Query Evaluation on Probabilistic Data
A Story of Dichotomies

Antoine Amarilli

May 19, 2020

Télécom Paris

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Uncertain data management

Relational databases manage data, represented here as a labeled graph

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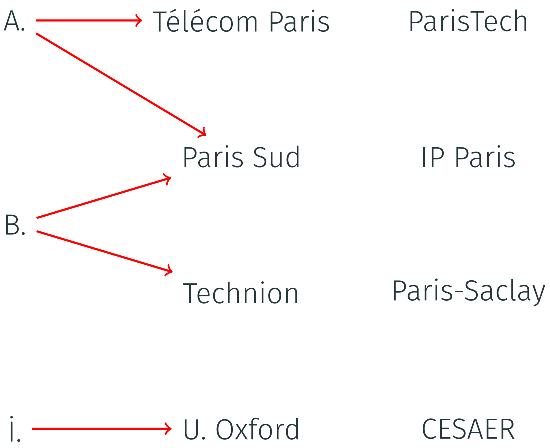

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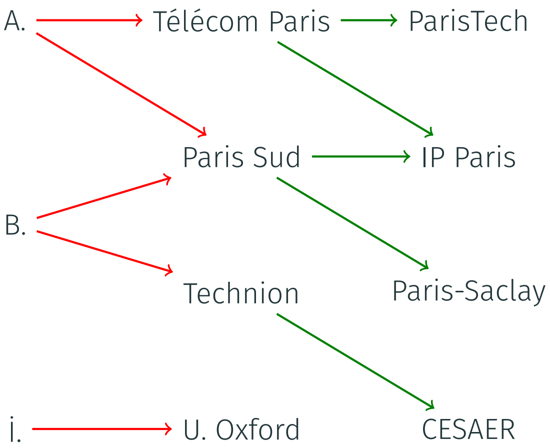

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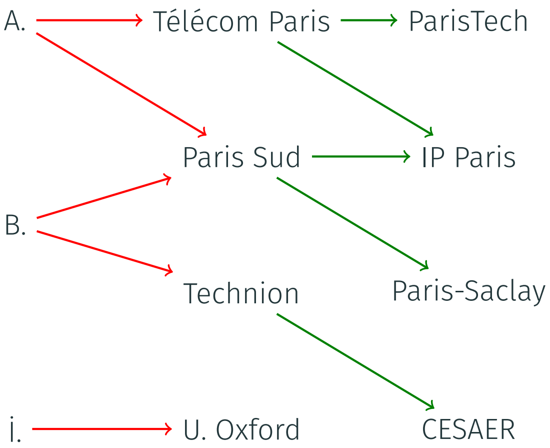
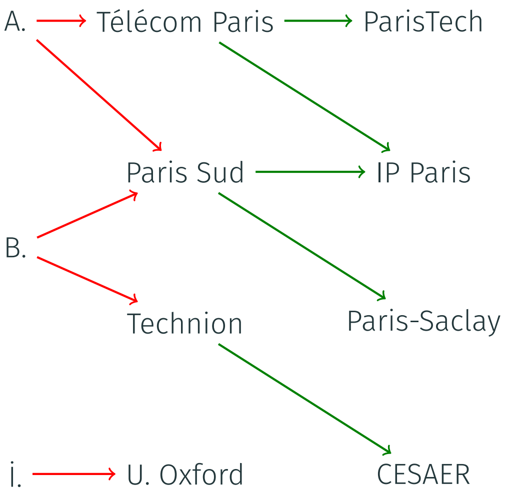


→ **Problem:** we are not **certain** about the true state of the data

Uncertain data model



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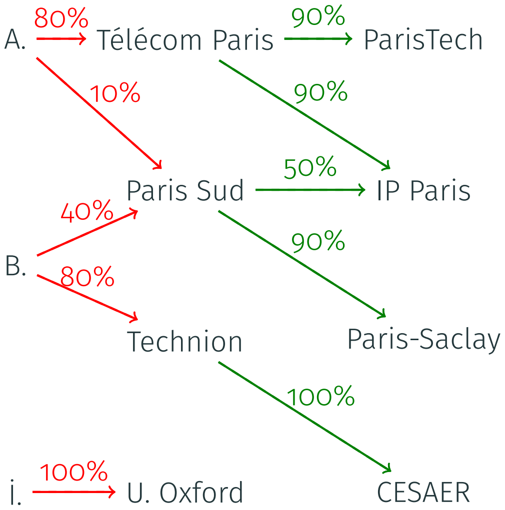

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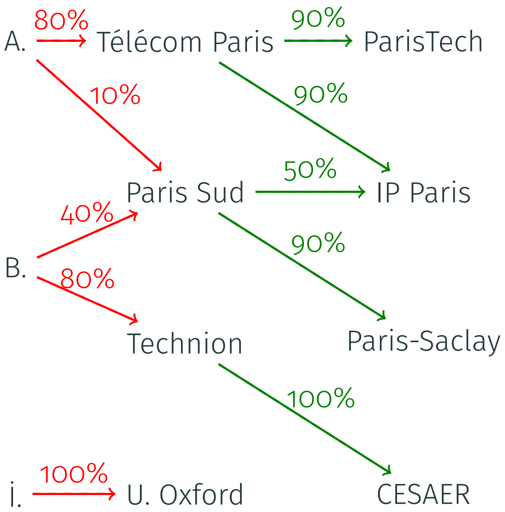

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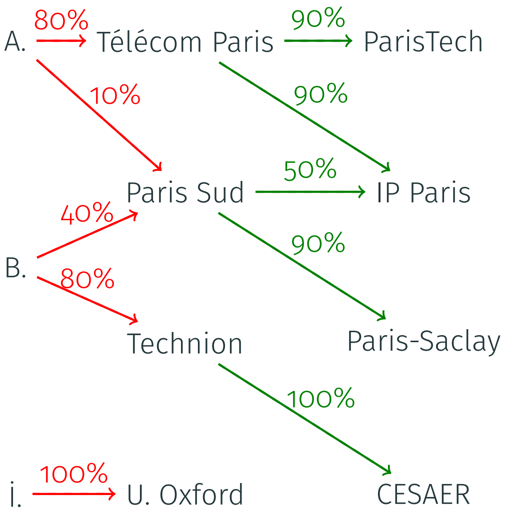

