Strings in Data Management: Enumeration and Incremental Maintenance

Antoine Amarilli

July 30, 2024

Télécom Paris

Introduction

Short presentation

- Associate prof at **Télécom Paris**
  - → Moving from Télécom Paris to **Inria Lille**

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- Core area: **database theory**
  - → Moving from **databases** to **theory**

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  - → Moving from Télécom Paris to **Inria Lille**
- Core area: **database theory**
  - → Moving from **databases** to **theory**
- Confession: **No prior stringology experience**!
  - → But attracted to the area via **database theory** problems

Talk overview

Main message: database theory problems sometimes lead to stringology problems

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Talk contents:

- Regular document spanners, a formalism motivated by information extraction

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Talk contents:

- Regular document spanners, a formalism motivated by information extraction
- Enumeration algorithms for regular document spanners on strings
- Incremental maintenance when the string is modified
- Directions for future research

Document spanners

Database motivation: Declarative information extraction

- **Standard setting in database research:** queries are posed over **relational tables**
- **In practice:** data is sometimes hidden in large textual documents

$\rightarrow$ **Information extraction** (IE): how to get from large textual data to structured data

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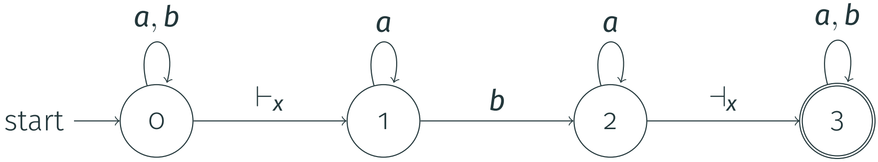
→ **Information extraction** (IE): how to get from large textual data to structured data

Guiding principle: **declarative** information extraction:

→ Specify **what** you want to extract, not **how** to extract it

Basic idea for declarative IE: Regular document spanners

A finite automaton with special transitions that extract substrings of the input string



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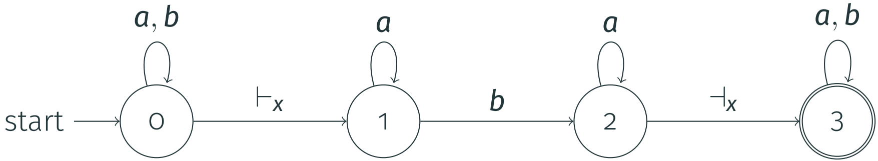

"Extract all couples of a nonempty factor with only ***a****'s and then a nonempty factor with only* ***b****'s"*

Formalizing document spanners

- **Document:** string over an alphabet

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$T =$	J	o	h	n	␣	4	5	6	1	2	3	
	0	1	2	3	4	5	6	7	8	9	10	11

Formalizing document spanners

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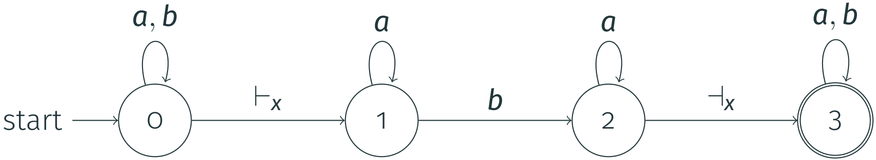
- **Mapping** over a set of variables X: function from X to spans
  $\rightarrow$ ex: for $X = \{x, y, z\}$, map x and y to $[0, 4\rangle$ and map z to $[11, 11\rangle$
- **Spanner**: function that maps each string to a set of mappings
  $\rightarrow$ ex: for $X = \{x, y, z\}$, each string is mapped to a **relational table** with columns x, y, z

Document spanner example

Take the spanner from before:



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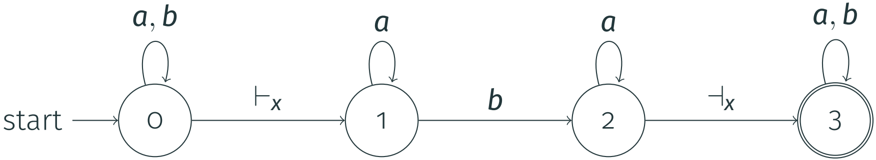

