On the optimization of recursive relational queries.

DIG Seminar

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The relational algebra

Generalities

The relational algebra [Cod70]

- a set of base relations

  the tables in SQL

- combined through operators

  union, projection, filter, join, etc.

- operates on named tuples

Syntax

φ	$::=$		term
	$\mid$	X	relation variable
	$\mid$	$\lvert c \to v \rvert$	constant
	$\mid$	$\emptyset$	empty set
	$\mid$	$\varphi_1 \cup \varphi_2$	union
	$\mid$	$\varphi_1 \bowtie \varphi_2$	join
	$\mid$	$\varphi_1 \triangleright \varphi_2$	antijoin
	$\mid$	$\sigma_{filter}(\varphi)$	filter
	$\mid$	$\rho_a^b(\varphi)$	rename
	$\mid$	$\pi_P(\varphi)$	projection

Figure 1: Syntax of the relational algebra

Examples

T

from	to
Lille	Paris
Lille	Saclay
Paris	Grenoble
Paris	Saclay
Saclay	Grenoble

$\pi_{to}(T)$

to
Paris
Saclay
Grenoble

Examples

T

from	to
Lille	Paris
Lille	Saclay
Paris	Grenoble
Paris	Saclay
Saclay	Grenoble

$\rho_{to}^{step}(T)$

from	step
Lille	Paris
Lille	Saclay
Paris	Grenoble
Paris	Saclay
Saclay	Grenoble

Examples

T

from	to
Lille	Paris
Lille	Saclay
Paris	Grenoble
Paris	Saclay
Saclay	Grenoble

$\rho_{to}^{step}(T) \bowtie \rho_{from}^{step}(T)$

from	step	to
Lille	Paris	Saclay
Lille	Paris	Grenoble
Lille	Saclay	Grenoble
Paris	Saclay	Grenoble

Examples

T

from	to
Lille	Paris
Lille	Saclay
Paris	Grenoble
Paris	Saclay
Saclay	Grenoble

$\sigma_{\text{from}=\text{Lille}}(T)$

from	to
Lille	Paris
Lille	Saclay

Examples

T

from	to
Lille	Paris
Lille	Saclay
Paris	Grenoble
Paris	Saclay
Saclay	Grenoble

$\sigma_{\text{from=Paris}}(T) \cup \sigma_{\text{from=Lille}}(T)$

from	to
Lille	Paris
Lille	Saclay
Paris	Saclay
Paris	Grenoble

Examples

T

from	to
Lille	Paris
Lille	Saclay
Paris	Grenoble
Paris	Saclay
Saclay	Grenoble

$\pi_{to}(T) \triangleright \sigma_{\text{from=Lille}}(T)$

to
Grenoble

Recursive relational algebra

Syntax

φ	$::=$		term
	$\mid$	X	relation variable
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	$\mid$	$\varphi_1 \triangleright \varphi_2$	antijoin
	$\mid$	$\sigma_{filter}(\varphi)$	filtering
	$\mid$	$\rho_a^b(\varphi)$	rename
	$\mid$	$\pi_{c_1,\ldots,c_n}(\varphi)$	projection

Figure 2: Syntax of our relational algebra

Syntax

φ	$::=$		term
	$\mid$	X	relation variable
	$\mid$	$\lvert c \rightarrow v \rvert$	constant
	$\mid$	$\emptyset$	empty set
	$\mid$	$\varphi_1 \cup \varphi_2$	union
	$\mid$	$\varphi_1 \bowtie \varphi_2$	join
	$\mid$	$\varphi_1 \triangleright \varphi_2$	antijoin
	$\mid$	$\sigma_{filter}(\varphi)$	filtering
	$\mid$	$\rho_a^b(\varphi)$	rename
	$\mid$	$\tilde{\pi}_c(\varphi)$	anti-projection

Figure 2: Syntax of our relational algebra

Syntax

$$
\begin{array}{rcll}
\varphi & ::= & & \text{term} \\
& | & X & \text{relation variable} \\
& | & |c \to v| & \text{constant} \\
& | & \emptyset & \text{empty set} \\
& | & \varphi_1 \cup \varphi_2 & \text{union} \\
& | & \varphi_1 \bowtie \varphi_2 & \text{join} \\
& | & \varphi_1 \triangleright \varphi_2 & \text{antijoin} \\
& | & \sigma_{filter}(\varphi) & \text{filtering} \\
& | & \rho_a^b(\varphi) & \text{rename} \\
& | & \tilde{\pi}_c(\varphi) & \text{anti-projection} \\
& | & \beta_a^b(\varphi) & \text{duplication}
\end{array}
$$

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	$\mid$	X	relation variable
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	$\mid$	$\tilde{\pi}_c(\varphi)$	anti-projection
	$\mid$	$\beta_a^b(\varphi)$	duplication
	$\mid$	$\mu(X = \varphi)$	fixpoint

Figure 2: Syntax of our relational algebra

Differences

Anti-projection
Remove a column

$$\tilde{\pi}_{d_1}\left(\dots\tilde{\pi}_{d_k}\left(\varphi\right)\dots\right) = \pi_{c_1\dots c_n}\left(\varphi\right)$$

Differences

Anti-projection
Remove a column

$$\tilde{\pi}_{d_1}\left(\ldots\tilde{\pi}_{d_k}\left(\varphi\right)\ldots\right) = \pi_{c_1\ldots c_n}\left(\varphi\right)$$

Duplication
Copy a column

$$\beta_a^b\left(\varphi\right) = \sigma_{a=b}\left(\varphi \bowtie \rho_a^b\left(\varphi\right)\right)$$

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Remove a column

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Duplication
Copy a column

$$\beta_a^b\left(\varphi\right) = \sigma_{a=b}\left(\varphi \bowtie \rho_a^b\left(\varphi\right)\right)$$

Fixpoints
Compute the least fixpoint of a function $S \to \varphi[X/S]$

$$[\![\mu(X = \varphi)]\!]_V = lim_{n \to \infty} U_n \qquad U_0 = \emptyset$$
$$U_{n+1} = U_n \cup [\![\varphi]\!]_{V[X/U_n]}$$

