Uncertain Data Management
Relational Probabilistic Database Models

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Table of contents

Uncertain instances

Remember from last class:

- Fix a finite set of possible tuples of same arity
- A possible world: a subset of the possible tuples
- A (finite) uncertain relation: set of possible worlds

Uncertain instances

Remember from last class:

- Fix a finite set of possible tuples of same arity
- A possible world: a subset of the possible tuples
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U_1

date	teacher	room
04	Silviu	C017
04	Antoine	C017
04	Antoine	C47
11	Silviu	C017
11	Silviu	C47
11	Antoine	C017

U_2

date	teacher	room
04	Silviu	C017
04	Antoine	C017
04	Antoine	C47
11	Silviu	C017
11	Silviu	C47
11	Antoine	C017

Probabilistic instances

- Support $\mathcal{U}$: uncertain relation

Probabilistic instances

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  - Function from $\mathcal{U}$ to reals in $[0, 1]$
  - It must sum up to 1: $\sum_{I \in \mathcal{U}} \pi(I) = 1$

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U_1

date	teacher	room
04	Silviu	C017
04	Antoine	C017
04	Antoine	C47
11	Silviu	C017
11	Silviu	C47
11	Antoine	C017

U_2

date	teacher	room
04	Silviu	C017
04	Antoine	C017
04	Antoine	C47
11	Silviu	C017
11	Silviu	C47
11	Antoine	C017

Probabilistic instances

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U_1

date	teacher	room
04	Silviu	C017
04	Antoine	C017
04	Antoine	C47
11	Silviu	C017
11	Silviu	C47
11	Antoine	C017

$\pi(U_1) = 0.8$

U_2

date	teacher	room
04	Silviu	C017
04	Antoine	C017
04	Antoine	C47
11	Silviu	C017
11	Silviu	C47
11	Antoine	C017

$\pi(U_2) = 0.2$

What about `NULL`s?

Remember that last time we saw:

- Codd-tables and v-tables and c-tables, with `NULL`s
- Boolean c-tables, with `NULL`s only in conditions
  - $\rightarrow$ Boolean variables

What about `NULL`s?

Remember that last time we saw:

- **Codd-tables** and **v-tables** and **c-tables**, with `NULL`s
- **Boolean c-tables**, with `NULL`s **only in conditions**
  - → Boolean variables

→ We focus for probabilities on models like **Boolean c-tables**

→ Easier to define probabilities on a **finite** space!

Relational algebra on uncertain instances

Remember from last class:

- Extend relational algebra operators to uncertain instances
- The possible worlds of the result should be...
  - take all possible worlds in the supports of the inputs
  - apply the operation and get the possible outputs

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U_1

04	S.	C017
11	S.	C47

U_2

11	A.	C017

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U_1

04	S.	C017
11	S.	C47

$\cup$

U_2

11	A.	C017

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U_1

04	S.	C017
11	S.	C47

U_2

11	A.	C017

$\bigcup$

V_1

V_2

11	A.	C017

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U_1

04	S.	C017
11	S.	C47

U_2

11	A.	C017

$\bigcup$

V_1

V_2

11	A.	C017

$=$

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04	S.	C017
11	S.	C47

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V_1

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$=$

04	S.	C017
11	S.	C47

04	S.	C017
11	S.	C47
11	A.	C017

11	A.	C017

Relational algebra on probabilistic instances

- Let's adapt relational algebra to probabilistic instances
- The possible worlds of the result should be…

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  - take all possible worlds of the inputs
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Relational algebra on probabilistic instances

- Let's adapt relational algebra to probabilistic instances
- The possible worlds of the result should be...
  - take all possible worlds of the inputs
  - apply the operation and get a possible output
- The probability of each possible world should be...
  - consider all input possible worlds that give it
  - sum up their probabilities

Example of relational algebra on probabilistic instances

Example of relational algebra on probabilistic instances

U_1

04	S.	C017
11	S.	C47

U_2

11	A.	C017

Example of relational algebra on probabilistic instances

U_1

04	S.	C017
11	S.	C47

$\bigcup$

U_2

11	A.	C017

Example of relational algebra on probabilistic instances

U_1

04	S.	C017
11	S.	C47

V_1

$\bigcup$

U_2

11	A.	C017

V_2

11	A.	C017

Example of relational algebra on probabilistic instances

U_1

04	S.	C017
11	S.	C47

$\pi(U_1) = 0.8$

U_2

11	A.	C017

$\pi(U_2) = 0.2$

$\bigcup$

V_1

$\pi(V_1) = 0.9$

V_2

11	A.	C017

$\pi(V_2) = 0.1$

Example of relational algebra on probabilistic instances

U_1

04	S.	C017
11	S.	C47

$\pi(U_1) = 0.8$

U_2

11	A.	C017

$\pi(U_2) = 0.2$

$\bigcup$

V_1

$\pi(V_1) = 0.9$

V_2

11	A.	C017

$\pi(V_2) = 0.1$

$=$

Example of relational algebra on probabilistic instances

U_1

04	S.	C017
11	S.	C47

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U_2

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$\cup$

V_1

$\pi(V_1) = 0.9$

V_2

11	A.	C017

$\pi(V_2) = 0.1$

$=$

W_1

04	S.	C017
11	S.	C47

Example of relational algebra on probabilistic instances

U_1

04	S.	C017
11	S.	C47

$\pi(U_1) = 0.8$

U_2

11	A.	C017

$\pi(U_2) = 0.2$

$\cup$

V_1

$\pi(V_1) = 0.9$

V_2

11	A.	C017

$\pi(V_2) = 0.1$

$=$

W_1

04	S.	C017
11	S.	C47

W_2

04	S.	C017
11	S.	C47
11	A.	C017

Example of relational algebra on probabilistic instances

U_1

04	S.	C017
11	S.	C47

$\pi(U_1) = 0.8$

U_2

11	A.	C017

$\pi(U_2) = 0.2$

$\cup$

V_1

(empty)

