

Structurally Tractable Uncertain Data

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Uncertain data management

Is data always **complete** and **certain**?

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- **Unreliable sources**
 - Crowdsourcing
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→ We need **uncertain data management**

Example model: TID

- Consider a **relational instance**

Date	Animal
Wed 3rd	Kangaroo
Wed 3rd	Tasmanian devil
Thu 4th	Kangaroo
Thu 4th	Tasmanian devil
Fri 5th	Kangaroo
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Example model: TID

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Wed 3rd	Tasmanian devil	0%
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- Assume **independence** between facts
 - Semantics: a **probability distribution** on regular instances
- What about **queries?** (Boolean CQs)
 - Semantics: compute the **probability** that the query holds

Big problem: Tractability

- Evaluate the **fixed Boolean CQ**: $\exists xy R(x) S(x, y) T(y)$
- Measure **data complexity**, i.e., as a function of the **instance**

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- **#P-hard** [Dalvi and Suciu, 2007] (instead of AC^0)

Existing approaches:

- Avoid **hard** queries [Dalvi and Suciu, 2012]
- Use **sampling** to get **approximate** answers

The general idea

Input instances are not **arbitrary!**

- Impose **structural restrictions** on instances
- Prove **fixed-parameter tractability results**

This talk

- Parameter: instance **treewidth**
 - Bound it by a **constant**
- MSO queries have **linear** data complexity [Courcelle, 1990]

This talk

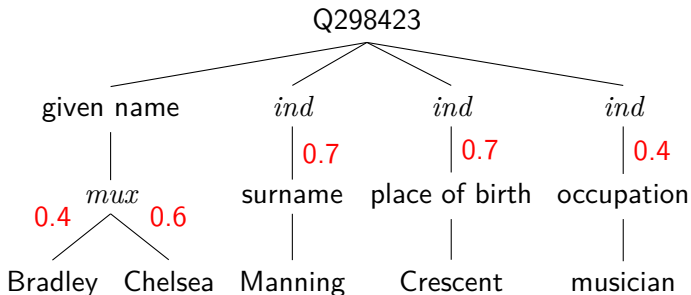
- Parameter: instance **treewidth**
- Bound it by a **constant**
- MSO queries have **linear** data complexity [Courcelle, 1990]
- Also holds on **TID instances** (with unit cost arithmetics)
(joint work with Pierre Bourhis and Pierre Senellart)

Table of contents

- 1 Introduction
- 2 Trees**
- 3 Treelike instances
- 4 Future work

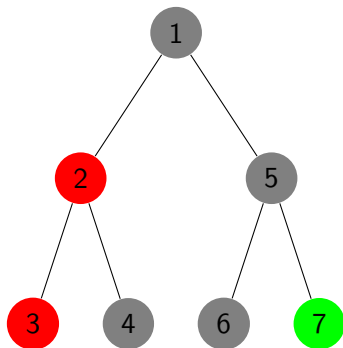
Uncertain tree example

- A possible **PrXML tree**, from Wikidata facts:



→ Probabilities reflect **contributor trustworthiness**

Formalizing uncertain trees



A **valuation** of a tree decides whether to **keep** or **discard** node labels.

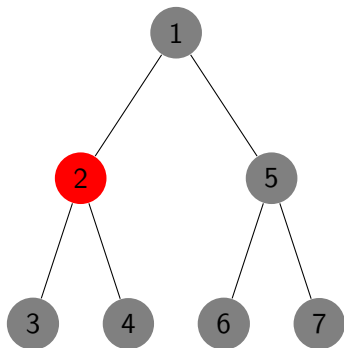
Example query:

“Is there both a red and green node?”

