Query Evaluation on Probabilistic Data: New Hard Cases

Antoine Amarilli[1], joint work with Benny Kimelfeld[2], İsmail İlkan Ceylan[3]

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Uncertain data management

- Databases: manage data and answer queries over it

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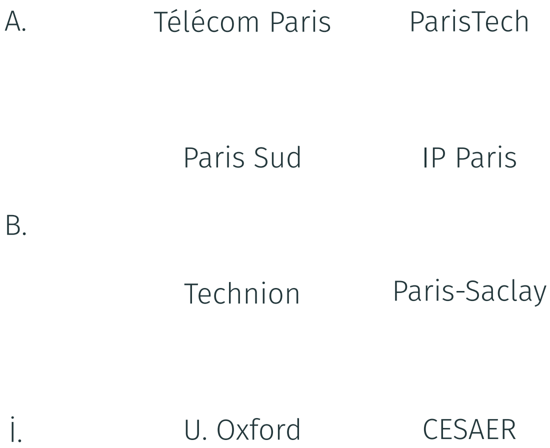

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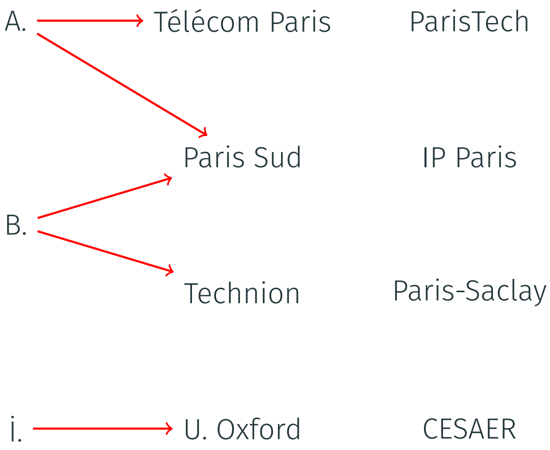

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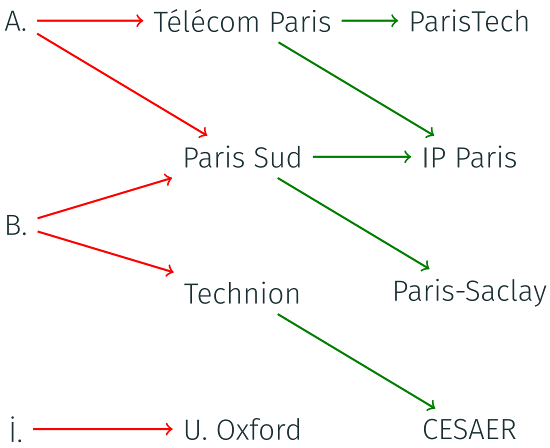

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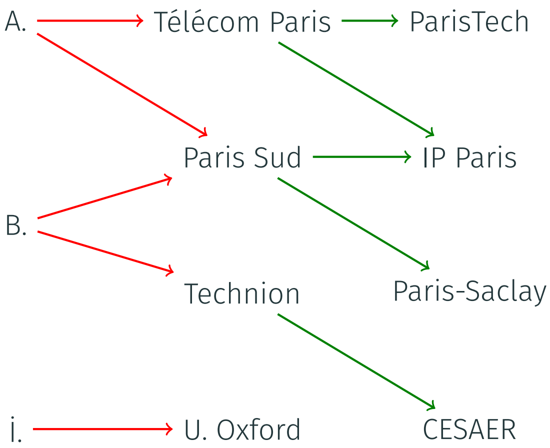
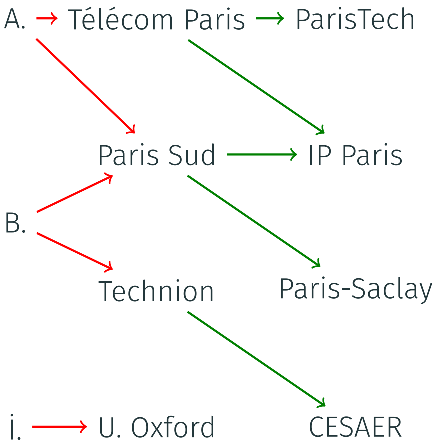