$\pi(V_1) = 0.9$

V_2

11	A.	C017

$\pi(V_2) = 0.1$

$=$

W_1

04	S.	C017
11	S.	C47

W_2

04	S.	C017
11	S.	C47
11	A.	C017

W_3

11	A.	C017

Example of relational algebra on probabilistic instances

U_1

04	S.	C017
11	S.	C47

$\pi(U_1) = 0.8$

U_2

11	A.	C017

$\pi(U_2) = 0.2$

$\cup$

V_1

$\pi(V_1) = 0.9$

V_2

11	A.	C017

$\pi(V_2) = 0.1$

$=$

W_1

04	S.	C017
11	S.	C47

$\pi(W_1) = 0.8 \cdot 0.9$

W_2

04	S.	C017
11	S.	C47
11	A.	C017

W_3

11	A.	C017

Example of relational algebra on probabilistic instances

U_1

04	S.	C017
11	S.	C47

$\pi(U_1) = 0.8$

U_2

11	A.	C017

$\pi(U_2) = 0.2$

$\cup$

V_1

(empty)

$\pi(V_1) = 0.9$

V_2

11	A.	C017

$\pi(V_2) = 0.1$

$=$

W_1

04	S.	C017
11	S.	C47

$\pi(W_1) = 0.8 \cdot 0.9$

W_2

04	S.	C017
11	S.	C47
11	A.	C017

$\pi(W_1) = 0.8 \cdot 0.1$

W_3

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Example of relational algebra on probabilistic instances

U_1

04	S.	C017
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U_2

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$\cup$

V_1

(empty)

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V_2

11	A.	C017

$\pi(V_2) = 0.1$

$=$

W_1

04	S.	C017
11	S.	C47

$\pi(W_1) = 0.8 \cdot 0.9$

W_2

04	S.	C017
11	S.	C47
11	A.	C017

$\pi(W_1) = 0.8 \cdot 0.1$

W_3

11	A.	C017

$\pi(W_1) = 0.2 \cdot 0.9$

Example of relational algebra on probabilistic instances

U_1

04	S.	C017
11	S.	C47

$\pi(U_1) = 0.8$

U_2

11	A.	C017

$\pi(U_2) = 0.2$

$\cup$

V_1

$\pi(V_1) = 0.9$

V_2

11	A.	C017

$\pi(V_2) = 0.1$

$=$

W_1

04	S.	C017
11	S.	C47

$\pi(W_1) = 0.8 \cdot 0.9$

W_2

04	S.	C017
11	S.	C47
11	A.	C017

$\pi(W_1) = 0.8 \cdot 0.1$

W_3

11	A.	C017

$\pi(W_1) = 0.2 \cdot 0.9$
$+0.2 \cdot 0.1$

Representation system

- Remember that if we have N possible tuples

Representation system

- Remember that if we have N possible tuples
  - $\rightarrow$ there are 2^N possible instances

Representation system

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  - $\rightarrow$ writing out an uncertain instance is exponential

Representation system

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→ Last time we saw Boolean c-tables as a concise way to represent uncertain instances

Representation system

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  - $\rightarrow$ there are 2^{2^N} possible uncertain instances
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- For probabilistic instances:
  - $\rightarrow$ there are infinitely many possible instances
  - $\rightarrow$ writing out a probabilistic instance is still exponential

Representation system

- Remember that if we have N possible tuples
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- For probabilistic instances:
  - $\rightarrow$ there are infinitely many possible instances
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$\rightarrow$ How to represent probabilistic instances?

Table of contents

Tuple-independent databases

- The simplest model: tuple-independent databases
- Annotate each instance fact with a probability

Tuple-independent databases

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U

date	teacher	room
04	Silviu	C017
04	Antoine	C017
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Tuple-independent databases

- The simplest model: tuple-independent databases
- Annotate each instance fact with a probability

U

date	teacher	room	
04	Silviu	C017	0.8
04	Antoine	C017	0.2
11	Silviu	C017	1

Tuple-independent databases

- The simplest model: tuple-independent databases
- Annotate each instance fact with a probability

U

date	teacher	room	
04	Silviu	C017	0.8
04	Antoine	C017	0.2
11	Silviu	C017	1

→ Assume independence between tuples
(Silviu and Antoine may teach at the same time)

Semantics of TID

- Each tuple is kept or discarded with the probability
- Probabilistic choices are independent across tuples

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U

date	teacher	room	
04	Silviu	C017	0.8
04	Antoine	C017	0.2
11	Silviu	C017	1

Semantics of TID

- Each tuple is kept or discarded with the probability
- Probabilistic choices are independent across tuples

U

date	teacher	room	
04	Silviu	C017	0.8
04	Antoine	C017	0.2
11	Silviu	C017	1

U

date	teacher	room

Semantics of TID

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U

date	teacher	room	
04	Silviu	C017	0.8
04	Antoine	C017	0.2
11	Silviu	C017	1

U

date	teacher	room
04	Silviu	C017

Semantics of TID

- Each tuple is kept or discarded with the probability
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U

date	teacher	room	
04	Silviu	C017	0.8
04	Antoine	C017	0.2
11	Silviu	C017	1

U

date	teacher	room
04	Silviu	C017
04	Antoine	C017

Semantics of TID

- Each tuple is kept or discarded with the probability
- Probabilistic choices are independent across tuples

U

date	teacher	room	
04	Silviu	C017	0.8
04	Antoine	C017	0.2
11	Silviu	C017	1

U

date	teacher	room
04	Silviu	C017
04	Antoine	C017
11	Silviu	C017

Semantics of TID

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U

date	teacher	room	
04	Silviu	C017	0.8
04	Antoine	C017	0.2
11	Silviu	C017	1

U

date	teacher	room
04	Silviu	C017
04	Antoine	C017
11	Silviu	C017

What's the probability of this outcome?

