A Dichotomy for Homomorphism-Closed Queries on Probabilistic Graphs

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October 29, 2020

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Uncertain data management

Relational databases manage data, represented here as a labeled graph

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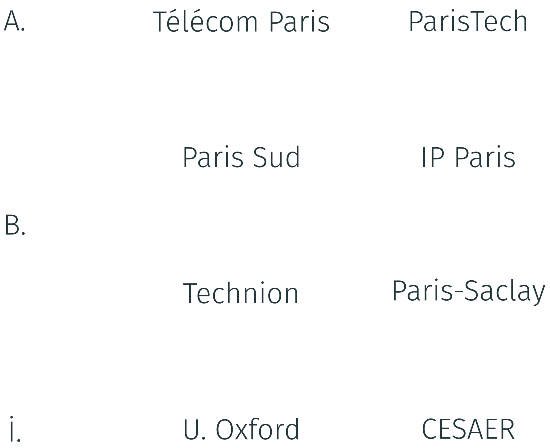

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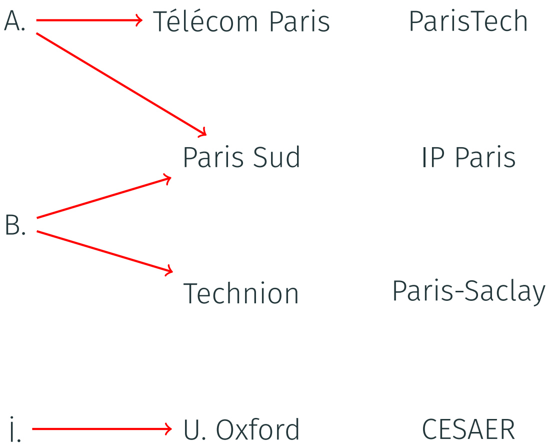

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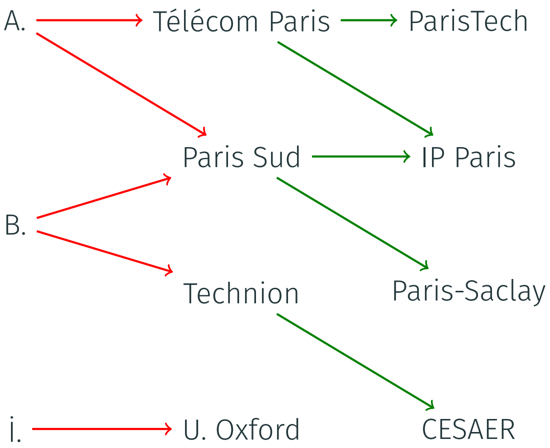

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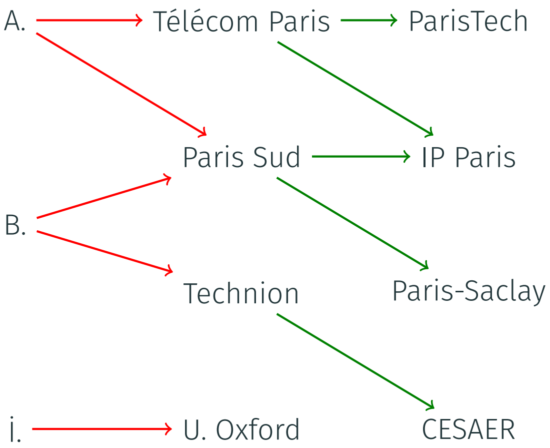
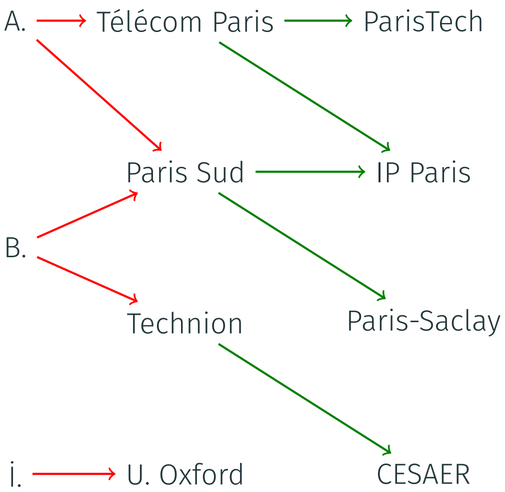


→ **Problem:** we are not **certain** about the true state of the data

Uncertain data model



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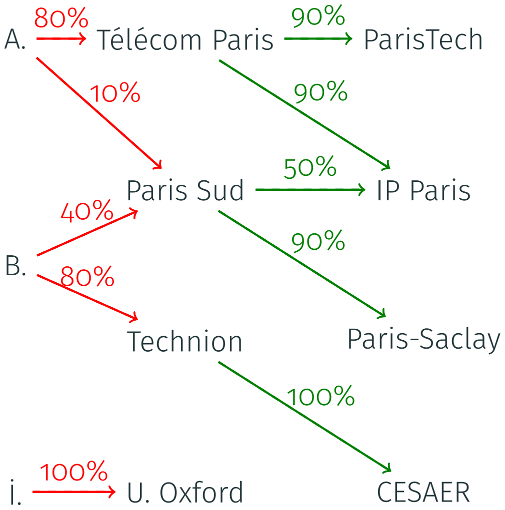