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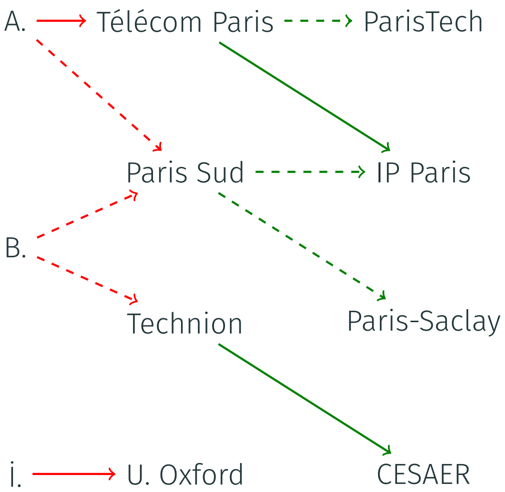

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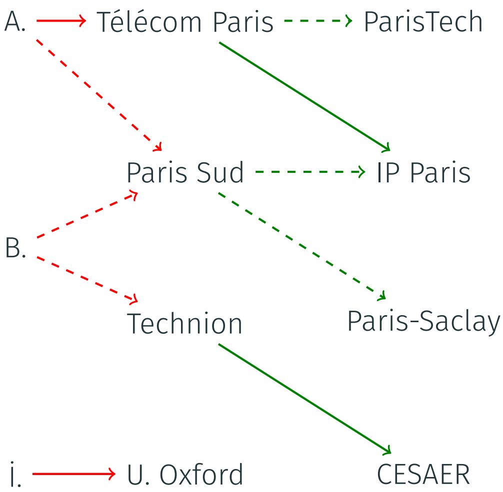

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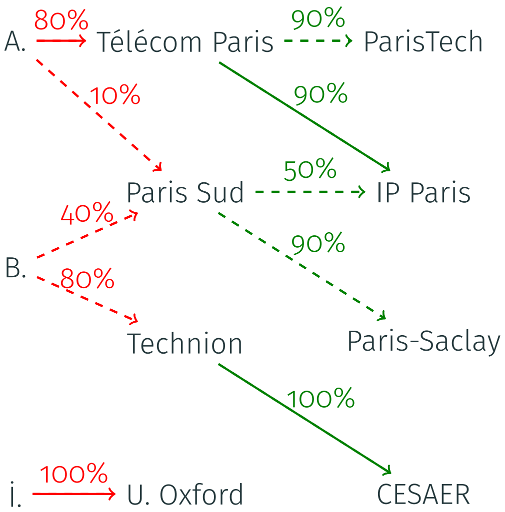

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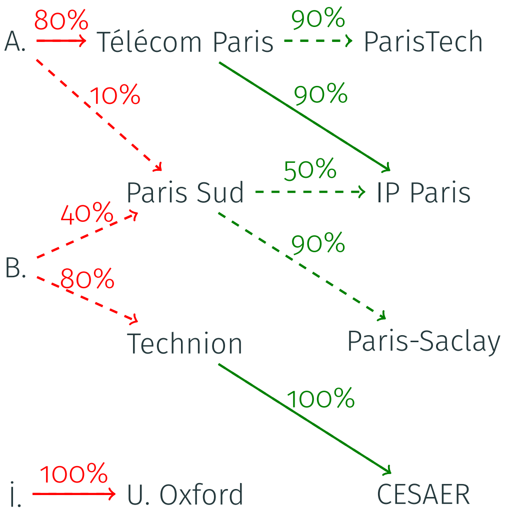

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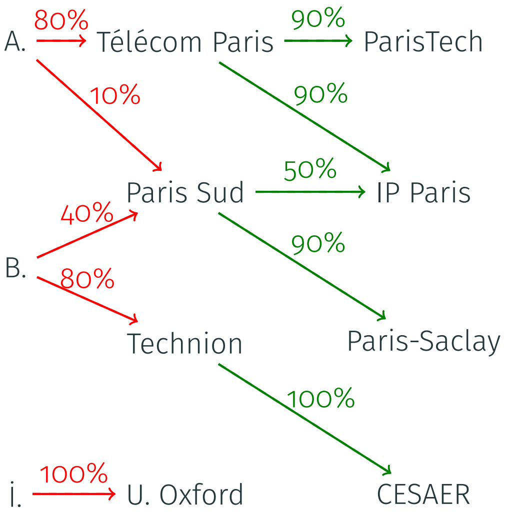

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$$\Pr(W) = \left(\prod_{F \in W} \Pr(F)\right) \times \left(\prod_{F \notin W} (1 - \Pr(F))\right)$$

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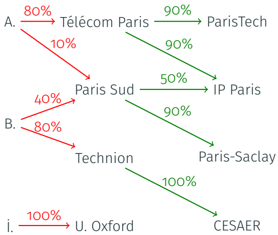

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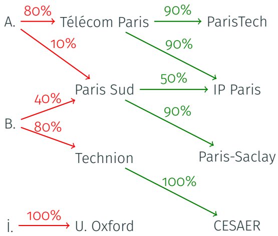

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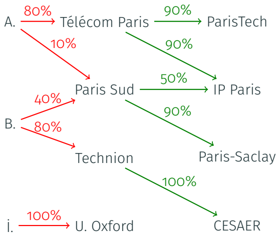

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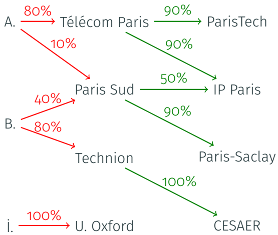

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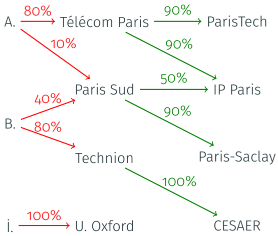

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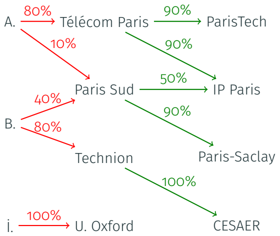

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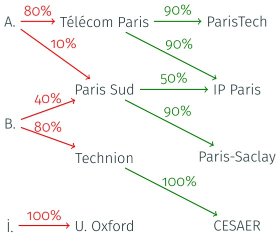

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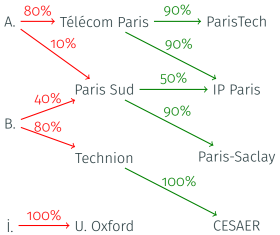

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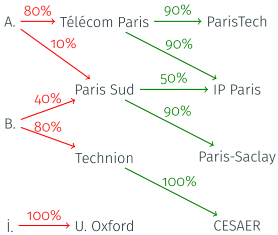

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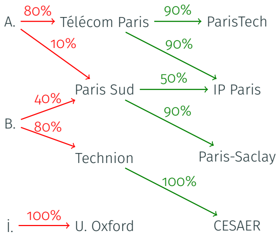