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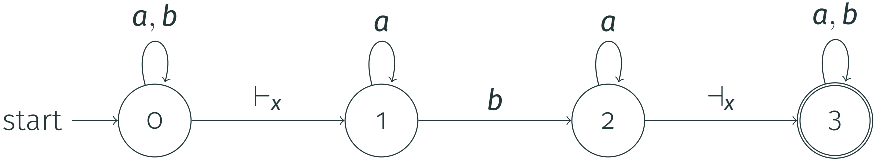

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$T =$ `b b a b`

0 1 2 3 4

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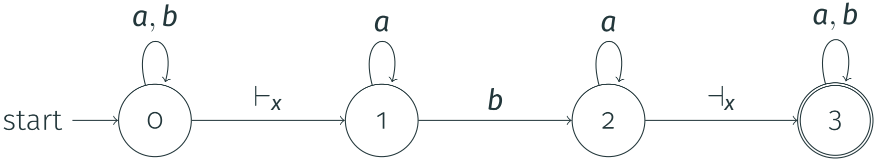

x
$[0, 1\rangle$
$[1, 2\rangle$
$[1, 3\rangle$
$[2, 4\rangle$
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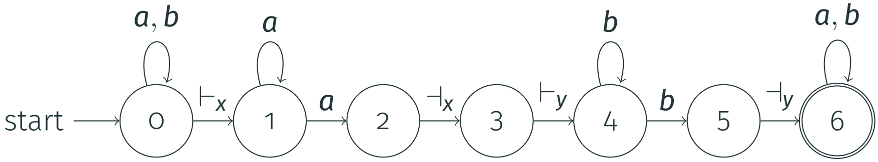

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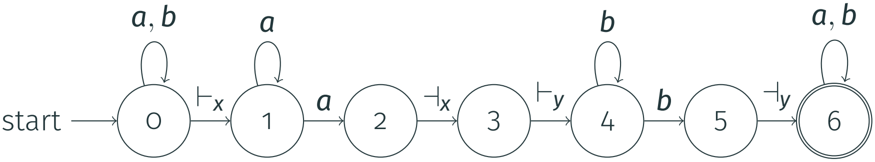

*"Extract all nonempty contiguous substrings of **a**'s followed by nonempty contiguous substrings of **b**'s"*

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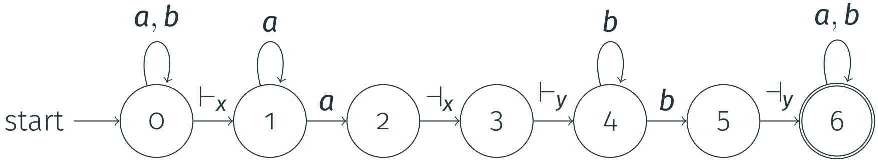

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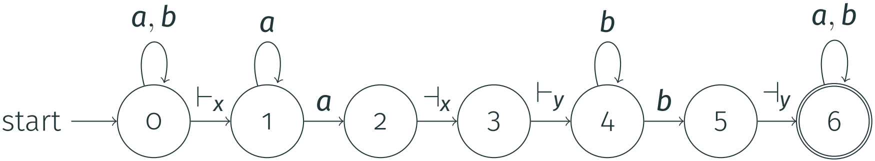

x	y
$[0,1\rangle$	$[1,2\rangle$
$[2,3\rangle$	$[3,4\rangle$
$[2,3\rangle$	$[3,5\rangle$

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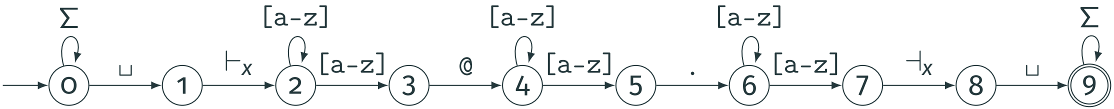
Formalizing regular document spanners

Regular spanners: those that can be expressed as variable-set automata (VAs)



