Open-World Finite Query Answering with Number Restrictions

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Open-world query answering (QA)

- Open-world query answering:
  - Relational instance I (ground facts), correct but incomplete
  - Boolean conjunctive query q
  - Consider all possible completions $J \supseteq I$
  - Is q certain, i.e., holds on all completions? If yes, $I \models_{\mathrm{unr}} q$

Constraints

- Impose constraints on the possible completions
  - Inclusion dependencies: some facts imply more facts:
    $\forall xd \text{ Employee}(x, d) \Rightarrow \exists y \text{ Advises}(x, y, d)$
    $\text{Employee}[1, 2] \subseteq \text{Advises}[1, 3]$

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→ Consider all $J \supseteq I$ that satisfy the constraints

→ Is q certain on all such completions? If yes, $I, \Sigma \models_{\mathrm{unr}} q$

Finite vs infinite

Instance: List of adviser–advisee pairs: Advises$(a, b), \ldots$

UID: Each advisee advises someone
Advises$[2] \subseteq$ Advises$[1]$

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→ Σ finitely controllable: $\forall I \, \forall q, \; I, \Sigma \models_{\mathrm{fin}} q$ iff $I, \Sigma \models_{\mathrm{unr}} q$

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- (Finite) open-world query answering is decidable for IDs and FDs with relations of arity ≤ 2 [Pratt-Hartmann, 2009, Ibáñez-García et al., 2014]

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Theorem

Finite open-world query answering for UIDs and FDs has PTIME data complexity and NP-complete combined complexity.

Main idea

- Find which UIDs and FDs are implied over finite instances
  - → This is PTIME [Cosmadakis et al., 1990]
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- Finite closure: add all finitely implied dependencies

→ Taking the finite closure ensure finite controllability?

Theorem

For any instance I, conjunctive query q, UIDs and FDs Σ, letting Σ^* be the finite closure of Σ, we have $I, \Sigma \models_{\mathrm{fin}} q$ iff $I, \Sigma^* \models_{\mathrm{unr}} q$.

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$\rightarrow$ Construct a finite completion J' satisfying Σ but not q:

- Start with I
- Follow J and add facts to satisfy the UIDs of Σ...
- ... using finitely many elements (reuse them)
- ... but respecting the FDs of Σ
- ... and without making q inadvertantly true

Constructing the finite completion

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$$\xrightarrow{\mathbf{R}} \qquad \xrightarrow{\mathbf{S}}$$

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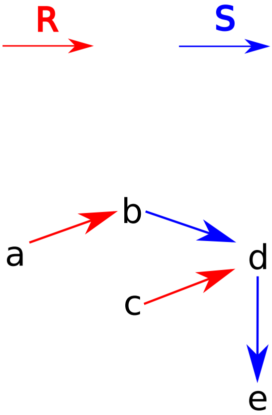

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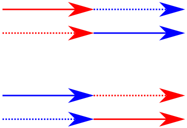

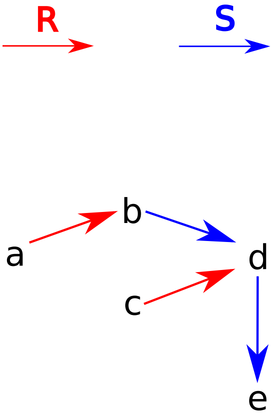

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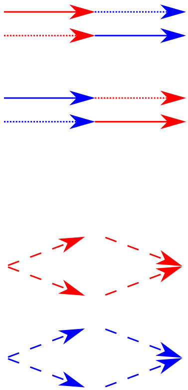

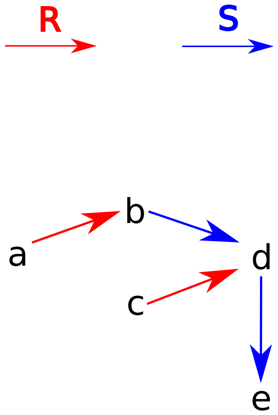

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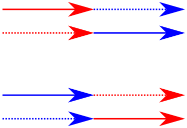

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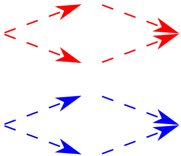

