Data Structures for Incremental Maintenance of String Properties under Updates

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Incremental maintenance on strings

The concept: **incremental maintenance**

- You have a **string**:
  - $\rightarrow$ e.g., `aaaabaabaca`

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- You want to maintain the property efficiently
  → e.g., with low running time or memory overhead

Naive vs efficient algorithms

How can we **efficiently maintain** the property “having at least one `a`” under **substitutions** on an input string w of length n?

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  $\rightarrow$ Complexity per update: **constant** (in the RAM model)

Structure of the talk

We focus on the dynamic membership problem:
incremental maintenance of membership to a regular language

- Dynamic membership under substitution updates
  - A general-purpose $O(\log n)$ algorithm
  - Better algorithms for specific languages: [A., Jachiet, Paperman, ICALP'21]

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- Beyond dynamic membership: incremental maintenance for enumeration

Regular languages and substitution updates

Problem: dynamic membership for regular languages under substitutions

- Fix a regular language L
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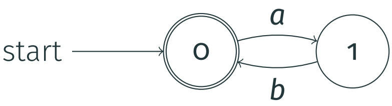
- Model: RAM model
  - Cell size in $\Theta(\log(n))$
  - Unit-cost arithmetics

A general-purpose $O(\log n)$ algorithm

Fix the language $L = (ab)^*$:



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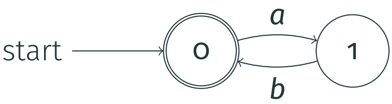

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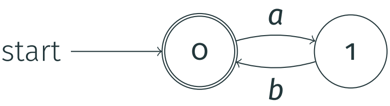

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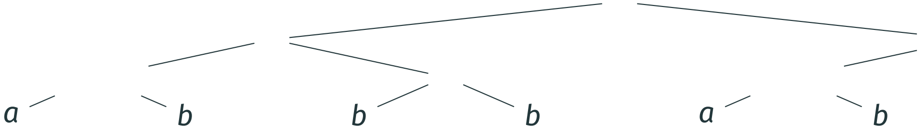

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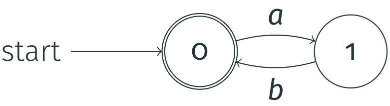

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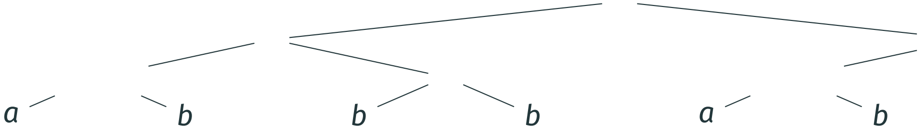

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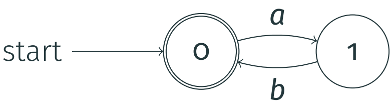

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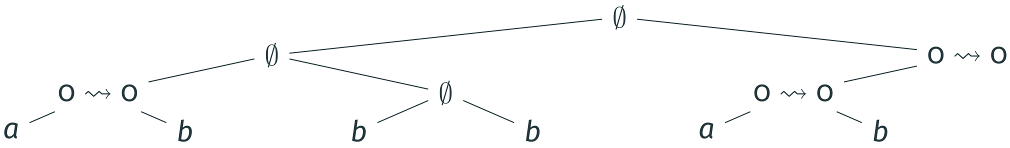

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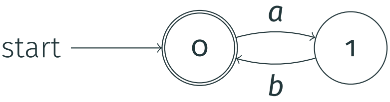

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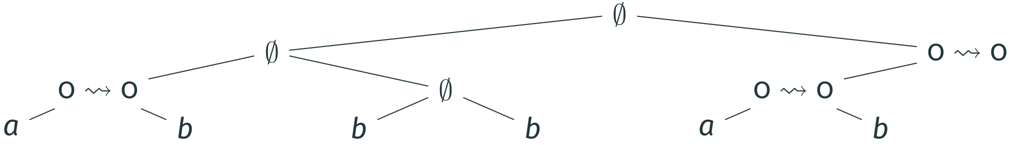

- The **tree root** describes if $w \in L$
- We can update the tree for each substitution **in $O(\log n)$**
- Can be improved to **$O(\log n / \log \log n)$** with a log-ary tree

Improving on $O(\log n)$ for some languages

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Question: what is the complexity of dynamic membership, depending on the fixed regular language L?

Summary of our results

QLZG: in $O(1)$

QSG: in $O(\log \log n)$
not in $O(1)$?