Valuation: $\{1, 2, 3, 4, 5, 6, 7\}$

The query is **true**

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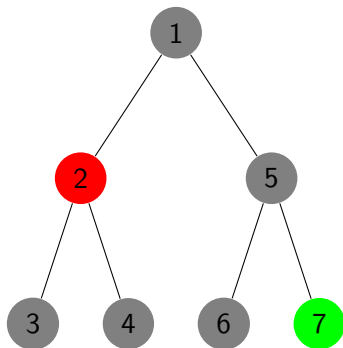
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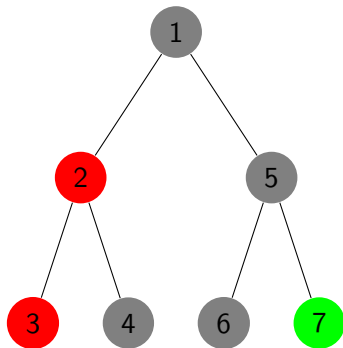
Example query:

“Is there both a red and green node?”

Valuation: $\{2, 7\}$

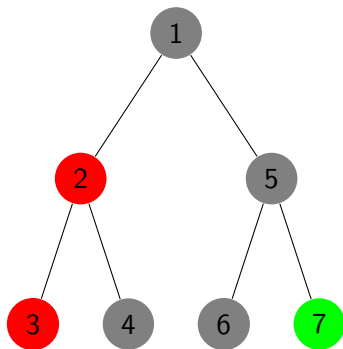
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Provenance formulae and circuits



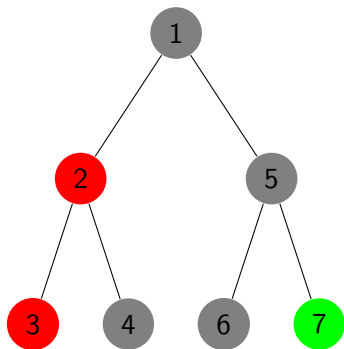
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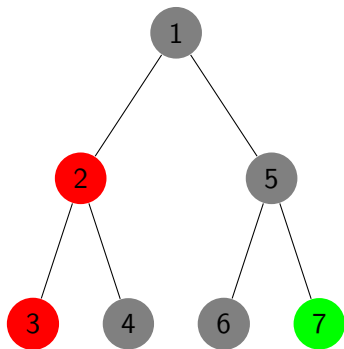
- Which **valuations** satisfy the query?
- **Provenance formula** of a query q on an uncertain tree T :
- **Boolean formula** ϕ
 - on **variables** $x_1 \dots x_7$
- $\nu(T)$ **satisfies** q iff $\nu(\phi)$ is **true**

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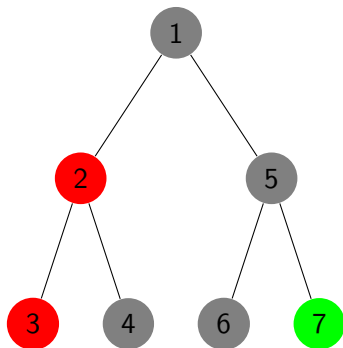
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 - $\nu(T)$ **satisfies** q iff $\nu(\phi)$ is **true**
- **Provenance circuit** of q on T [Deutch et al., 2014]
 - **Boolean circuit** C
 - with **input gates** $g_1 \dots g_7$
 - $\nu(T)$ **satisfies** q iff $\nu(C)$ is **true**

Example



Is there both a **red** and a **green** node?

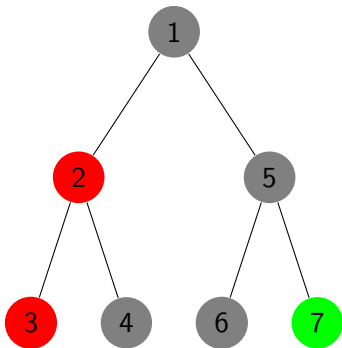
Example



Is there both a **red** and a **green** node?

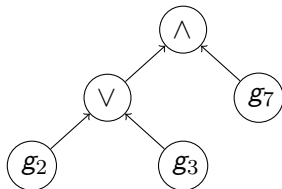
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Is there both a **red** and a **green** node?