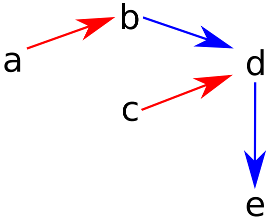

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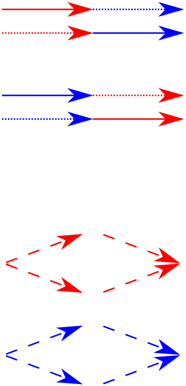

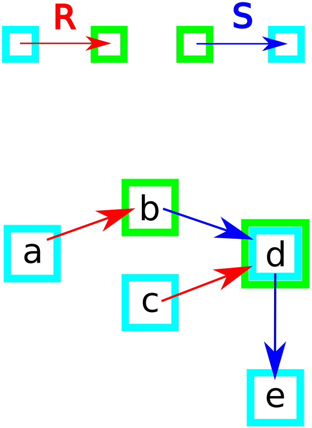

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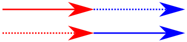

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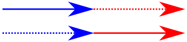

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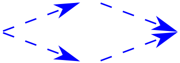

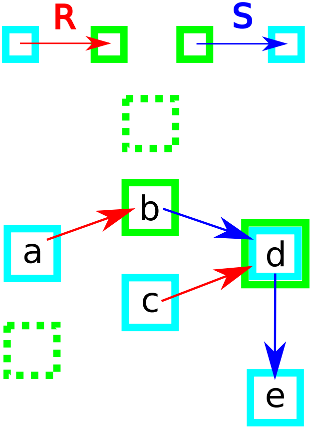

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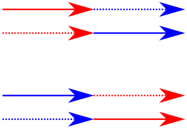

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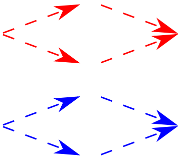

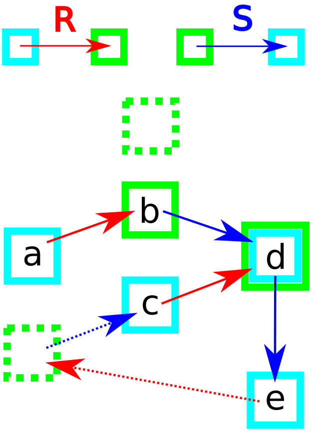

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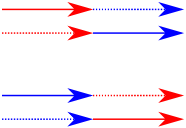

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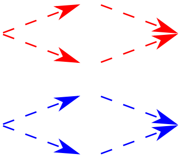

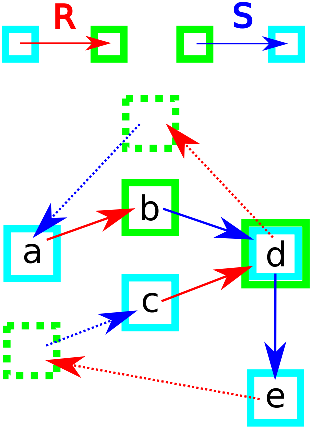

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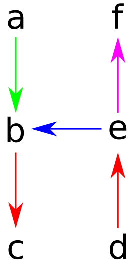

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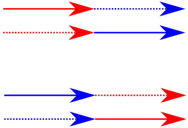

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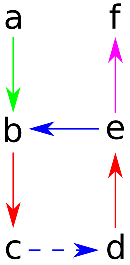

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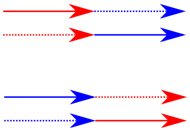

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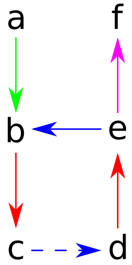

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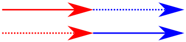

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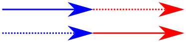
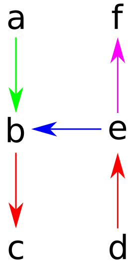


and UFDs...

2. Adding acyclic queries



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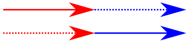
UIDs:

$R[2] \mathrel{\substack{\subseteq \\ \supseteq}} S[1]$



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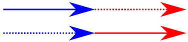

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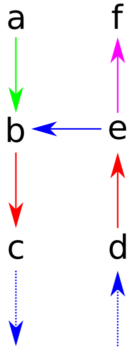

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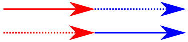

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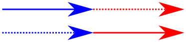

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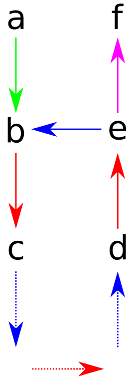

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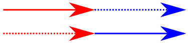

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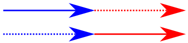

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  - conjunctive query

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Thanks for your attention!

