Tractable Lineages on Treelike Instances: Limits and Extensions

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Tuple independent databases (TID)

- Probabilistic databases: model uncertainty about data
- Simplest model: tuple independent databases (TID)
  - A relational database I
  - A probability valuation π mapping each fact of I to $[0, 1]$
- Semantics of a TID (I, π): a probability distribution on $I' \subseteq I$:
  - Each fact $F \in I$ is either present or absent with probability $\pi(F)$
  - Assume independence across facts

Example: TID

	S	
a	a	1
b	v	.5
b	w	.2

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Probabilistic query evaluation (PQE)

Let us fix:

- Relational signature σ
- Class $\mathcal{I}$ of relational instances on σ (e.g., acyclic, treelike)
- Class $\mathcal{Q}$ of Boolean queries (e.g., CQs, acyclic CQs)

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Probabilistic query evaluation (PQE) problem for $\mathcal{Q}$ and $\mathcal{I}$:

- Fix a query $q \in \mathcal{Q}$
- Read an instance $I \in \mathcal{I}$ and a probability valuation π
- Compute the probability that (I, π) satisfies q
- Data complexity: measured as a function of (I, π)

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  - $q : \exists x\, y\ R(x) \wedge S(x, y) \wedge T(y)$ is unsafe!

Instance-based dichotomy

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→ **This talk:** PQE is intractable for $\mathcal{Q}$ on any unbounded-treewidth class $\mathcal{I}$ (under some assumptions)

→ Treewidth measures how much data is close to a tree

- Trees have treewidth 1
- Cycles have treewidth 2
- k-cliques and $(k-1)$-grids have treewidth $k-1$

Main result: hardness of PQE

Theorem

For any arity-2 signature σ, there is a first-order query q such that for any constructible unbounded-treewidth class $\mathcal{I}$, the PQE problem for $\mathcal{Q} = \{q\}$ and $\mathcal{I}$ is #P-hard under RP reductions

- Arity-2: Relations have arity ≤ 2 (and one has arity 2), i.e., graphs
- Unbounded-treewidth: for all $k \in \mathbb{N}$, there is $I_k \in \mathcal{I}$ of treewidth $\geq k$
- Constructible: given k, we can compute such an instance I_k in PTIME
- #P-hard under RP reductions: reduce in PTIME with high probability from the problem of counting accepting paths of a PTIME machine

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Idea: Topological minors

- G is a topological minor of H if:

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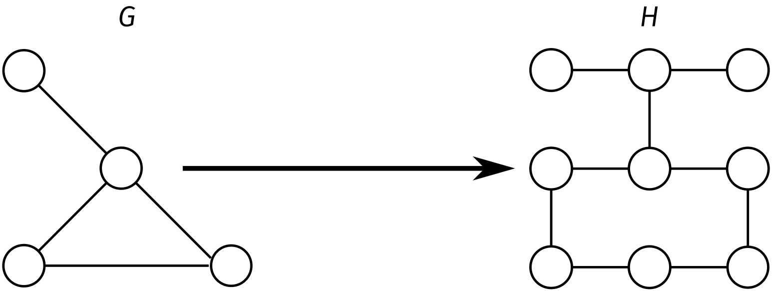

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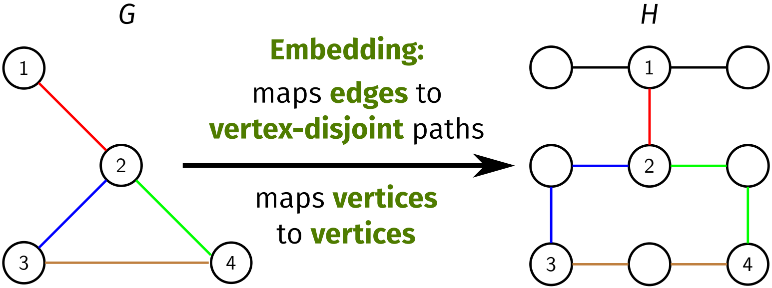

Topological minor extraction results

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More recently:

Theorem [Chekuri and Chuzhoy, 2014]
There is a certain constant $c \in \mathbb{N}$ such that
for any planar graph G of degree ≤ 3 and graph H of treewidth $\geq |G|^c$,
we can embed G as a topological minor of H in PTIME with high proba

Proof sketch

- Choose a problem from which to reduce:
  - Must be #P-hard on planar degree-3 graphs
  - Must be encodable to an FO query q

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- Compute in PTIME an instance I_k of $\mathcal{I}$ of treewidth $\geq k$
- Compute in randomized PTIME an embedding of G in I_k
- Construct a probability valuation π of I_k such that:
  - Unneccessary edges of I_k are removed
  - PQE for q gives the answer to the hard problem

  $\rightarrow$ Easy for MSO but trickier for FO

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Non-probabilistic evaluation

Apply [Chekuri and Chuzhoy, 2014] to method of [Ganian et al., 2014] for MSO non-probabilistic query evaluation (QE):

Theorem

For any arity-2 signature σ and level Σ_i^P of the polynomial hierarchy, there is a MSO query q_i such that, for any constructible, subinstance-closed, unbounded-tw class $\mathcal{I}$, the QE problem for $\mathcal{Q} = \{q_i\}$ and $\mathcal{I}$ is Σ_i^P-hard under RP reductions

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Variant: we also show a result like in [Ganian et al., 2014], with:

- weaker constructibility requirement on $\mathcal{I}$:
  - $\rightarrow$ $\mathcal{I}$ is densely unbounded poly-logarithmically
- stronger complexity hypothesis:
  - $\rightarrow$ PH does not admit $2^{o(n)}$-sized circuits

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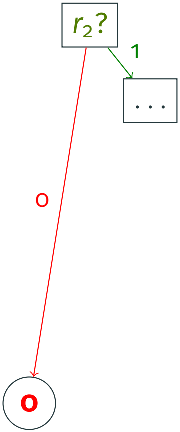

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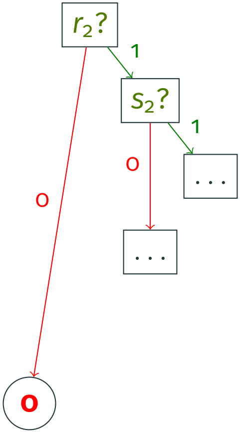

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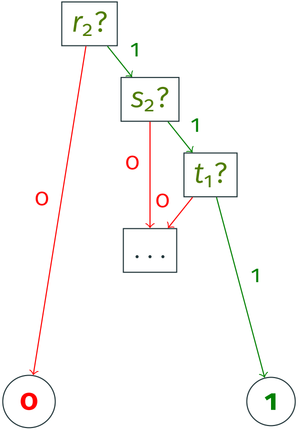

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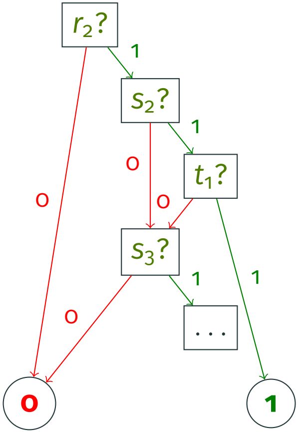

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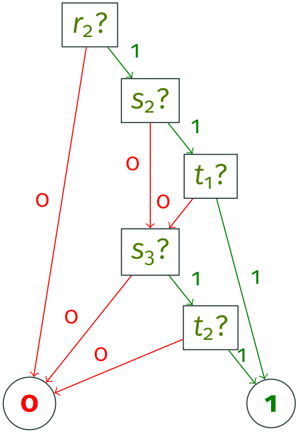

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$\rightarrow$ Meta-dichotomy on the connected UCQ$^{\neq}$ for which this holds

Also in paper

- Hardness results for MSO match counting:
  - $\rightarrow$ Count how many subsets X are such that I satisfies $q(X)$
- Connect the tractability result of MSO on treelike TID to the study of tractable lineages:
  - $\rightarrow$ d-DNNFs, OBDDs, formulae, etc.
- Connect the same result to tractability of safe queries:
  - Inversion-free queries: subclass of safe queries, tractable OBDDs
  - $\rightarrow$ We can always rewrite their input instances to treelike instances in a lineage-preserving way (hence, probability-preserving)

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Summary of our dichotomy

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instance family is #P-hard under RP reductions

Summary of our dichotomy

Upper. PQE for MSO on treelike instances has linear data complexity up to arithmetic costs

Lower. PQE for FO on any constructible, arity-2, unbounded-tw instance family is #P-hard under RP reductions

$\rightarrow$ Bounded treewidth is the right notion for tractability of PQE

Future work

- Can we show #P-hardness under usual P reductions?
  - $\rightarrow$ Depends on [Chekuri and Chuzhoy, 2014]

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Thanks for your attention!

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Encoding treelike instances [Chaudhuri and Vardi, 1992]

Instance:

N	
a	b
b	c
c	d
d	e
e	f

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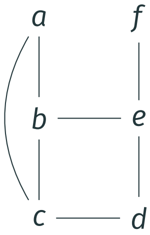
Gaifman graph:



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b	*c*
c	*d*
d	*e*
e	*f*

S	
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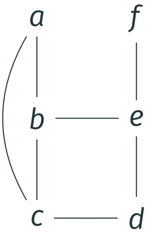
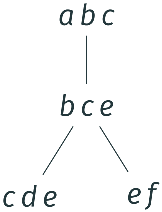
Gaifman graph:



Tree decomp.:



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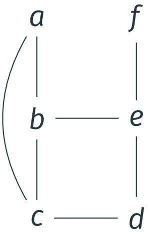

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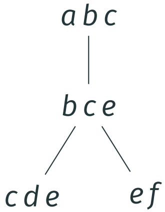
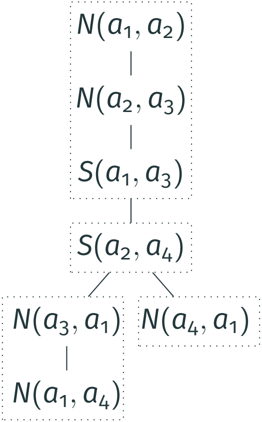
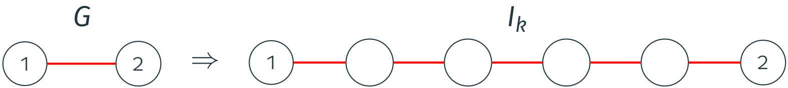


Tree encoding:



Defining the query



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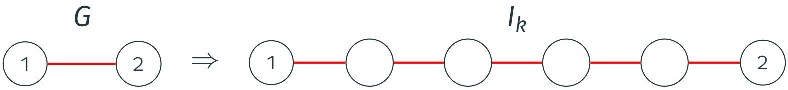

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→ Our q restricts to a subset of the worlds of known weight and gives the right answer up to renormalization