- Provenance formula: $(x_2 \vee x_3) \wedge x_7$
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Our main result on trees

Theorem

For any fixed *MSO query* q (first order + quantify on sets) we can compute a *provenance circuit* C for any input tree T in *linear time* in the input T .

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→ Key ideas:

- Compile q to a *tree automaton* [Thatcher and Wright, 1968]
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If tree nodes have a *probability* of being independently kept, we can compute the *query probability* in linear time.

- Relates to *message passing* [Lauritzen and Spiegelhalter, 1988]
- *Already known* [Cohen et al., 2009]

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Treewidth intuition

Generalize from **trees** to **treelike instances**:

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- **Treewidth**: measure on instances
 - **Trees** have treewidth **1**
 - **Cycles** have treewidth **2**
 - **k -cliques** and **k -grids** have treewidth **$k - 1$**
- **Treelike**: the **treewidth** is bounded by a **constant**

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 - **Treelike**: the **treewidth** is bounded by a **constant**
- Treelike instances can be **encoded** to trees

Treewidth formal definition

Instance:

N

a b

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c d

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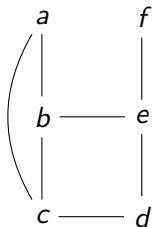
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Gaifman graph:



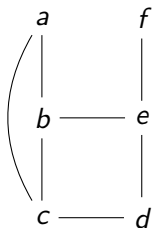
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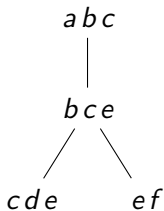
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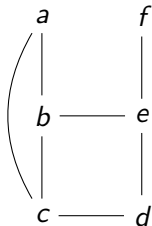
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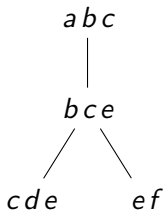
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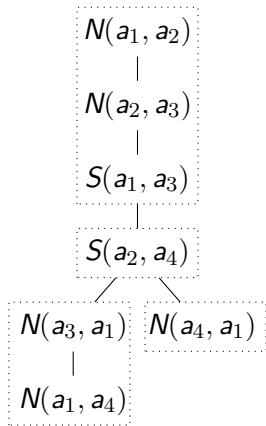
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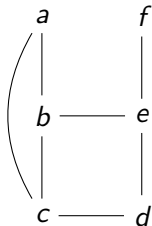
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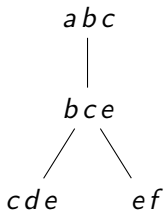
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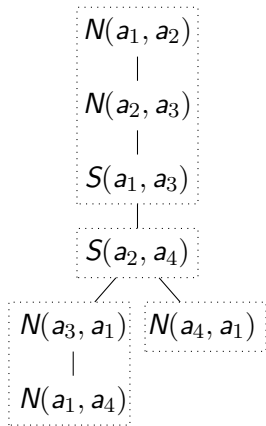
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→ Treelike: constant bound on the maximal bag size

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For any fixed *MSO query* q and bound $k \in \mathbb{N}$,
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Corollary

MSO queries have *linear* data complexity on *treelike TID instances*.

Further results

- Support **other models** with **dependencies** between facts:
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- Support **other models** with **dependencies** between facts:
 - **Block-independent disjoint** (BID): mutually exclusive facts
 - **pc-tables**: events and Boolean annotations
- Support **other tasks**:
 - **Counting query results** encodes to probabilistic evaluation
 - General connection to **semiring provenance** [Green et al., 2007]

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- **Structural restrictions:**
 - Are real-world instances **tree-like?**
 - Are there other possible **restrictions?**
 - **Experiments?**

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- What about incorporating **new evidence**?
 - Connect to work on **conditioning** [Tang et al., 2012]

Other projects and directions

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 - **Certainty** of a Boolean CQ when completing under constraints
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Thanks for your attention!