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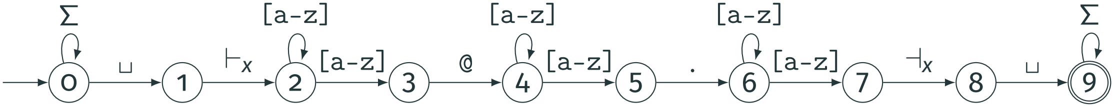

In practice, often more convenient to write in the subclass of regex-formulas:

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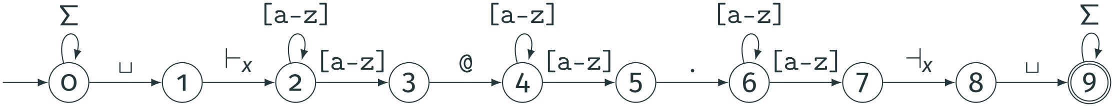

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Other **more general classes**:

- **Core spanners**: featuring **string equality selection**
- **Generalized core spanners**: featuring **difference**

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Is there a result for $\vdash_x e \dashv_x$?

- Find all matches of pattern with variables xax, using string equality selection

$$\Sigma^* \vdash_x \Sigma^* \dashv_x a \vdash_{x'} \Sigma^* \dashv_{x'} \Sigma^* \text{ with string equality } x = x'$$

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→ **Efficient evaluation**: can we efficiently compute the result of a spanner on a string?

Enumeration

Database motivation: Enumeration algorithms

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→ Research area (in databases and outside): **enumeration algorithms**

A simpler problem: Enumerating results of regular spanners

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  - Input:
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We mostly focus on a **simpler case**: $P = \Sigma^* \vdash_x e \dashv_x \Sigma^*$ where e is a regular expression

$\rightarrow$ Find all substrings of T satisfying e

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  - The **number of matches** is $\Omega(|T|^2)$

$\rightarrow$ We need a **different way** to measure complexity

Formalizing enumeration algorithms

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Regular spanner P

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Regular spanner P

Phase 1: Preprocessing

Formalizing enumeration algorithms

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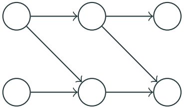

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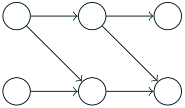

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Phase 2: Enumeration

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of 2017.  Auth OpenPGP. OpenId.  Bitcoin.
Contact Email and XMPP a3nm@a3nm.net
Affiliation Associate professor ...
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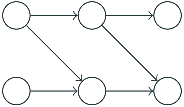
String T

$\Sigma^* \vdash_x$ `[a-z]`$^+$ `@` `[a-z]`$^+$ `.` `[a-z]`$^+$ $\dashv_x \Sigma^*$

Regular spanner P

Phase 1: **Preprocessing**



Index structure

Phase 2: **Enumeration**

$\{x : [42, 57\rangle\},$

Results

Formalizing enumeration algorithms

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Antoine Amarilli Description Name Antoine
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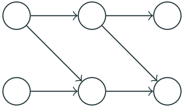

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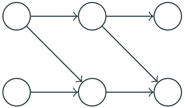

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Results

Two ways to measure performance:

- Total time for phase 1
- Delay between two results in phase 2

... as a function of the string T and regular spanner P

Results for enumerating regexp matches and regular spanner mappings

For the problem of enumerating regexp matches, we can show:

Theorem (follows from [Florenzano et al., 2018])

Given a string T and a deterministic automaton A, we can enumerate the subword occurrences in T that match A with preprocessing $O(|T| \times |A|)$ and constant delay.

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We can show (with more effort):

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For a nondeterministic automaton A, we can enumerate the subword occurrences in T that match A with preprocessing $O(|T| \times Poly(|A|))$ and delay $O(Poly(|A|))$.

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- Remark: if the regexp is just **a word** or a **set of words**, the complexity is $O(|T| \times |A|)$: not helpful compared to Knuth-Morris-Pratt / Aho-Corasick.
- We can achieve the **same complexity** for regular spanners (not just regexps)

Incremental maintenance

Database motivation: Incremental validation

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$\rightarrow$ **Forget trees**: what about incremental validation on **strings**?

