Exam Re-Take

Uncertain Data Management
Université Paris-Saclay, M2 Data&Knowledge

May 20th, 2017

This is the re-take of the final exam for the Uncertain Data Management class. The grade in this exam will replace your grade of the first session of the final exam, and will become your final grade for the class. The exam consists of two independent exercises. Each exercise *must* be answered on a separate sheet of paper. Each sheet *must* be numbered and carry your name on the top right.

No additional explanations will be given during the exam, and no questions will be answered. If you think you have found an error in the problem statement, you should report on your answer sheet what you believe to be the error, and how you chose to interpret the intent of the question to recover from the alleged error.

You are allowed up to two A4 sheets of personal notes (i.e., four page sides), printed or written by hand, with font size of 10 points at most. If you use such personal notes, you must hand them in along with your answers. You may not use any other written material.

The exam is strictly personal: any communication or influence between students, or use of outside help, is prohibited. No electronic devices such as calculators, computers, or mobile phones, are permitted. Any violation of the rules may result in a grade of 0 and/or disciplinary action.

Exercise 1: Constructing probabilistic relations (10 points)

We consider one relation $T(\mathsf{pupil}, \mathsf{teacher})$ indicating which pupil follows classes by which teacher, and one relation $M(\mathsf{manager}, \mathsf{subordinate})$ indicating which teacher is a direct manager of which teacher. The relation are given below as tuple-independent (TID) instances, with probabilities expressed as $p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}$:

T

pupil	teacher	
Jane	Alice	p_a
Jane	Bob	p_b
Jane	Carol	p_c

M

manager	subordinate	
Alice	Bob	q_{ab}
Alice	Carol	q_{ac}
Bob	Carol	q_{bc}

We consider the Boolean conjunctive query Q asking whether there is a pupil who is taught by some teacher and who is also taught by some manager of that teacher.

We will be interested in the probability of the query Q on T and M, as a function of the values of $p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}$. We denote this probability by $f(p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc})$. Here are some examples:

- If $p_a = p_b = p_c = q_{ab} = q_{ac} = q_{bc} = 0$ then we have $f(p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}) = 0$.
- Whenever $q_{ac} = 0$ and $p_b = 0$ then we always have $f(p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}) = 0$.
- Whenever $p_a = p_b = q_{ab} = 1$ then we always have $f(p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}) = 1$.

- Whenever $p_c = 0$, $p_a = p_b = 1$, and $q_{ab} = 0.5$, then we always have $f(p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}) = 0.5$.

All questions are independent except when indicated otherwise. No justification is expected for your answers unless indicated otherwise.

Question 1 (1 point). Express the query Q in the relational algebra.

Question 2 (1 point). Express the query Q in the relational calculus.

Question 3 (1 point). Give a choice of values for $p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}$ such that $f(p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}) = 0.42$.

Question 4 (1 point). Give a choice of values for $p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}$ such that *all these values are in* $\{0, 0.5, 1\}$ and $f(p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}) = 3/4$.

Question 5 (1 point). The *denominator* of a rational value $0 \leq v \leq 1$ is the smallest positive integer p such that pv is an integer. In other words, p is the denominator of v when written as a fraction in irreducible terms. For instance, the denominator of 0 and 1 are 1, the denominator of 0.5 is 2, the denominator of $3/7$ is 7, the denominator of $2/8$ is 4, and the denominator of 0.42 is 50.

Explain briefly why, for any rational values $0 \leq v \leq 1$ and $0 \leq v' \leq 1$, if the denominator of v is d and the denominator of v' is d', then the denominator of vv' is a divisor of dd'. Give an example of *positive* v and v' where the denominator of vv' is equal to dd'. Give another example where it is strictly less than dd'.

Question 6 (2 point). Using the previous question, show that, for any choice of $p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}$ where all values are in $\{0, 0.5, 1\}$, the denominator of $f(p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc})$ is at most 64.

Question 7 (2 point). Show that, for any choice of values $p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}$ that are all in $\{0, 0.5, 1\}$, we have either $f(p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}) = 0$ or $f(p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}) \geq 1/8$.

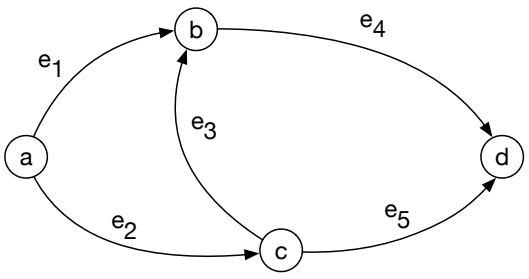
Give an example of a choice of $p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}$ in $\{0, 0.5, 1\}$, such that $f(p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}) = 1/8$.

Question 8 (1 point). Give a choice of values $p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}$ that are all in $\{0, 0.5, 1\}$ such that $f(p_a, p_b, p_c, q_{ab}, q_{ac}, q_{bc}) = 7/16$.

Exercise 2: Reachability Queries on Probabilistic Graphs (10 points)

In this exercise, we will consider reachability queries on probabilistic graphs. A *probabilistic* graph is a directed graph in which edges are annotated with probabilistic events. In this exercise we consider *independent edge events*: each edge is annotated by a specific probabilistic event e, which is associated to a probability $P(e)$ that the edge exists, assuming independence between events.

For the following questions, we will work with the following probabilistic graph, where edges are annotated with events e_1, e_2, e_3, e_4, e_5:



Question 1 (2 points). Consider the following function P assigning probabilities to events: $P(e_1) = 0.5$, $P(e_2) = 1$, $P(e_3) = 0.5$, $P(e_4) = 1$, and $P(e_5) = 1$. How many *distinct possible worlds*, having non-zero probability, are there for the probabilistic graph $\mathcal{G}$? Draw each of them, and write down their probabilities. (Hint: possible worlds for a probabilistic graph have the same semantics as in TID databases.)

Question 2 (3 points). On a probabilistic graph, a reachability query $r(s, t)$ asks whether there exists at least one path in the graph between the nodes s and t. Moreover, the lineage $\Phi(s, t)$ is the logical expression (where terms are the edge events described above) which encodes the condition under which there exists at least one path between s and t.

For instance, for $r(a, b)$ on $\mathcal{G}$, the corresponding lineage is:

$$\Phi(a, b) = e_1 \vee (e_2 \wedge e_3),$$

signifying that at least one of two paths, $a \to b$ or $a \to c \to b$, should exist for $r(a, b)$ to be true.

Draw the *read-once circuit* corresponding to $\Phi(a, b)$. Write down the probability formula for $r(a, b)$, as a function of edge events e_1, e_2, e_3, e_4, e_5. Compute the final probability using the probabilities given in Question 1.

Question 3 (3 points). Consider now $r(a, d)$ on $\mathcal{G}$. Write down the corresponding lineage, $\Phi(a, d)$. Write down its probability formula, as a function of edge events e_1, e_2, e_3, e_4, e_5, and compute the probability, using the probabilities given in Question 1.

Question 4 (2 points). Can $\Phi(a, d)$ be rewritten as a *read-once* formula? Give a read-once rewriting, or explain why none exists. Draw an OBDD corresponding to $\Phi(a, d)$.