→ **Problem:** we may be **uncertain** about the data

Uncertain data model



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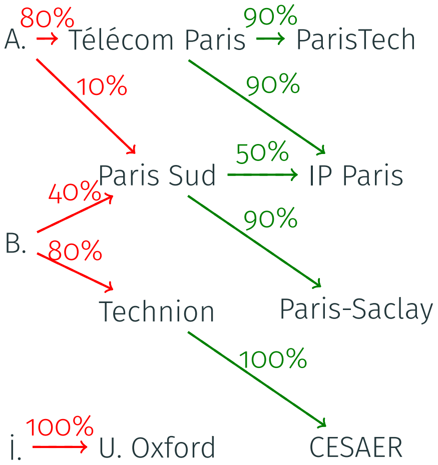

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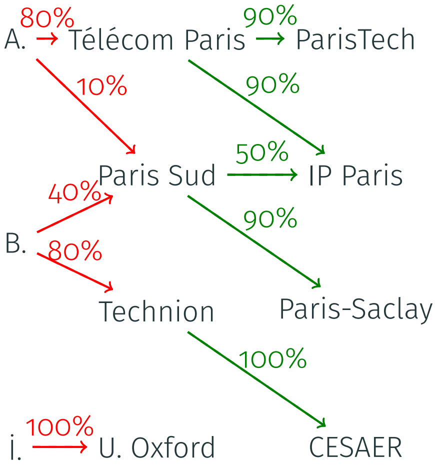

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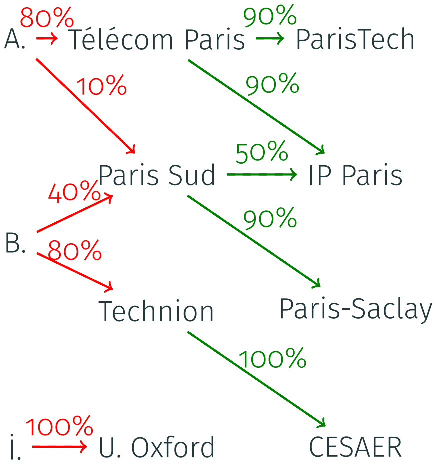

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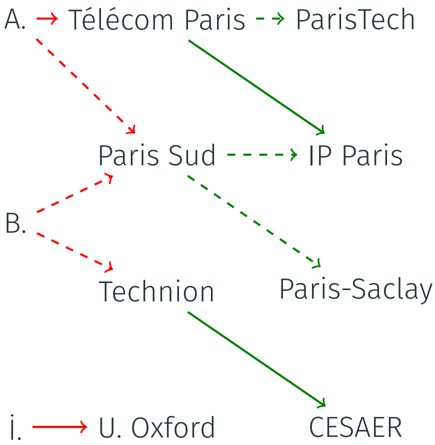

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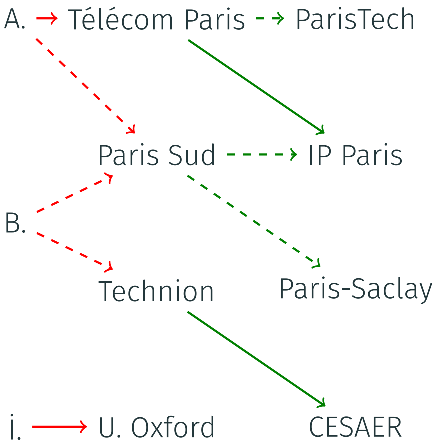