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Theorem

For any regular language L recognized by an automaton A, given a string T, we can maintain dynamic membership of T to L under substitution updates in $O(Poly(|A|) \times \log |T|)$ per update.

Proof of the $O(\log n)$ algorithm

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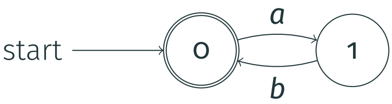

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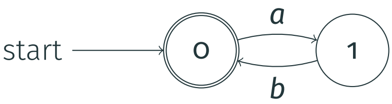

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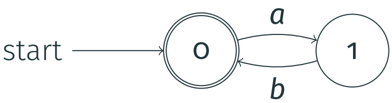
Fix the language $L = (ab)^*$: 

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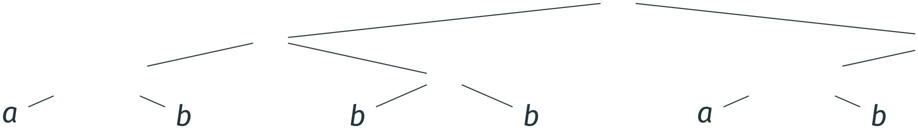

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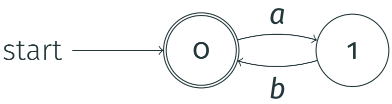

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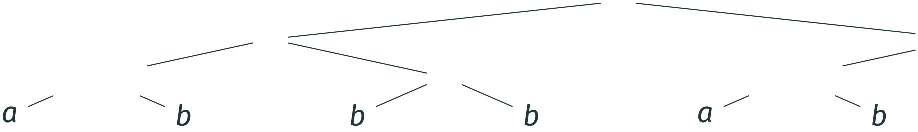

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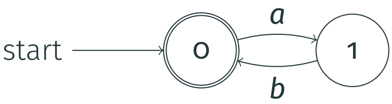

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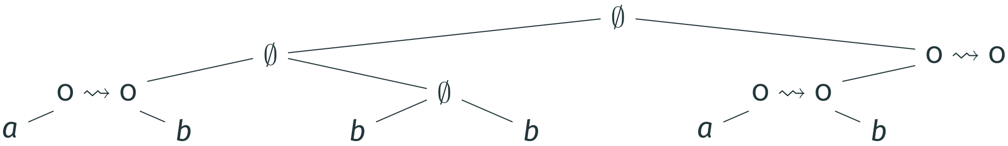

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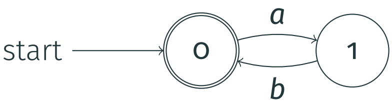

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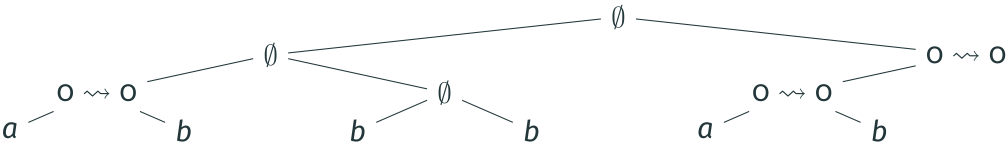

- The tree root describes if $T \in L$
- We can update the tree for each substitution in $O(\log n)$
- Can be improved to $O(\log n / \log \log n)$ with a log-ary tree

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Question: what is the complexity of dynamic membership, depending on the fixed regular language L?

Summary of our results (joint work with Louis Jachiet and Charles Paperman)

QLZG: in $O(1)$

QSG: in $O(\log\log n)$
not in $O(1)$?

All: in $\Theta(\log n / \log\log n)$

- We identify a class **QLZG** of regular languages:
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- The problem is always in $O(\log n / \log \log n)$

Combining incremental maintenance and enumeration

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Also generalizes to **trees** for a suitable notion of tree automata with captures

Conclusion

Summary, ongoing research, and open problems

- The message: some database theory questions are better answered by stringology
  - **Regular spanners** as a formalism for expressive pattern matching tasks on strings
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Thanks for your attention!

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