References I

Calì, A., Gottlob, G., and Pieris, A. (2012).
Towards more expressive ontology languages: The query answering problem.
Artif. Intel., 193.

Calì, A., Lembo, D., and Rosati, R. (2003).
On the decidability and complexity of query answering over inconsistent and incomplete databases.
In *PODS*.

Cosmadakis, S. S., Kanellakis, P. C., and Vardi, M. Y. (1990).
Polynomial-time implication problems for unary inclusion dependencies.
JACM, 37(1).

References II

Ibáñez-García, Y., Lutz, C., and Schneider, T. (2014).
Finite model reasoning in Horn description logics.
In *KR*.

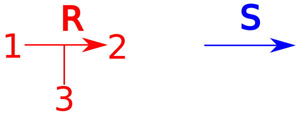
Otto, M. (2002).
Modal and guarded characterisation theorems over finite transition systems.
In *LICS*.

Pratt-Hartmann, I. (2009).
Data-complexity of the two-variable fragment with counting quantifiers.
Inf. Comput., 207(8).

References III

Rosati, R. (2006).
On the decidability and finite controllability of query processing in databases with incomplete information.
In *PODS*.

1. Supporting arbitrary arity



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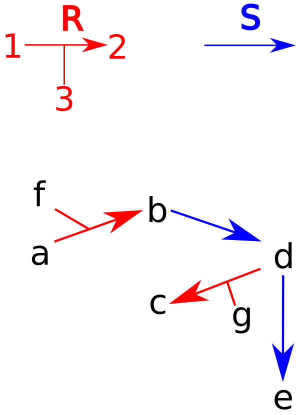

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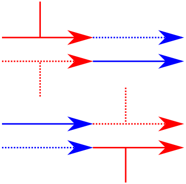

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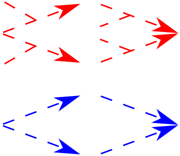

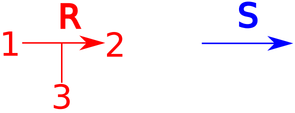

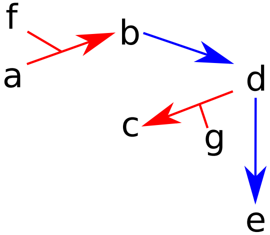

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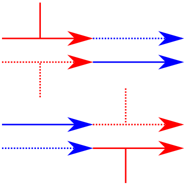

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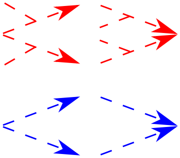

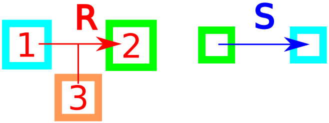

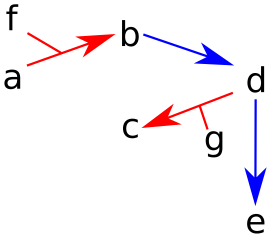

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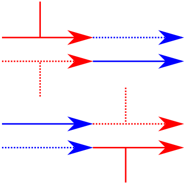

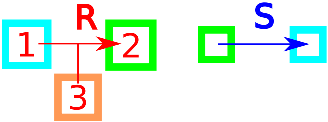

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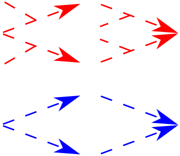

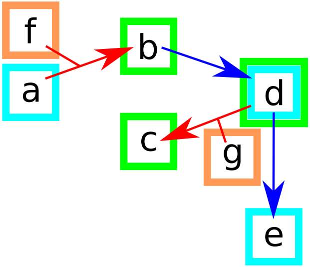

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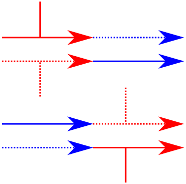

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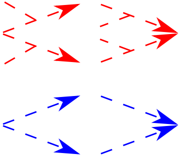