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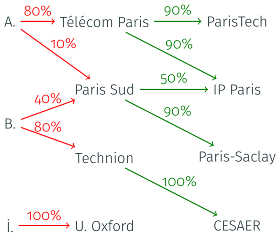

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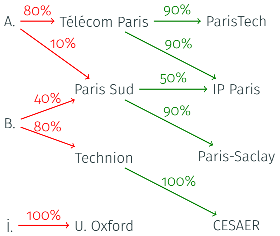

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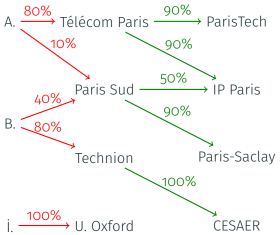

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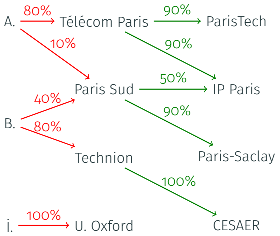

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Research goal: Understanding the complexity of PQE

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- **Restricted instances:** $\mathrm{PQE}(Q)$ for MSO queries...
  - Is **in PTIME** if the input data is restricted to have **bounded treewidth**
  - Is **intractable otherwise** under some assumptions

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  - $\mathrm{PQE}(Q)$ is **#P-hard** for all homomorphism-closed queries not equivalent to a safe UCQ
- **Restricted instances:** $\mathrm{PQE}(Q)$ for MSO queries...
  - Is **in PTIME** if the input data is restricted to have **bounded treewidth**
  - Is **intractable otherwise** under some assumptions
- **Restricted instances:** if all probabilities are **50%** then the complexity is the same

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  $\rightarrow$ e.g., single-atom CQs

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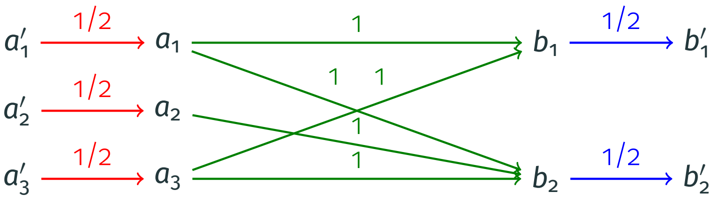

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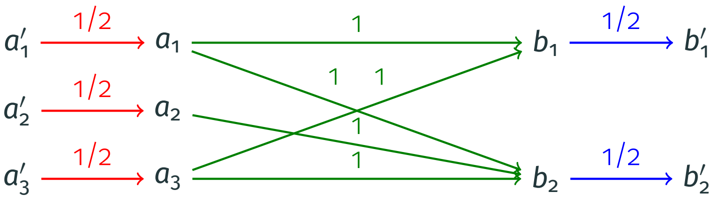

Idea: Satisfying valuations of ϕ correspond to **possible worlds** with a **match** of Q

The "small" Dalvi and Suciu dichotomy

- Self-join-free CQ: only one edge of each color (no repeated color)

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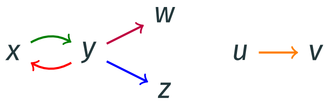

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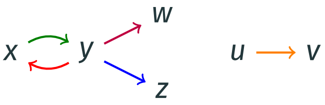

- The dichotomy generalizes to higher-arity data (hierarchical queries)

Proving the small dichotomy (upper bound)

$$\begin{tikzcd} & & w \\ x \arrow[r, bend left] & y \arrow[l, bend left] \arrow[ur] \arrow[dr] & \\ & & z \end{tikzcd} \qquad u \longrightarrow v$$

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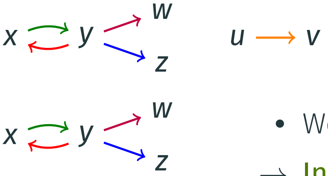

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$\rightarrow$ Independent conjunction over the connected components

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- We can test all possible values of the **separator variable**

$\rightarrow$ **Independent disjunction** over the values of the separator

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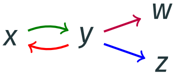

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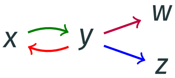

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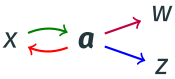

- We can test all possible values of the separator variable

$\rightarrow$ Independent disjunction over the values of the separator

- For every match, we consider every other variable separately

$\rightarrow$ Independent conjunction over the variables

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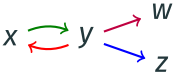

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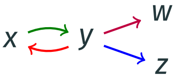

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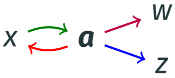

- We can test all possible values of the **separator variable**

$\rightarrow$ **Independent disjunction** over the values of the separator

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$\rightarrow$ **Independent conjunction** over the variables

- We consider every value for the **other variable**

$\rightarrow$ **Independent disjunction** over the possible assignments

$\rightarrow$ **Independent conjunction** over the facts

Proving the small dichotomy (lower bound)

Every non-star self-join-free CQ contains a pattern essentially like:

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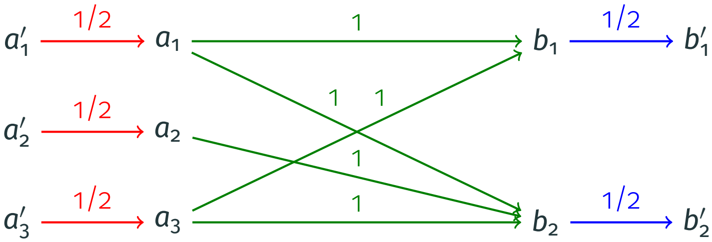
We can use this to reduce from #SAT like before:



The "big" Dalvi and Suciu dichotomy

Full dichotomy on the unions of conjunctive queries (UCQs):

Theorem (Dalvi and Suciu 2012)

Let Q be a UCQ:

- *If Q is handled by a complicated algorithm $\mathrm{PQE}(Q)$ is in PTIME*
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This result is **far more complicated** (but still generalizes to higher arity)

- **Upper bound:**
  - an algorithm generalizing the previous case with **inclusion-exclusion**
  - many **unpleasant details** (e.g., a ranking transformation)
- **Lower bound:** hardness proof on minimal cases where the algorithm does not work

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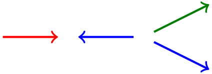
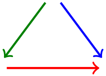
We study the case of **queries closed under homomorphisms**

Homomorphism-closed queries

- A homomorphism from a graph G to a graph G' maps the vertices of G to those of G' while preserving the edges

 has a homomorphism to 

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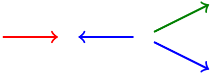
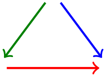
 has a homomorphism to 

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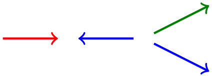
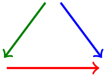
 has a homomorphism to 