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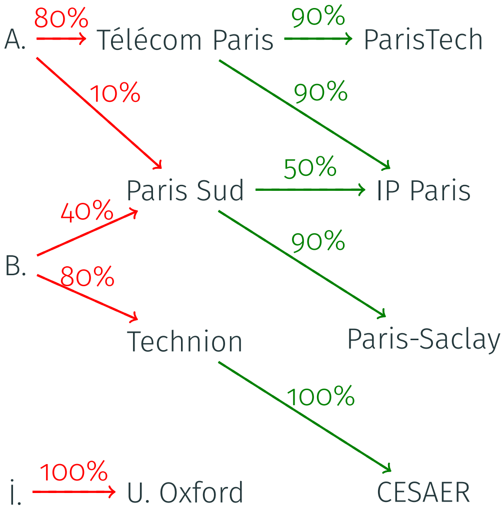

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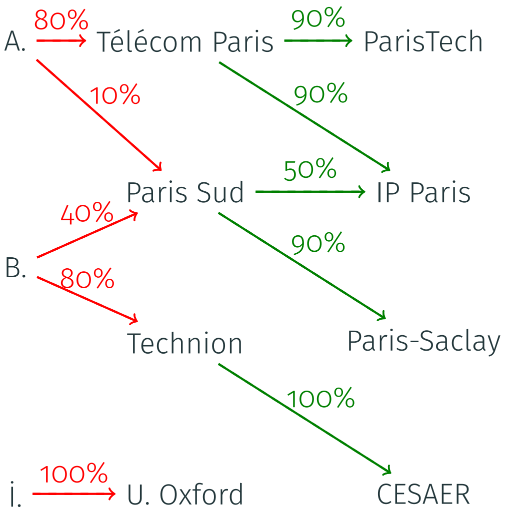

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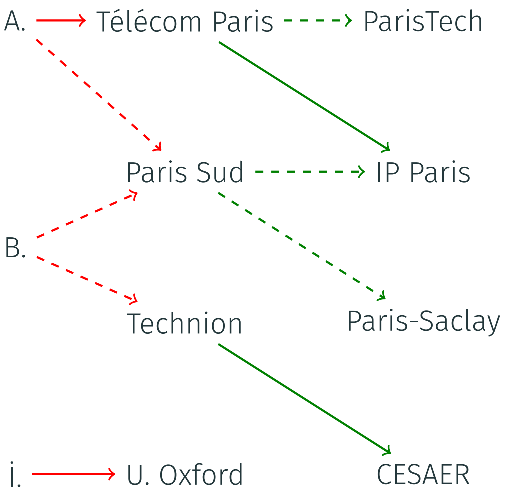

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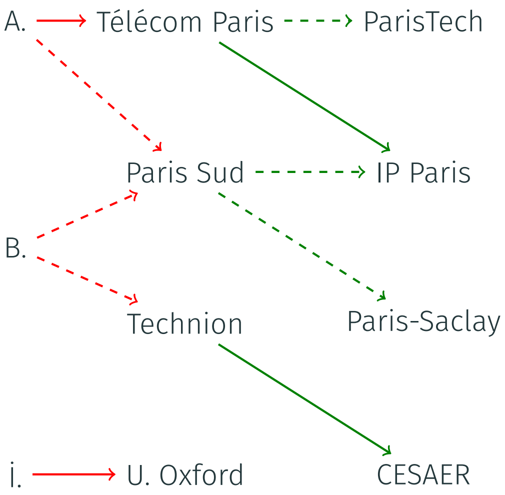

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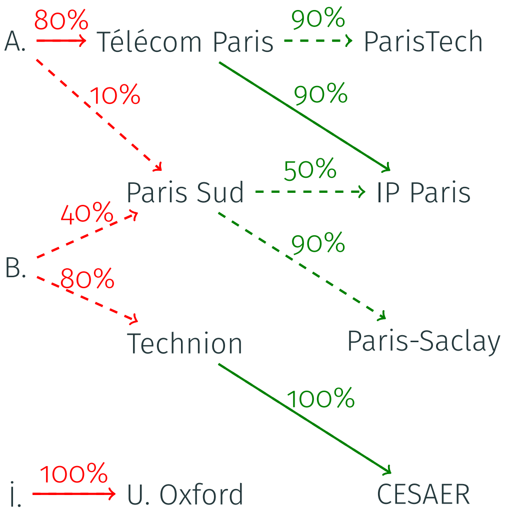

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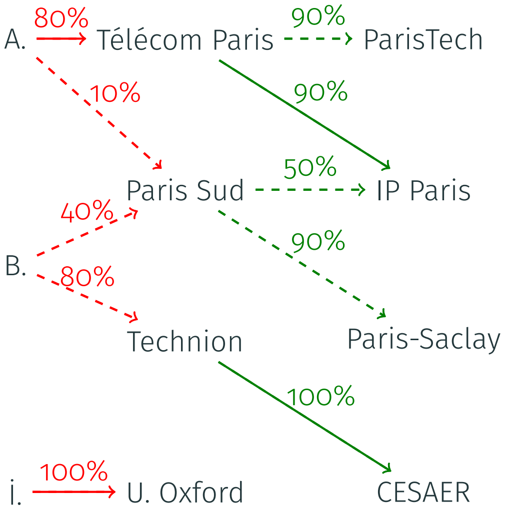

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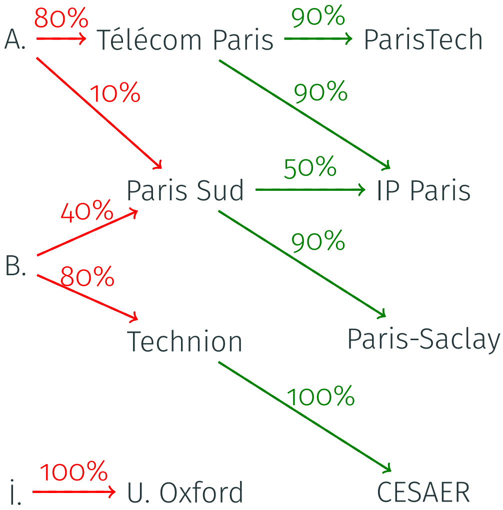