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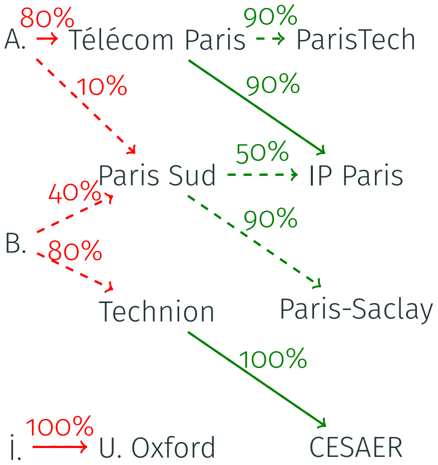

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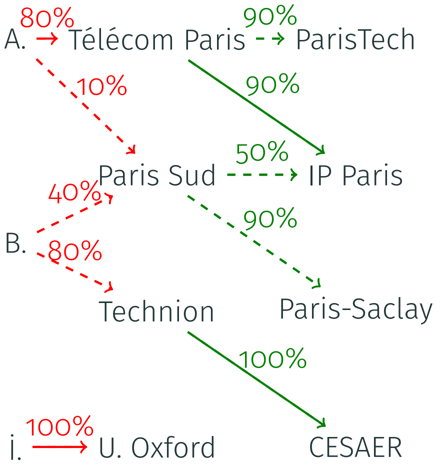

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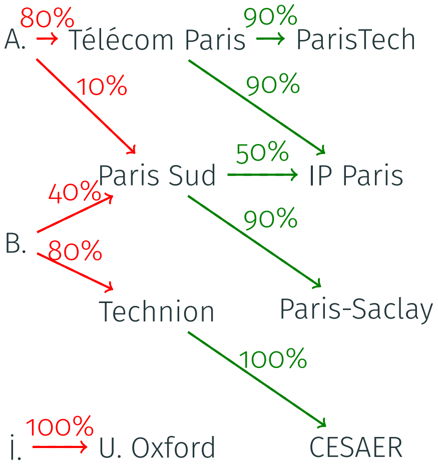

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→ This model is **simplistic**, but already challenging to understand

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- **Union of conjunctive queries**: does one of the patterns match?
- **Homomorphism-closed query Q**: if G satisfies Q and G has a homomorphism to G' then G' also satisfies Q

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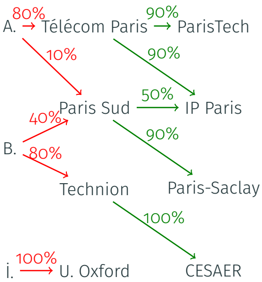

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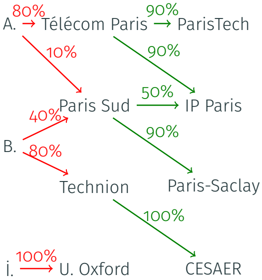

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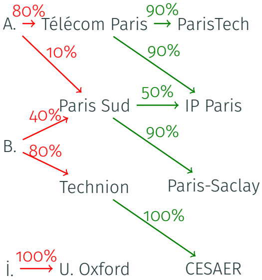

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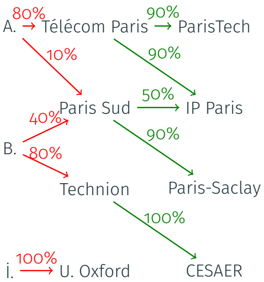

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→ What is the **complexity** of the problem $\mathrm{PQE}(Q)$, depending on the query *Q*?

