the text T: linear, i.e., $O(|T|)$
  - Combined complexity in T and P: polynomial

Actual problem: Extracting all patterns

- **Problem:** This only tells us **if** the pattern is in the text!
  $\rightarrow$ ``YES''

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$\rightarrow$ One match: $\langle \alpha : 20, \beta : 32 \rangle$

Formal problem statement

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$$P := \textvisiblespace^{+}\ {\color{red}\alpha}\ \texttt{[a-z0-9.]}^{*}\ \texttt{@}\ \texttt{[a-z0-9.]}^{*}\ {\color{red}\beta}\ \textvisiblespace^{+}$$

  - Output: the list of matches of P on T

$$\langle {\color{red}\alpha} : 187, {\color{red}\beta} : 199\rangle, \dots$$

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  - Output: the list of matches of P on T

$$\langle \alpha : 187, \beta : 199 \rangle, \ldots$$

- We measure the complexity of the problem:
  - In data complexity, as a function of T
  - In combined complexity, as a function of P and T

Measuring the complexity

- Naive algorithm: Consider all ways to assign capture variables and test for each of them if it satisfies the pattern

l	o	l

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$\alpha\beta$ `l    o    l`

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$\rightarrow$ We need a **different way** to measure complexity

Enumeration algorithms

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→ Formalization: enumeration algorithms

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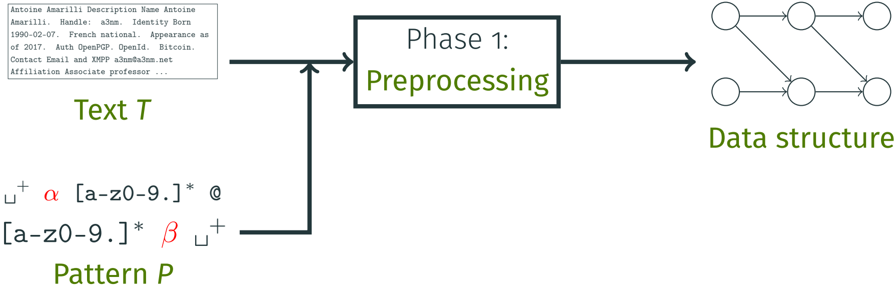
Pattern P

Phase 1:
Preprocessing

Formalizing enumeration algorithms



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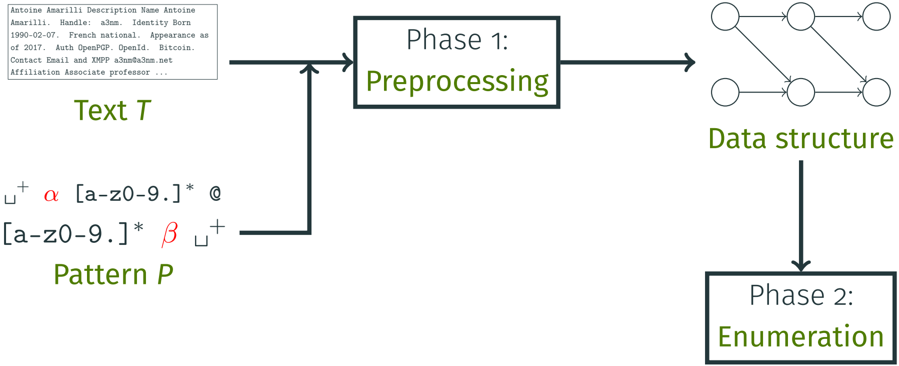

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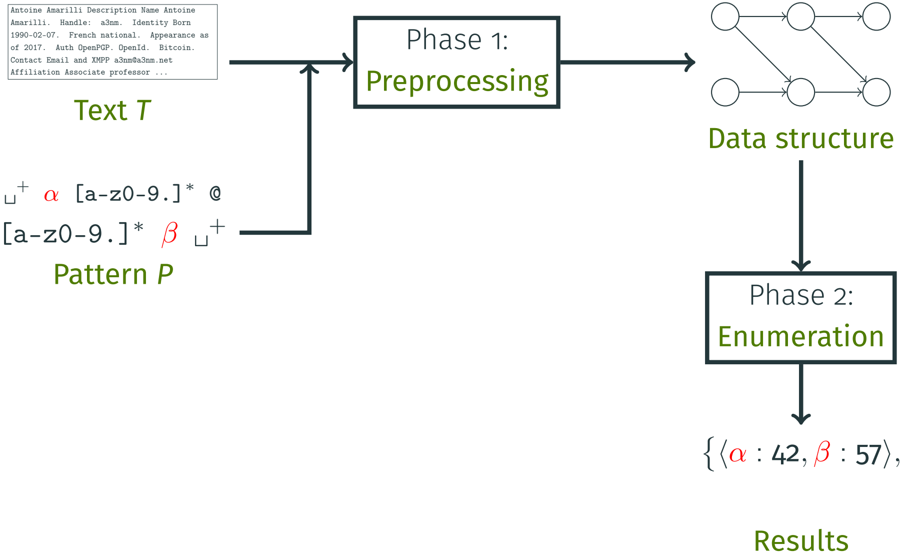