Semantics of TID

- Each tuple is kept or discarded with the probability
- Probabilistic choices are independent across tuples

U

date	teacher	room	
04	Silviu	C017	0.8
04	Antoine	C017	0.2
11	Silviu	C017	1

U

date	teacher	room
04	Silviu	C017
04	Antoine	C017
11	Silviu	C017

What's the probability of this outcome?

0.8

Semantics of TID

- Each tuple is kept or discarded with the probability
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U

date	teacher	room	
04	Silviu	C017	0.8
04	Antoine	C017	0.2
11	Silviu	C017	1

U

date	teacher	room
04	Silviu	C017
04	Antoine	C017
11	Silviu	C017

What's the probability of this outcome?

$$0.8 \times$$

Semantics of TID

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U

date	teacher	room	
04	Silviu	C017	0.8
04	Antoine	C017	0.2
11	Silviu	C017	1

U

date	teacher	room
04	Silviu	C017
04	Antoine	C017
11	Silviu	C017

What's the probability of this outcome?

$$0.8 \times (1 - 0.2)$$

Semantics of TID

- Each tuple is kept or discarded with the probability
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U

date	teacher	room	
04	Silviu	C017	0.8
04	Antoine	C017	0.2
11	Silviu	C017	1

U

date	teacher	room
04	Silviu	C017
04	Antoine	C017
11	Silviu	C017

What's the probability of this outcome?

$$0.8 \times (1 - 0.2) \times 1$$

Getting a probability distribution

The semantics of a TID instance is a probabilistic instance...

$\rightarrow$ the possible worlds are the subsets

Getting a probability distribution

The semantics of a TID instance is a probabilistic instance...

→ the possible worlds are the subsets
  → always keeping tuples with probability 1

Getting a probability distribution

The semantics of a TID instance is a probabilistic instance...

- $\rightarrow$ the possible worlds are the subsets
  - $\rightarrow$ always keeping tuples with probability 1

Formally, for a TID instance I, the probability of J:

Getting a probability distribution

The semantics of a TID instance is a probabilistic instance…

$\rightarrow$ the possible worlds are the subsets

$\rightarrow$ always keeping tuples with probability 1

Formally, for a TID instance I, the probability of J:

- we must have $J \subseteq I$
- product of $p_{\mathbf{t}}$ for each tuple $\mathbf{t}$ kept in J
- product of $1 - p_{\mathbf{t}}$ for each tuple $\mathbf{t}$ not kept in J

Is it a probability distribution?

Do the probabilities always sum to 1?

Is it a probability distribution?

Do the probabilities always sum to 1?

- 0 tuples: vacuous

Is it a probability distribution?

Do the probabilities always sum to 1?

- 0 tuples: vacuous
- 1 tuple: $p + (1 - p) = 1$

Is it a probability distribution?

Do the probabilities always sum to 1?

- 0 tuples: vacuous
- 1 tuple: $p + (1 - p) = 1$
- 2 tuples: $p_1 p_2 + p_1(1 - p_2) + (1 - p_1)p_2 + (1 - p_1)(1 - p_2)$

Is it a probability distribution?

Do the probabilities always sum to 1?

- 0 tuples: vacuous
- 1 tuple: $p + (1 - p) = 1$
- 2 tuples: $p_1 p_2 + p_1(1 - p_2) + (1 - p_1)p_2 + (1 - p_1)(1 - p_2)$
  $\rightarrow \; p_1(p_2 + (1 - p_2)) + (1 - p_1)(p_2 + (1 - p_2))$

Is it a probability distribution?

Do the probabilities always sum to 1?

- 0 tuples: vacuous
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- 2 tuples: $p_1 p_2 + p_1(1 - p_2) + (1 - p_1)p_2 + (1 - p_1)(1 - p_2)$
  $\rightarrow\ p_1(p_2 + (1 - p_2)) + (1 - p_1)(p_2 + (1 - p_2))$
  $\rightarrow\ p_1 \times 1 + (1 - p_1) \times 1$

Is it a probability distribution?

Do the probabilities always sum to 1?

- 0 tuples: vacuous
- 1 tuple: $p + (1 - p) = 1$
- 2 tuples: $p_1 p_2 + p_1(1 - p_2) + (1 - p_1)p_2 + (1 - p_1)(1 - p_2)$
  - $\rightarrow\ p_1(p_2 + (1 - p_2)) + (1 - p_1)(p_2 + (1 - p_2))$
  - $\rightarrow\ p_1 \times 1 + (1 - p_1) \times 1$
  - $\rightarrow\ 1$

Is it a probability distribution?

Do the probabilities always sum to 1?

- 0 tuples: vacuous
- 1 tuple: $p + (1 - p) = 1$
- 2 tuples: $p_1 p_2 + p_1(1 - p_2) + (1 - p_1)p_2 + (1 - p_1)(1 - p_2)$
  - $\rightarrow\ p_1(p_2 + (1 - p_2)) + (1 - p_1)(p_2 + (1 - p_2))$
  - $\rightarrow\ p_1 \times 1 + (1 - p_1) \times 1$
  - $\rightarrow\ 1$
- More tuples: $p_1(\cdots) + (1 - p_1)(\cdots)$ and induction hypothesis

Strong representation system

Remember from last class:

Uncertain instance: set of possible worlds

Strong representation system

Remember from last class:

Uncertain instance: set of possible worlds

Uncertainty framework: concise way to represent uncertain instances

Strong representation system

Remember from last class:

Uncertain instance: set of possible worlds

Uncertainty framework: concise way to represent uncertain instances

Query language: here, relational algebra

Strong representation system

Remember from last class:

Uncertain instance: set of possible worlds

Uncertainty framework: concise way to represent uncertain instances

Query language: here, relational algebra

Definition (Strong representation system)

For any query in the language,

Strong representation system

Remember from last class:

Uncertain instance: set of possible worlds

Uncertainty framework: concise way to represent uncertain instances

Query language: here, relational algebra

Definition (Strong representation system)

For any query in the language,
on uncertain instances represented in the framework,

Strong representation system

Remember from last class:

Uncertain instance:	set of possible worlds
Uncertainty framework:	concise way to represent uncertain instances
Query language:	here, relational algebra

Definition (Strong representation system)

For any query in the language,
on uncertain instances represented in the framework,
the uncertain instance obtained by evaluating the query

Strong representation system

Remember from last class:

Uncertain instance:	set of possible worlds
Uncertainty framework:	concise way to represent uncertain instances
Query language:	here, relational algebra

Definition (Strong representation system)
For any query in the language,
on uncertain instances represented in the framework,
the uncertain instance obtained by evaluating the query
can also be represented in the framework.