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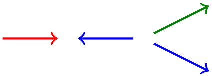

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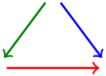

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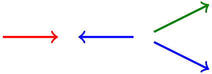
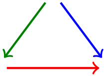
 has a homomorphism to 

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- Homomorphism-closed queries can equivalently be seen as **infinite unions of CQs** (corresponding to their models)

Our result

We show:

Theorem (Amarilli and Ceylan 2020)

For any query Q closed under homomorphisms:

- *Either Q is equivalent to a tractable UCQ and $\mathrm{PQE}(Q)$ is in PTIME*
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Basic idea: finding a tight pattern

The challenging part is to show:

Theorem

For any query Q *closed under homomorphisms and* ***unbounded****,* $\mathrm{PQE}(Q)$ *is* ***#P-hard***

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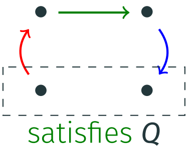

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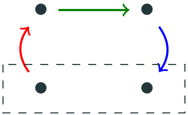

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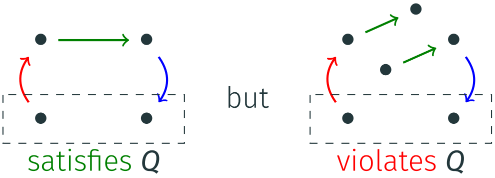

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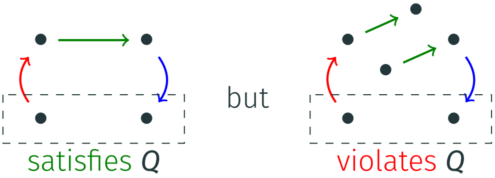

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- Unbounded queries have arbitrarily large minimal models
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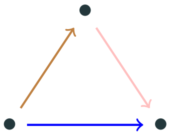

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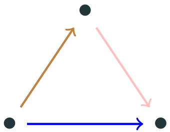

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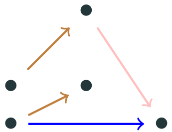

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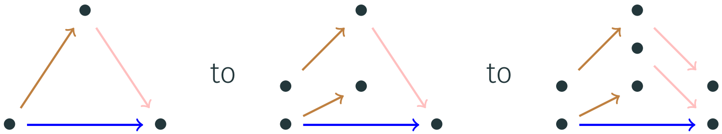

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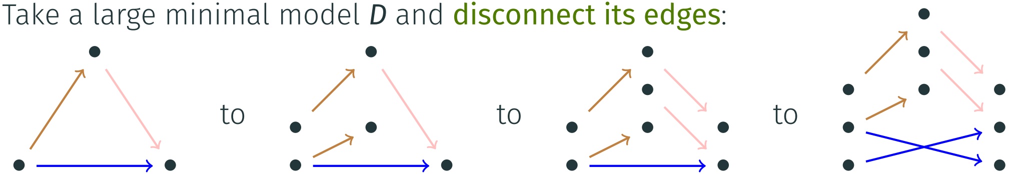

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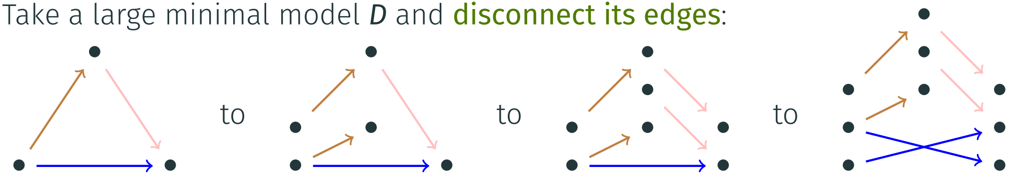

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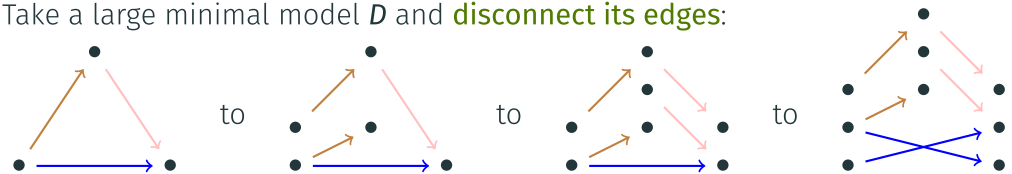

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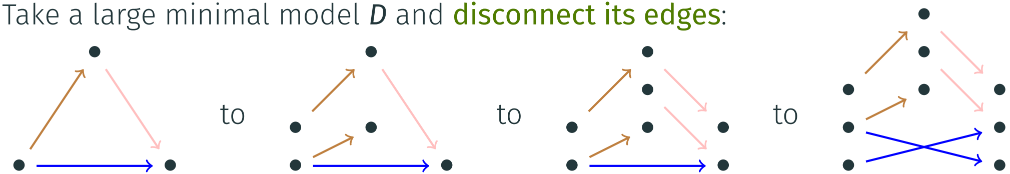

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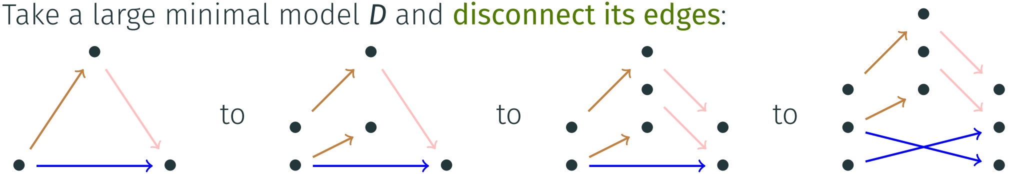

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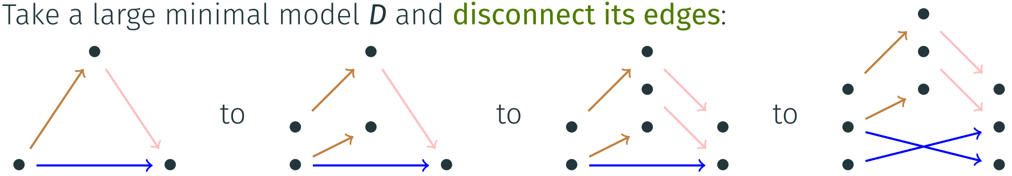