Examples

T

from	to
Lille	Paris
Lille	Saclay
Paris	Grenoble
Paris	Saclay
Saclay	Grenoble

$\tilde{\pi}_{from}(T)$

to
Paris
Saclay
Grenoble

Examples

T

from	to
Lille	Paris
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$\beta^{to}_{from}(\tilde{\pi}_{to}(T))$

from	to
Lille	Lille
Saclay	Saclay
Paris	Paris

Examples

$$T^{+} = \mu(X = T \cup T/X)$$

T

from	to
Lille	Paris
Paris	Saclay
Saclay	Lyon
Lyon	Grenoble

from	to

Examples

$$T^+ = \mu(X = T \cup \tilde{\pi}_s\,(\rho^s_{to}\,(X) \bowtie \rho^s_{from}\,(T)))$$

T

from	to
Lille	Paris
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from	to
Lille	Paris
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Examples

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from	to
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Paris	Saclay
Saclay	Lyon
Lyon	Grenoble

from	to
Lille	Paris
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Lyon	Grenoble
Lille	Saclay
Paris	Lyon
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from	to
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Saclay	Lyon
Lyon	Grenoble
Lille	Saclay
Paris	Lyon
Saclay	Grenoble
Lille	Lyon
Paris	Grenoble

Examples

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from	to
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Lyon	Grenoble
Lille	Saclay
Paris	Lyon
Saclay	Grenoble
Lille	Lyon
Paris	Grenoble
Lille	Grenoble

Limitations

Limitations on fixpoints

- recursive variables must appear positively

$$\textit{No } \mu(X = R \triangleright X)$$

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$$\textit{No } \mu(X = \mu(Y = X \cup Y))$$

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- no mutually recursive fixpoints

$$\textit{No } \mu(X = \mu(Y = X \cup Y))$$

→ *corresponds to linear datalog!*

→ *superset of `WITH RECURSIVE` in SQL!*

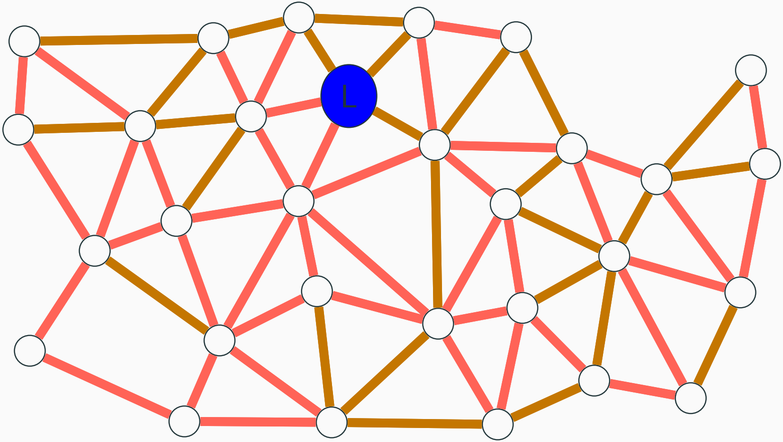
Performance of recursive queries

An example

:Lille `:TGV`/`:Bus*` `?o`

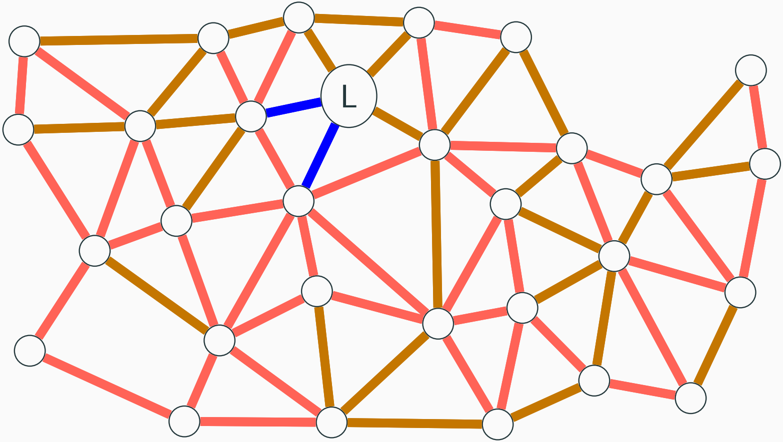
An example

:Lille `:TGV`/`:Bus*` `?o`



An example

:Lille :TGV/:Bus* ?o



An example

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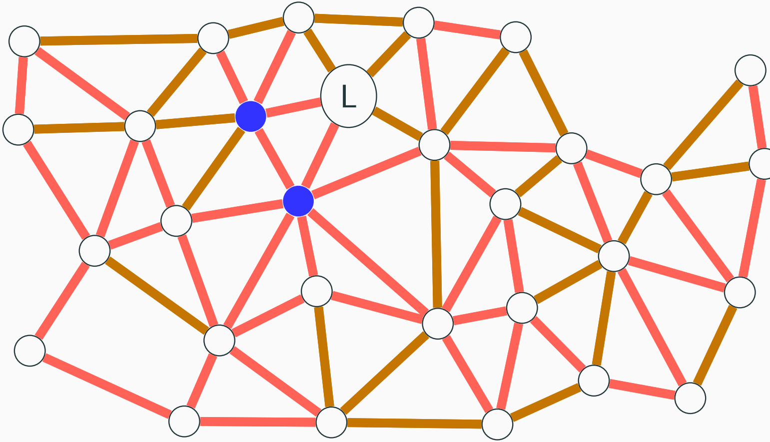

An example

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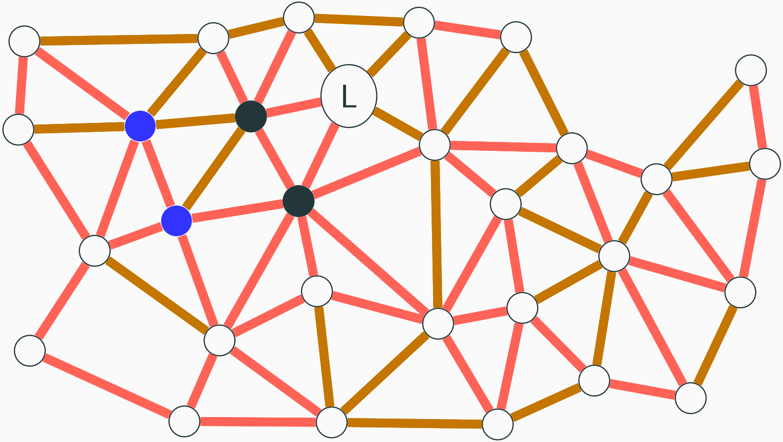