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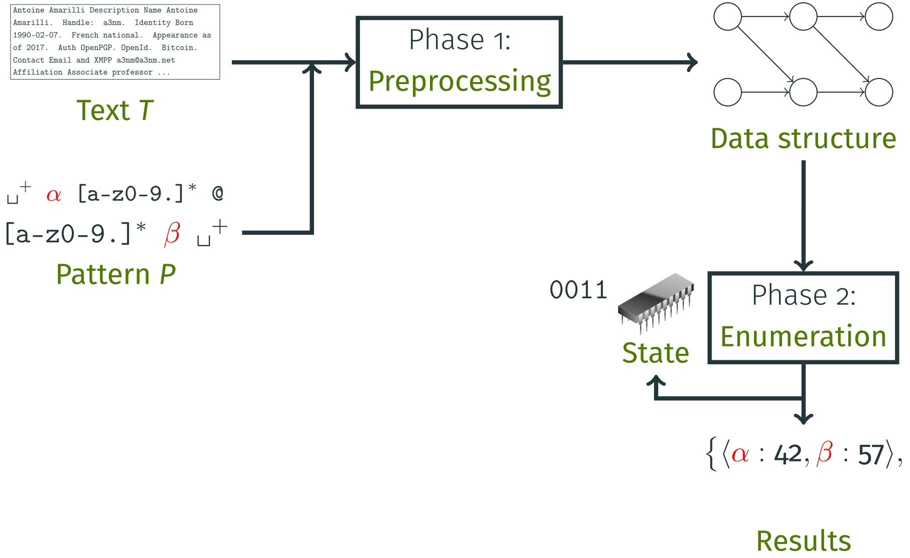

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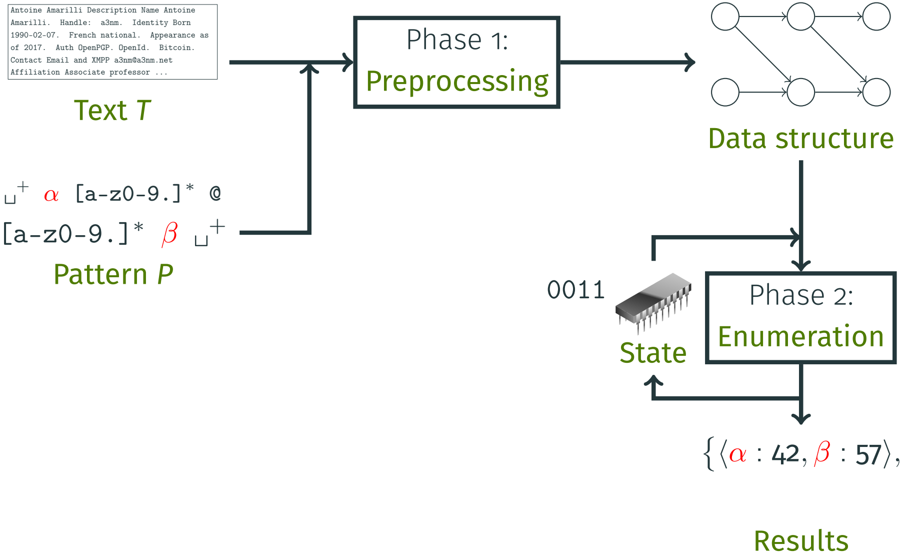