All: in $\Theta(\log n / \log \log n)$

- We identify a class **QLZG** of regular languages:
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- The problem is always in $O(\log n / \log\log n)$

Regular languages
and more expressive updates

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Proof: regular languages are closed under reversal

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- Whenever the updates shift the string too much and the guardian is far from the current middle, create a new guardian at the new middle

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Theorem (Folklore?)

Dynamic membership to any fixed regular language under insertion, substitution, deletion, split, join is possible in $O(\log n)$ time

Proof: use balancing binary trees (AVL trees) instead of the fixed complete binary tree of the $O(\log n)$ algorithm for substitutions

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Theorem (Question by Louis Jachiet, result by Kasper Green Larsen, mentioned by David Eppstein, CStheory (TCS.SE), 2020)

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→ Open question: combination of substitutions + endpoint updates

→ Open question: different models, e.g., doubly linked lists?

Incremental maintenance for enumeration structures

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  - → "compute an index to **enumerate** efficiently the factors ab^*c"

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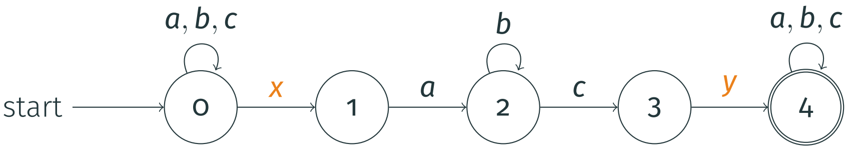
A robust notion: **automata with captures**



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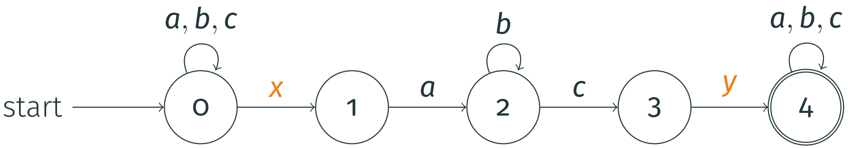
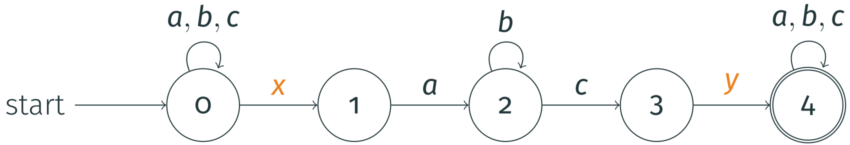
A robust notion: automata with captures



- Equivalently: monadic second-order queries with free variables
- Special case: document spanners studied in information extraction

Semantics of an automaton with captures

Consider the automaton with captures A on an input string w:



	0	1	2	3	4	5	6	7	8	9
w =	b	a	c	b	a	b	b	c	c	

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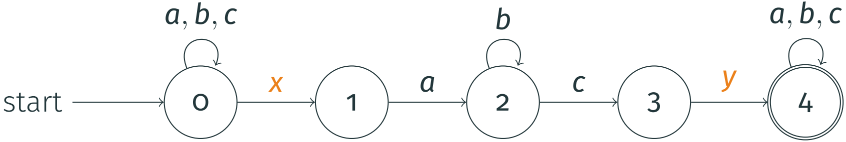

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Set of results of A on w:

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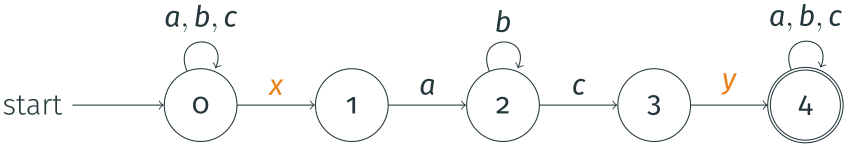

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Set of results of A on w: positions where to insert x and y in w such that A accepts

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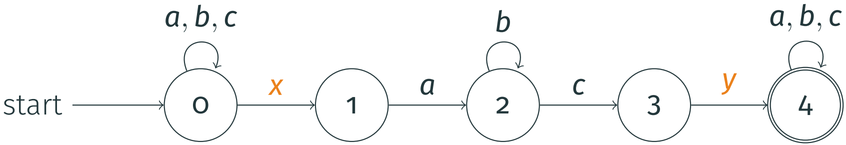

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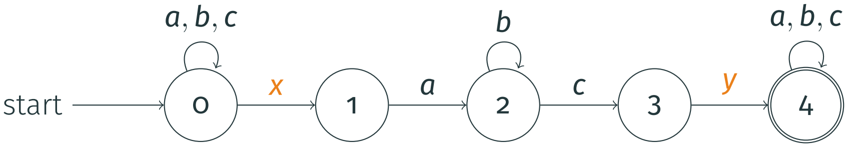