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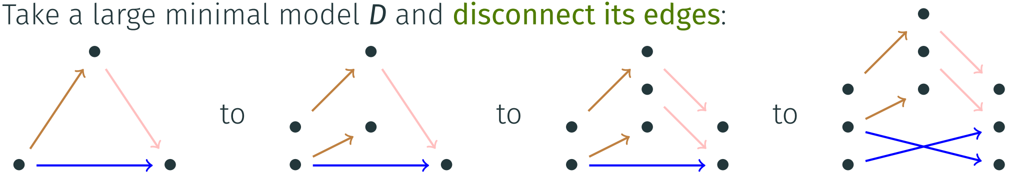

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Using tight patterns to show hardness of PQE

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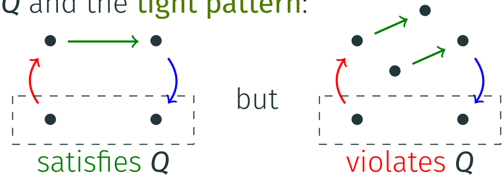

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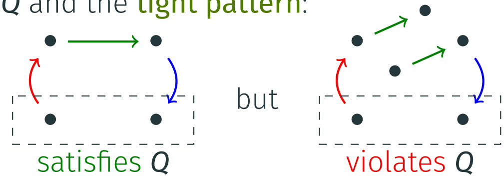

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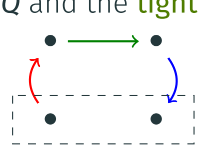

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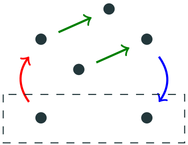

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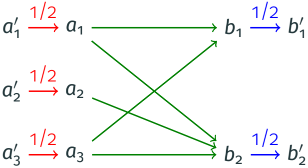

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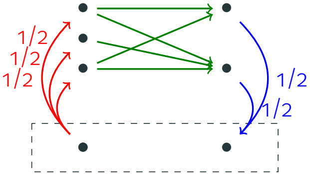

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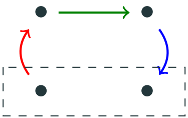

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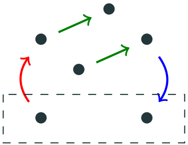

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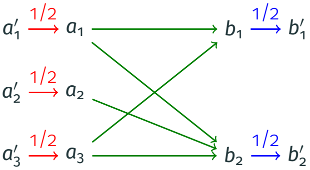

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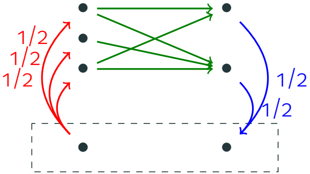

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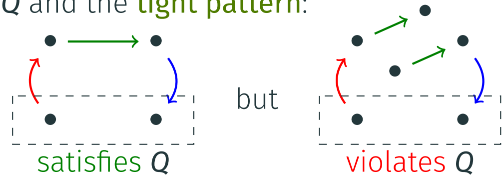

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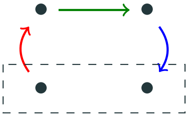

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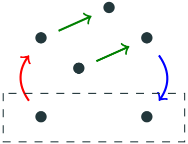

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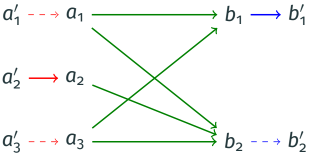

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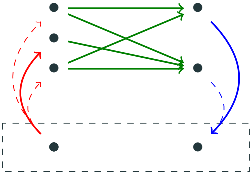

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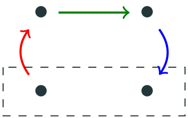

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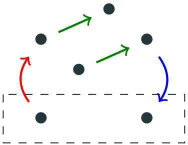

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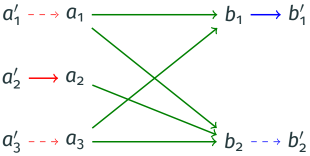

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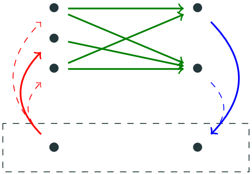

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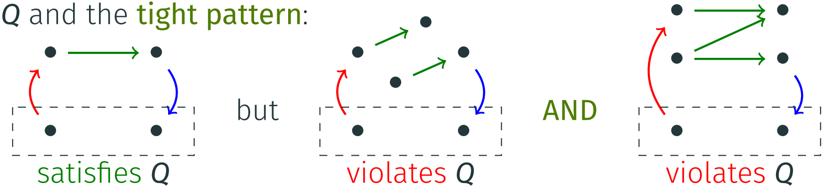

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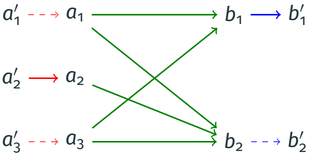

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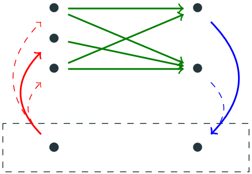

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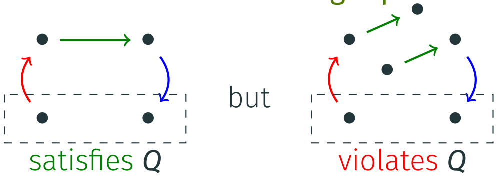

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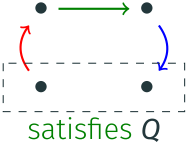

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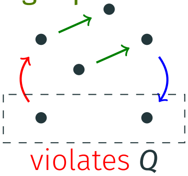

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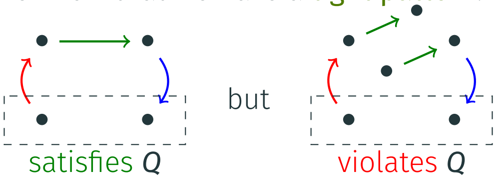

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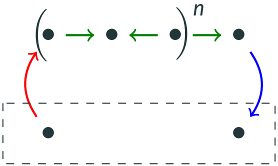

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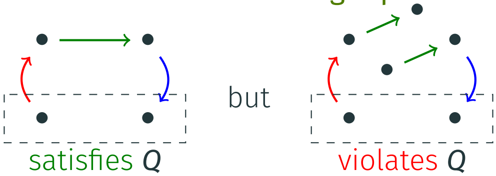

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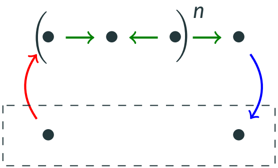

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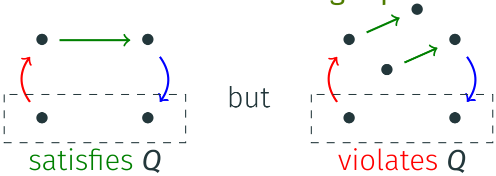

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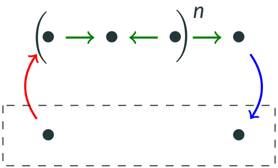