An example

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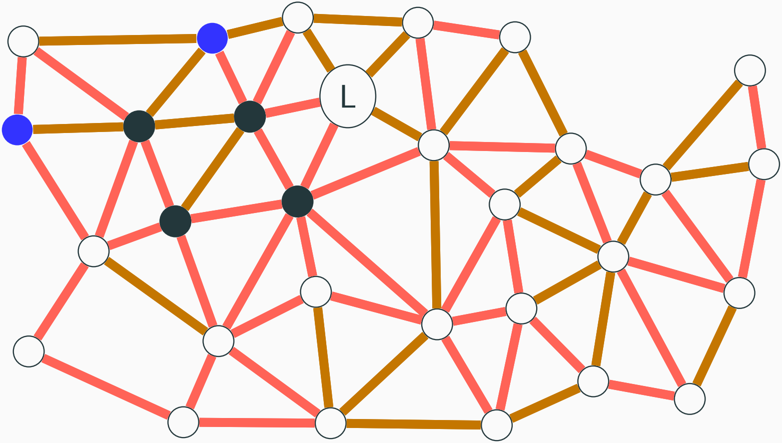

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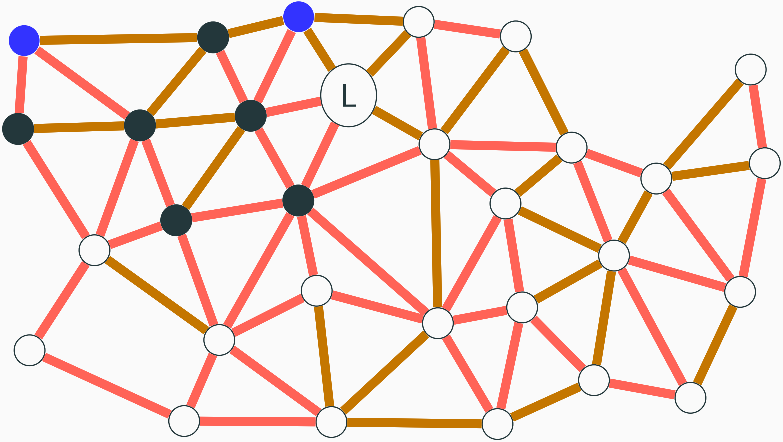

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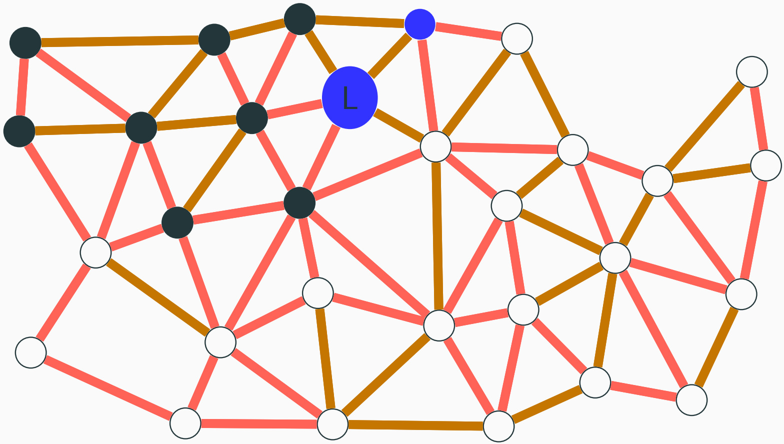

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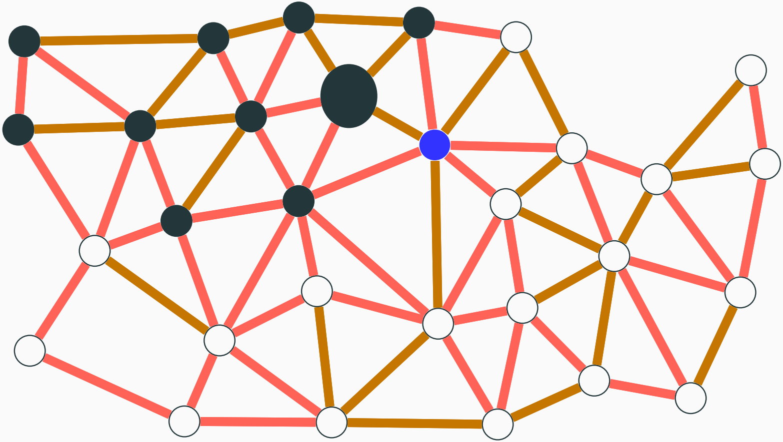

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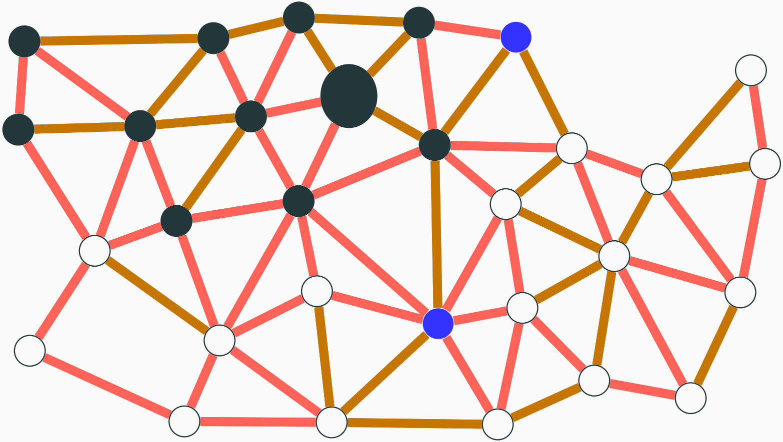