Strong representation system

Remember from last class:

Uncertain instance: set of possible worlds

Uncertainty framework: concise way to represent uncertain instances

Query language: here, relational algebra

Definition (Strong representation system)

For any query in the language,
on uncertain instances represented in the framework,
the uncertain instance obtained by evaluating the query
can also be represented in the framework.

$\rightarrow$ Are TID instances a strong representation system?

Implementing select

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2
11	Silviu	C47	1

Implementing select

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2
11	Silviu	C47	1

$\sigma_{\text{teacher}=\text{“Silviu”}}(U)$

date	teacher	room

Implementing select

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2
11	Silviu	C47	1

$\sigma_{\text{teacher}=\text{“Silviu”}}(U)$

date	teacher	room	
04	Silviu	C47	0.8
11	Silviu	C47	1

Implementing select

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2
11	Silviu	C47	1

$\sigma_{\text{teacher}=\text{“Silviu”}}(U)$

date	teacher	room	
04	Silviu	C47	0.8
11	Silviu	C47	1

→ Is this correct? ...

Implementing select

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2
11	Silviu	C47	1

$\sigma_{\text{teacher}=\text{“Silviu”}}(U)$

date	teacher	room	
04	Silviu	C47	0.8
11	Silviu	C47	1

$\rightarrow$ Is this correct? … So far, so good.

Implementing project

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2
11	Silviu	C47	1
11	Antoine	C47	0.1
18	Silviu	C47	0.9

Implementing project

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2
11	Silviu	C47	1
11	Antoine	C47	0.1
18	Silviu	C47	0.9

$\pi_{\text{date}}(U)$

date

Implementing project

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2
11	Silviu	C47	1
11	Antoine	C47	0.1
18	Silviu	C47	0.9

$\pi_{\text{date}}(U)$

date
04
11
18

Implementing project

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2
11	Silviu	C47	1
11	Antoine	C47	0.1
18	Silviu	C47	0.9

$\pi_{\text{date}}(U)$

date	
04	
11	
18	0.9

Implementing project

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2
11	Silviu	C47	1
11	Antoine	C47	0.1
18	Silviu	C47	0.9

$\pi_{\text{date}}(U)$

date	
04	
11	1
18	0.9

Implementing project

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2
11	Silviu	C47	1
11	Antoine	C47	0.1
18	Silviu	C47	0.9

$\pi_{\text{date}}(U)$

date	
04	$1 - (1 - 0.2) \cdot (1 - 0.8)$
11	1
18	0.9

Implementing project

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2
11	Silviu	C47	1
11	Antoine	C47	0.1
18	Silviu	C47	0.9

$\pi_{\text{date}}(U)$

date	
04	$1-(1-0.2)\cdot(1-0.8)$
11	1
18	0.9

→ Is this correct? ...

Implementing project

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2
11	Silviu	C47	1
11	Antoine	C47	0.1
18	Silviu	C47	0.9

$\pi_{\text{date}}(U)$

date	
04	$1 - (1 - 0.2) \cdot (1 - 0.8)$
11	1
18	0.9

→ Is this correct? ... So far, so good.

Implementing product

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Implementing product

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Repair

room	cause	
C47	leopard	0.1

Implementing product

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Repair

room	cause	
C47	leopard	0.1

$U \times$ Repair

date	teacher	room	cause

Implementing product

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Repair

room	cause	
C47	leopard	0.1

$U \times$ Repair

date	teacher	room	cause
04	Silviu	C47	leopard
04	Antoine	C47	leopard

Implementing product

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Repair

room	cause	
C47	leopard	0.1

$U \times$ Repair

date	teacher	room	cause	
04	Silviu	C47	leopard	0.8×0.1
04	Antoine	C47	leopard	0.2×0.1

Implementing product

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Repair

room	cause	
C47	leopard	0.1

$U \times$ Repair

date	teacher	room	cause	
04	Silviu	C47	leopard	0.8×0.1
04	Antoine	C47	leopard	0.2×0.1

$\rightarrow$ Is this **correct?**

Implementing product ... OR NOT!

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Repair

room	cause	
C47	leopard	0.1

$U \times$ Repair

date	teacher	room	cause	
04	Silviu	C47	leopard	0.8×0.1
04	Antoine	C47	leopard	0.2×0.1

$\rightarrow$ Is this correct?

$\rightarrow$ It's **WRONG**!

Why is it wrong?

U

date	teacher	room	
04	Silviu	C47	1
04	Antoine	C47	1

Why is it wrong?

U

date	teacher	room	
04	Silviu	C47	1
04	Antoine	C47	1

Repair

room	cause	
C47	leopard	1/2

Why is it wrong?

U

date	teacher	room	
04	Silviu	C47	1
04	Antoine	C47	1

Repair

room	cause	
C47	leopard	1/2

$U \times$ Repair

date	teacher	room	cause
04	Silviu	C47	leopard
04	Antoine	C47	leopard

Why is it wrong?

U

date	teacher	room	
04	Silviu	C47	1
04	Antoine	C47	1

Repair

room	cause	
C47	leopard	1/2

$U \times$ Repair

date	teacher	room	cause	
04	Silviu	C47	leopard	1/2
04	Antoine	C47	leopard	1/2

Why is it wrong?