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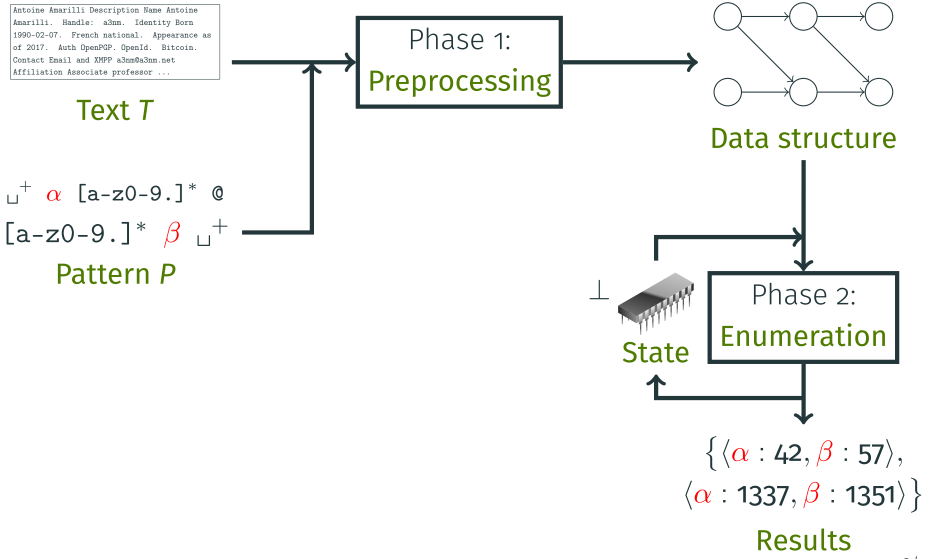

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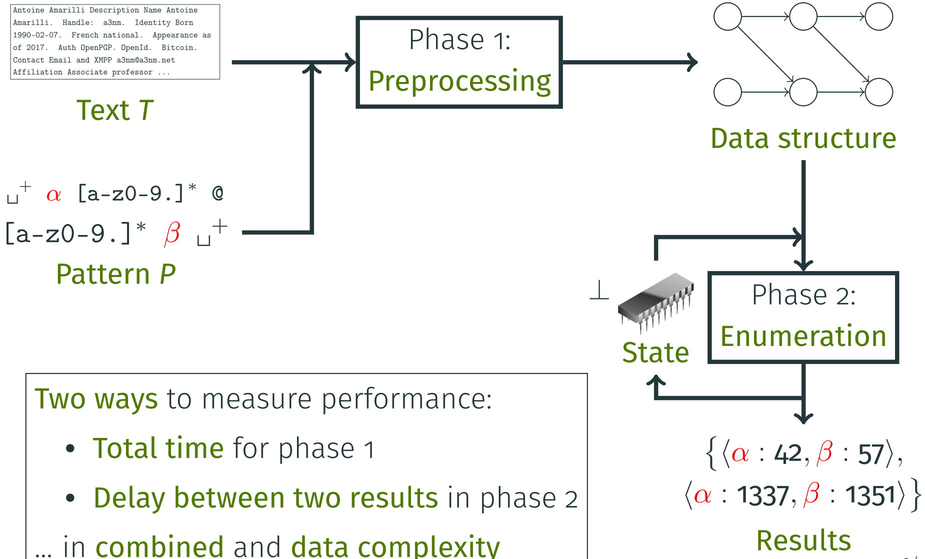

Two ways to measure performance:

- Total time for phase 1
- Delay between two results in phase 2

... in combined and data complexity

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  - In terms of **preprocessing**...
    - **Combined complexity** is... **polynomial**: convert P to an automaton A

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    - Data complexity is...

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  - In terms of delay...

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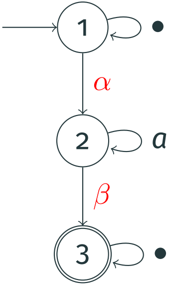

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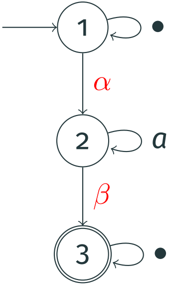

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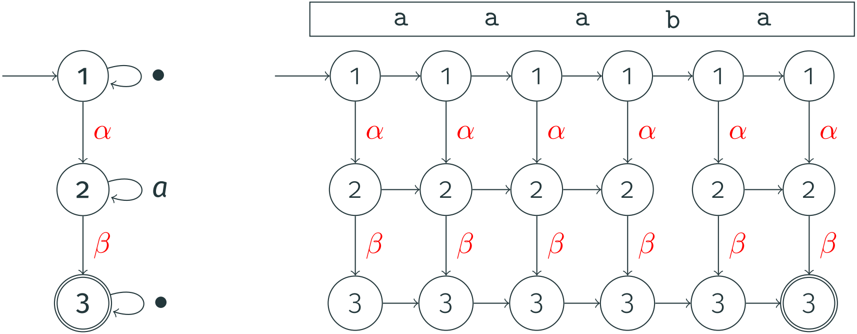