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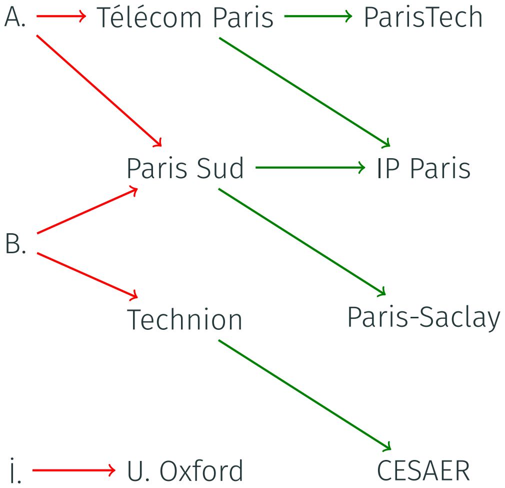
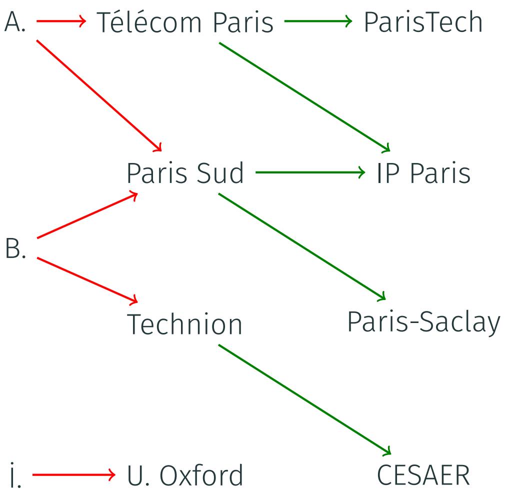
$$\Pr(W) = \left(\prod_{F \in W} \Pr(F)\right) \times \left(\prod_{F \notin W} (1 - \Pr(F))\right)$$

Queries



Central database task: **evaluate queries**

Queries



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"Is there some person ***x*** *employed in an institution who is part of a consortium* ***z****?"*

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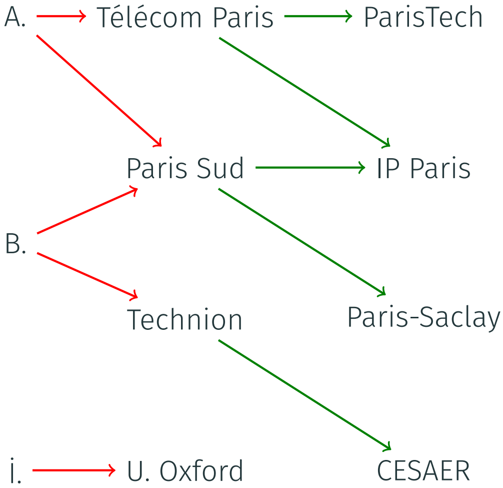

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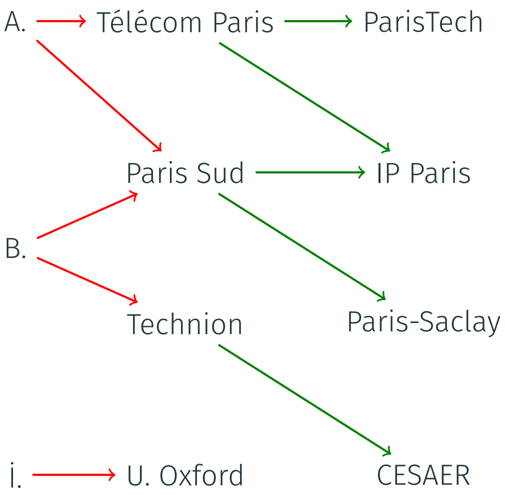

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Result on this graph:

x	z
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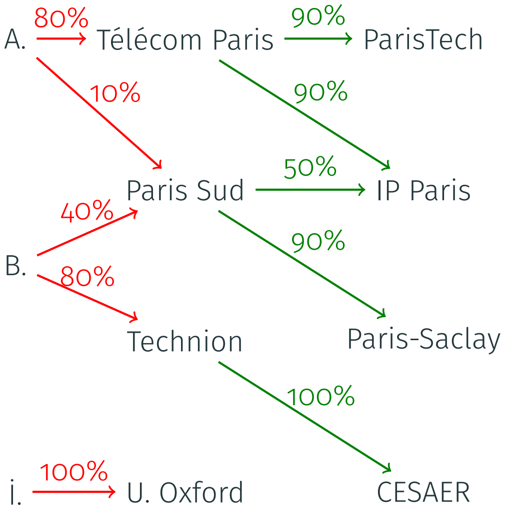

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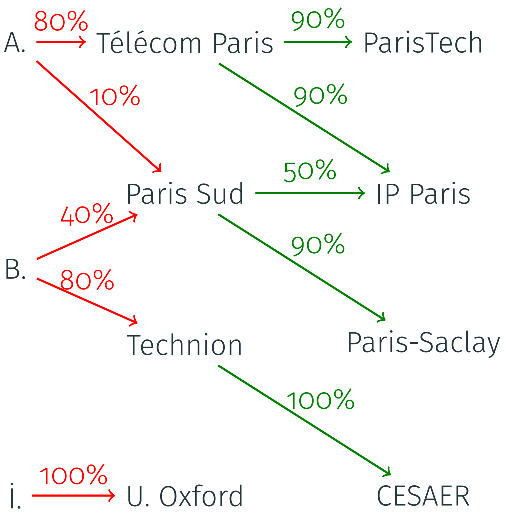

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Result on this graph:

x	z	
A.	ParisTech	72%
A.	IP Paris	99.1%
A.	Paris-Saclay	9%
B.	IP Paris	20%
B.	Paris-Saclay	36%
B.	CESAER	80%

Restricting to YES/NO queries

To make the problem simpler to study, we will restrict to YES/NO queries:

- Query: maps a graph to YES/NO

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→ From now on, all queries are **YES/NO queries**,
so we have just one YES/NO answer to compute, or **just one** probability

Query languages

Which kinds of queries do we want to express?