U

date	teacher	room	
04	Silviu	C47	1
04	Antoine	C47	1

Repair

room	cause	
C47	leopard	1/2

$U \times$ Repair

date	teacher	room	cause	
04	Silviu	C47	leopard	1/2
04	Antoine	C47	leopard	1/2

$\rightarrow$ The two tuples are **not independent!**

$\rightarrow$ The first is there **iff** the second is there.

Why does it matter?

$U \times$ Repair

date	teacher	room	cause	
04	Silviu	C47	leopard	1/2
04	Antoine	C47	leopard	1/2

Why does it matter?

$U \times \text{Repair}$

date	teacher	room	cause	
04	Silviu	C47	leopard	1/2
04	Antoine	C47	leopard	1/2

$\pi_{\textbf{room}}(U \times \text{Repair})$

room
C47

Why does it matter?

$U \times \text{Repair}$

date	teacher	room	cause	
04	Silviu	C47	leopard	$1/2$
04	Antoine	C47	leopard	$1/2$

$\pi_{\textbf{room}}(U \times \text{Repair})$

room	
C47	$1-(1-1/2)\times(1-1/2)$

Why does it matter?

$U \times \text{Repair}$

date	teacher	room	cause	
04	Silviu	C47	leopard	1/2
04	Antoine	C47	leopard	1/2

$\pi_{\textbf{room}}(U \times \text{Repair})$

room	
C47	$1-(1-1/2)\times(1-1/2)$

$\rightarrow$ Probability of 3/4...

Why does it matter?

$U \times \text{Repair}$

date	teacher	room	cause	
04	Silviu	C47	leopard	1/2
04	Antoine	C47	leopard	1/2

$\pi_{\textbf{room}}(U \times \text{Repair})$

room	
C47	$1-(1-1/2)\times(1-1/2)$

→ Probability of 3/4...

→ But the leopard had probability 1/2!

TID are not a strong representation system

- Remember how Codd tables required named nulls?
- The result of a query on TID may not be a TID

→ We will see that the correlations can be complex

TID are not a strong representation system

- Remember how Codd tables required named nulls?
- The result of a query on TID may not be a TID

$\rightarrow$ We will see that the correlations can be complex

- How to evaluate queries on a TID then?

$\rightarrow$ List all possible worlds and count the probabilities

Query evalation done right

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Query evalation done right

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Repair

room	cause	
C47	leopard	0.1

Query evalation done right

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Repair

room	cause	
C47	leopard	0.1

$\pi_{\textbf{room}}(U \times \text{Repair})$

room
C47

Query evalation done right

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Repair

room	cause	
C47	leopard	0.1

$\pi_{\textbf{room}}(U \times \text{Repair})$

room	
C47	???

Query evalation done right

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

U

room	cause
C47	leopard 0.1

Repair

$\pi_{\textbf{room}}(U \times \text{Repair})$

room	
C47	???

- Either there is no leopard and then no result...

Query evalation done right

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Repair

room	cause	
C47	leopard	0.1

$\pi_{\textbf{room}}(U \times \text{Repair})$

room	
C47	???

- Either there is no leopard and then no result...
- Or there is a leopard and then...

Query evalation done right

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Repair

room	cause	
C47	leopard	0.1

$\pi_{\textbf{room}}(U \times \text{Repair})$

room	
C47	???

- Either there is no leopard and then no result...
- Or there is a leopard and then...
    - Non-empty result:

Query evalation done right

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Repair

room	cause	
C47	leopard	0.1

$\pi_{\textbf{room}}(U \times \text{Repair})$

room	
C47	???

- Either there is no leopard and then no result...
- Or there is a leopard and then...
  - Non-empty result: $1 - (1 - 0.8)(1 - 0.2)$

Query evalation done right

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Repair

room	cause	
C47	leopard	0.1

$\pi_{\textbf{room}}(U \times \text{Repair})$

room	
C47	???

- Either there is no leopard and then no result...
- Or there is a leopard and then...
  - Non-empty result: $1 - (1 - 0.8)(1 - 0.2) = 0.84$

Query evalation done right

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Repair

room	cause	
C47	leopard	0.1

$\pi_{\textbf{room}}(U \times \text{Repair})$

room
C47

- Either there is no leopard and then no result...
- Or there is a leopard and then...
  - Non-empty result: $1 - (1 - 0.8)(1 - 0.2) = 0.84$
- The query probability is:

Query evalation done right

U

date	teacher	room	
04	Silviu	C47	0.8
04	Antoine	C47	0.2

Repair

room	cause	
C47	leopard	0.1

$\pi_{\textbf{room}}(U \times \text{Repair})$

room	
C47	0.084

- Either there is no leopard and then no result...
- Or there is a leopard and then...
    - Non-empty result: $1 - (1 - 0.8)(1 - 0.2) = 0.84$
- The query probability is: 0.1×0.84

Expressiveness of TID

Can we represent all probabilistic instances with TID?

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Can we represent all probabilistic instances with TID?

"The class is taught by Antoine or Silviu or no one but not both"

Expressiveness of TID

Can we represent all probabilistic instances with TID?

"The class is taught by Antoine or Silviu or no one but not both"

U_1
teacher
Silviu
$\pi(U_1) = 0.8$

Expressiveness of TID

Can we represent **all** probabilistic instances with TID?

*"The class is taught by Antoine or Silviu or no one but **not both**"*

U_1

teacher
Silviu
$\pi(U_1) = 0.8$

U_2

teacher
Antoine
$\pi(U_2) = 0.1$

Expressiveness of TID

Can we represent **all** probabilistic instances with TID?

*"The class is taught by Antoine or Silviu or no one but **not both**"*

U_1

teacher
Silviu

$\pi(U_1) = 0.8$

U_2

teacher
Antoine

$\pi(U_2) = 0.1$

U_3

teacher

$\pi(U_3) = 0.1$

Expressiveness of TID

Can we represent all probabilistic instances with TID?