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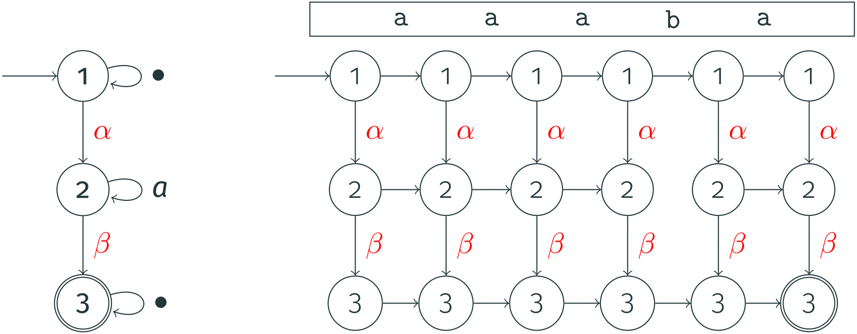

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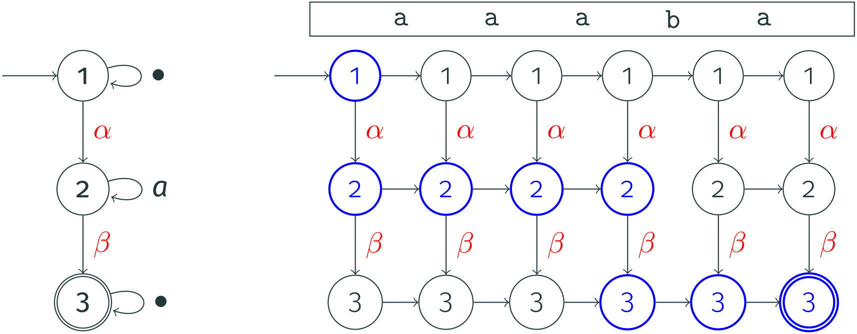

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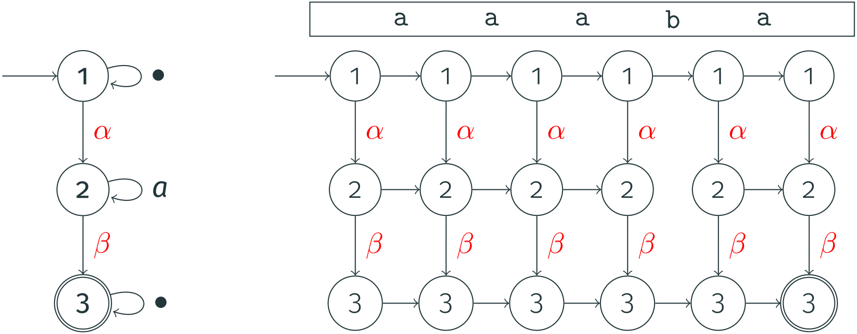

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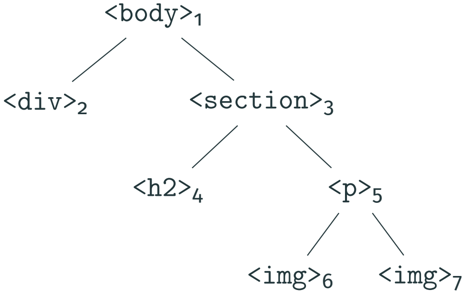
$\rightarrow$ **Challenge:** Enumerate paths but avoid **duplicate matches** and do not **waste time** to ensure constant delay

Extension: From Text to Trees

- The data T is no longer text but is now a tree:



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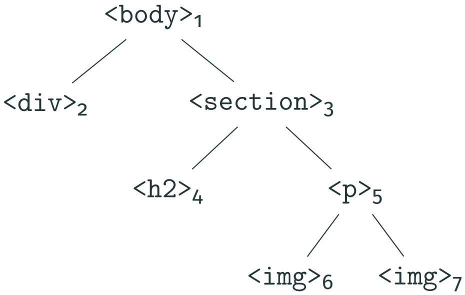