- Conjunctive query (CQ): can I find a match of a pattern?
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- **Regular path queries** (RPQ): can I find a match of a **regular path**?
  - e.g., $\exists x\, y \quad x \ \longrightarrow (\longrightarrow)^{*} \longrightarrow y$

Problem statement: Probabilistic query evaluation (PQE)

- We fix a query Q, for instance the CQ: $\exists x\, y\, z \quad x \longrightarrow y \longrightarrow z$

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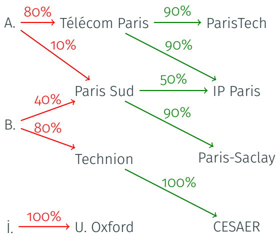

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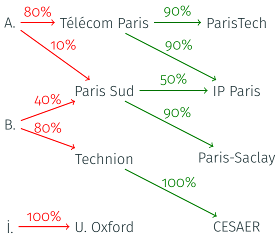

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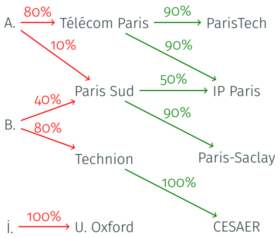

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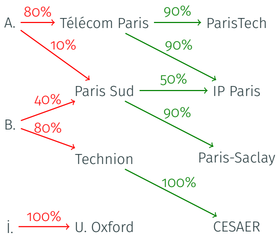

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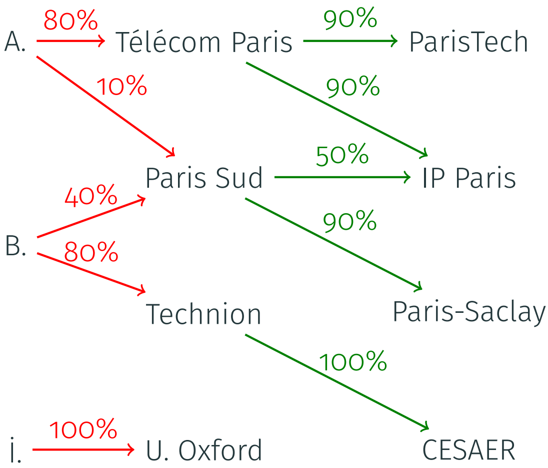
- We can always compute the probability in exponential time (go over all possibilities)

PQE: a simple example



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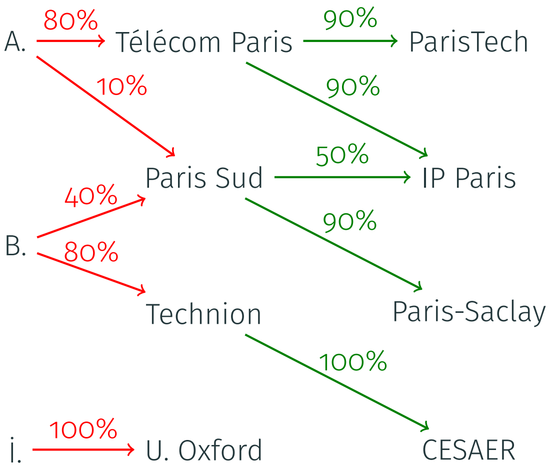

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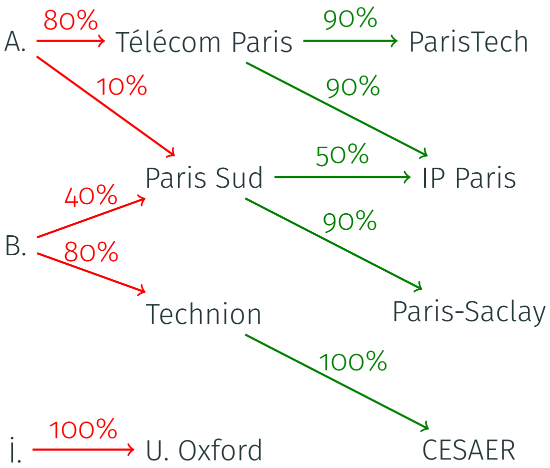

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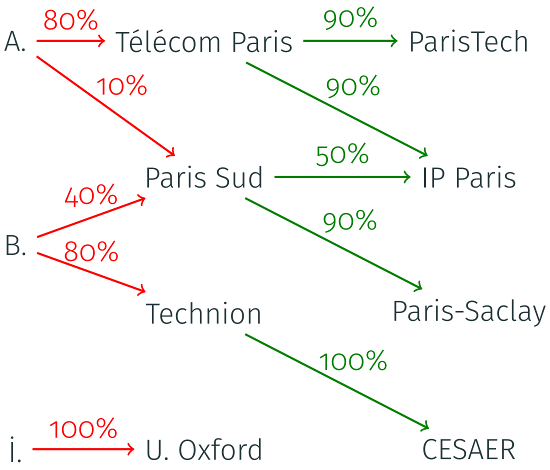

