



Uncertain Data Management Reminders on Relational Algebra and Calculus

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Basics

Relational algebra

SQL

Relations

- Relation name and arity
- Attribute names (optional)

Relation Class, arity 5					
date	teacher	resp	name	num	

Relations

- Relation name and arity
- Attribute names (optional)

date	teacher	resp	name	num
2017-11-21	Antoine	Fabian	Uncert. Data Mgmt	1
2017-11-28	Antoine	Fabian	Uncert. Data Mgmt	2
2017-12-05	Antoine	Fabian	Uncert. Data Mgmt	3
2017-12-12	Silviu	Fabian	Uncert. Data Mgmt	4

• Set of rows (tuples) on a domain (no duplicates)

- Signature σ : set of relation names and attributes, e.g.:
 - Class(date, teacher, resp, name, num)
 - Student(id, name)
 - Member(student, classname)
- Database instance: one relation for each name in σ

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 - Class(date, teacher, resp, name, num)
 - Student(id, name)
 - Member(student, classname)
- Database instance: one relation for each name in σ
- \rightarrow Query:
 - Input: database
 - Output: relation

Two languages to write queries:

- The relational algebra:
 - operational way to define queries
 - based on operators to construct new relations
- The relational calculus:
 - **declarative** way to define queries
 - based on first-order logic

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- The relational algebra:
 - operational way to define queries
 - based on operators to construct new relations
- The relational calculus:
 - **declarative** way to define queries
 - based on first-order logic
- \rightarrow Codd's theorem: both have the same expressive power
 - SQL, the practical language used by databases

Basics

Relational algebra

SQL

• Basic relations:

- \cdot the relation names in the signature
- constant relations, e.g., the empty relation
- Projection П
- Selection σ
- Renaming ρ
- Union \cup
- Product \times and join \bowtie
- Difference -

Class					
date	teacher	resp	name	num	
2017-11-21	Antoine	Fabian	Uncert. Data Mgmt	1	
2017-11-28	Antoine	Fabian	Uncert. Data Mgmt	2	
2017-12-05	Antoine	Fabian	Uncert. Data Mgmt	3	
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Π_{teacher,resp}(Class) teacher resp

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$\Pi_{\text{teacher}, \text{resp}}(\text{Class})$				
teacher	resp			
Antoine	Fabian			
Silviu	Fabian			

Class					
date	teacher	resp	name	num	
2017-11-21	Antoine	Fabian	Uncert. Data Mgmt	1	
2017-11-28	Antoine	Fabian	Uncert. Data Mgmt	2	
2017-12-05	Antoine	Fabian	Uncert. Data Mgmt	3	
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$\Pi_{\text{teacher}, \text{resp}}(\text{Class})$				
teacher	resp			
Antoine	Fabian			
Silviu	Fabian			

 $\rightarrow~$ Duplicates are removed

Class					
date	teacher	resp	name	num	
2017-11-21	Antoine	Fabian	Uncert. Data Mgmt	1	
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$\sigma_{ t teacher="Silviu"}(t class)$					
date	teacher	resp	name	num	

	Class						
date	teacher	resp	name	num			
2017-11-21	Antoine	Fabian	Uncert. Data Mgmt	1			
2017-11-28	Antoine	Fabian	Uncert. Data Mgmt	2			
2017-12-05	Antoine	Fabian	Uncert. Data Mgmt	3			
2017-12-12	Silviu	Fabian	Uncert. Data Mgmt	4			

$\sigma_{\text{teacher}="Silviu"}(Class)$	
---	--

date	teacher	resp	name	num
2017-12-12	Silviu	Fabian	Uncert. Data Mgmt	4

Class						
date	teacher	resp	name	num		
2017-11-21	Antoine	Fabian	Uncert. Data Mgmt	1		
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date	teacher	resp	name	num	
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 $\rho_{\text{resp} \rightarrow \text{boss}}(\text{Class})$

	Class						
date	teacher	resp	name	num			
2017-11-21	Antoine	Fabian	Uncert. Data Mgmt	1			
2017-11-28	Antoine	Fabian	Uncert. Data Mgmt	2			
2017-12-05	Antoine	Fabian	Uncert. Data Mgmt	3			
2017-12-12	Silviu	Fabian	Uncert. Data Mgmt	4			

$ ho_{resp o boss}$	(Cl	lass)
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date	teacher	boss	name	num
2017-11-21	Antoine	Fabian	Uncert. Data Mgmt	1
2017-11-28	Antoine	Fabian	Uncert. Data Mgmt	2
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- Take tuples occurring in **one of** the input tables
- Applies to two tables with the same attributes



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Students1

id name

- 1 Arthur Dent
- 2 Ford Prefect



- Take tuples occurring in **one of** the input tables
- Applies to two tables with the same attributes

Students1

- **id name**
 - 2 Ford Prefect



- Take tuples occurring in **one of** the input tables
- Applies to two tables with the same attributes

	Students1		52
id	name	id	name
1	Arthur Dent	10	Zanhad D
2	Ford Prefect	42	Zарпой В.