Existing results

Dichotomy on the unions of conjunctive queries (UCQs):

Theorem [Dalvi and Suciu, 2012]

- *Some UCQs Q are safe and $\mathrm{PQE}(Q)$ is in PTIME*
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- *There is a query Q such that $\mathrm{PQE}(Q)$ is #P-hard on any TID family of unbounded treewidth (with several technical assumptions)*

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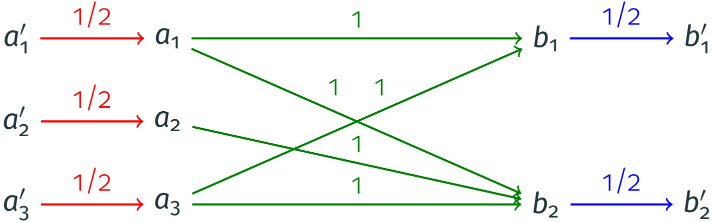

New results in this talk

We present more cases where PQE is #P-hard:

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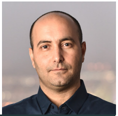

- With Benny Kimelfeld, in the unweighted case:

Theorem [Amarilli and Kimelfeld, 2019]

For any CQ Q without self-joins (every edge has a different color), if Q *is unsafe then* $\mathrm{PQE}(Q)$ *is #P-hard even if all probabilities are* $1/2$

Hardness for queries closed under homomorphisms

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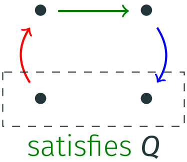
- The query WA/MO$^+$ is **equivalent** to WA/MO which is a **safe UCQ**
- The query WA/MO$^+$/IN is **not equivalent to a UCQ** so PQE is **#P-hard**

Proof idea: finding hard patterns

- Fix the query Q and find a tight pattern, i.e,. a graph such that:



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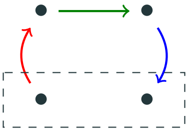

satisfies Q

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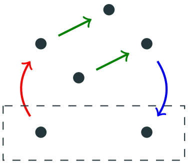

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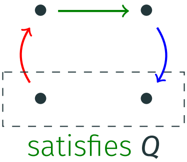

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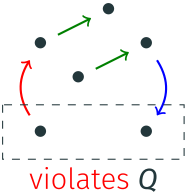

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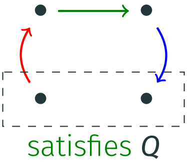

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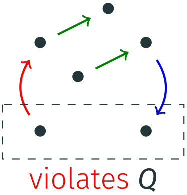

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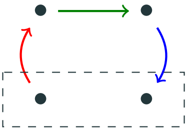

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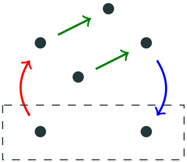

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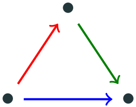

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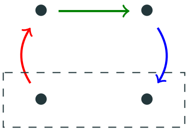

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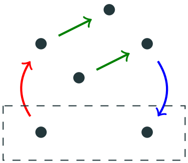

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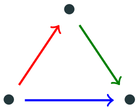

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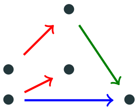

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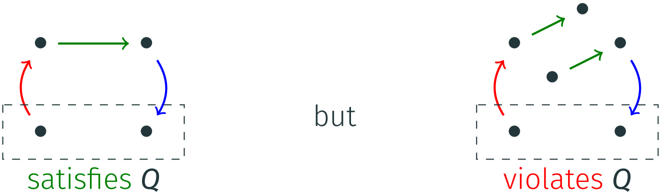

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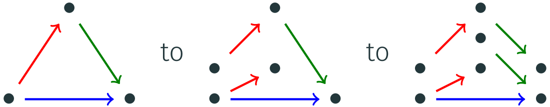

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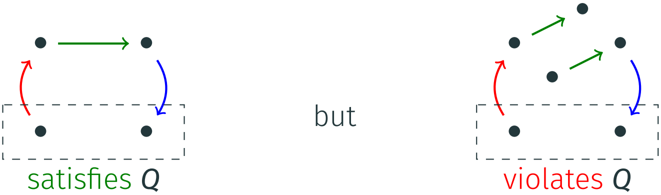

