

# Exercise sheet for Session 4

## Uncertain data management

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### 1 Exercise 1: From BID to pc-instances

The Bureau for Invasion and Domination in Westeros, using cutting-edge machine learning techniques, has determined an uncertain forecast of the future political state of key cities in the Seven Kingdoms, represented as the following BID instance. (The absence of a city in the table means that the city was sacked and burnt to the ground.)

| Westeros       |              |     |
|----------------|--------------|-----|
| <u>city</u>    | <u>house</u> |     |
| Winterfell     | Stark        | 0.3 |
| Winterfell     | Greyjoy      | 0.2 |
| Winterfell     | Bolton       | 0.4 |
| King's Landing | Lannister    | 0.5 |
| King's Landing | Baratheon    | 0.2 |
| King's Landing | Targaryen    | 0.2 |
| King's Landing | Stark        | 0.1 |

**Question 1.** Suppose that we know that King's Landing fell to house Stark. What is the probability, knowing this information, that Winterfell is also controlled by house Stark?

**Question 2.** Using the technique shown in class, write a pc-table that represents the same probabilistic relation.

**Question 3.** Describe in plain English the meaning of the variables occurring in the resulting pc-instance for the tuples containing "Winterfell".

**Question 4.** Write in the relational algebra, then in the relational calculus, a query  $Q_1$  that tests whether there is some house that controls two different cities.

**Question 5.** What is the probability that the pc-instance satisfies  $Q_1$ ? How many possible worlds of the pc-instance instance satisfy  $Q_1$ ? How many valuations of the pc-table satisfy  $Q_1$ ?

**Question 6.** Write in the relational algebra, then in the relational calculus, a query  $Q_2$  that computes which houses control at least one city.

**Question 7.** Construct a pc-table to represent the output of the query  $Q_2(\text{Westeros})$ . How many possible worlds does it have? Compute the probability that house Stark holds some city.

**Question 8.** Prove that  $Q_2(\text{Westeros})$  cannot be represented by a TID instance.

**Question 9.** Prove that  $Q_2(\text{Westeros})$  cannot be represented by a BID instance.

## 2 Exercise 2: Adding probabilities to a c-table

Consider the (slightly modified) Boolean c-table obtained in the first question of Exercise 3 for Session 2:

| Classes |        |         |      |                             |
|---------|--------|---------|------|-----------------------------|
| session | date   | prof    | room |                             |
| 2       | Nov 30 | Antoine | C017 |                             |
| 3       | Dec 7  | Antoine | C47  | $\neg x_1$                  |
| 4       | Dec 14 | Silviu  | C47  | $\neg x_1 \wedge \neg x'_3$ |
| 5       | Jan 4  | Silviu  | C47  | $\neg x_1 \wedge x_2$       |
| 6       | Jan 11 | Silviu  | C47  | $\neg x_1 \wedge x_2$       |

Recall also the semantics of the events:

- $x_1$ : Room C47 collapses. All UDM classes in room C47 must be canceled.
- $x_2$ : D&K students accept to return from vacation. If this does *not* happen, all UDM classes in January are cancelled.
- $x'_3$ : Silviu is sick on December 14, we must cancel this class.

**Question 1.** Assign the following probabilities that the variables are true:

- $x_1$ : 0.01
- $x_2$ : 0.2
- $x'_3$ : 0.1

Consider the resulting pc-instance. For each tuple, compute the probability that this tuple is present, i.e., the total mass of the possible worlds where it occurs. Construct the TID instance `Classes2` on the same tuples, where each tuple carries this probability.

**Question 2.** Consider the Boolean query  $Q$  that asks whether Silviu teaches a class. Write it in the relational algebra and in the relational calculus.

**Question 3.** Consider the result of evaluating  $Q$  on the pc-table. What would be the Boolean formula that would annotate the one empty tuple of this result?

**Question 4.** What is the probability of this Boolean formula? What to conclude about  $Q$  on the pc-instance?

**Question 5.** How does this compare to the probability that the TID instance satisfies query  $Q$ ? What to conclude about the pc-instance and the TID instance?

**Question 6.** We observe that the December 7th class has taken place. We formally define the distribution on possible worlds *conditioned* by this observation, where the universe of possible worlds consists of those consistent with the observation, and the probability of each world is its probability in the initial instance, divided by the total probability of the possible world of the initial instance that satisfy the observation.

Write a pc-instance `Classes3` describing the conditioned distribution.