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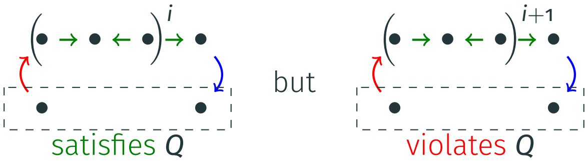

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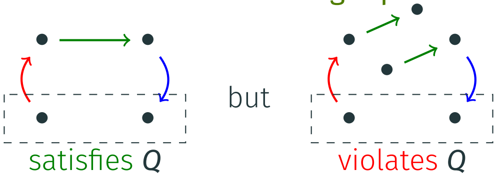

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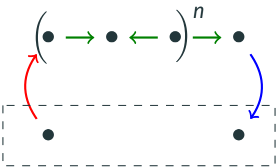

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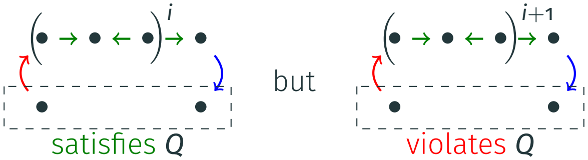

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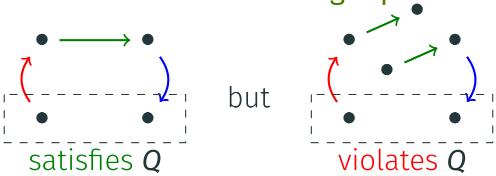

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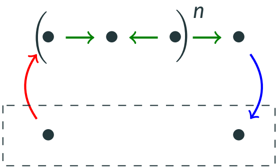

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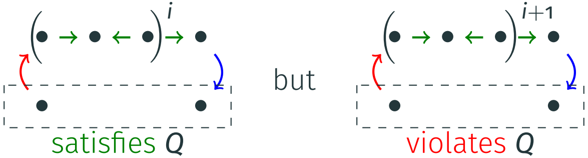

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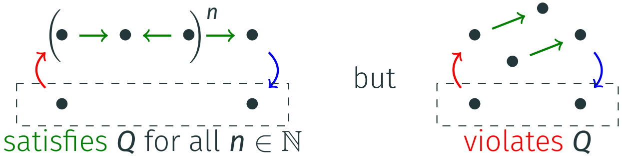

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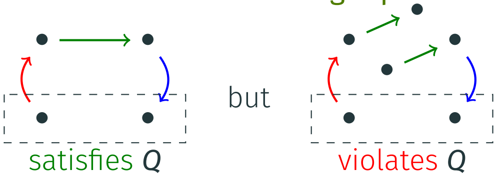

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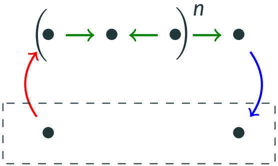

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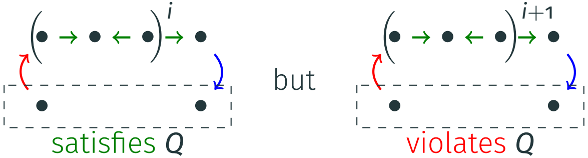

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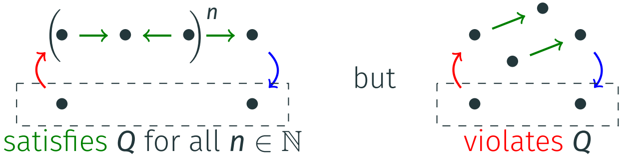
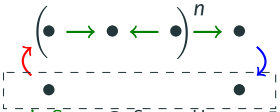
Case 2: all iterates **satisfy** the query:



$\rightarrow$ Call this an **iterable pattern**

Using iterable patterns to show hardness of PQE

We have an **iterable pattern**:



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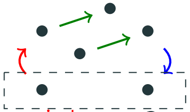

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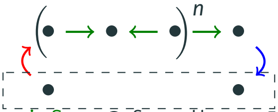

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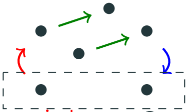

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Idea: reduce from the **#P-hard** problem **source-to-target connectivity**:

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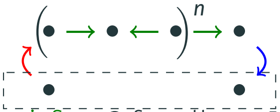

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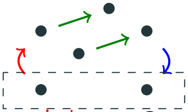

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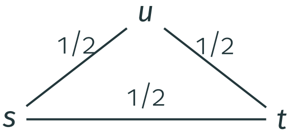

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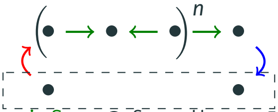

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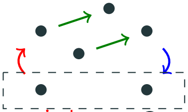

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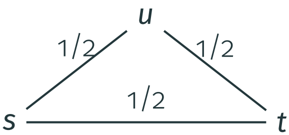

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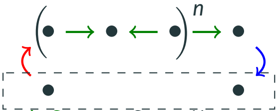

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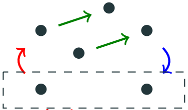

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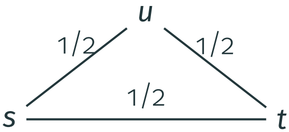

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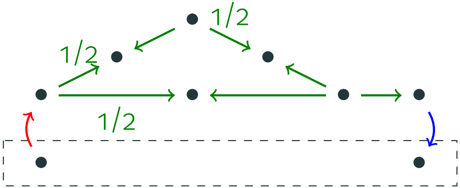

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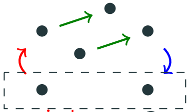

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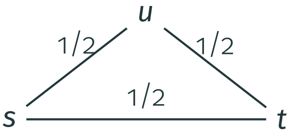

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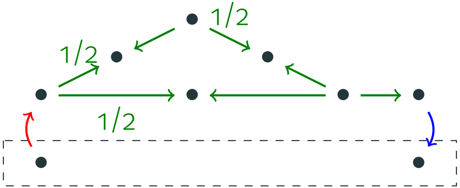

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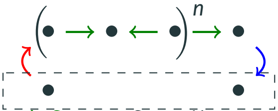

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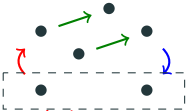

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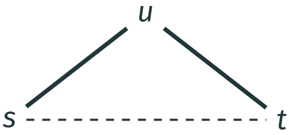

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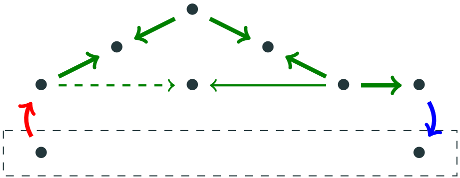