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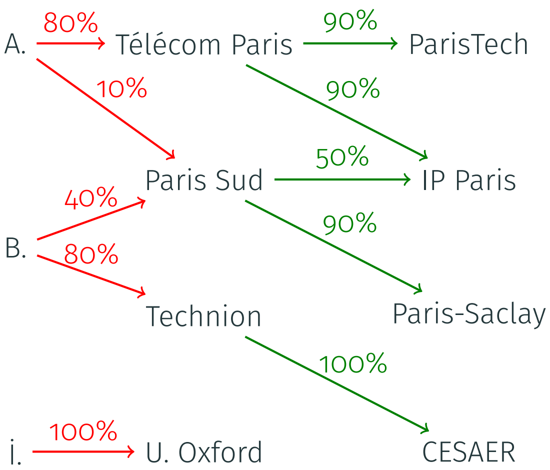

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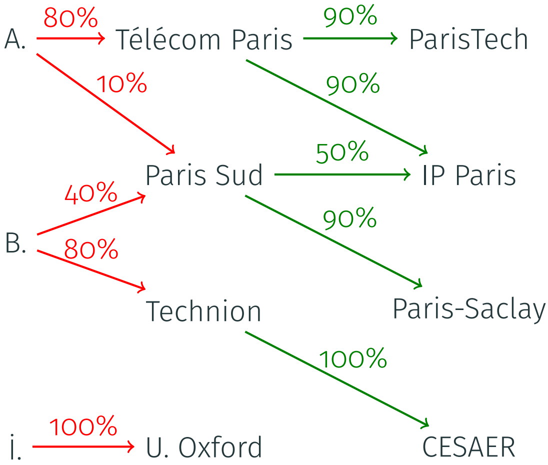

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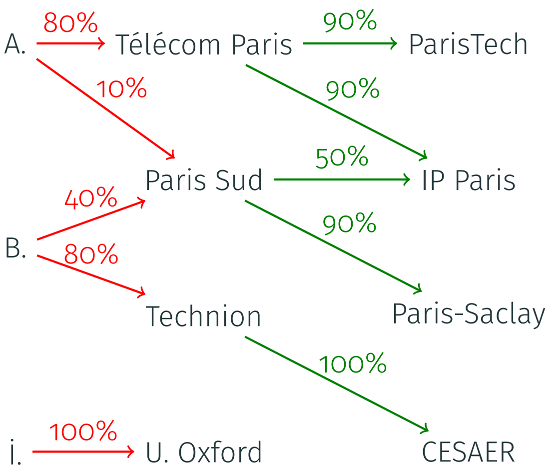

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- These choices are independent, so x is: $(1 - 80\%) \times (1 - 10\%) \times (1 - 40\%) \times (1 - 80\%) \times (1 - 100\%)$

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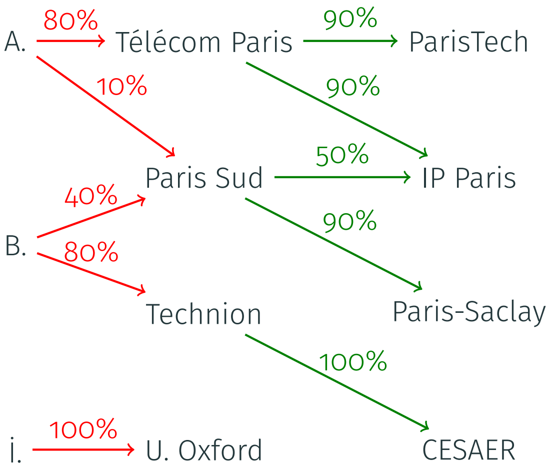

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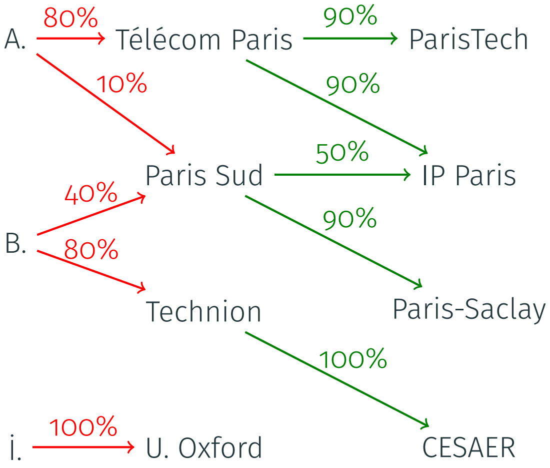
- These choices are independent, so x is: $(1 - 80\%) \times (1 - 10\%) \times (1 - 40\%) \times (1 - 80\%) \times (1 - 100\%)$
- This gives **$x = 0\%$**, so the query has probability **100%**

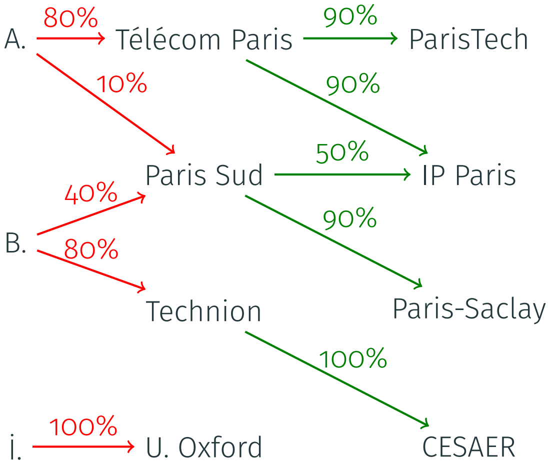
PQE: a simple example



Find the probability of: $\exists x\, y \quad x \longrightarrow y$

- It's easier to compute the probability x that there is **no match of the query**
  - $\rightarrow$ The probability we want is $\mathbf{1 - x}$
- There is no match of the query iff **every red edge is not kept**
- These choices are independent, so x is: $(1 - 80\%) \times (1 - 10\%) \times (1 - 40\%) \times (1 - 80\%) \times (1 - 100\%)$
- This gives $\mathbf{x = 0\%}$, so the query has probability **100%**
- This process is in **polynomial time**

