Query Evaluation with Model Counting and Knowledge Compilation: A Survey of Old and Recent Results

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Explanations for Boolean queries

- We focus on **query evaluation**: evaluate a **query** on **data**

$$Q(x, z) : \exists y \; R(x, y) \wedge S(y, z)$$

Query

R	
a	b
a	b'

S	
b	c
b'	c'

Instance

x	z
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    - **Probabilistic query evaluation** (PQE): add probabilities (weights) to each fact
    - **Shapley values**: another kind of aggregate

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    - **Shapley values**: another kind of aggregate
  - **Knowledge compilation**: compute a **representation** of the **Boolean provenance**, i.e., the set of possible worlds satisfying Q (here: $(r_1 \wedge s_1) \vee (r_2 \wedge s_2)$)
    - Depends on the **class of representations**: circuits, diagrams, d-SDNNFs...
    - **Intensional approach**: we can use knowledge compilation for model counting

Roadmap

Goal of this talk: give an overview of results on query evaluation for:

- **Model counting:** (approximate) counting algorithms / hardness bounds
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- **Combined complexity**

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$\rightarrow$ **For which queries Q** are these problems $\mathrm{UR}(Q)$ and $\mathrm{PQE}(Q)$ solvable in PTIME?

Data complexity of model counting for SJFCQs

A **self-join free conjunctive query** (SJFCQ) is a CQ without repeated relations

Theorem (Dalvi, Suciu, VLDB'04, VLDBJ'07)

The following dichotomy holds on SJFCQs Q:

- *If Q is **hierarchical**, then $\mathrm{PQE}(Q)$ is in **PTIME***
- *Otherwise, $\mathrm{PQE}(Q)$ is **#P-hard***

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Theorem (A., Kimelfeld, ICDT'21, LMCS'22)

The same dichotomy holds for uniform reliability $(\mathrm{UR}(Q))$

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For the class of **unions of conjunctive queries** (UCQs):

Theorem (Dalvi, Suciu, JACM'12)

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- *For some unsafe UCQs, hardness already holds for uniform reliability*

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Is $\mathrm{UR}(Q)$ #P-hard for every unsafe UCQ, like $\mathrm{PQE}(Q)$? Open

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Do these hardness results extend to **higher-arity signatures**? **Open**

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$\rightarrow$ Is treelikeness the only condition on the instance that makes PQE tractable?

Data complexity: Counting matchings on unbounded-treewidth graphs

- Fix a binary relation R, let Q_2 be the query asking if there are two incident facts
  - $\rightarrow$ Q_2 is a UCQ with inequalities: $(R(x,y) \vee R(y,x)) \wedge (R(y,z) \vee R(z,y)) \wedge x \neq z$

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- For which other queries than Q_2 does this hold? What about higher arity? Open

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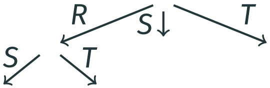

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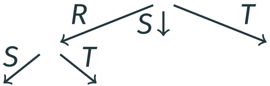

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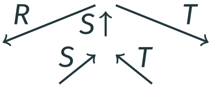

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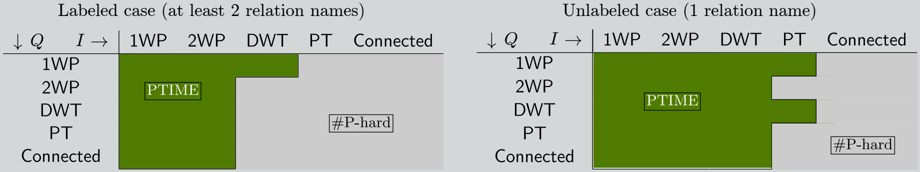

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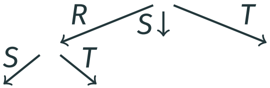

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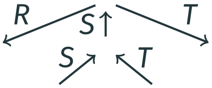

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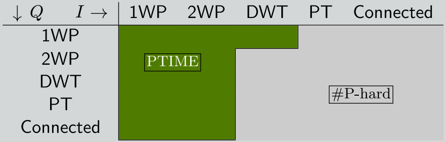
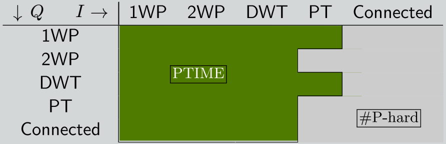
Labeled case (at least 2 relation names)



Unlabeled case (1 relation name)



Do hardness results still hold for **UR**? **Open** (we haven't thought about it)

Model counting: Approximation

Model counting problems (UR/PQE) ask for an exact answer. Is it easier to get an approximate answer?