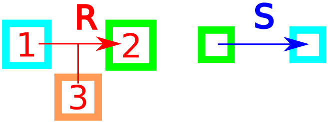

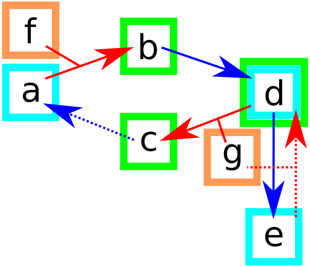

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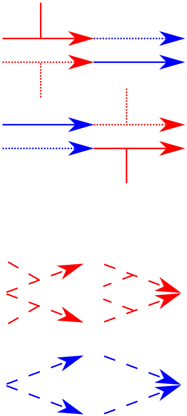

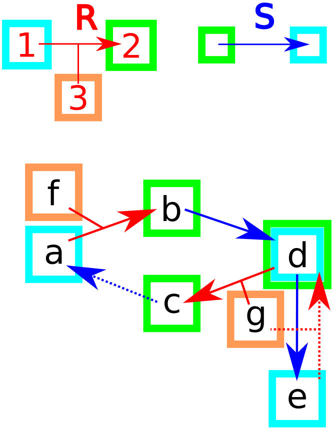

- When no UFDs: reuse elements from suitable positions

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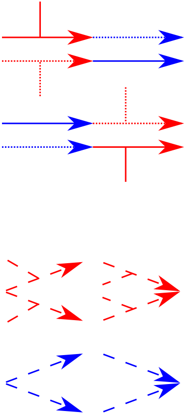

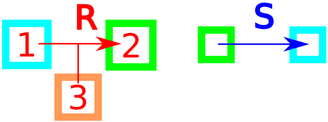

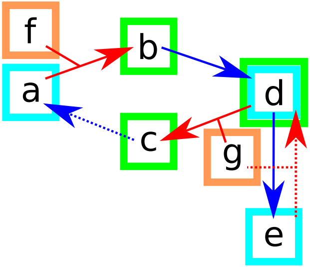

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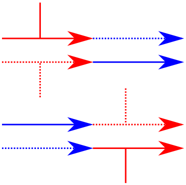

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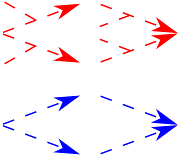
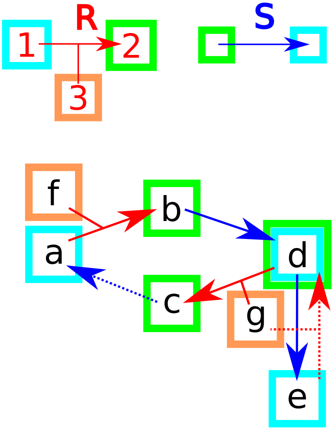
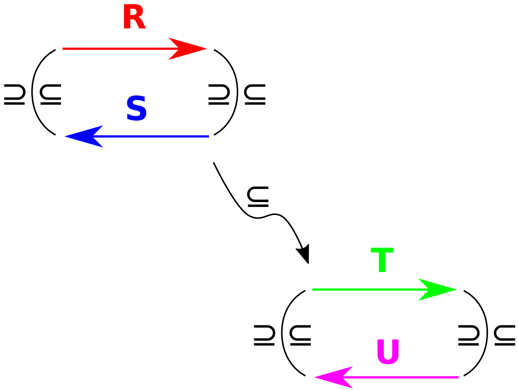




- When no UFDs: reuse elements from suitable positions
  - $\rightarrow$ Must saturate initially to make sure such elements exist
- With UFDs: bijective maps within $\rightleftarrows$-classes

3. From reversible to arbitrary UIDs and UFDs



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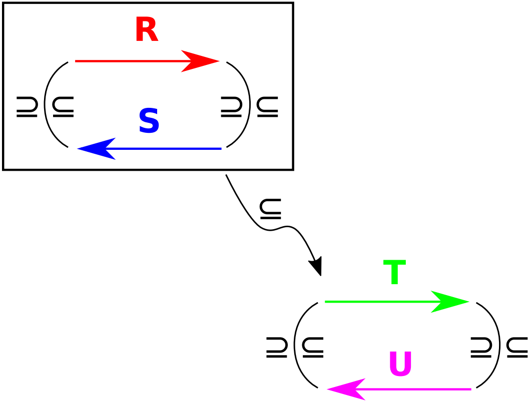

