Uncertain Data Management
Reminders on Relational Algebra and Calculus

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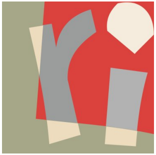

Thanks to Pierre Senellart for proofreading these slides.

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Relations

- Relation name and arity
- Attribute names (optional)

Relation Class, arity 5

date	**teacher**	**resp**	**name**	**num**

Relations

- Relation name and arity
- Attribute names (optional)

Relation Class, arity 5

date	**teacher**	**resp**	**name**	**num**
2015-11-23	Antoine	Fabian	Uncert. Data Mgmt	1
2015-11-30	Antoine	Fabian	Uncert. Data Mgmt	2
2015-12-07	Antoine	Fabian	Uncert. Data Mgmt	3
2015-11-14	Silviu	Fabian	Uncert. Data Mgmt	4

- Set of rows (tuples) on a domain (no duplicates)

Relational signature

- Signature σ: set of relation names and attributes
  - Example: Class(**date**, **teacher**, **resp**, **name**, **num**), Student(**id**, **name**), Member(**student**, **classname**)
- Database instance: one relation for each name in σ

Relational signature

- Signature σ: set of relation names and attributes
  - Example: Class(**date**, **teacher**, **resp**, **name**, **num**), Student(**id**, **name**), Member(**student**, **classname**)
- Database instance: one relation for each name in σ

$\rightarrow$ Query:

  - Input: database, and produces a relation

Languages

Two languages to write queries:

- The relational algebra:
  - operational way to define queries
  - based on operators to construct new relations
- The relational calculus:
  - declarative way to define queries
  - based on first-order logic

Languages

Two languages to write queries:

- The relational algebra:
  - operational way to define queries
  - based on operators to construct new relations
- The relational calculus:
  - declarative way to define queries
  - based on first-order logic

→ Codd's theorem: both have the same expressive power

Languages

~~Two~~ Three languages to write queries:

- The relational algebra:
  - operational way to define queries
  - based on operators to construct new relations
- The relational calculus:
  - declarative way to define queries
  - based on first-order logic

$\rightarrow$ Codd's theorem: both have the same expressive power

- SQL, the practical language used by databases

Table of contents

Relational algebra

- Basic relations:
  - the relation names in the signature
  - constant relations, e.g., the empty relation
- Projection Π
- Selection σ
- Renaming ρ
- Union $\cup$
- Product $\times$ and join $\bowtie$
- Difference $-$

Projection

Keep a subsequence of the attributes:

Class

date	teacher	resp	name	num
2015-11-23	Antoine	Fabian	Uncert. Data Mgmt	1
2015-11-30	Antoine	Fabian	Uncert. Data Mgmt	2
2015-12-07	Antoine	Fabian	Uncert. Data Mgmt	3
2015-11-14	Silviu	Fabian	Uncert. Data Mgmt	4

Projection

Keep a subsequence of the attributes:

Class

date	teacher	resp	name	num
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2015-11-14	Silviu	Fabian	Uncert. Data Mgmt	4

$\Pi_{\mathbf{teacher,resp}}(\text{Class})$

teacher	resp

Projection

Keep a subsequence of the attributes:

Class

date	teacher	resp	name	num
2015-11-23	Antoine	Fabian	Uncert. Data Mgmt	1
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2015-11-14	Silviu	Fabian	Uncert. Data Mgmt	4

$\Pi_{\mathbf{teacher},\mathbf{resp}}(\text{Class})$

teacher	resp
Antoine	Fabian
Silviu	Fabian

Projection

Keep a subsequence of the attributes:

Class

date	teacher	resp	name	num
2015-11-23	Antoine	Fabian	Uncert. Data Mgmt	1
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2015-12-07	Antoine	Fabian	Uncert. Data Mgmt	3
2015-11-14	Silviu	Fabian	Uncert. Data Mgmt	4