PQE: a more complicated example



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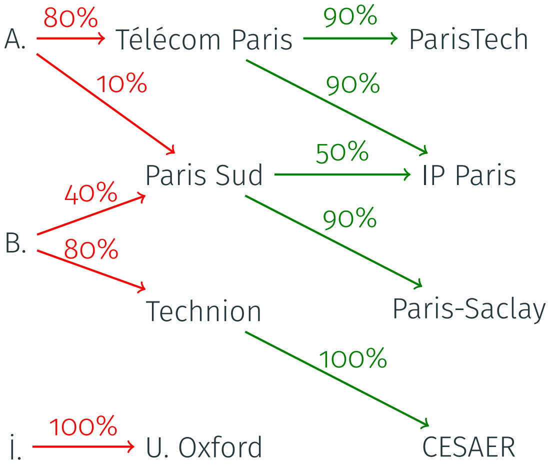

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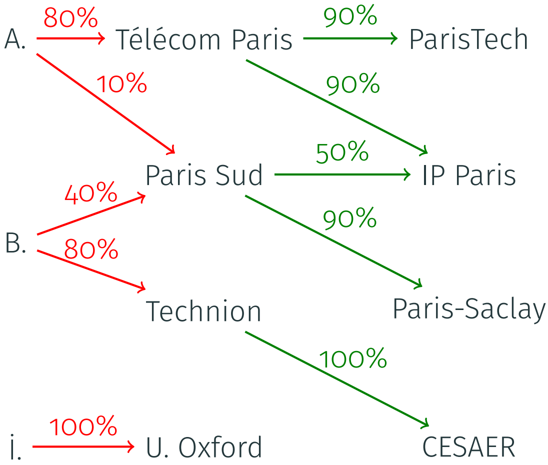
- Key insight: consider all possible choices for the middle variable y

PQE: a more complicated example



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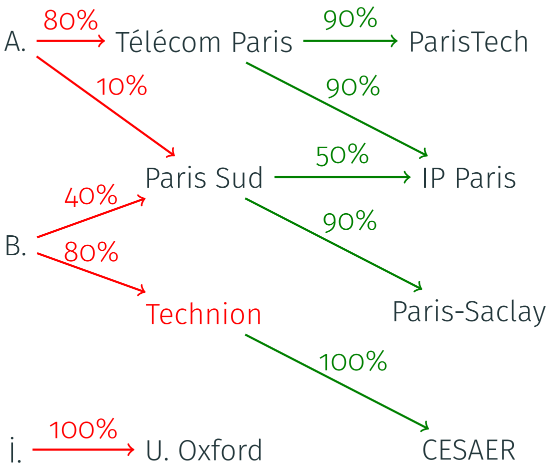

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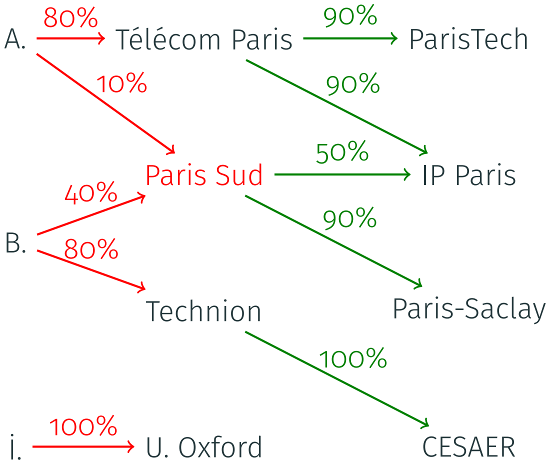

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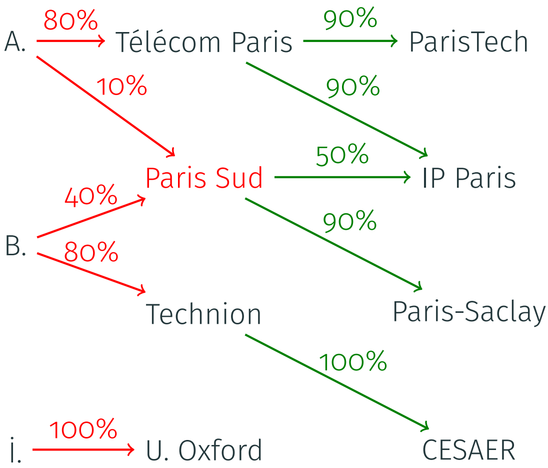

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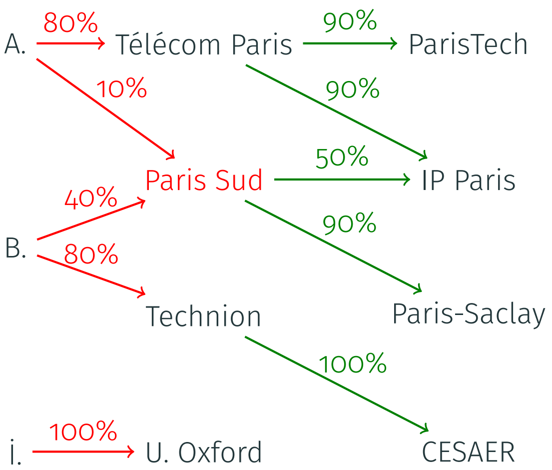