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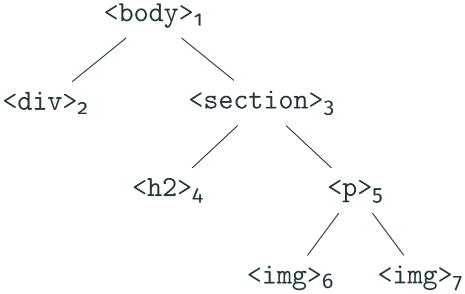

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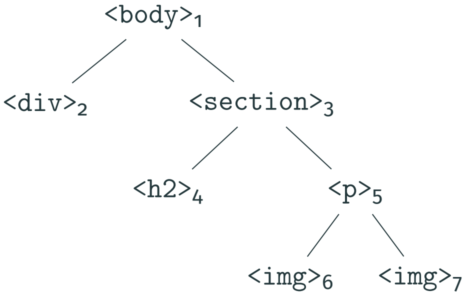

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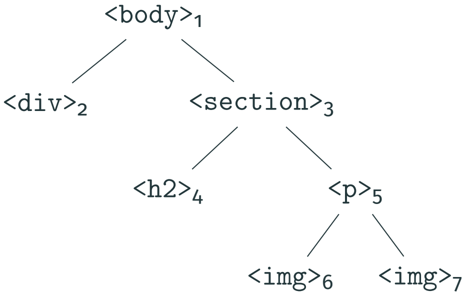

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- **Results:** $\langle \alpha : 4, \beta : 6 \rangle, \langle \alpha : 4, \beta : 7 \rangle$

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- We **conjecture** the following bounds for this task (ongoing work):

Conjecture

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Extension: Handling Updates

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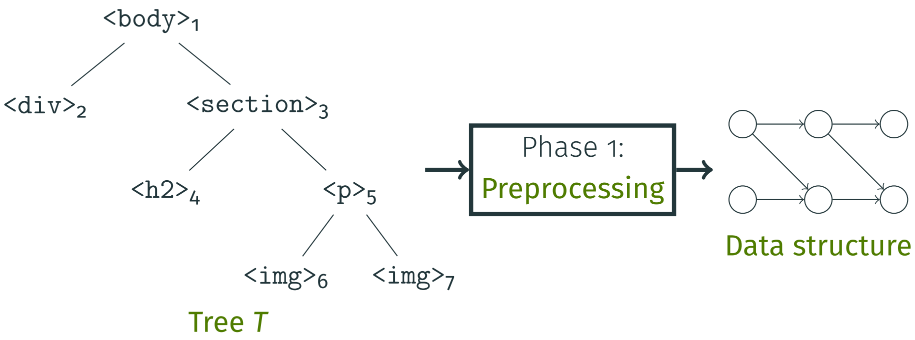

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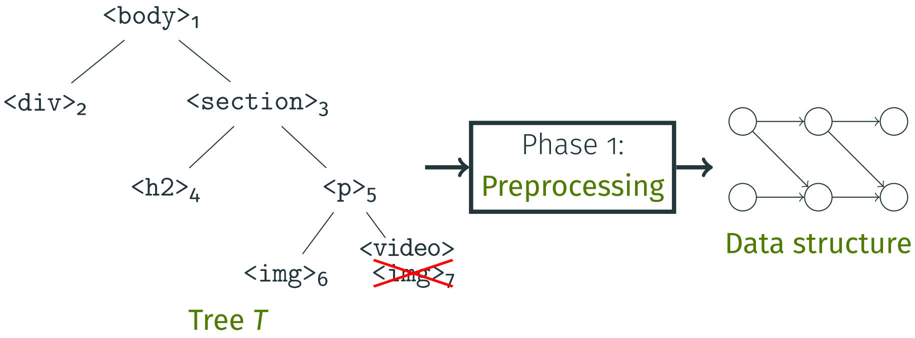