$\Pi_{\mathbf{teacher},\mathbf{resp}}(\text{Class})$

teacher	resp
Antoine	Fabian
Silviu	Fabian

→ Duplicates are removed

Selection

- Keep a subset of the tuples

Class

date	teacher	resp	name	num
2015-11-23	Antoine	Fabian	Uncert. Data Mgmt	1
2015-11-30	Antoine	Fabian	Uncert. Data Mgmt	2
2015-12-07	Antoine	Fabian	Uncert. Data Mgmt	3
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Selection

- Keep a subset of the tuples

Class

date	teacher	resp	name	num
2015-11-23	Antoine	Fabian	Uncert. Data Mgmt	1
2015-11-30	Antoine	Fabian	Uncert. Data Mgmt	2
2015-12-07	Antoine	Fabian	Uncert. Data Mgmt	3
2015-11-14	Silviu	Fabian	Uncert. Data Mgmt	4

$\sigma_{\mathbf{teacher}=\text{“Silviu”}}(\text{Class})$

date	teacher	resp	name	num

Selection

- Keep a subset of the tuples

Class

date	**teacher**	**resp**	**name**	**num**
2015-11-23	Antoine	Fabian	Uncert. Data Mgmt	1
2015-11-30	Antoine	Fabian	Uncert. Data Mgmt	2
2015-12-07	Antoine	Fabian	Uncert. Data Mgmt	3
2015-11-14	Silviu	Fabian	Uncert. Data Mgmt	4

$\sigma_{\textbf{teacher}=\text{“Silviu”}}(\text{Class})$

date	**teacher**	**resp**	**name**	**num**
2015-11-14	Silviu	Fabian	Uncert. Data Mgmt	4

Rename

- Change the name of attributes

Class

date	teacher	resp	name	num
2015-11-23	Antoine	Fabian	Uncert. Data Mgmt	1
2015-11-30	Antoine	Fabian	Uncert. Data Mgmt	2
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Rename

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$$\rho_{\mathbf{resp} \to \mathbf{boss}}(\text{Class})$$

Rename

- Change the name of attributes

Class

date	teacher	resp	name	num
2015-11-23	Antoine	Fabian	Uncert. Data Mgmt	1
2015-11-30	Antoine	Fabian	Uncert. Data Mgmt	2
2015-12-07	Antoine	Fabian	Uncert. Data Mgmt	3
2015-11-14	Silviu	Fabian	Uncert. Data Mgmt	4

$\rho_{\mathbf{resp} \to \mathbf{boss}}$(Class)

date	teacher	boss	name	num
2015-11-23	Antoine	Fabian	Uncert. Data Mgmt	1
2015-11-30	Antoine	Fabian	Uncert. Data Mgmt	2
2015-13-07	Antoine	Fabian	Uncert. Data Mgmt	3
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Union

- Take tuples occurring in one of the input tables
- Applies to two tables with the same attributes

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Students1

id	name
1	Arthur Dent
2	Ford Prefect

Union

- Take tuples occurring in one of the input tables
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Students1

id	name
1	Arthur Dent
2	Ford Prefect

$\bigcup$

Union

- Take tuples occurring in one of the input tables
- Applies to two tables with the same attributes

Students1

id	name
1	Arthur Dent
2	Ford Prefect

$\bigcup$

S2

id	name
42	Zaphod B.

Union

- Take tuples occurring in one of the input tables
- Applies to two tables with the same attributes

Students1

id	name
1	Arthur Dent
2	Ford Prefect

$\cup$

S2

id	name
42	Zaphod B.

$=$

Union

- Take tuples occurring in one of the input tables
- Applies to two tables with the same attributes

Students1

id	name
1	Arthur Dent
2	Ford Prefect

$\bigcup$

S2

id	name
42	Zaphod B.

$=$

Students1 $\cup$ S2

id	name
1	Arthur Dent
2	Ford Prefect
42	Zaphod B.