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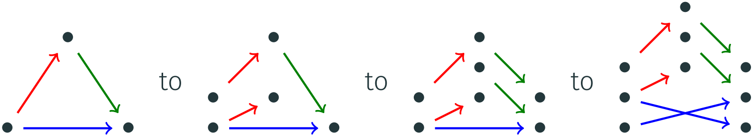

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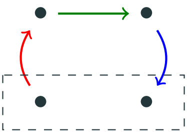

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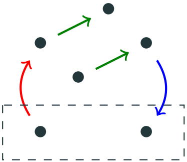

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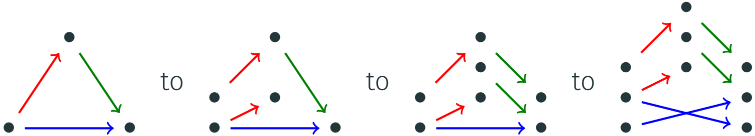
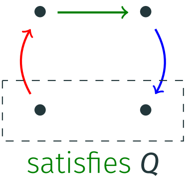


  - If Q is still true then the model is "explained" by a **union of stars**

Using hard patterns for #P-hardness



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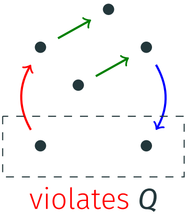
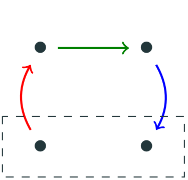


We can reduce from #PP2DNF like before:



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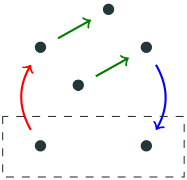

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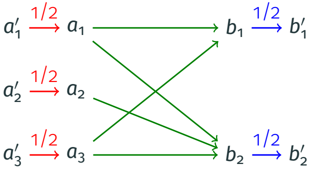

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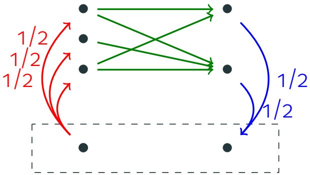

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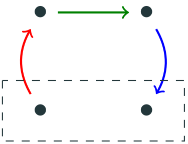

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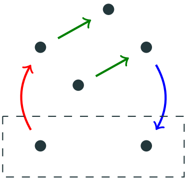

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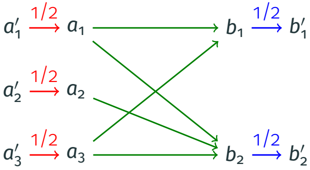

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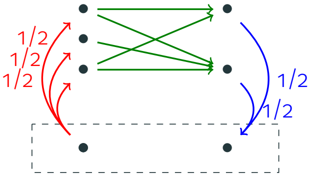

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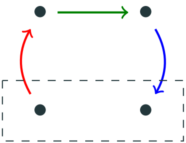

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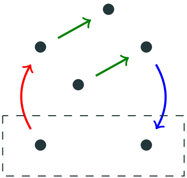

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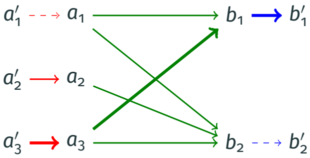

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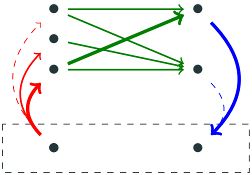

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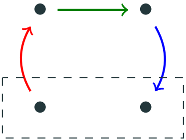

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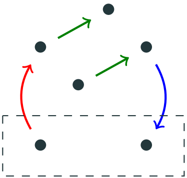

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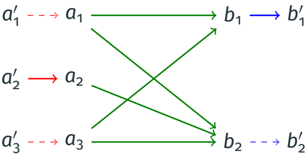

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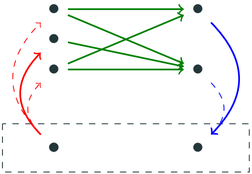