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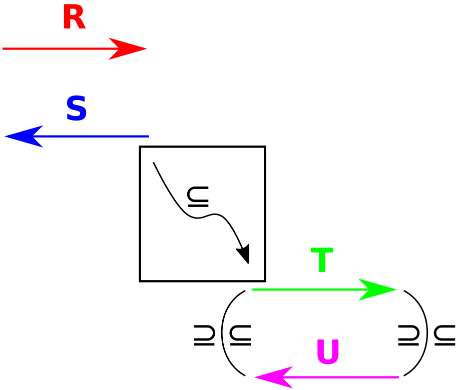

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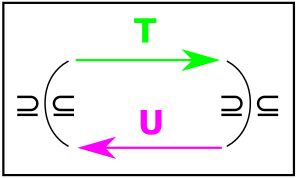
$$\xrightarrow{\mathbf{R}}$$

$$\xleftarrow{\mathbf{S}}$$



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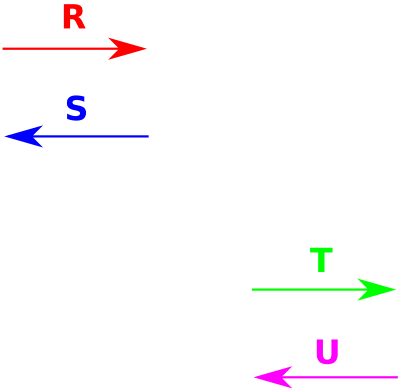

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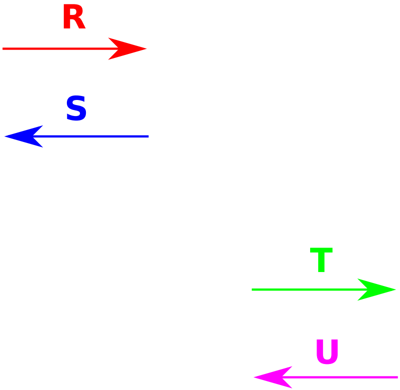

- UIDs can be decomposed in reversible subsets
- The subsets can be solved independently
- Relies on the finite implication construction [Cosmadakis et al., 1990]

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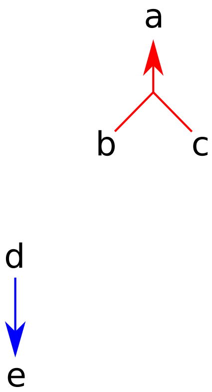

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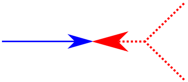

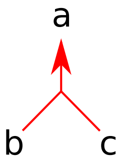

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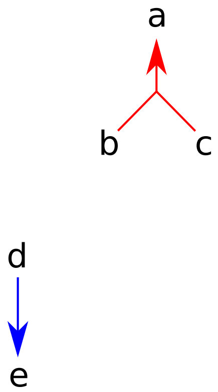

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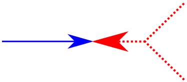
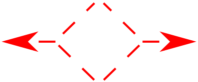


FD:

R[1,3] → R[2]



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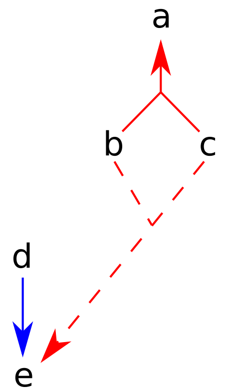

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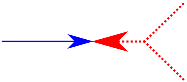

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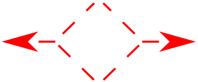

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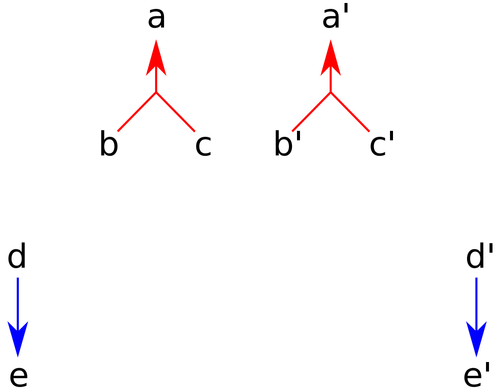

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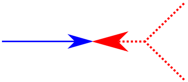

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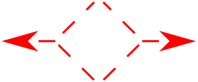