Product

- Take all combinations of the input tables

Product

- Take all combinations of the input tables

Students

id	name
1	Arthur Dent
2	Ford Prefect

Product

- Take all combinations of the input tables

Students

id	name
1	Arthur Dent
2	Ford Prefect

$\times$

Product

- Take all combinations of the input tables

Students

id	name
1	Arthur Dent
2	Ford Prefect

$\times$

Rooms

room
E200
E242

Product

- Take all combinations of the input tables

Students

id	name
1	Arthur Dent
2	Ford Prefect

$\times$

Rooms

room
E200
E242

$=$

Product

- Take all combinations of the input tables

Students

id	name
1	Arthur Dent
2	Ford Prefect

$\times$

Rooms

room
E200
E242

$=$

Students $\times$ Rooms

id	name	room
1	Arthur Dent	E200
1	Arthur Dent	E242
2	Ford Prefect	E200
2	Ford Prefect	E242

Join

→ Product is useful to express joins:

Join

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Member

id	class
1	UDM
2	UDM

Join

→ Product is useful to express joins:

Member

id	class
1	UDM
2	UDM

$\bowtie$

Join

→ Product is useful to express joins:

Member

id	class
1	UDM
2	UDM

$\bowtie$

Class

class	date
UDM	Nov 23
ABC	Nov 27
UDM	Nov 30

Join

→ Product is useful to express joins:

Member

id	class
1	UDM
2	UDM

$\bowtie$

Class

class	date
UDM	Nov 23
ABC	Nov 27
UDM	Nov 30

$=$

Join

→ Product is useful to express joins:

Member

id	class
1	UDM
2	UDM

$\bowtie$

Class

class	date
UDM	Nov 23
ABC	Nov 27
UDM	Nov 30

=

Member $\bowtie$ Class

id	class	date
1	UDM	Nov 23
1	UDM	Nov 30
2	UDM	Nov 23
2	UDM	Nov 30

Join

→ Product is useful to express joins:

Member

id	class
1	UDM
2	UDM

$\bowtie$

Class

class	date
UDM	Nov 23
ABC	Nov 27
UDM	Nov 30

=

Member $\bowtie$ Class

id	class	date
1	UDM	Nov 23
1	UDM	Nov 30
2	UDM	Nov 23
2	UDM	Nov 30

Express Member $\bowtie$ Class with the previous operators:

Join

→ Product is useful to express joins:

Member

id	class
1	UDM
2	UDM

$\bowtie$

Class

class	date
UDM	Nov 23
ABC	Nov 27
UDM	Nov 30

=

Member $\bowtie$ Class

id	class	date
1	UDM	Nov 23
1	UDM	Nov 30
2	UDM	Nov 23
2	UDM	Nov 30

Express Member $\bowtie$ Class with the previous operators:

Member $\times$ Class

Join

→ Product is useful to express joins:

Member

id	class
1	UDM
2	UDM

$\bowtie$

Class

class	date
UDM	Nov 23
ABC	Nov 27
UDM	Nov 30

=

Member $\bowtie$ Class

id	class	date
1	UDM	Nov 23
1	UDM	Nov 30
2	UDM	Nov 23
2	UDM	Nov 30

Express Member $\bowtie$ Class with the previous operators:

$$\rho_{\textbf{class}\to\textbf{class2}}(\text{Member}) \times \text{Class}$$

Join

→ Product is useful to express joins:

Member

id	class
1	UDM
2	UDM

$\bowtie$

Class

class	date
UDM	Nov 23
ABC	Nov 27
UDM	Nov 30

$=$

Member $\bowtie$ Class

id	class	date
1	UDM	Nov 23
1	UDM	Nov 30
2	UDM	Nov 23
2	UDM	Nov 30

Express Member $\bowtie$ Class with the previous operators:

$$\sigma_{\text{class}=\text{class2}} \left(\rho_{\text{class}\to\text{class2}}(\text{Member}) \times \text{Class} \right)$$

Join

→ Product is useful to express joins:

Member

id	class
1	UDM
2	UDM

$\bowtie$

Class

class	date
UDM	Nov 23
ABC	Nov 27
UDM	Nov 30

=

Member $\bowtie$ Class

id	class	date
1	UDM	Nov 23
1	UDM	Nov 30
2	UDM	Nov 23
2	UDM	Nov 30

Express Member $\bowtie$ Class with the previous operators:

$$\Pi_{\text{id,class,date}}\left(\sigma_{\text{class}=\text{class2}}\left(\rho_{\text{class}\to\text{class2}}(\text{Member}) \times \text{Class}\right)\right)$$

Difference

- Take tuples that are in one table but not in the other
- Applies to two tables with same attributes

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Difference

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- Applies to two tables with same attributes

Students1

id	name
1	Arthur Dent
2	Ford Prefect

—

Difference

- Take tuples that are in one table but not in the other
- Applies to two tables with same attributes

Students1

id	name
1	Arthur Dent
2	Ford Prefect

−

S3

id	name
1	Arthur Dent
42	Zaphod B.