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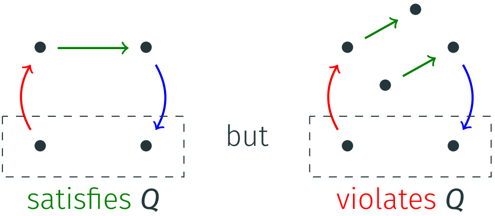

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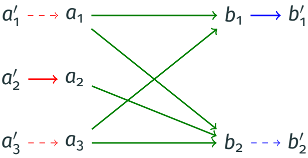

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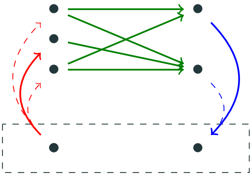

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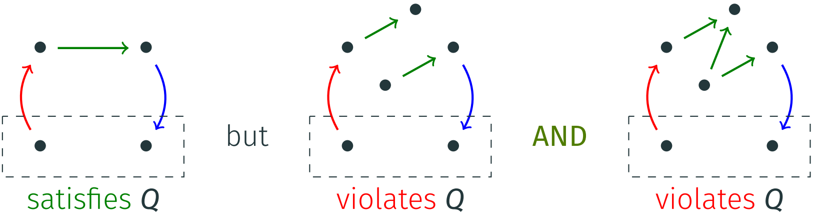

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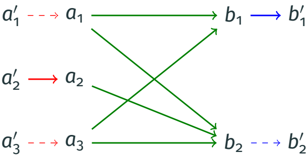

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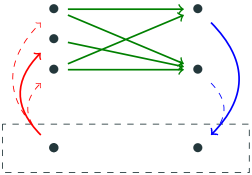

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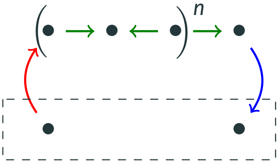

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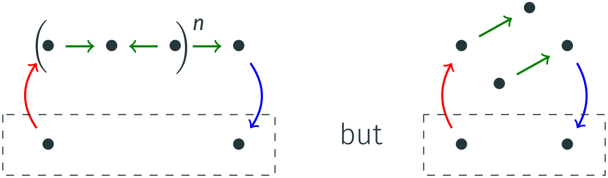

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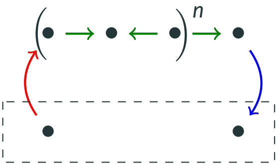

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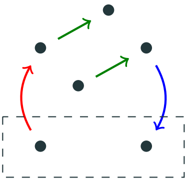

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Idea: use iterable patterns to reduce from the **#P-hard** problem **source-to-target connectivity**:

- Given a TID with a **source** and **sink**, what is the **probability** that the sink is **reachable** from the sink?

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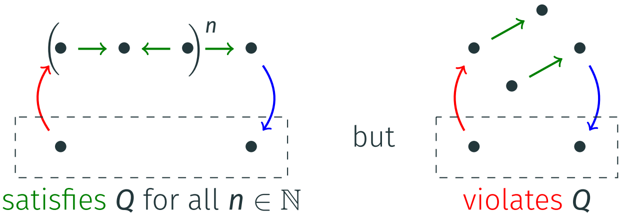

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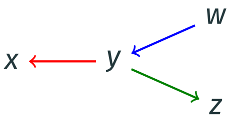
Technically challenging to get a correct reduction!

Hardness for unweighted PQE

Problem statement

- We restrict back to CQs
- We impose self-join-freeness: every edge color is different



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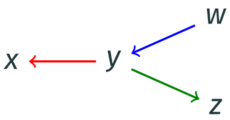

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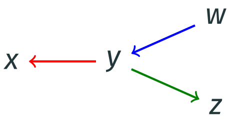

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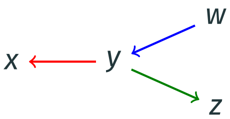

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$\rightarrow$ Equivalently: given a graph G, how many subgraphs satisfy Q

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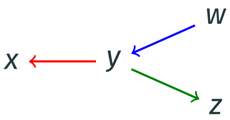

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Theorem

For any self-join-free CQ Q, if Q is unsafe then $MC(Q)$ is #P-hard.