"The class is taught by Antoine or Silviu or no one but not both"

U_1

teacher
Silviu

$\pi(U_1) = 0.8$

U_2

teacher
Antoine

$\pi(U_2) = 0.1$

U_3

teacher

$\pi(U_3) = 0.1$

U

teacher
Antoine
Silviu

Expressiveness of TID

Can we represent **all** probabilistic instances with TID?

*"The class is taught by Antoine or Silviu or no one but **not both**"*

U_1

teacher
Silviu

$\pi(U_1) = 0.8$

U_2

teacher
Antoine

$\pi(U_2) = 0.1$

U_3

teacher

$\pi(U_3) = 0.1$

U

teacher	
Antoine	0.1
Silviu	

Expressiveness of TID

Can we represent **all** probabilistic instances with TID?

*"The class is taught by Antoine or Silviu or no one but **not both**"*

U_1

teacher
Silviu

$\pi(U_1) = 0.8$

U_2

teacher
Antoine

$\pi(U_2) = 0.1$

U_3

teacher

$\pi(U_3) = 0.1$

U

teacher	
Antoine	0.1
Silviu	0.8

Expressiveness of TID

Can we represent all probabilistic instances with TID?

"The class is taught by Antoine or Silviu or no one but not both"

U_1

teacher
Silviu

$\pi(U_1) = 0.8$

U_2

teacher
Antoine

$\pi(U_2) = 0.1$

U_3

teacher

$\pi(U_3) = 0.1$

U

teacher	
Antoine	0.1
Silviu	0.8

$\rightarrow$ We cannot forbid that both teach the class!

Table of contents

Block-independent disjoint instances

- A more expressive framework than TID
- Call some attributes the key (underlined)

Block-independent disjoint instances

- A more expressive framework than TID
- Call some attributes the key (underlined)

U

<u>mon</u>	<u>day</u>	teacher	room
Jan	04	Silviu	C017
Jan	04	Antoine	C017
Jan	11	Silviu	C47
Jan	11	Antoine	C017

Block-independent disjoint instances

- A more expressive framework than TID
- Call some attributes the key (underlined)

U

mon	day	teacher	room
Jan	04	Silviu	C017
Jan	04	Antoine	C017
Jan	11	Silviu	C47
Jan	11	Antoine	C017

- The blocks are the sets of tuples with the same key

Block-independent disjoint instances

- A more expressive framework than TID
- Call some attributes the key (underlined)

U

mon	day	teacher	room
Jan	04	Silviu	C017
Jan	04	Antoine	C017
Jan	11	Silviu	C47
Jan	11	Antoine	C017

- The blocks are the sets of tuples with the same key
- Each tuple has a probability

Block-independent disjoint instances

- A more expressive framework than TID
- Call some attributes the key (underlined)

U

<u>mon</u>	<u>day</u>	teacher	room	
Jan	04	Silviu	C017	0.9
Jan	04	Antoine	C017	0.1
Jan	11	Silviu	C47	0.8
Jan	11	Antoine	C017	0.1

- The blocks are the sets of tuples with the same key
- Each tuple has a probability

Block-independent disjoint instances

- A more expressive framework than TID
- Call some attributes the key (underlined)

U

<u>mon</u>	<u>day</u>	teacher	room	
Jan	04	Silviu	C017	0.9
Jan	04	Antoine	C017	0.1
Jan	11	Silviu	C47	0.8
Jan	11	Antoine	C017	0.1

- The blocks are the sets of tuples with the same key
- Each tuple has a probability
- Probabilities must sum to ≤ 1 in each block

BID semantics

U

mon	day	teacher	room	
Jan	04	Silviu	C017	0.9
Jan	04	Antoine	C017	0.1
Jan	11	Silviu	C47	0.8
Jan	11	Antoine	C017	0.1

BID semantics

U

<u>mon</u>	<u>day</u>	teacher	room	
Jan	04	Silviu	C017	0.9
Jan	04	Antoine	C017	0.1
Jan	11	Silviu	C47	0.8
Jan	11	Antoine	C017	0.1

- For each block:

BID semantics

U

<u>mon</u>	<u>day</u>	teacher	room	
Jan	04	Silviu	C017	0.9
Jan	04	Antoine	C017	0.1
Jan	11	Silviu	C47	0.8
Jan	11	Antoine	C017	0.1

- For each block:
    - Pick one tuple according to probabilities

BID semantics

U

<u>**mon**</u>	<u>**day**</u>	**teacher**	**room**	
Jan	04	Silviu	C017	0.9
Jan	04	Antoine	C017	0.1
Jan	11	Silviu	C47	0.8
Jan	11	Antoine	C017	0.1

- For each block:
    - Pick one tuple according to probabilities
    - Possibly no tuple if probabilities are < 1

BID semantics

U

mon	day	teacher	room	
Jan	04	Silviu	C017	0.9
Jan	04	Antoine	C017	0.1
Jan	11	Silviu	C47	0.8
Jan	11	Antoine	C017	0.1

- For each block:
  - Pick one tuple according to probabilities
  - Possibly no tuple if probabilities are < 1

$\rightarrow$ Do choices independently in each block

BID semantics

U

mon	day	teacher	room	
Jan	04	Silviu	C017	0.9
Jan	04	Antoine	C017	0.1
Jan	11	Silviu	C47	0.8
Jan	11	Antoine	C017	0.1

U

mon	day	teacher	room

- For each **block**:
  - Pick **one** tuple according to probabilities
  - Possibly **no** tuple if probabilities are < 1

→ Do choices **independently** in each block

BID semantics

U

mon	day	teacher	room	
Jan	04	Silviu	C017	0.9
Jan	04	Antoine	C017	0.1
Jan	11	Silviu	C47	0.8
Jan	11	Antoine	C017	0.1

U

mon	day	teacher	room
Jan	04	Silviu	C017
Jan	04	Antoine	C017

- For each **block**:
  - Pick **one** tuple according to probabilities
  - Possibly **no** tuple if probabilities are < 1

$\rightarrow$ Do choices **independently** in each block

BID semantics

U

mon	day	teacher	room	
Jan	04	Silviu	C017	0.9
Jan	04	Antoine	C017	0.1
Jan	11	Silviu	C47	0.8
Jan	11	Antoine	C017	0.1

U

mon	day	teacher	room
Jan	04	Silviu	C017
Jan	04	Antoine	C017
Jan	11	Silviu	C47
Jan	11	Antoine	C017

- For each block:
  - Pick one tuple according to probabilities
  - Possibly no tuple if probabilities are < 1

→ Do choices independently in each block

BID captures TID

- Each TID can be expressed as a BID...