Difference

- Take tuples that are in one table but not in the other
- Applies to two tables with same attributes

Students1

id	name
1	Arthur Dent
2	Ford Prefect

−

S3

id	name
1	Arthur Dent
42	Zaphod B.

=

Difference

- Take tuples that are in one table but not in the other
- Applies to two tables with same attributes

Students1

id	name
1	Arthur Dent
2	Ford Prefect

−

S3

id	name
1	Arthur Dent
42	Zaphod B.

=

Students1 − S3

id	name
2	Ford Prefect

Table of contents

Basic relations

```
CREATE TABLE Students(id INT(6), name VARCHAR(30));
INSERT INTO Students VALUES (1, 'Arthur Dent');
INSERT INTO Students VALUES (2, 'Ford Prefect');
```

Basic relations

```
CREATE TABLE Students(id INT(6), name VARCHAR(30));
INSERT INTO Students VALUES (1, 'Arthur Dent');
INSERT INTO Students VALUES (2, 'Ford Prefect');

SELECT * FROM Students;
```

Basic relations

```
CREATE TABLE Students(id INT(6), name VARCHAR(30));
INSERT INTO Students VALUES (1, 'Arthur Dent');
INSERT INTO Students VALUES (2, 'Ford Prefect');

SELECT * FROM Students;

+------+--------------+
| id   | name         |
+------+--------------+
|    1 | Arthur Dent  |
|    2 | Ford Prefect |
+------+--------------+
2 rows in set (0.00 sec)
```

Projection and rename

```
SELECT name FROM Students;
```

Projection and rename

```
SELECT name FROM Students;
```

```
+---------------+
| name          |
+---------------+
| Arthur Dent   |
| Ford Prefect  |
+---------------+
```

Projection and rename

```
SELECT name FROM Students;

+--------------+
| name         |
+--------------+
| Arthur Dent  |
| Ford Prefect |
+--------------+

SELECT name, id, id AS identifier FROM Students;
```

Projection and rename

```
SELECT name FROM Students;

+--------------+
| name         |
+--------------+
| Arthur Dent  |
| Ford Prefect |
+--------------+

SELECT name, id, id AS identifier FROM Students;

+--------------+------+------------+
| name         | id   | identifier |
+--------------+------+------------+
| Arthur Dent  |    1 |          1 |
| Ford Prefect |    2 |          2 |
+--------------+------+------------+
```

Selection

```
SELECT * FROM Students;
```

Selection

```
SELECT * FROM Students;

+------+--------------+
| id   | name         |
+------+--------------+
|    1 | Arthur Dent  |
|    2 | Ford Prefect |
+------+--------------+
```

Selection

```
SELECT * FROM Students;

+------+--------------+
| id   | name         |
+------+--------------+
|    1 | Arthur Dent  |
|    2 | Ford Prefect |
+------+--------------+

SELECT * FROM Students WHERE id='2';
```

Selection

```
SELECT * FROM Students;

+------+---------------+
| id   | name          |
+------+---------------+
|    1 | Arthur Dent   |
|    2 | Ford Prefect  |
+------+---------------+

SELECT * FROM Students WHERE id='2';

+------+---------------+
| id   | name          |
+------+---------------+
|    2 | Ford Prefect  |
+------+---------------+
```

Union

```
SELECT * FROM Students;

+------+--------------+
| id   | name         |
+------+--------------+
|    1 | Arthur Dent  |
|    2 | Ford Prefect |
+------+--------------+
```