- Take tuples occurring in **one of** the input tables
- Applies to two tables with the same attributes





- Take tuples occurring in **one of** the input tables
- Applies to two tables with the same attributes

	Students1					Stu	dents1 U S2
• •	Studentsi			S2	_	id	name
Id	name	\cup	id	name	=	1	Arthur Dent
1 2	Arthur Dent Ford Prefect		42	Zaphod B.	-	2	Ford Prefect
~					-	42	Zaphod B.



- Take tuples occurring in **one of** the input tables
- Applies to two tables with the same attributes

	Studentsi					Stu	dents1 U S2
	Studentsi			S2		id	name
id	name		id	namo			
1	Arthur Dont	\mathbf{O}	IU	name	. —	1	Arthur Dent
I			42	Zaphod B.		2	Ford Prefect
2	Ford Prefect			•		12	Zanhod B
						44	Zapiloa D.

 \rightarrow Duplicates are **removed** here as well

Students

id name

- 1 Arthur Dent
- 2 Ford Prefect

Students

id name ×

2 Ford Prefect

	Students		Rooms
id	name	X	room
1	Arthur Dent		E200
2	Ford Prefect		E242



					Students × Rooms		
Students			Rooms	_	id	name	room
id	name	X	room	=	1	Arthur Dent	E200
1	Arthur Dent		E200		1	Arthur Dent	E242
2	Ford Prefect		E242		2	Ford Prefect	E200
					2	Ford Prefect	E242

Ctudanta ... Daama

 \rightarrow Product is useful to express **joins**:

$\rightarrow\,$ Product is useful to express joins:

Member					
id	class				
1	UDM				
2	UDM				
Me	_				
----	-------	--			
id	class				
1	UDM				
2	UDM				

Member		Class			
id class		class	date		
1		UDM	Nov 21		
2		ABC	Nov 24		
	ODM	UDM	Nov 28		

Member		Class	
id	class	class	date
		UDM	Nov 21
1		ABC	Nov 24
	UDM	UDM	Nov 28

Member			Class		Member 🖂 Class			
						id	class	date
id	class		CldSS			1	UDM	Nov 21
1	UDM		UDM	Nov 21		1	UDM	Nov 28
2	UDM		ABC Nov 24 UDM Nov 28	Nov 24		2	UDM	Nov 21
				INOV 28	NUV 28	2	UDM	Nov 28

Member		Class			Member 🖂 Class			
		. –	class date		id	class	date	
id	class	\bowtie	class uale	uate		1	UDM	Nov 21
1	UDM		UDM	Nov 21		1	UDM	Nov 28
2	UDM		ABC	Nov 24		2	UDM	Nov 21
		-	UDM	NOV 28		2	UDM	Nov 28

Express Member \bowtie Class with the previous operators:

Member			Class			Member 🖂 Class			
		. –	class data		id	class	date		
id	class	\bowtie	class			1	UDM	Nov 21	
1	UDM		UDM	Nov 21		1	UDM	Nov 28	
2	UDM		ABC	Nov 24		2	UDM	Nov 21	
			UDM	NOV 28		2	UDM	Nov 28	

Express Member \bowtie Class with the previous operators:

Member \times Class

Member		Class			Member 🖂 Class			
			class	date		id	class	date
id	class		class	uate		1	UDM	Nov 21
1	UDM		UDM	Nov 21		1	UDM	Nov 28
2	UDM		ABC	Nov 24		2	UDM	Nov 21
			UDM	NOV 28		2	UDM	Nov 28

Express Member \bowtie Class with the previous operators:

 $\rho_{class \rightarrow class2}$ (Member) × Class

			Class		Me	mber 🛛	Class
Member		. –	class	date	id	class	date
id	class		class	uate	 1	UDM	Nov 21
1	UDM	M UDM Nov 21 ABC Nov 24	1	UDM	Nov 28		
2	UDM		ABC No	Nov 24	2	UDM	Nov 21
			UDM	NOV 28	2	UDM	Nov 28

Express Member \bowtie Class with the previous operators:

 $\sigma_{\text{class}=\text{class2}} (\rho_{\text{class}\rightarrow\text{class2}}(\text{Member}) \times \text{Class})$

Member		Class			Member 🖂 Class			
			class data		id	class	date	
id	class	M	class	uate	· ·	1	UDM	Nov 21
1	UDM		UDM	Nov 21		1	UDM	Nov 28
2	UDM		ABC	Nov 24		2	UDM	Nov 21
			UDM	NOV 28		2	UDM	Nov 28

Express Member \bowtie Class with the previous operators:

$$\Pi_{\mathsf{id},\mathsf{class},\mathsf{date}} \left(\sigma_{\mathsf{class}=\mathsf{class2}} \left(\rho_{\mathsf{class}\to\mathsf{class2}}(\mathsf{Member}) \times \mathsf{Class} \right) \right)$$

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Students1

id name

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- 2 Ford Prefect

- Take tuples that are in one table but **not** in the other
- Applies to two tables with same attributes

	Students1		S3
id	name	id	name
1	Arthur Dent	1	Arthur Dent
2	Ford Prefect	42	Zaphod B.

- Take tuples that are in one table but **not** in the other
- Applies to two **tables** with same **attributes**

	Students1		S3	
id	name	id	name	_
1	Arthur Dent	1	Arthur Dent	
2	Ford Prefect	42	Zaphod B.	

- Take tuples that are in one table but **not** in the other
- Applies to two **tables** with same **attributes**

Students1		S3			Students1 – S3		
id	name	id	name	_	id	name	
1	Arthur Dent	1	Arthur Dent			Ford Drofact	
2	Ford Prefect	42	Zaphod B.		2		

Basics

Relational algebra

SQL

CREATE TABLE Students(id INT(6), name VARCHAR(30)); INSERT INTO Students VALUES (1, 'Arthur Dent'); INSERT INTO Students VALUES (2, 'Ford Prefect'); CREATE TABLE Students(id INT(6), name VARCHAR(30)); INSERT INTO Students VALUES (1, 'Arthur Dent'); INSERT INTO Students VALUES (2, 'Ford Prefect');

SELECT * FROM Students;

CREATE TABLE Students(id INT(6), name VARCHAR(30)); INSERT INTO Students VALUES (1, 'Arthur Dent'); INSERT INTO Students VALUES (2, 'Ford Prefect');

SELECT * FROM Students;

+----+ | id | name | +----+ | 1 | Arthur Dent | | 2 | Ford Prefect | +----+ 2 rows in set (0.00 sec)

SELECT name FROM Students;

SELECT name FROM Students;

+----+ | name | +----+ | Arthur Dent | | Ford Prefect | +----+

SELECT name FROM Students;

+-----+ | name | +----+ | Arthur Dent | | Ford Prefect | +----+

SELECT name, id, id AS identifier FROM Students;

SELECT name FROM Students;

+----+ | name | +----+ | Arthur Dent | | Ford Prefect | +----+

SELECT name, id, id AS identifier FROM Students;

+		.+.			.+.		-+
I	name	I	id		I	identifier	I
+		+-			+-		-+
I	Arthur Dent	I		1	I	1	I
I	Ford Prefect	I		2	I	2	I
+		.+.			.+.		-+

SELECT * FROM Students;

Selection

SELECT * FROM Students;

+----+ | id | name | +----+ | 1 | Arthur Dent | | 2 | Ford Prefect | +----+

Selection

SELECT * FROM Students;

+----+ | id | name | +----+ | 1 | Arthur Dent | | 2 | Ford Prefect | +----+

SELECT * FROM Students WHERE id='2';

Selection

SELECT * FROM Students;

+-----+ | id | name | +----+ | 1 | Arthur Dent | | 2 | Ford Prefect | +----+

SELECT * FROM Students WHERE id='2';



SELECT * FROM Students;

+----+ | id | name | +----+ | 1 | Arthur Dent | | 2 | Ford Prefect | +----+

SELECT * FROM Students;

+----+ | id | name | +----+ | 1 | Arthur Dent | | 2 | Ford Prefect | +----+

SELECT * FROM S2;