BID captures TID

- Each TID can be expressed as a BID...
  - → Take all attributes as key
  - → Each block contains a single tuple

BID captures TID

- Each TID can be expressed as a BID...
  - → Take all attributes as key
  - → Each block contains a single tuple

U

date	teacher	room	
04	Silviu	C017	0.8
04	Antoine	C017	0.2
11	Silviu	C017	1

Expressiveness of BID

Can we represent all probabilistic instances with BID?

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"The class is taught by exactly two among Antoine, Silviu, Fabian."

Expressiveness of BID

Can we represent all probabilistic instances with BID?

"The class is taught by exactly two among Antoine, Silviu, Fabian."

U_1

teacher
Silviu
Fabian

$\pi(U_1) = 0.8$

Expressiveness of BID

Can we represent **all** probabilistic instances with BID?

"The class is taught by exactly two among Antoine, Silviu, Fabian."

U_1

teacher
Silviu
Fabian
$\pi(U_1) = 0.8$

U_2

teacher
Antoine
Fabian
$\pi(U_2) = 0.1$

Expressiveness of BID

Can we represent **all** probabilistic instances with BID?

"The class is taught by exactly two among Antoine, Silviu, Fabian."

U_1

teacher
Silviu
Fabian
$\pi(U_1) = 0.8$

U_2

teacher
Antoine
Fabian
$\pi(U_2) = 0.1$

U_3

teacher
Antoine
Silviu
$\pi(U_3) = 0.1$

Expressiveness of BID

Can we represent all probabilistic instances with BID?

"The class is taught by exactly two among Antoine, Silviu, Fabian."

U_1

teacher
Silviu
Fabian
$\pi(U_1) = 0.8$

U_2

teacher
Antoine
Fabian
$\pi(U_2) = 0.1$

U_3

teacher
Antoine
Silviu
$\pi(U_3) = 0.1$

$\rightarrow$ If **teacher** is a key **teacher**, then TID

Expressiveness of BID

Can we represent all probabilistic instances with BID?

"The class is taught by exactly two among Antoine, Silviu, Fabian."

U_1

teacher
Silviu
Fabian
$\pi(U_1) = 0.8$

U_2

teacher
Antoine
Fabian
$\pi(U_2) = 0.1$

U_3

teacher
Antoine
Silviu
$\pi(U_3) = 0.1$

→ If **teacher** is a key **<u>teacher</u>**, then TID

→ If **teacher** is not a key, then only one tuple

Expressiveness of BID

Can we represent all probabilistic instances with BID?

"The class is taught by exactly two among Antoine, Silviu, Fabian."

U_1

teacher
Silviu
Fabian
$\pi(U_1) = 0.8$

U_2

teacher
Antoine
Fabian
$\pi(U_2) = 0.1$

U_3

teacher
Antoine
Silviu
$\pi(U_3) = 0.1$

→ If **teacher** is a key **teacher**, then TID

→ If **teacher** is not a key, then only one tuple

→ We cannot represent this probabilistic instance as a BID

Table of contents

Boolean c-tables

Remember Boolean c-tables:

- Set of Boolean variables $x_1, x_2, \ldots$
- Each tuple has a condition: Variables, Boolean operators

Boolean c-tables

Remember Boolean c-tables:

- Set of Boolean variables $x_1, x_2, \ldots$
- Each tuple has a condition: Variables, Boolean operators

date	teacher	room	
04	Silviu	C42	$\neg x_1$
04	Antoine	C42	x_1
11	Silviu	C017	$x_2 \land \neg x_1$
11	Antoine	C017	$x_2 \land x_1$
11	Silviu	C47	$\neg x_2 \land \neg x_1$
11	Antoine	C47	$\neg x_2 \land x_1$

x_1 Silviu is sick

x_2 Projector in C017 is working

pc-tables

A (Boolean) ***pc-table*** *is a Boolean c-table*
plus a ***probability*** p_i *for each* x_i
indicating the ***independent probability*** *that* x_i *is true.*

pc-tables

*A (Boolean) **pc-table** is a Boolean c-table*
*plus a **probability** p_i for each x_i*
*indicating the **independent probability** that x_i is true.*

Formally:

- A Boolean valuation ν of the x_i maps each to 0 or 1
  - Possible world of the Boolean c-instance under ν
  - The possible worlds are the worlds over all valuations

pc-tables

*A (Boolean) **pc-table** is a Boolean c-table plus a **probability** p_i for each x_i indicating the **independent probability** that x_i is true.*

Formally:

- A Boolean **valuation** ν of the x_i maps each to 0 or 1
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    - $\rightarrow$ Yeah, it's like TID instances!

pc-table example

date	teacher	room	
04	Silviu	C42	$\neg x_1$
04	Antoine	C42	x_1
11	Silviu	C017	$x_2 \wedge \neg x_1$
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x_1 Silviu is sick

x_2 Projector in C017 is working

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$\rightarrow$ Probability 0.1

x_2 Projector in C017 is working

$\rightarrow$ Probability 0.2

pc-table semantics example

date	**teacher**	**room**	$x_1 : 0.1, x_2 : 0.2$
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→ Probability of possible world: sum over the valuations

→ Here: only this valuation, 0.18

pc-tables capture TID

Give each tuple its own variable:

U

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→ Give each variable the probability of the tuple

pc-tables capture mutually exclusive

- Remember non-Boolean c-tables

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mon	day	teacher	room	
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- Give a probability to each value of x, summing up to 1

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  → Example: x has probability:
  - 0.8 to be 1
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  → Example: x has probability:
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- Remember our rewriting from non-Boolean to Boolean...