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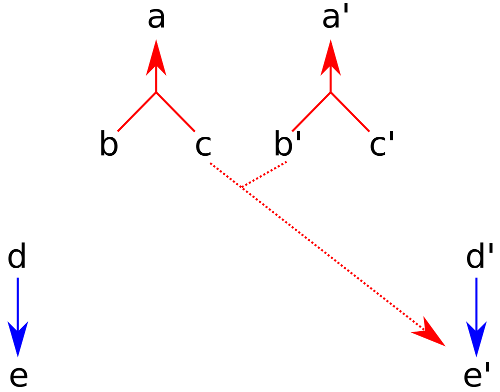

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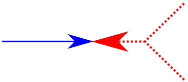

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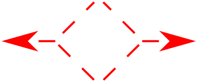

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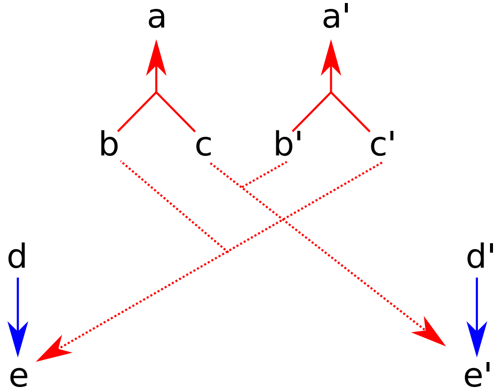
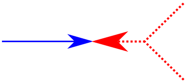


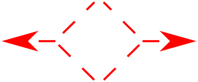
UID:

S[2] ⊆ R[2]

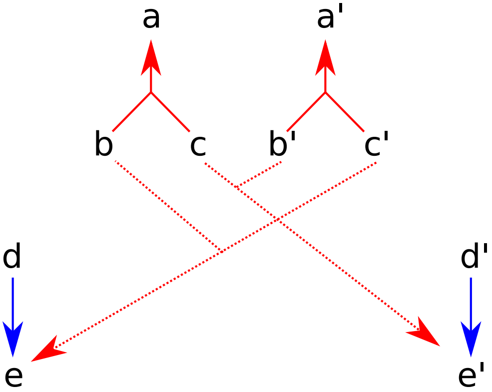


FD:

R[1,3] → R[2]



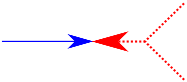
4. From UFDs to FDs



- Create initially many reusable elements
- Reuse them in new combinations

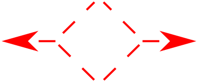
UID:

S[2] ⊆ R[2]

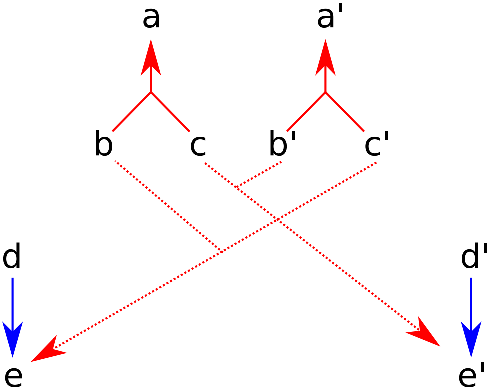


FD:

R[1,3] → R[2]

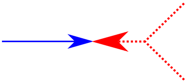


4. From UFDs to FDs



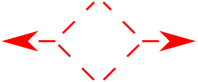
UID:

S[2] ⊆ R[2]



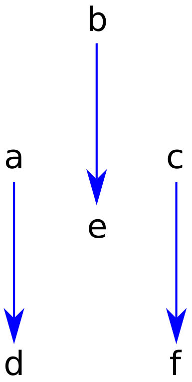
FD:

R[1,3] → R[2]

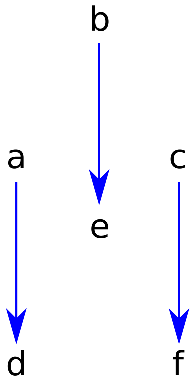


- Create initially many reusable elements
- Reuse them in new combinations
- Dense interpretations lemma:
  linearly many elements give super-linearly many combinations

5. From acyclic queries to arbitrary queries



5. From acyclic queries to arbitrary queries

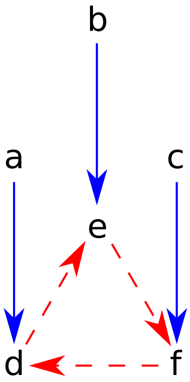


UID:

$S[2] \subseteq R[1]$

$S[2] \subseteq R[2]$

5. From acyclic queries to arbitrary queries

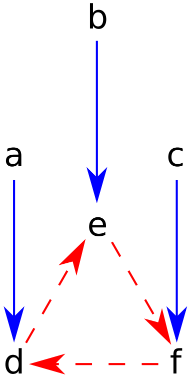


UID:

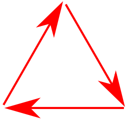
$S[2] \subseteq R[1]$

$S[2] \subseteq R[2]$

5. From acyclic queries to arbitrary queries



query:

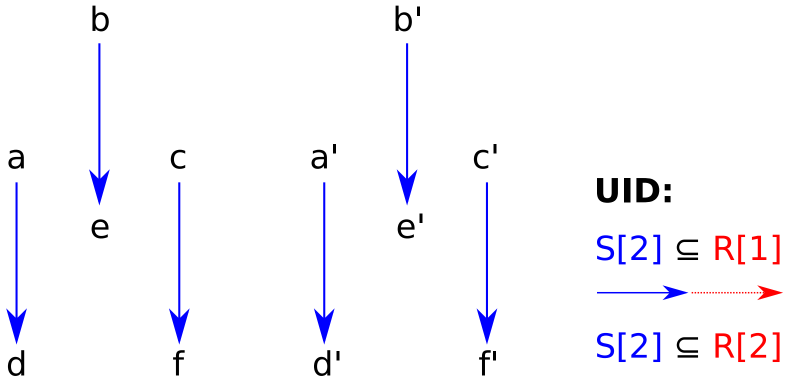


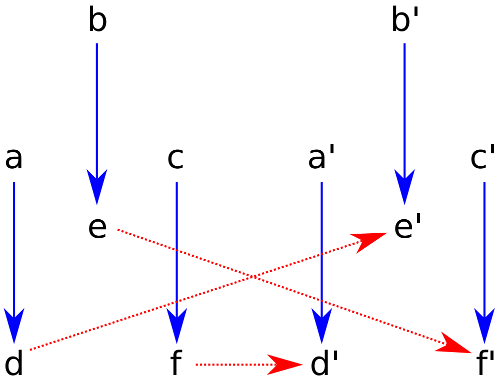
UID:

$S[2] \subseteq R[1]$

$S[2] \subseteq R[2]$

5. From acyclic queries to arbitrary queries



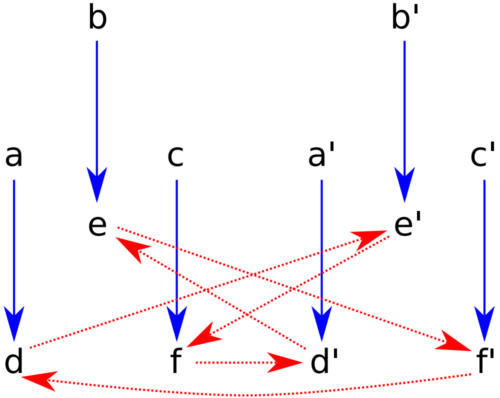


UID:

S[2] ⊆ R[1]

S[2] ⊆ R[2]

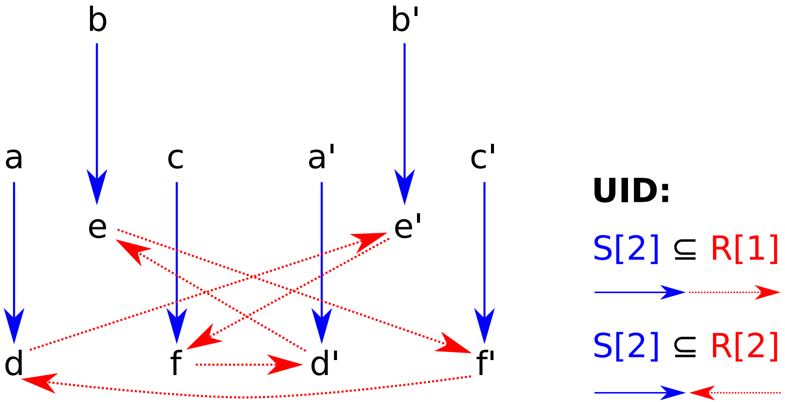
5. From acyclic queries to arbitrary queries



UID:

S[2] ⊆ R[1]

S[2] ⊆ R[2]



- Product with a group of high girth [Otto, 2002]
- Must be tweaked to avoid violating FDs