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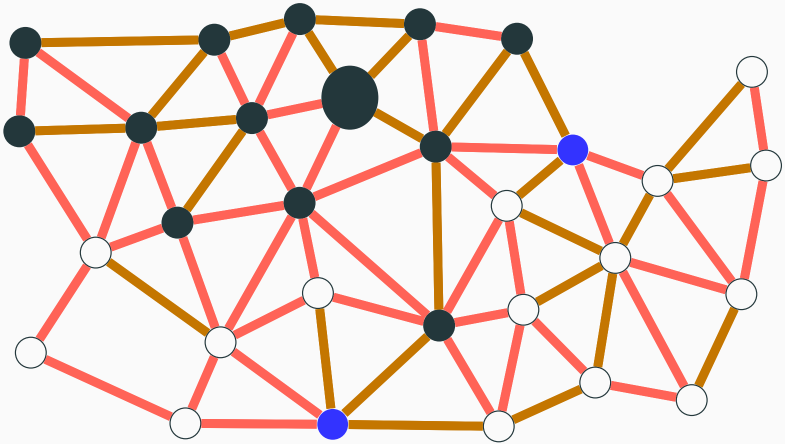

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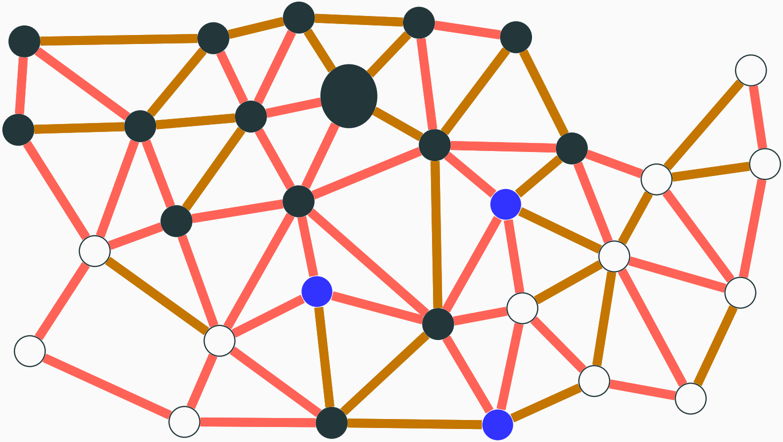

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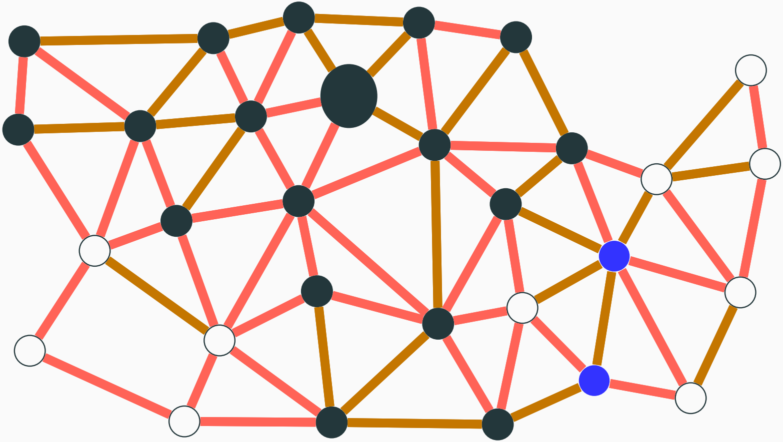

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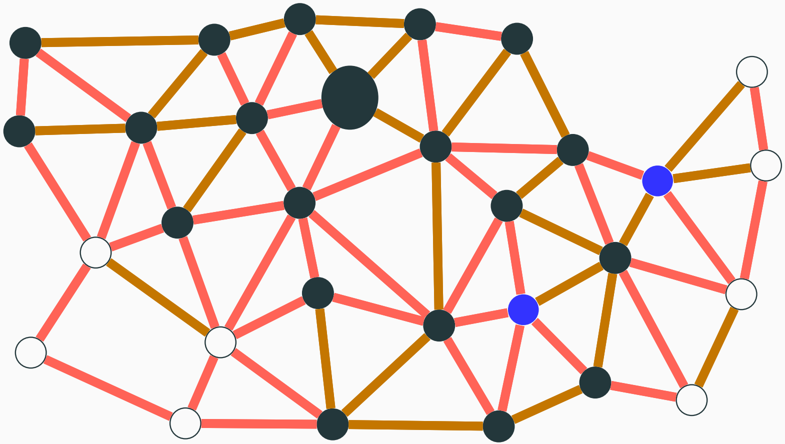
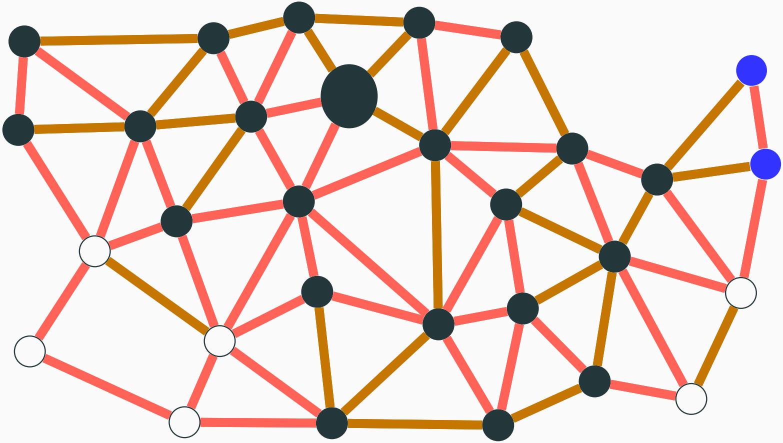


An example

:Lille `:TGV/:Bus* ?o`



An example

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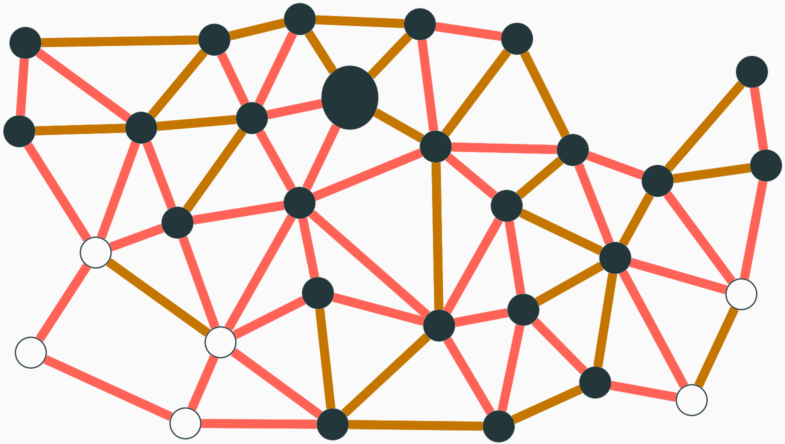