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FPRAS:

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  - → **Rest of the talk only covers this notion**

Model counting: Easy approximation results

When can we have an FPRAS for model counting?

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  - $\rightarrow$ When can we obtain such an algorithm?

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There is a combined FPRAS for PQE with SJFCQs Q of bounded hypertreewidth and arbitrary instances I with probabilities on facts

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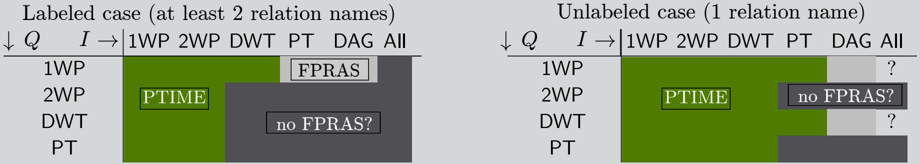

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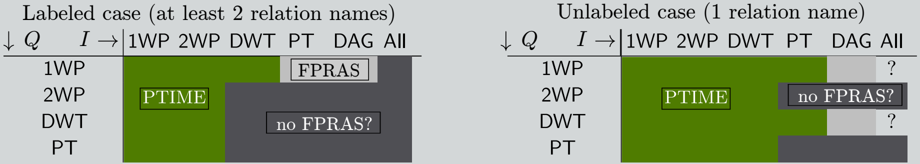

(Using **FPRAS for #NFA** for word and tree automata (Arenas et al., JACM'21, PODS'21))

FPRAS inexistence results assume $\mathrm{RP} \neq \mathrm{NP}$

Table of contents

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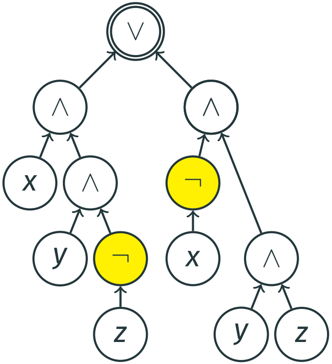
Goal: provenance representation in **tractable circuit classes** from knowledge compilation

Tractable circuits: d-DNNF and d-SDNNF

Tractable circuit class: **d-DNNF:**

- $\neg$ are only applied to variables



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The inputs are mutually exclusive
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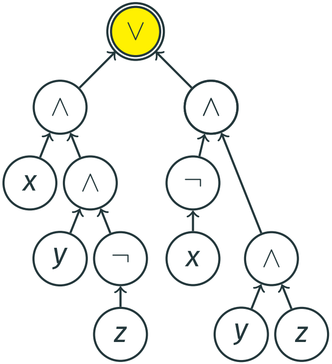

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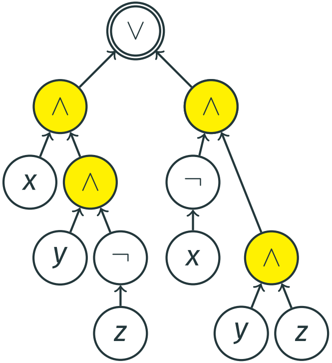

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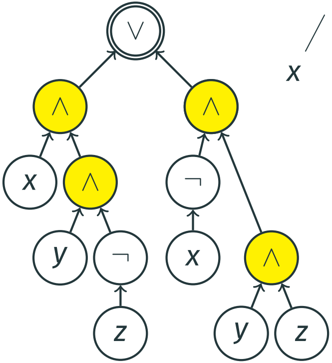
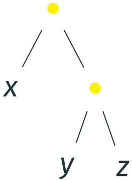
Frequent extra requirement: **structuredness** (following a **vtree**), aka **d-SDNNF**





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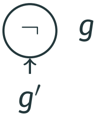

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$$g' \rightarrow \neg \; g$$

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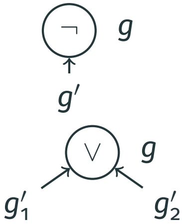

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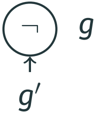

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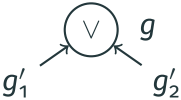