References I

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References IV



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Semiring provenance [Green et al., 2007]

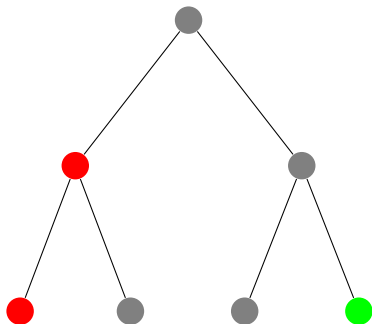
- **Semiring** $(K, \oplus, \otimes, 0, 1)$
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 - 0 absorptive for \otimes
- Idea: Maintain **annotations** on tuples while evaluating:
 - **Union**: annotation is the **sum** of union tuples
 - **Select**: select as usual
 - **Project**: annotation is the **sum** of projected tuples
 - **Product**: annotation is the **product**

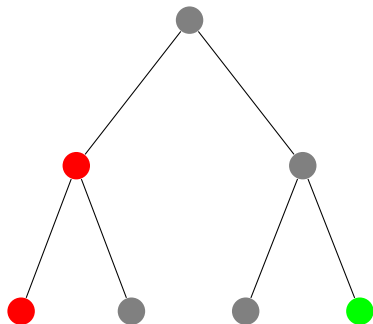
Tree automata

Tree alphabet: ● ● ●



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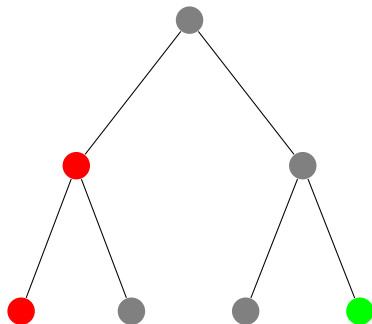
Tree alphabet: ● ● ●



- **bNTA**: bottom-up nondeterministic tree automaton
- “Is there both a red and green node?”

Tree automata

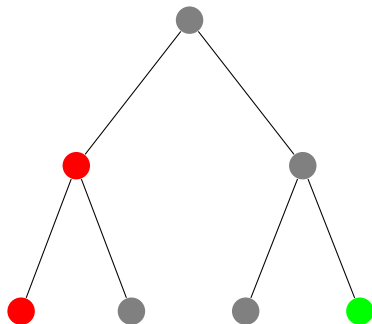
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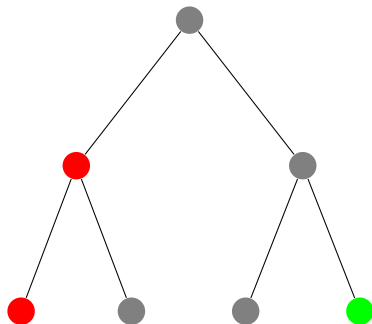
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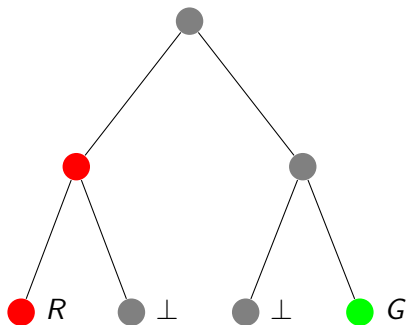
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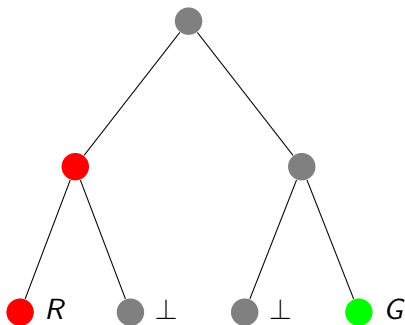