An example

`:L :TGV/:Bus* ?o`

An example

```
:L :TGV/:Bus* ?o
```

$$\tilde{\pi}_{?s}\left(\sigma_{?s=:L}\left(:\mathsf{TGV}/\mu(X = \beta_s^o\left(AllNodes\right) \cup X/:\mathsf{Bus})\right)\right)$$

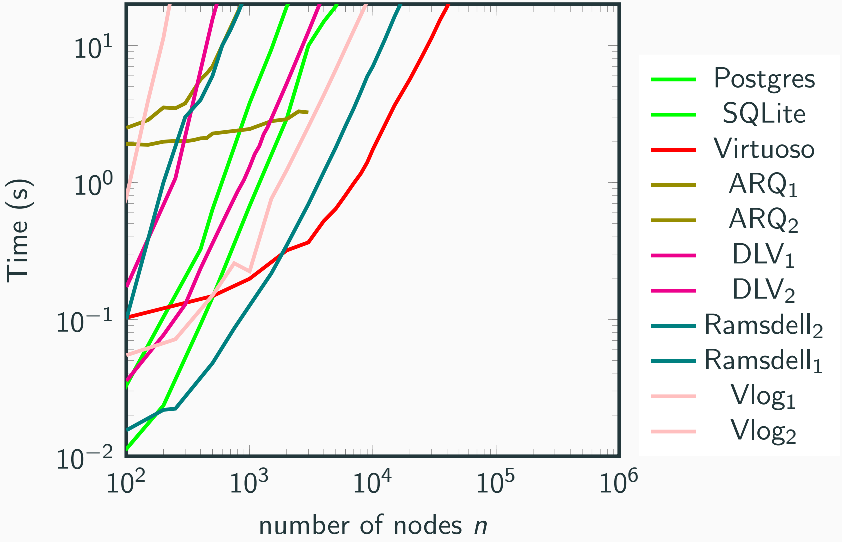
An example

:L `:TGV`/`:Bus`* ?o

$$\tilde{\pi}_{?s}\left(\sigma_{?s=:L}\left(:\mathrm{TGV}/\mu(X = \beta^{o}_{s}\left(AllNodes\right) \cup X/:\mathrm{Bus})\right)\right)$$

$$\tilde{\pi}_{?s}\left(\sigma_{?s=:L}\left(:\mathrm{TGV}/\mu(X = \beta^{o}_{s}\left(AllNodes\right) \cup :\mathrm{Bus}/X)\right)\right)$$

Benchmarking



Benchmarking

Logicblox

- Materialization of all intermediate predicate
- ... except for “on demand” predicate
- therefore manual optimization

Rewrite rules

Rewrite rules for fixpoints

- pushing filters?

$$\sigma_{filter}\,(\mu(X = \varphi)) \stackrel{?}{=} \mu(X = \sigma_{filter}\,(\varphi))$$

Rewrite rules

Rewrite rules for fixpoints

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$$\sigma_{\mathit{filter}}\,(\mu(X = \varphi)) \stackrel{?}{=} \mu(X = \sigma_{\mathit{filter}}\,(\varphi))$$

- pushing joins?

$$\psi \bowtie \mu(X = \varphi) \stackrel{?}{=} \mu(X = \psi \bowtie \varphi)$$

Rewrite rules

Rewrite rules for fixpoints

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$$\psi \bowtie \mu(X = \varphi) \stackrel{?}{=} \mu(X = \psi \bowtie \varphi)$$

- pushing antijoins?

$$\mu(X = \varphi) \triangleright \psi \stackrel{?}{=} \mu(X = \varphi \triangleright \psi)$$

Rewrite rules

Rewrite rules for fixpoints

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$$\mu(X = \varphi) \triangleright \psi \stackrel{?}{=} \mu(X = \varphi \triangleright \psi)$$

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$$\tilde{\pi}_p(\mu(X = \varphi)) \stackrel{?}{=} \mu(X = \tilde{\pi}_p(\varphi))$$

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Rewrite rules for fixpoints

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- pushing anti-projections?

$$\tilde{\pi}_p\,(\mu(X = \varphi)) \stackrel{?}{=} \mu(X = \tilde{\pi}_p\,(\varphi))$$

- combine fixpoints?

$$\mu(X = \psi \cup \kappa) \bowtie \mu(X = \varphi \cup \xi) \stackrel{?}{=} \mu(X = \psi \bowtie \varphi \cup \xi \cup \kappa)$$

Rewrite rules for fixpoints

- Reverse fixpoints?

An example

:L :TGV/:Bus* ?o

An example

:L `:TGV`/`:Bus`* ?o

$$\tilde{\pi}_{?s}\,(\sigma_{?s=:L}\,(:\mathsf{TGV}/\mu(X = \beta_{s}^{o}\,(AllNodes) \cup X/:\mathsf{Bus})))$$

An example

:L `:TGV/:Bus* ?o`

$$\tilde{\pi}_{?s}\left(\sigma_{?s=:L}\left(:TGV/\mu(X = \beta_s^o\left(AllNodes\right) \cup X/:Bus)\right)\right)$$

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An example

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$$\tilde{\pi}_{?s}\left(\sigma_{?s=:L}\left(:TGV/\mu(X = \beta_s^o\left(AllNodes\right) \cup X/:Bus)\right)\right)$$

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$$\tilde{\pi}_{?s}\left(\sigma_{?s=:L}\left(\mu(X = :TGV \cup X/:Bus)\right)\right)$$

$$\tilde{\pi}_{?s}\left(\mu(X = \sigma_{?s=:L}\left(:TGV\right) \cup X/:Bus)\right)$$

An example

:L :TGV/:Bus* ?o

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$$\mu(X = \tilde{\pi}_{?s}\left(\sigma_{?s=:L}\left(:\mathrm{TGV}\right)\right) \cup X/:\mathrm{Bus})$$

Other methods of optimization

Datalog?
No combination of fixpoints

Automata based techniques?
Step by Step and no conjunction

$(a/b/c)^+ \quad vs \quad ((a/b)/c)^+ \quad vs \quad ((a/b/c))^+$

Special joins (RDF-3X ferari)?
Compute efficiently $A \bowtie (B)*$ but same problem...