Union

```
SELECT * FROM Students;

+------+--------------+
| id   | name         |
+------+--------------+
|    1 | Arthur Dent  |
|    2 | Ford Prefect |
+------+--------------+

SELECT * FROM S2;

+------+----------+
| id   | name     |
+------+----------+
|   42 | Zaphod B |
+------+----------+
```

Union

```
SELECT * FROM Students;

+------+--------------+
| id   | name         |
+------+--------------+
|    1 | Arthur Dent  |
|    2 | Ford Prefect |
+------+--------------+

SELECT * FROM S2;

+------+----------+
| id   | name     |
+------+----------+
|   42 | Zaphod B |
+------+----------+
```

```
(SELECT * FROM Students)
UNION
(SELECT * FROM S2);
```

Union

```
SELECT * FROM Students;

+------+---------------+
| id   | name          |
+------+---------------+
|    1 | Arthur Dent   |
|    2 | Ford Prefect  |
+------+---------------+
```

```
SELECT * FROM S2;

+------+-----------+
| id   | name      |
+------+-----------+
|   42 | Zaphod B  |
+------+-----------+
```

```
(SELECT * FROM Students)
UNION
(SELECT * FROM S2);

+------+---------------+
| id   | name          |
+------+---------------+
|    1 | Arthur Dent   |
|    2 | Ford Prefect  |
|   42 | Zaphod B      |
+------+---------------+
```

Product

```
SELECT * FROM Students;

+------+--------------+
| id   | name         |
+------+--------------+
|    1 | Arthur Dent  |
|    2 | Ford Prefect |
+------+--------------+
```

Product

```
SELECT * FROM Students;

+------+--------------+
| id   | name         |
+------+--------------+
|    1 | Arthur Dent  |
|    2 | Ford Prefect |
+------+--------------+

SELECT * FROM Rooms;

+------+
| room |
+------+
| E200 |
| E242 |
+------+
```

Product

```
SELECT * FROM Students;

+------+--------------+
| id   | name         |
+------+--------------+
|    1 | Arthur Dent  |
|    2 | Ford Prefect |
+------+--------------+
```

```
SELECT * FROM Rooms;

+------+
| room |
+------+
| E200 |
| E242 |
+------+
```

```
SELECT * FROM Students, Rooms;
```

Product

```
SELECT * FROM Students;

+-------+---------------+
| id    | name          |
+-------+---------------+
|     1 | Arthur Dent   |
|     2 | Ford Prefect  |
+-------+---------------+
```

```
SELECT * FROM Rooms;

+-------+
| room  |
+-------+
| E200  |
| E242  |
+-------+
```

```
SELECT * FROM Students, Rooms;

+-------+---------------+-------+
| id    | name          | room  |
+-------+---------------+-------+
|     1 | Arthur Dent   | E200  |
|     2 | Ford Prefect  | E200  |
|     1 | Arthur Dent   | E242  |
|     2 | Ford Prefect  | E242  |
+-------+---------------+-------+
```

Join

```
SELECT * FROM Member;
+------+-------+
| id   | class |
+------+-------+
|    1 | UDM   |
|    2 | UDM   |
+------+-------+
```

Join

```
SELECT * FROM Member;
+------+-------+
| id   | class |
+------+-------+
|    1 | UDM   |
|    2 | UDM   |
+------+-------+

SELECT * FROM Classes;
+-------+--------+
| class | date   |
+-------+--------+
| UDM   | Nov 23 |
| ABC   | Nov 27 |
| UDM   | Nov 30 |
+-------+--------+
```

Join

```
SELECT * FROM Member;

+------+-------+
| id   | class |
+------+-------+
|    1 | UDM   |
|    2 | UDM   |
+------+-------+

SELECT * FROM Classes;

+-------+--------+
| class | date   |
+-------+--------+
| UDM   | Nov 23 |
| ABC   | Nov 27 |
| UDM   | Nov 30 |
+-------+--------+
```

```
SELECT * FROM
  Member NATURAL JOIN Classes;
```