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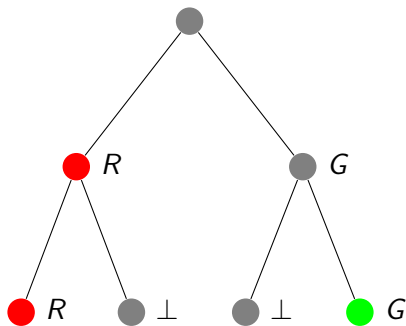
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Tree automata

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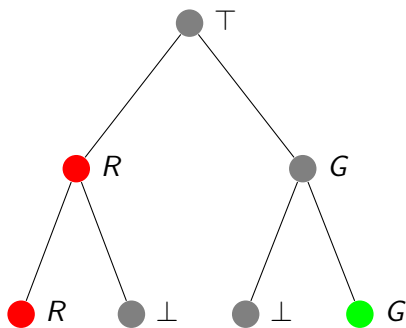


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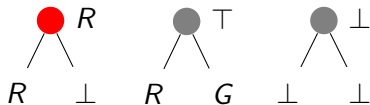
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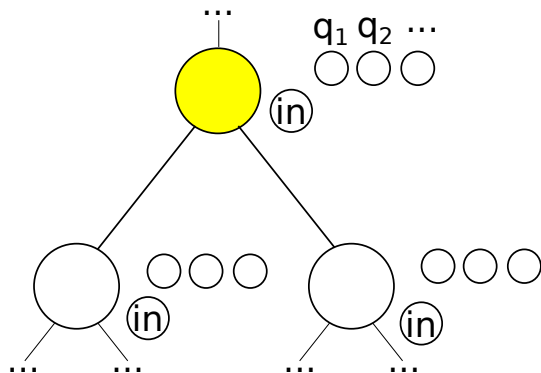
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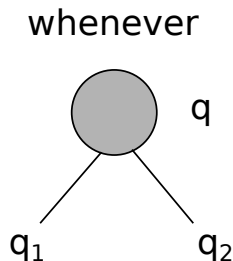
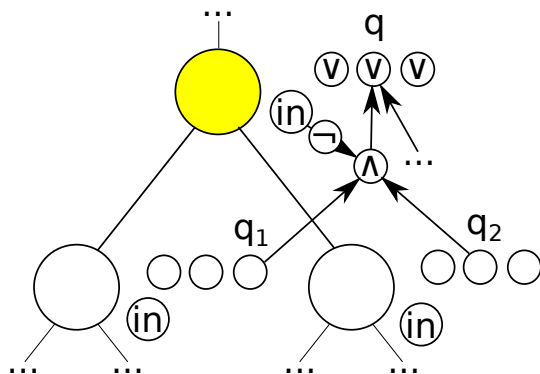
Constructing the provenance circuit

→ Construct a Boolean provenance circuit **bottom-up**



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Example: block-independent disjoint (BID) instances

<u>name</u>	city	iso	<i>p</i>
pods	melbourne	au	0.8
pods	sydney	au	0.2
icalp	tokyo	jp	0.1
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- Evaluating a fixed CQ is **#P-hard** in general
→ For a **treelike** instance, **linear time**!

Supporting coefficients

- In the world of **trees**
 - The same **valuation** can be accepted **multiple times**
 - Number of **accepting runs** of the bNTA
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 - The same **match** can be the image of **multiple homomorphisms**
- Add **assignment facts** to represent possible assignments
- Encode to a bNTA that **guesses them**

Supporting exponents

- In the world of **trees**
 - The same **fact** can be used **multiple times**
 - Annotate nodes with a **multiplicity**
 - The bNTA is **monotone** for that **multiplicity**
 - Use each **input gate** as many times as we read its fact
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 - In the world of **treelike instances**
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 - **Maximal multiplicity** is query-dependent but **instance-independent**
- Encodes CQs to bNTAs that read **multiplicities**
- Consider all possible CQ **self-homomorphisms**
 - Count the multiplicities of **identical atoms**
 - Rewrite relations to **add multiplicities**
 - Usual compilation on the **modified signature**