Waveguide
Efficient on a single RPQ but cannot optimize across RPQ.

Theoretical framework

Decomposed fixpoints
Given a fixpoint $\mu(X = \varphi)$ it can be rewritten to $\mu(X = \varphi_{con} \cup \varphi_{rec})$ with:

- φ_{con} constant, *i.e.* $[\![\varphi_{con}]\!]_{V[X/\emptyset]} = [\![\varphi_{con}]\!]_{V[X/S]}$
- φ_{rec} recursive, *i.e.* $[\![\varphi_{con}]\!]_{V[X/\emptyset]} = \emptyset$

Linearity of fixpoints
Given a fixpoint $\mu(X = \varphi)$:

$$[\![\varphi]\!]_{V[X/S]} = [\![\varphi]\!]_{V[X/\emptyset]} \bigcup_{w \in S} [\![\varphi]\!]_{V[X/\{w\}]}$$

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Given a fixpoint $\mu(X = \varphi)$:

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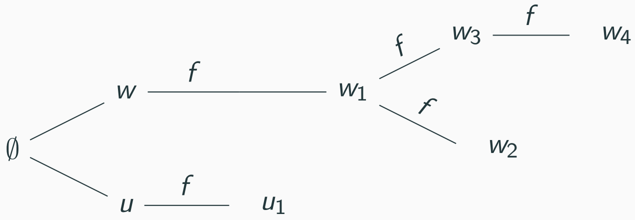
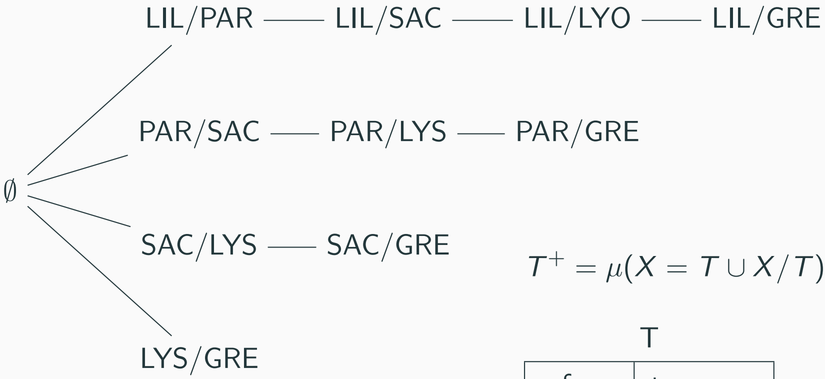
Lineage
For each $m \in U_{i+1} \setminus U_i$ we can find $w \in U_i$ such that $m \in f(w)$ with $f(w) = [\![\varphi]\!]_{V[X/\{w\}]} \setminus [\![\varphi]\!]_{V[X/\emptyset]}$.



Examples



$$T^+ = \mu(X = T \cup X/T)$$

T

from	to
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Paris	Saclay
Saclay	Lyon
Lyon	Grenoble

Invariant

How the elements of $f(w)$ depend on w?

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Stabilizers
For each w, $m \in f(w)$ and $c \in stab(\varphi)$: $m(c) = w(c)$.

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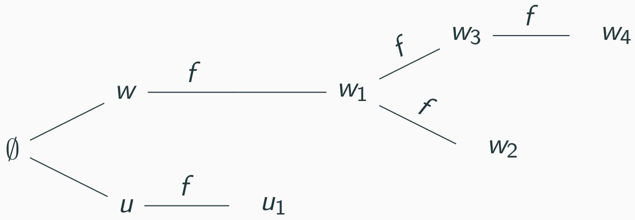
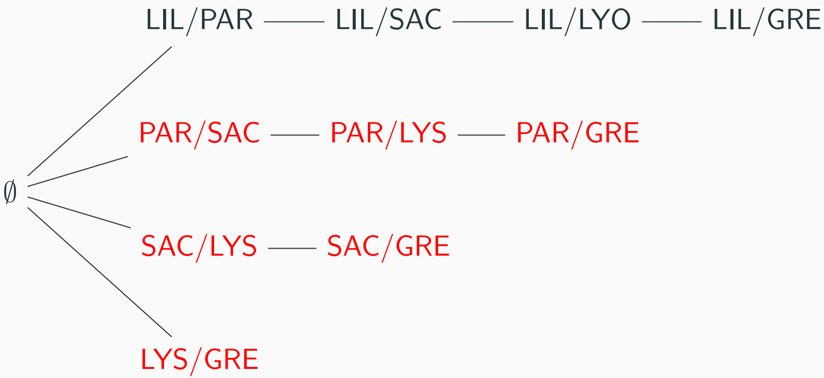
For each w, $m \in f(w)$ and $c \in stab(\varphi)$: $m(c) = w(c)$.



$$\sigma_{filter}(\mu(X = \varphi)) = \mu(X = \sigma_{filter}(\varphi))$$

when *filter* operates on $stab(\varphi)$

Examples



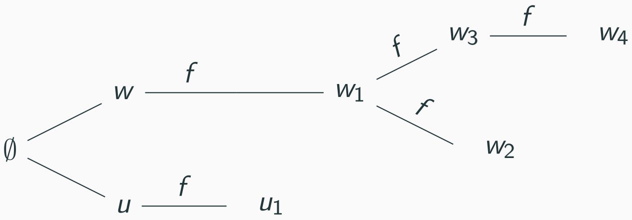
$$\sigma_{from=Lille}\left(T^{+}\right) = \mu(X = \sigma_{from=Lille}(T) \cup X/T)$$

$$\sigma_{to=Lille}\left(T^{+}\right) \neq \mu(X = \sigma_{from=Lille}(T) \cup X/T)$$

Invariant

How the elements of $f(w)$ depend on w?