Join

```
SELECT * FROM Member;

+------+-------+
| id   | class |
+------+-------+
|    1 | UDM   |
|    2 | UDM   |
+------+-------+
```

```
SELECT * FROM Classes;

+-------+--------+
| class | date   |
+-------+--------+
| UDM   | Nov 23 |
| ABC   | Nov 27 |
| UDM   | Nov 30 |
+-------+--------+
```

```
SELECT * FROM
  Member NATURAL JOIN Classes;

+-------+------+--------+
| class | id   | date   |
+-------+------+--------+
| UDM   |    1 | Nov 23 |
| UDM   |    2 | Nov 23 |
| UDM   |    1 | Nov 30 |
| UDM   |    2 | Nov 30 |
+-------+------+--------+
```

Difference

```
SELECT * FROM Students;

+------+--------------+
| id   | name         |
+------+--------------+
|    1 | Arthur Dent  |
|    2 | Ford Prefect |
+------+--------------+
```

Difference

```
SELECT * FROM Students;

+------+--------------+
| id   | name         |
+------+--------------+
|    1 | Arthur Dent  |
|    2 | Ford Prefect |
+------+--------------+

SELECT * FROM S3;

+------+-------------+
| id   | name        |
+------+-------------+
|    1 | Arthur Dent |
|   42 | Zaphod B.   |
+------+-------------+
```

Difference

```
SELECT * FROM Students;

+------+--------------+
| id   | name         |
+------+--------------+
|    1 | Arthur Dent  |
|    2 | Ford Prefect |
+------+--------------+

SELECT * FROM S3;

+------+-------------+
| id   | name        |
+------+-------------+
|    1 | Arthur Dent |
|   42 | Zaphod B.   |
+------+-------------+
```

```
SELECT * FROM Students
WHERE (id, name) NOT IN
  (SELECT * FROM S3);
```

Difference

```
SELECT * FROM Students;

+------+--------------+
| id   | name         |
+------+--------------+
|    1 | Arthur Dent  |
|    2 | Ford Prefect |
+------+--------------+

SELECT * FROM S3;

+------+-------------+
| id   | name        |
+------+-------------+
|    1 | Arthur Dent |
|   42 | Zaphod B.   |
+------+-------------+
```

```
SELECT * FROM Students
WHERE (id, name) NOT IN
  (SELECT * FROM S3);

+------+--------------+
| id   | name         |
+------+--------------+
|    2 | Ford Prefect |
+------+--------------+
```

Composing operations

Our translation of:

```
SELECT * FROM
  Member NATURAL JOIN Classes;
```

can be expressed as:

Composing operations

Our translation of:

```
SELECT * FROM
  Member NATURAL JOIN Classes;
```

can be expressed as:

```
SELECT id, class, date FROM
  (SELECT * FROM
    (SELECT id, class AS class2 FROM Member) sub1,
    Classes
  ) sub2
  WHERE class = class2;
```

Composing operations

Our translation of:

```
SELECT * FROM
  Member NATURAL JOIN Classes;
```

can be expressed as:

```
SELECT id, class, date FROM
  (SELECT * FROM
    (SELECT id, class AS class2 FROM Member) sub1,
    Classes
  ) sub2
  WHERE class = class2;
```

→ SQL can express the relational algebra

Composing operations

Our translation of:

```
SELECT * FROM
  Member NATURAL JOIN Classes;
```

can be expressed as:

```
SELECT id, class, date FROM
  (SELECT * FROM
    (SELECT id, class AS class2 FROM Member) sub1,
    Classes
  ) sub2
  WHERE class = class2;
```

→ SQL can express the relational algebra

- ... but please, never write queries like this!