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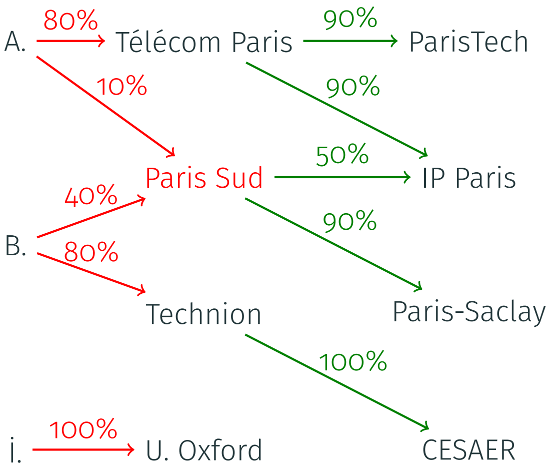

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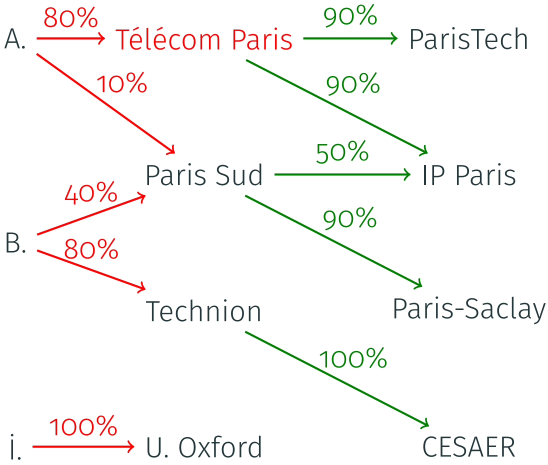

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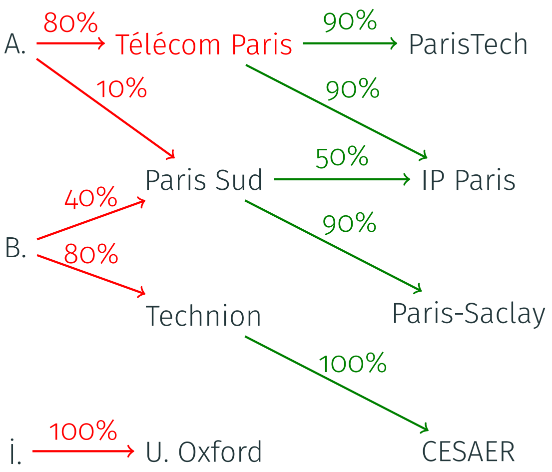

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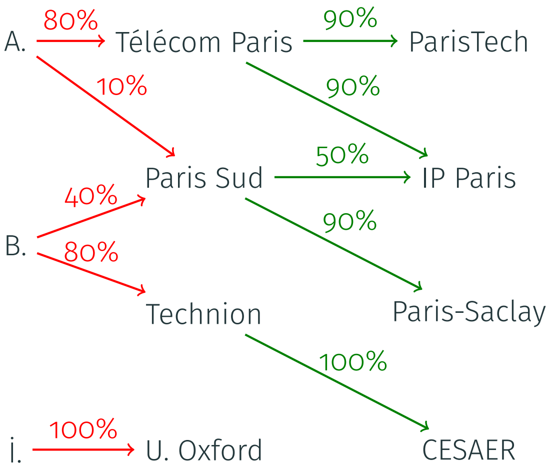

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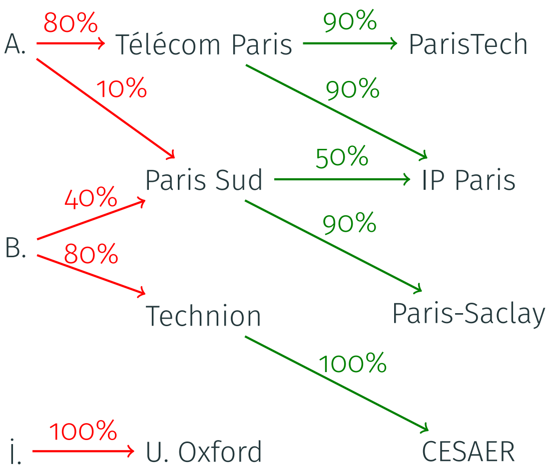

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- This is scary but **polynomial time**

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What is the complexity of $\mathrm{PQE}(Q)$ depending on the query Q?

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- **Existing results** on UCQ:
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- I'll also mention some of my work on **restricted graph classes**

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More restricted instances: Words, trees and bounded treewidth (1 slide)

More restricted instances: Unweighted instances (1 slide)

Conclusion and open problems

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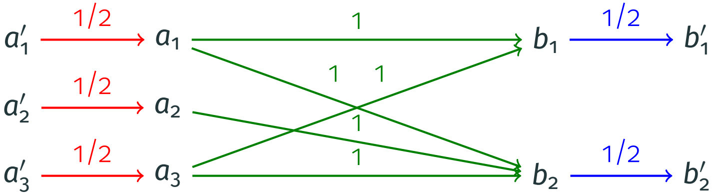

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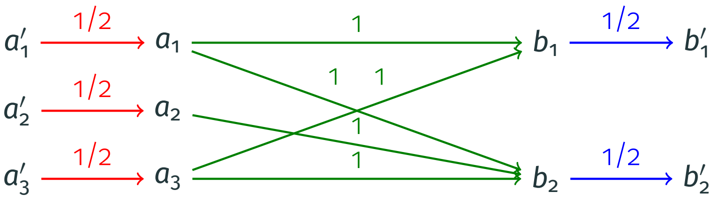