Added columns
For each $c \in add(\varphi)$: $f(w) \bowtie |c \to v| = f(w \bowtie |c \to v|)$



$$\psi \bowtie \mu(X = \varphi) = \mu(X = \psi \bowtie \varphi)$$

when $sort(\psi) \subseteq stab(\varphi)$

and $sort(\psi) \subseteq add(\varphi) \cup sort(\mu(X = \varphi))$

Rewrite rules

Rewrite rules for fixpoints

- pushing filters

$$\sigma_{filter}\,(\mu(X=\varphi)) \stackrel{?}{=} \mu(X=\sigma_{filter}\,(\varphi))$$

- pushing joins

$$\psi \bowtie \mu(X=\varphi) \stackrel{?}{=} \mu(X=\psi \bowtie \varphi)$$

- pushing antijoins

$$\mu(X=\varphi) \triangleright \psi \stackrel{?}{=} \mu(X=\varphi \triangleright \psi)$$

- pushing anti-projections

$$\tilde{\pi}_p\,(\mu(X=\varphi)) \stackrel{?}{=} \mu(X=\tilde{\pi}_p\,(\varphi))$$

- combine fixpoints

$$\mu(X=\psi \cup \kappa) \bowtie \mu(X=\varphi \cup \xi) \stackrel{?}{=} \mu(X=\psi \bowtie \varphi \cup \xi \cup \kappa)$$

Streams

Streams are one-way communication channels

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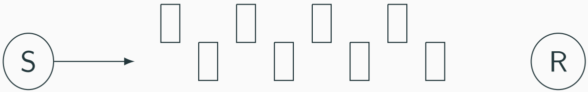
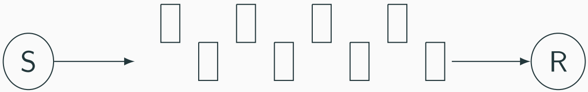
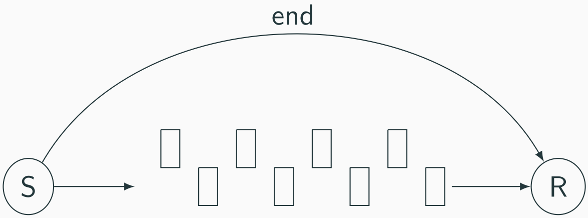


Streams are one-way communication channels



Streams

Streams are one-way communication channels



Streams: a a good abstraction for iterative distributed execution

- no order of messages

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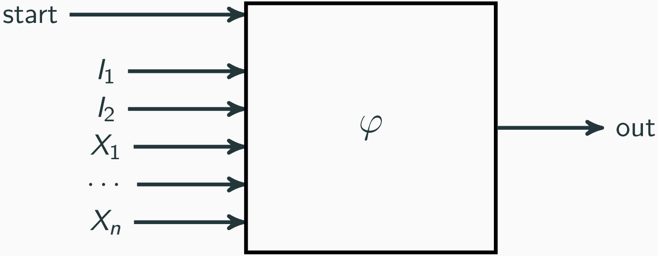
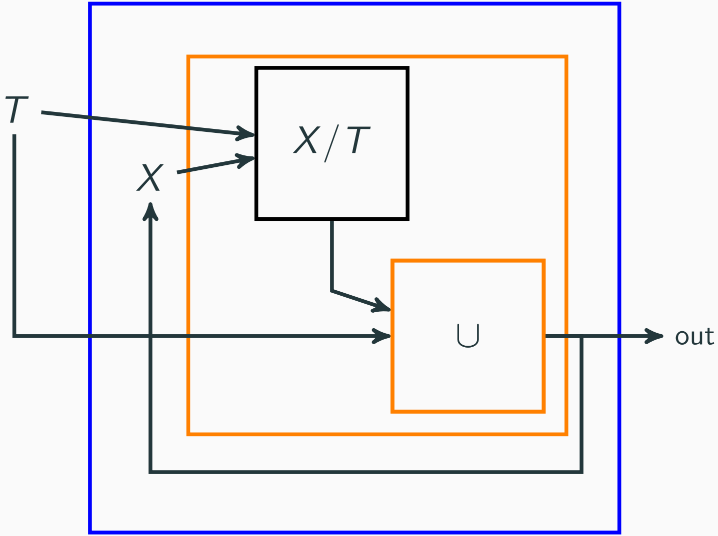
- no order of messages
- not necessarily a DAG
- fast single machine communication and slow inter-machine communication
- partial typing of message content (for fast serialization)

Execution of μ-algebra terms with streams



Streams for $\mu(X = X/T \cup T)$



Benchmarking

Q2

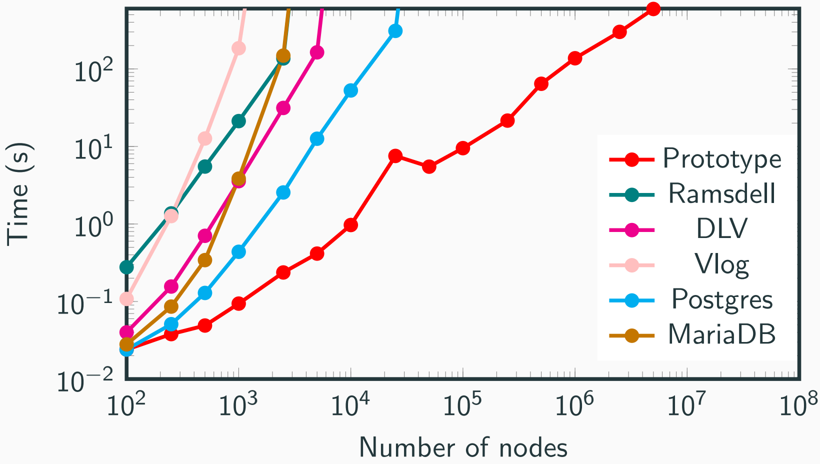


Figure 4: $?a\ \ (P1+)/(P5+)\ \ ?b$.