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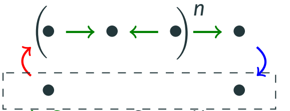

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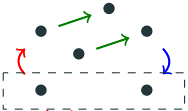

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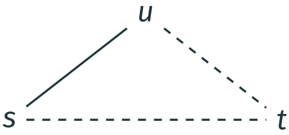

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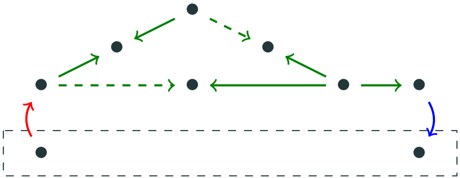

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Table of contents

Going back to more restricted instances

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*Let $k \in \mathbb{N}$ be a constant bound, and let Q be a Boolean **monadic second-order** query. Then $\mathrm{PQE}(Q)$ is in **PTIME** on input TID instances with **treewidth** $\leq k$*

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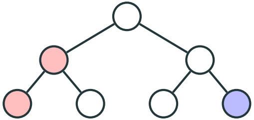
*Conversely, there is a query Q for which $\mathrm{PQE}(Q)$ is intractable on **any** input instance family of unbounded treewidth (under some technical assumptions)*

Reminder: Non-probabilistic query evaluation on trees

Database: a **tree** T where nodes have a color from an alphabet 



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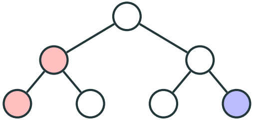

Query Q: in monadic second-order logic (MSO)

- $P_{\bigcirc}(x)$ means "x is blue"
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"Is there both a pink and a blue node?"

$\exists x\, y\; P_{\bigcirc}(x) \wedge P_{\bigcirc}(y)$

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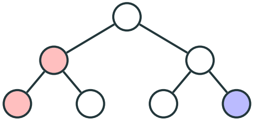

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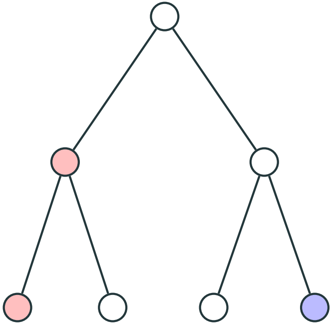
Result: YES/NO indicating if the tree T satisfies the query Q

Tree automata

Tree alphabet: 



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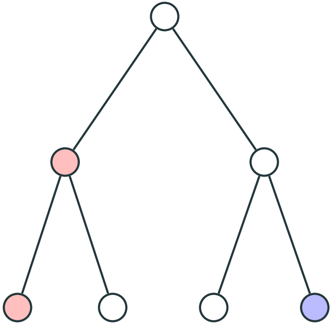

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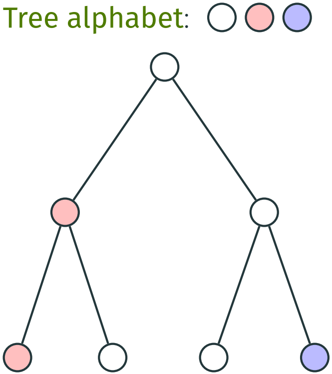

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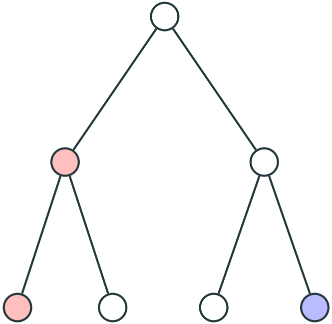

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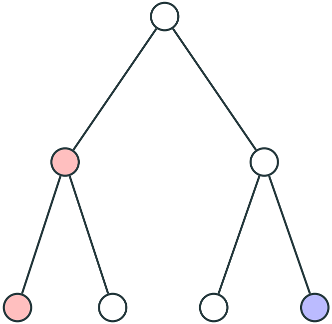

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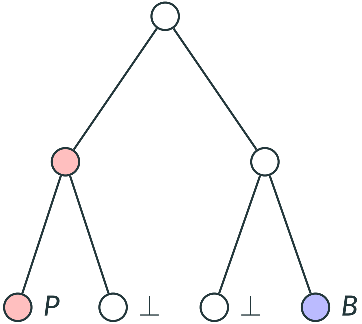

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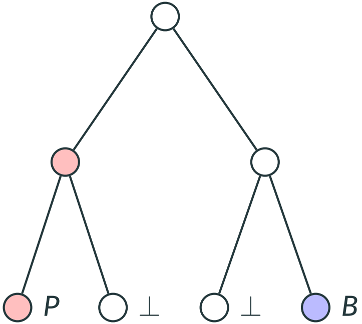

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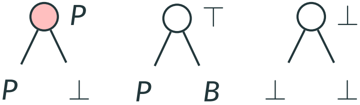

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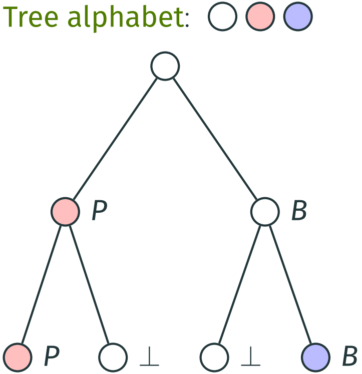

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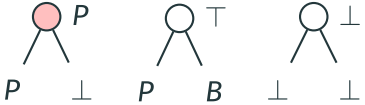

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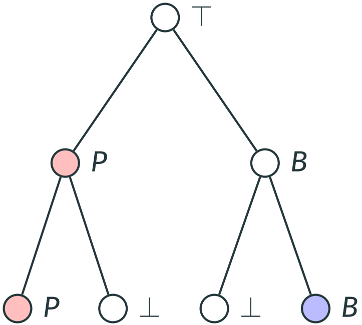

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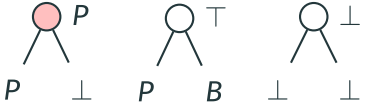

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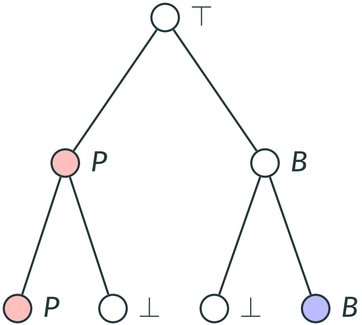

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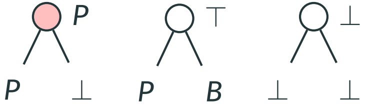

Theorem (Thatcher and Wright 1968)