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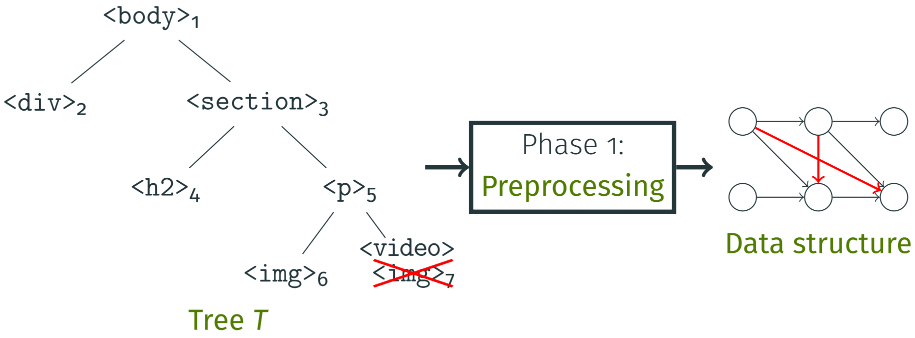

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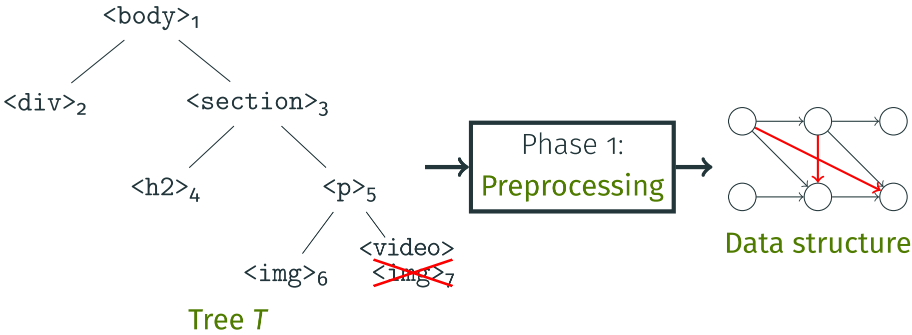

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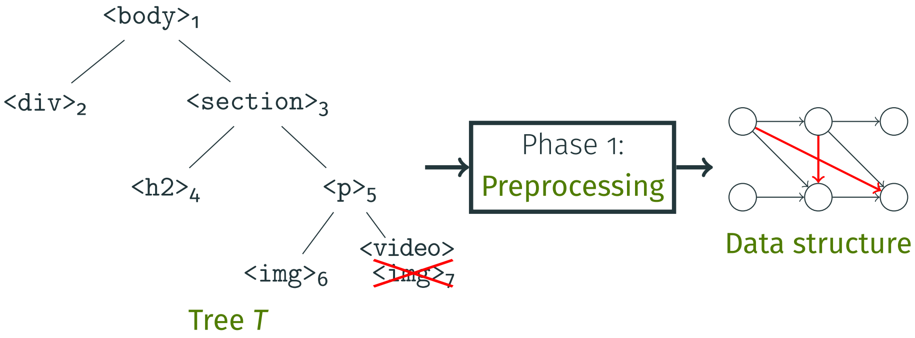

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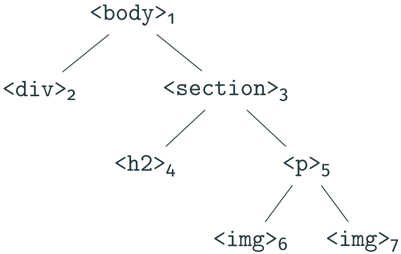

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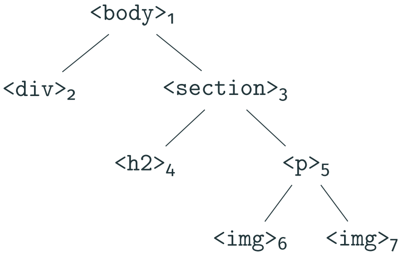

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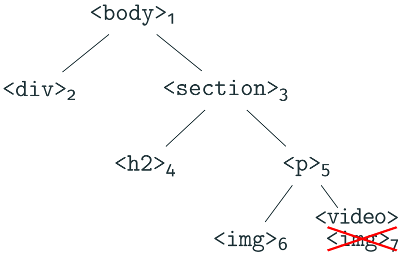

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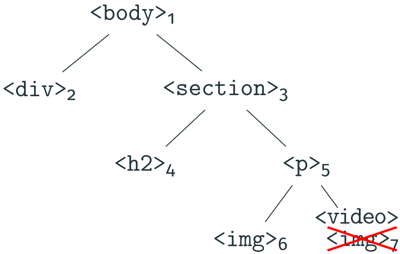

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Remaining open questions:

→ Does this hold for more general updates (insert/delete, etc.)?

→ Can we also achieve tractable combined complexity?

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Thanks for your attention!

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To appear.