Reminder: rewriting non-Boolean to Boolean

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mon	day	teacher	room	
Jan	04	Silviu	C017	$x = 00$
Jan	04	Antoine	C017	$x = 01$
Jan	04	Fabian	C017	$x = 10$

Reminder: rewriting non-Boolean to Boolean

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$\rightarrow$ How to choose the probabilities?

Choosing the probabilities

- We start with the probabilities:
  - $x = 00$ has probability 0.8
  - $x = 01$ has probability 0.1
  - $x = 10$ has probability 0.1
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  Either the first bit is 0 or it is 1:
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Converting mutually exclusive to pc-tables

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- Probabilities: x has proba 0.8 to be 1, 0.1 to be 2, 0.1 to be 3

→ Rewriting:

Converting mutually exclusive to pc-tables

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Jan	04	Fabian	C017	$x_1 \wedge \neg x_2'$

→ x_1 has proba $1/9$, x_2 has proba $1/2$, x_2' has proba 0

Capturing BID with pc-tables

- This process generalizes: create decision trees

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day	teacher	room	
04	Silviu	C017	0.9
04	Antoine	C017	0.1
11	Silviu	C47	0.8
11	Antoine	C017	0.1

Capturing BID with pc-tables

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x_1 has probability 0.1

y_1 has probability 0.1

y_2 has probability 1/9

Strong representation system

- Remember from last class:
  Boolean c-tables are a strong representation system
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→ pc-tables are a strong representation system

Capturing all probabilistic instances

- Remember:
  - Support $\mathcal{U}$: uncertain relation
    - Here, set of subsets of a finite set of tuples
  - Probability distribution π on $\mathcal{U}$

Capturing all probabilistic instances

- Remember:
  - Support $\mathcal{U}$: uncertain relation
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$\rightarrow$ Can any probabilistic instance be represented by a pc-table?

Capturing uncertain instances with Boolean c-tables (1)

Remember from last time:

- Number the possible worlds in binary
- For each tuple, write the possible worlds where it appears

Capturing uncertain instances with Boolean c-tables (1)

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00		01		10		11	
v	**w**	**v**	**w**	**v**	**w**	**v**	**w**
a	d	a	d	a	d	a	d
b	e	b	e	b	e	b	e
c	f	c	f	c	f	c	f

Capturing uncertain instances with Boolean c-tables (1)

Remember from last time:

- Number the possible worlds in binary
- For each tuple, write the possible worlds where it appears

00

v	w
a	d
b	e
c	f

01

v	w
a	d
b	e
c	f

10

v	w
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b	e
c	f

11

v	w
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b	e
c	f

v	w	
a	d	$x = 00 \lor x = 01 \lor x = 10 \lor x = 11$
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Capturing uncertain instances with Boolean c-tables (1)

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a	d	a	d	a	d	a	d
b	e	b	e	b	e	b	e
c	f	c	f	c	f	c	f

V	**W**	
a	d	$x = 00 \lor x = 01 \lor x = 10 \lor x = 11$
b	e	$x = 01$
c	f	$x = 01 \lor x = 10 \lor x = 11$

→ We can also do this with pc-tables

Capturing uncertain instances with Boolean c-tables (2)

Remember: the second step was to reduce to binary:

v	**w**	
a	d	$x = 00 \lor x = 01 \lor x = 10 \lor x = 11$
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Capturing uncertain instances with Boolean c-tables (2)

Remember: the second step was to reduce to binary:

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v	**w**	
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b	e	$x = 01$
c	f	$x = 01 \vee x = 10 \vee x = 11$

v	w	
a	d	$\neg x_1 \wedge \neg x_2 \vee \neg x_1 \wedge x_2 \vee x_1 \wedge \neg x_2 \vee x_1 \wedge x_2$
b	e	$\neg x_1 \wedge x_2$
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- For pc-instances, how to choose the probabilities?

Capturing uncertain instances with Boolean c-tables (2)

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a	d	$\neg x_1 \wedge \neg x_2 \vee \neg x_1 \wedge x_2 \vee x_1 \wedge \neg x_2 \vee x_1 \wedge x_2$
b	e	$\neg x_1 \wedge x_2$
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- For pc-instances, how to choose the probabilities?

$\rightarrow$ We have seen this: this is encoding a mutually exclusive choice

Table of contents

Summary

We have seen relational formalisms for probabilistic instances:

- TID, a simple model with independent probabilities on tuples
- BID, adding blocks with mutually exclusive choices
- pc-tables, i.e., Boolean c-tables with probabilities on variables

$\rightarrow$ pc-tables can capture any probabilistic instance

Summary

We have seen relational formalisms for probabilistic instances:

- TID, a simple model with independent probabilities on tuples
- BID, adding blocks with mutually exclusive choices
- pc-tables, i.e., Boolean c-tables with probabilities on variables

$\rightarrow$ pc-tables can capture any probabilistic instance

- In the next class: how to evaluate queries efficiently
- Let's see a few advanced topics

Conditioning

- With probabilities, conditioning is a common operation
  - *What is the probability that it rains given that the grass is wet?*

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$\rightarrow$ Easy to condition by $x_1 = 1$

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Conditioning

- With probabilities, conditioning is a common operation
  - *What is the probability that it rains given that the grass is wet?*
- Conditioning a pc-table (the distribution on possible worlds):
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→ Semantics: ignore valuations that violate the global condition

→ Easier to add things to the global condition

Other models

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- Continuous distributions: impose conditions like "this value follows a normal distribution"
  - Usually intractable to reason with
  - MCDBs: Monte Carlo DataBases: use sampling

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- TID instances plus UCQ do not suffice
  - → Always a maximal world for inclusion

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