First step: Restricting to a simpler query

For any unsafe query, we can reduce from simpler queries, essentially:

$$x \longrightarrow y \longrightarrow z \longrightarrow w$$

$\rightarrow$ We must show that $\mathrm{MC}(Q)$ is #P-hard for this query

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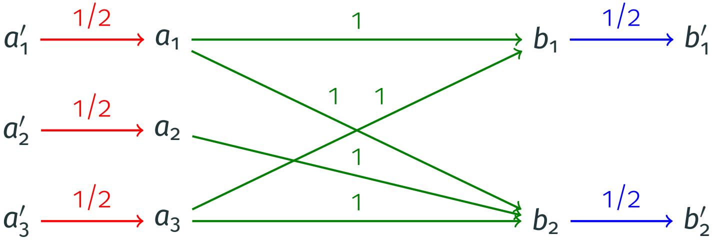
Can we use our earlier reduction for #P-hardness of PQE?



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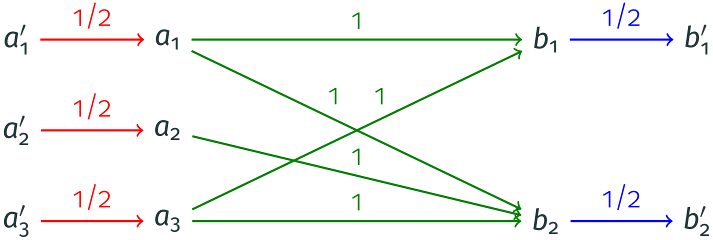
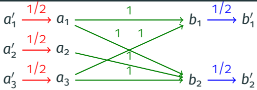
Can we use our **earlier reduction** for #P-hardness of PQE?



$\rightarrow$ Problem: this reduction crucially uses **probability 1**

Getting to an equation system

We want to reduce from $\mathrm{PQE}(Q)$, on some graph G with probabilities



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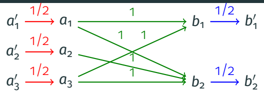

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- Split the subsets on some parameter e.g., the number of nodes
  $\rightarrow\ X = X_1, \ldots, X_k$

Getting to an equation system

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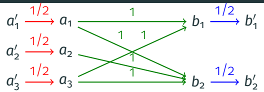

Task: count the number X of red-blue edge subsets that violate Q

- Split the subsets on some parameter e.g., the number of nodes
  $\rightarrow X = X_1, \ldots, X_k$
- Create unweighted copies of G modified with some gadgets e.g., replace each edge by multiple copies of a path
  $\rightarrow$ Created $G_1, \ldots, G_k$
  $\rightarrow$ Call the oracle for $\mathrm{MC(Q)}$ on each to get $N_1, \ldots, N_k$

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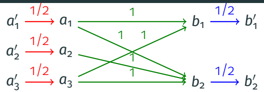

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  $\rightarrow$ Call the oracle for $\mathrm{MC}(\mathrm{Q})$ on each to get $N_1, \ldots, N_k$
- Show that each N_i is a linear function of $X_1, \ldots, X_k$, so:

$$\begin{pmatrix} N_1 \\ \vdots \\ N_k \end{pmatrix} = \begin{pmatrix} \alpha_{1,1} & \cdots & \alpha_{1,k} \\ \vdots & \ddots & \vdots \\ \alpha_{k,1} & \cdots & \alpha_{k,k} \end{pmatrix} \cdot \begin{pmatrix} X_1 \\ \vdots \\ X_k \end{pmatrix}$$

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We can choose gadgets and parameters to get a Vandermonde matrix, and show invertibility via several arithmetical tricks

Conclusion and open problems

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Thanks for your attention!

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