Table of contents

Signatures and instances

Remember the signature σ:

- relations having a name and an arity
  - e.g., Class has arity 5
- no more attribute names; attributes are $(1, 2, 3, 4, 5)$

Signatures and instances

Remember the signature σ:

- relations having a name and an arity
  - e.g., Class has arity 5
- no more attribute names; attributes are $(1, 2, 3, 4, 5)$

An interpretation $\mathcal{I}$:

- domain $\mathcal{D}$ of values
- for each relation name R with arity n, a set $R^{\mathcal{I}}$ of n-tuples
- we write each n-tuple as a fact: $R(a, b, c)$

Signatures and instances

Remember the signature σ:

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An interpretation $\mathcal{I}$:

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Example: $\text{Class}^{\mathcal{I}} = \{$

$$\begin{aligned}
&\text{Class}(\text{2015-11-23}, \text{Antoine}, \text{Fabian}, \text{UDM}, 1),\\
&\text{Class}(\text{2015-11-30}, \text{Antoine}, \text{Fabian}, \text{UDM}, 2),\\
&\text{Class}(\text{2015-12-07}, \text{Antoine}, \text{Fabian}, \text{UDM}, 3),\\
&\text{Class}(\text{2015-12-14}, \text{Silviu}, \text{Fabian}, \text{UDM}, 4),
\end{aligned}$$

$\}$

First-order logic

First-order logic (FO) on a signature σ:

- atoms $R(x, y, z)$
  - each x, y, z can be a variable
  - each x, y, z can be a constant (e.g., "Silviu")
- Boolean connectives: AND, OR, NOT
- existential quantification $\exists x\ \phi(x)$
- universal quantification $\forall x\ \phi(x)$
- variables can be free or bound

Satisfaction of a formula

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- Interpretation $\mathcal{I}$
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→ e.g., in $\mathcal{I} = \{S(a,b,c)\}$, for $\phi : \exists z\; S(x,y,z)$,
$\mathcal{I}$ satisfies $\phi(a,b)$.

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- etc.

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- An FO formula ϕ with free variables x, y, z can be seen as defining a query
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→ Let's translate the relational algebra operators!

Base relations and renames

- For a relation Student with arity 2, take the formula ϕ : Student(x, y)
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Base relations and renames

- For a relation Student with arity 2, take the formula ϕ : Student(x, y)
  - $\rightarrow$ free variables: x, y
- We no longer have attribute names, only positions
- We can swap variables around:
  - $\rightarrow$ If $\phi(x, y)$ defines a relation, we can define $\psi(x, y)$ as $\phi(y, x)$

Selections

If $\phi(x, y, z)$ defines a relation, we can do:

- $\sigma_{1=2}$: take $\phi(x, x, z)$ with free variables x, z
- $\sigma_{1=\text{"Silviu"}}$: take $\phi(\text{"Silviu"}, y, z)$
  - free variables y, z
  - "Silviu" is a constant

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→ This defines a relation of arity 2

→ $\psi(a, b)$ holds iff $\phi(a, b, c)$ holds for some c

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$$\psi(x, y, z) := \phi(x, y, z) \lor \phi'(x, y, z)$$

$\rightarrow$ This defines a relation with same arity

$\rightarrow$ $\psi(a, b, c)$ holds for all (a, b, c)
such that $\phi(a, b, c)$ holds or $\phi'(a, b, c)$ holds

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→ The arity is the sum of the original arities

→ $\psi(a, b, c, d)$ holds iff $\phi(a, b)$ and $\phi'(b, c)$ hold

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$\rightarrow$ In general, restrictions on when to use $\neg$

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“What are the date–name triples of classes taught by Silviu?”

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→ Easier to reason about Boolean queries in the calculus

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Standard query evaluation

Using the relational calculus, but it works the same for the algebra.

- We have an interpretation $\mathcal{I}$ ("database")
- We have a conjunctive query Q
- Our task:

  Boolean. Does $\mathcal{I}$ satisfies Q? (Yes/no, written $\mathcal{I} \models Q$)

  Non-Bool. For which values a, b, c does $\mathcal{I}$ satisfy $Q(a, b, c)$?

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- Goal: be as efficient as possible

  $\rightarrow$ computational complexity

Complexity definition

Several notions of complexity:

- Data. The query is fixed, the input is the instance
- Query. The instance is fixed, the input is the query (rare)
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By default: data complexity: for each query Q we ask

- given $\mathcal{I}$ as input
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→ Queries are usually small and data usually large

→ Scaling in the data more important than in the query

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$\rightarrow$ Highly parallelizable: with polynomial number of processors execution is in constant time

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