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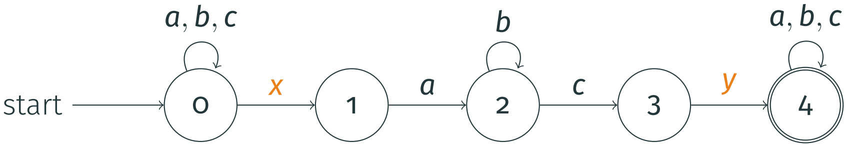

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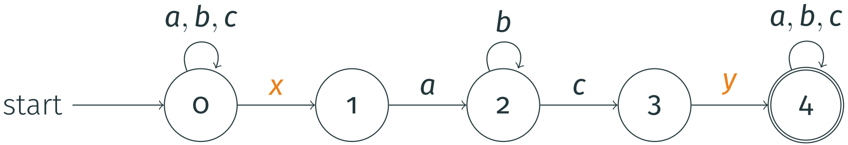

$$
\begin{array}{cccccccccccc}
 & & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
w & = & \mathtt{b} & \mathtt{a} & \mathtt{c} & \mathtt{b} & x\mathtt{a} & \mathtt{b} & \mathtt{b} & \mathtt{c} & y\mathtt{c} &
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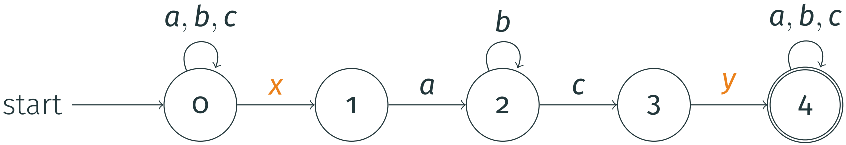

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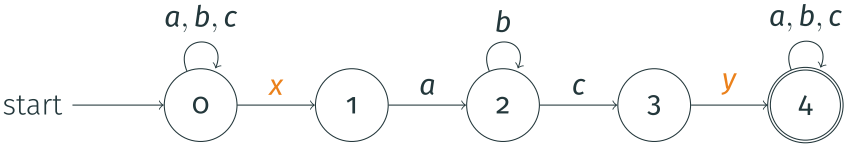

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In this case: endpoints of the factors which are in language ab^*c

Enumeration algorithms

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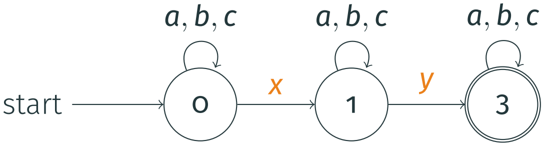

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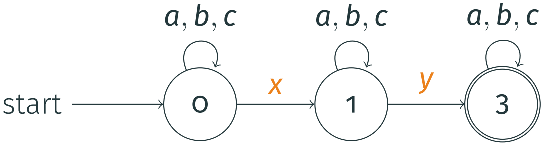

Goal: constant-delay, independent from the string length. Several uses:

- We can check if there is at least one result, in constant time
- We can produce all results in output-linear time

Enumeration without updates

How can we enumerate the results of an automaton with captures on a string (without updates)?

Theorem ([Florenzano et al., 2018])

For a fixed automaton with captures A, given a string w, we can prepare in $O(w)$ a data structure to enumerate the results with constant-delay

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$\rightarrow$ Can we incrementally maintain enumeration structures under updates?

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We can maintain a constant-delay enumeration structure for automata with captures under insertion, substitution, and deletion updates in time $O(\log n)$

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Conjecture

*Both are doable: support **join** and **split** in time $O(\log n)$ and **constant-delay***

Also: support **endpoint updates** with constant time and constant-delay

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$\rightarrow$ **Open research question!**

Conclusion and perspectives

High-level summary

- We want to **incrementally maintain** information on a string under updates
- Simple Boolean problem: **dynamic membership** to a regular language
- More expressive problem: **maintaining an enumeration structure** for an automaton with captures
- **General case:** everything should always be in $O(\log n)$ (?)
- Better cases:
  - **Endpoint updates:** everything is in $O(1)$ (?)
  - **Substitution updates** for **dynamic membership**: $O(1)$ or $O(\log \log n)$ or $\Theta(\log n / \log \log n)$ (... or?) depending on the language
- **Future research:** identify more cases below $O(\log n)$

Future directions

- Maintaining a structure for **infix testing**, membership testing, etc.
  - → Without updates: factorization forests, or structure of [Bojańczyk, 2009]
  - → With **substitutions**: amounts to **incremental maintenance** for another language
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Thanks for your attention!

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Other research themes

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