Q3

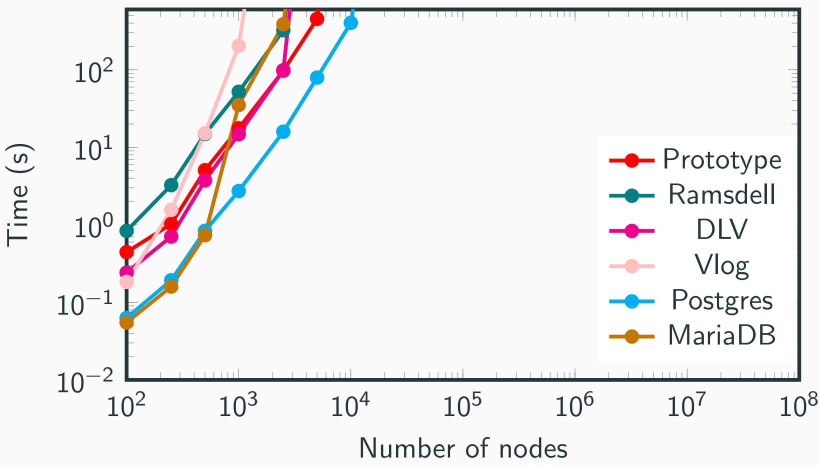


Figure 5: $?a\ \ (P1+)/P2\ \ ?b\ .\ ?b\ \ P3+\ \ ?c.$

Q7

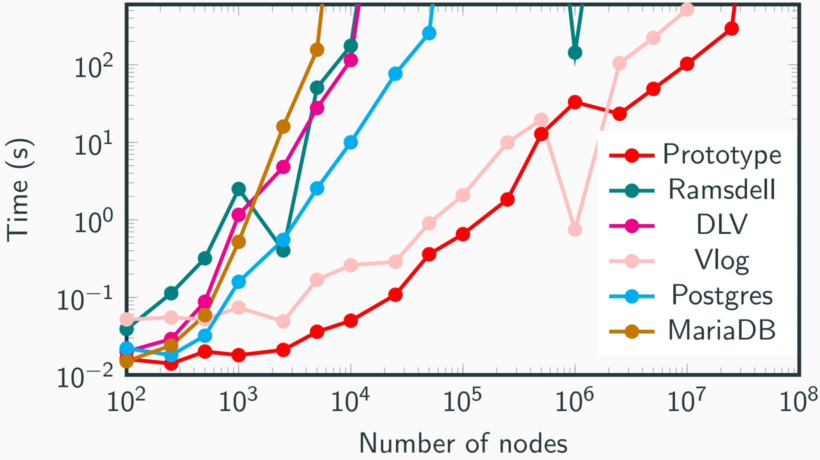


Figure 6: $N0\ P1/(P2+)\ ?a$

Q10

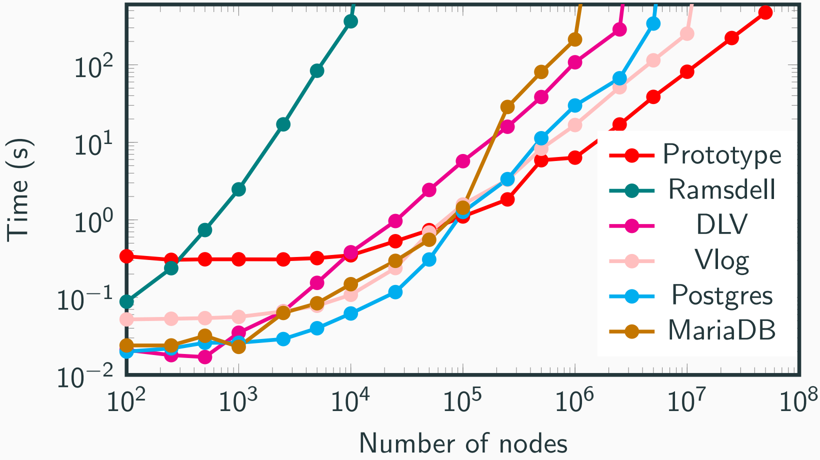
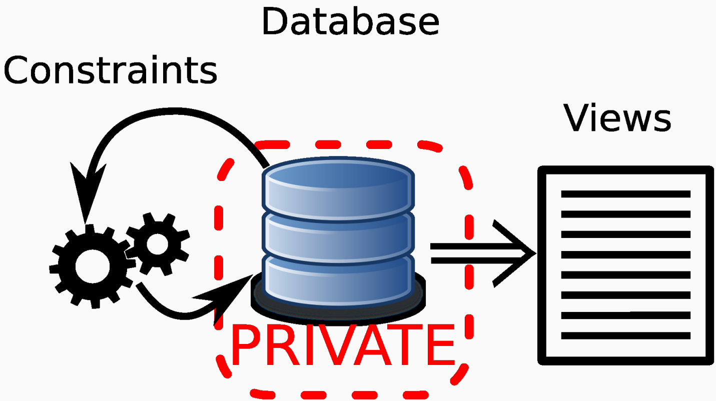


Figure 7: $?a\ \ (P4+)/(P5+)/(P3+)\ \ ?b$

Why recursive queries?

- Evaluate property paths
- Evaluate general recursive queries
- OBDA without rewriting nor materialization

What am I doing now?



Questions?

Edgar F Codd.
A relational model of data for large shared data banks.
Communications of the ACM, 13(6):377–387, 1970.

Semantics

Semantics

$$
\begin{aligned}
[\![\varphi_1 \bowtie \varphi_2]\!]_V &= \{m_1 + m_2 \mid m_1 \in [\![\varphi_1]\!]_V \wedge m_2 \in [\![\varphi_2]\!]_V \wedge m_1 \sim m_2\} \\
[\![\varphi_1 \cup \varphi_2]\!]_V &= [\![\varphi_1]\!]_V \cup [\![\varphi_2]\!]_V \\
[\![\varphi_1 \triangleright \varphi_2]\!]_V &= \{m \in [\![\varphi_1]\!]_V \mid \forall m' \in [\![\varphi_2]\!]_V \ \neg(m' \sim m)\} \\
[\![\tilde{\pi}_a(\varphi)]\!]_V &= \left\{\{c \to v \in m \mid c \neq a\} \;\middle|\; m \in [\![\varphi]\!]_V\right\} \\
[\![X]\!]_V &= V(X) \\
[\![\beta_a^b(\varphi)]\!]_V &= \Big\{\{c \to v \in m \mid c \neq b\} \cup \{b \to v \mid a \to v \in m\} \\
&\qquad \Big| \; m \in [\![\varphi]\!]_V\Big\} \\
[\![\sigma_{filter}(\varphi)]\!]_V &= \{m \mid m \in [\![\varphi]\!]_V \wedge filter(m) = \top\} \\
[\![\mu(X = \varphi)]\!]_V &= [\![X]\!]_{V[X/U_\infty]}, \ U_0 = \emptyset, \ U_{i+1} = U_i \cup [\![\varphi]\!]_{V[X/U_i]}
\end{aligned}
$$