Idea: Satisfying valuations of ϕ correspond to **possible worlds** with a **match** of Q

The “small” Dalvi and Suciu dichotomy

- Self-join-free CQ: only one edge of each color (no repeated color)

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Let Q be a self-join-free CQ:

- *If Q is a **star**, then $\mathrm{PQE}(Q)$ is in **PTIME***
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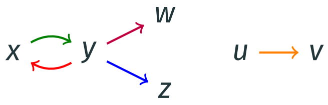
- A **star** is a CQ where each connected component has a **separator variable** that occurs in every edge of the component



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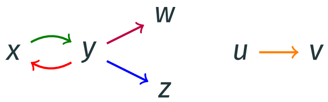
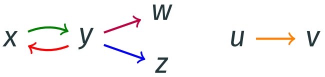


- The dichotomy generalizes to higher-arity data (hierarchical queries)

Proving the small dichotomy (upper bound)



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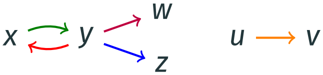

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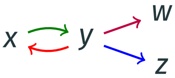

- We consider each connected component separately

$\rightarrow$ **Independent conjunction** over the connected components

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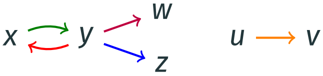

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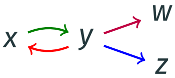

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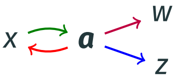

- We can test all possible values of the **separator variable**

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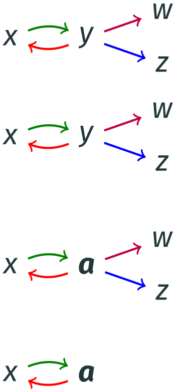

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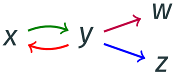
- We can test all possible values of the **separator variable**

$\rightarrow$ **Independent disjunction** over the values of the separator

- For every match, we consider every **other variable** separately

$\rightarrow$ **Independent conjunction** over the variables

Proving the small dichotomy (upper bound)



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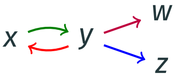

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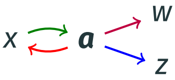
$\rightarrow$ **Independent conjunction** over the connected components



- We can test all possible values of the **separator variable**

$\rightarrow$ **Independent disjunction** over the values of the separator

- For every match, we consider every **other variable** separately

$\rightarrow$ **Independent conjunction** over the variables

- We consider every value for the **other variable**

$\rightarrow$ **Independent disjunction** over the possible assignments

$\rightarrow$ **Independent conjunction** over the facts

Proving the small dichotomy (lower bound)

Every non-star self-join-free CQ contains a pattern essentially like:

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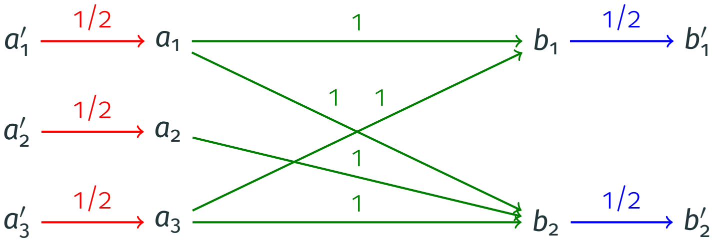
We can use this to reduce from #SAT like before:



The "big" Dalvi and Suciu dichotomy

Full dichotomy on the **unions of conjunctive queries** (UCQs):

Theorem (Dalvi and Suciu 2012)

Let Q be a UCQ:

- *If Q is handled by a complicated algorithm $\mathrm{PQE}(Q)$ is in PTIME*
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This result is **far more complicated** (but still generalizes to higher arity)

- **Upper bound:**
  - an algorithm generalizing the previous case with **inclusion-exclusion**
  - many **unpleasant details** (e.g., a ranking transformation)
- **Lower bound:** hardness proof on minimal cases where the algorithm does not work

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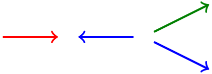
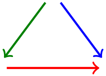
We study the case of **queries closed under homomorphisms**

Homomorphism-closed queries

- A **homomorphism** from a graph G to a graph G' maps the vertices of G to those of G' while preserving the edges

 has a homomorphism to 

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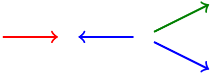
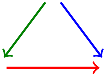
 has a homomorphism to 

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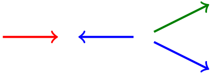
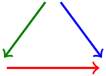
 has a homomorphism to 

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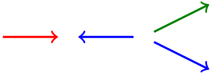

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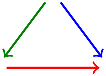

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- Queries with **negations** or **inequalities** are not homomorphism-closed
- Homomorphism-closed queries can equivalently be seen as **infinite unions of CQs** (corresponding to their models)

Our result

We show:

Theorem (Amarilli and Ceylan 2020)

For any query Q closed under homomorphisms:

- *Either Q is equivalent to a tractable UCQ and $\mathrm{PQE}(Q)$ is in PTIME*
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  - Hence, $\mathrm{PQE}(Q)$ is #P-hard

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*Conversely, there is a query Q for which $\mathrm{PQE}(Q)$ is intractable on **any** input instance family of unbounded treewidth (under some technical assumptions)*

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We study to **self-join-free CQs** and extend the "small" Dalvi and Suciu dichotomy to SC:

Theorem (Amarilli and Kimelfeld 2020)

Let Q be a self-join-free CQ:

- *If Q is a **star**, then $\mathrm{PQE}(Q)$ is in **PTIME***
- *Otherwise, even $\mathrm{SC}(Q)$ is **#P-hard***

→ This also extends **beyond arity two** (hierarchical queries)

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Thanks for your attention!

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