+-----+ | id | name | +-----+ | 42 | Zaphod B | +-----+

SELECT * FROM Students;



SELECT * FROM S2;

+-----+ | id | name | +-----+ | 42 | Zaphod B | +-----+ (SELECT * FROM Students) UNION (SELECT * FROM S2);

SELECT * FROM Students;



+----+ | id | name | +----+ | 42 | Zaphod B | +----+

```
(SELECT * FROM Students)
UNION
(SELECT * FROM S2);
```

+-		-+-		+
I	id	Ι	name	I
+-		-+-		+
I	1	I	Arthur Dent	I
L	2	Ι	Ford Prefect	I
L	42	Ι	Zaphod B	I
+-		-+-		+







| E200 |

| E242 |

+---+

SELECT * FROM Students, Rooms;



SELECT * FROM Students, Rooms;

+.			.+.		.+.		+
 +.	id		.+.	name	 .+.	room	 +
I		1	1	Arthur Dent	1	E200	
I		2	I	Ford Prefect	I	E200	۱
I		1	I	Arthur Dent	I	E242	١
I		2	I	Ford Prefect	I	E242	۱
+.			.+.		.+.		+

Join

SELECT * FROM Member;

+-----+ | id | class | +----+ | 1 | UDM | | 2 | UDM | +----+
Join

SELECT * FROM Member;

+----+ | id | class | +----+ | 1 | UDM | | 2 | UDM | +----+

SELECT * FROM Classes;

+-----+ | class | date |

+----+

| UDM | Nov 21 |

| ABC | Nov 24 |

| UDM | Nov 28 |

+----+

Join

SELECT * FROM Member;

+-----+ | id | class | +-----+ | 1 | UDM | | 2 | UDM | +----+

SELECT * FROM Member NATURAL JOIN Classes;

SELECT * FROM Classes;

+----+

| class | date |

+----+

| UDM | Nov 21 |

| ABC | Nov 24 |

| UDM | Nov 28 |

+----+

Join

SELECT * FROM Member;

+----+ | id | class | +----+ | 1 | UDM | | 2 | UDM | +----+

SELECT * FROM Classes;

+----+ | class | date | +----+ | UDM | Nov 21 | | ABC | Nov 24 | | UDM | Nov 28 | +---+

SELECT * FROM Member NATURAL JOIN Classes;

+-		+			.+.			+
I	class	I	id			date		I
+		+ •			-+-			+
I	UDM	I		1	I	Nov	21	I
I	UDM	I		2	I	Nov	21	I
I	UDM	I		1	I	Nov	28	I
I	UDM	I		2	I	Nov	28	I
+.		.+.			.+.			+







```
SELECT * FROM Students
WHERE (id, name) NOT IN
  (SELECT * FROM S3);
```



SELECT * FROM Students
WHERE (id, name) NOT IN
 (SELECT * FROM S3);

+-			+.			-+
I	id		I	name		I
+-			+-			-+
I		2	I	Ford H	Prefect	
+-			+-			-+

Composing operations

Our translation of:

SELECT * FROM Member NATURAL JOIN Classes;

can be expressed as:

```
Our translation of:
```

```
SELECT * FROM
Member NATURAL JOIN Classes;
```

```
can be expressed as:
```

```
SELECT id, class, date FROM
 (SELECT * FROM
  (SELECT id, class AS class2 FROM Member) sub1,
   Classes
 ) sub2
```

WHERE class = class2;

```
Our translation of:
```

```
SELECT * FROM
Member NATURAL JOIN Classes;
```

```
can be expressed as:
```

```
SELECT id, class, date FROM
  (SELECT * FROM
    (SELECT id, class AS class2 FROM Member) sub1,
    Classes
  ) sub2
  WHERE class = class2;
```

ightarrow SQL can express the relational algebra

```
Our translation of:
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```
SELECT * FROM
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```
can be expressed as:
```

```
SELECT id, class, date FROM
  (SELECT * FROM
    (SELECT id, class AS class2 FROM Member) sub1,
    Classes
  ) sub2
```

```
WHERE class = class2;
```

- ightarrow SQL can express the relational algebra
 - ... but please, **never** write queries like this!



Abiteboul, S., Hull, R., and Vianu, V. (1995). *Foundations of Databases.*

Addison-Wesley.

http://webdam.inria.fr/Alice/pdfs/all.pdf.



ISO 9075:2008: SQL.