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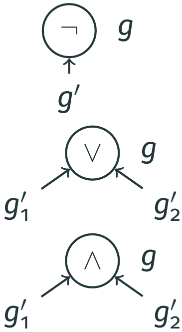

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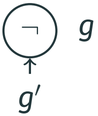

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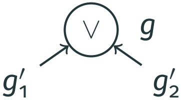

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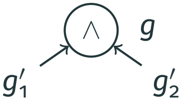

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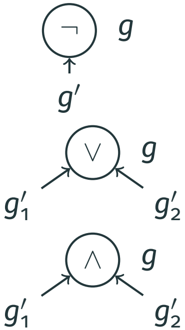

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$\rightarrow$ If you can build a d-DNNF provenance representation in PTIME, then PQE is in PTIME

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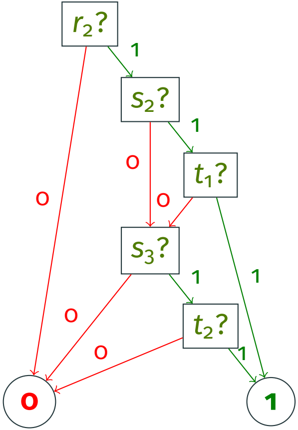

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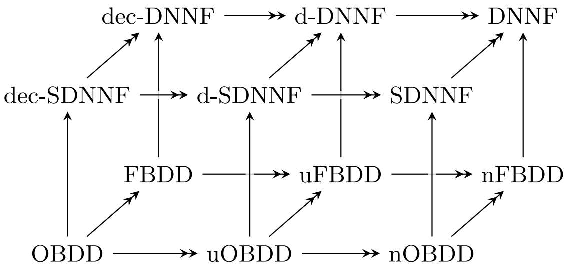

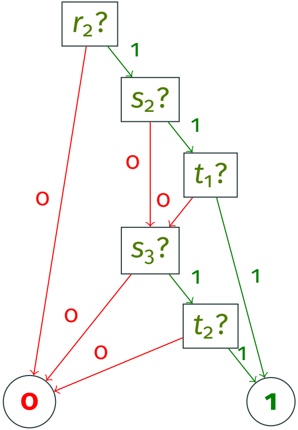

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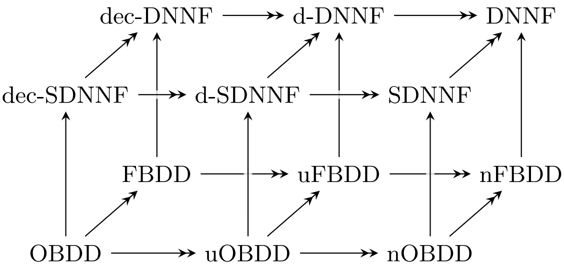

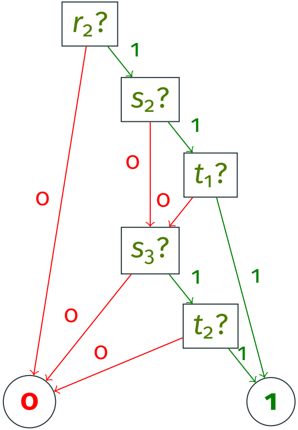

- Generalize **d-DNNFs** to **d-Ds**: allow arbitrary negations

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Data complexity of provenance computation for UCQs and hom-closed queries

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- For homomorphism-closed queries: open

Data complexity of provenance computation for treelike data

Going back to the setting of restricted instance classes, we have:

Theorem (A., Bourhis, Senellart, ICALP'15, PODS'16)

For any fixed MSO query Q *and treelike instance family* $\mathcal{I}$*, given an instance* I *in* $\mathcal{I}$*, we can compute the provenance of* Q *on* I *in PTIME as a d-SDNNF circuit*

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For the "two incident facts" query Q_2, given an instance I, any d-SDNNF representation of the provenance of Q_2 on I is exponential: in $\Omega\left(2^{tw(I)^{1/d}}\right)$ for some $d \geq 1$

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What about more expressive circuit formalisms (d-DNNF)? **Open**

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We have studied this in several settings:

- Data complexity for SJFCQs, UCQs, hom-closed queries, etc.
- Data complexity on restricted instance families (treelike or not)
- Combined complexity

Open problems

General research directions on these topics:

- Connections between these two frameworks (intensional-extensional conjecture? lower bound techniques?)
- Connections to other aggregate queries? (Shapley value, etc.)
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Thanks for your attention!

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