MSO and tree automata have the same expressive power on trees

Probabilistic query evaluation on trees

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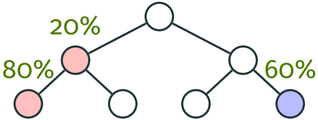

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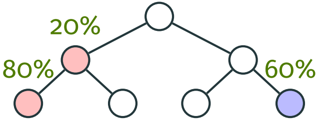

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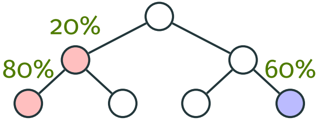

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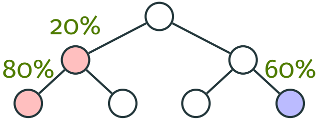


Query Q: in monadic second-order logic (MSO)

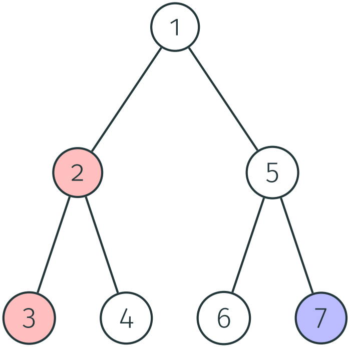
$\exists x\, y\; P_{\bigcirc}(x) \wedge P_{\bigcirc}(y)$

Result: **probability** that the probabilistic tree T satisfies the query Q

Theorem

*For any fixed **MSO query** Q, the problem $\mathrm{PQE}(Q)$ on trees is in **PTIME***

Uncertain trees: capturing how the query result depends on the choices



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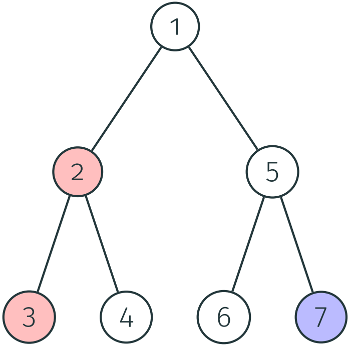

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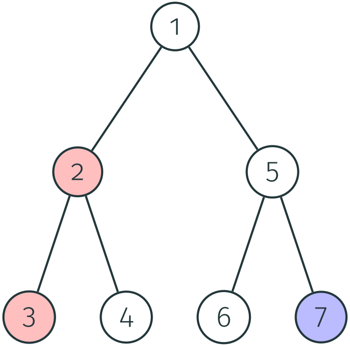

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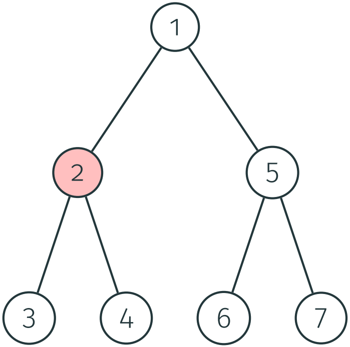

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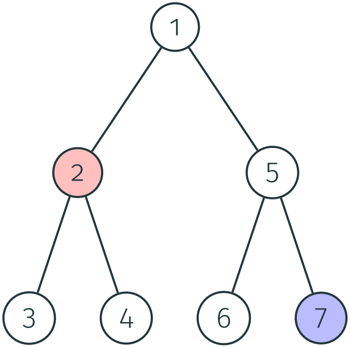

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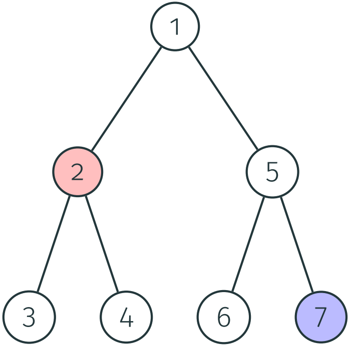

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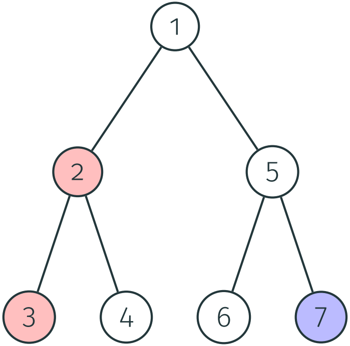

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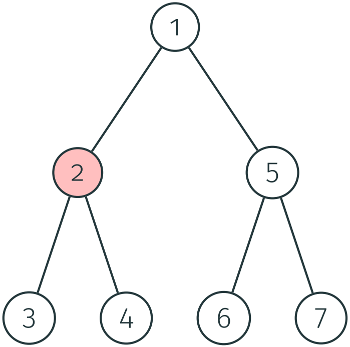

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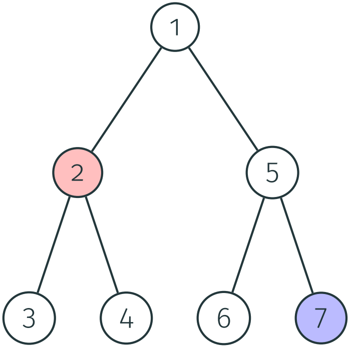

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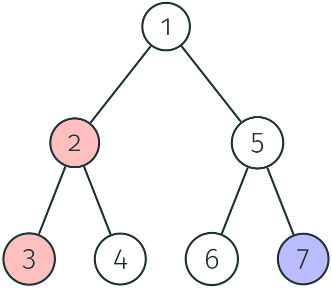
Example: Provenance circuit



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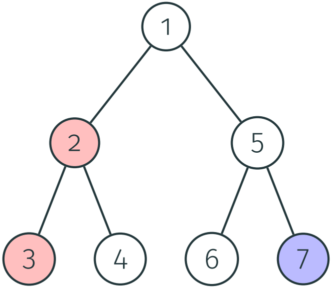

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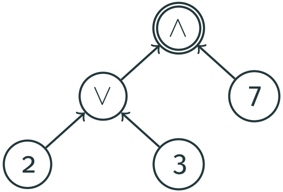

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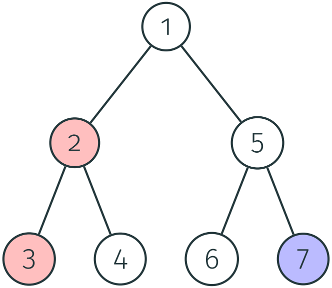

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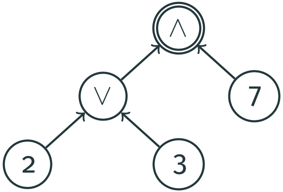

Formal definition of provenance circuits:

- Boolean query Q, uncertain tree T, circuit C
- **Variable gates** of C: nodes of T
- **Condition:** Let ν be a valuation of T, then $\nu(C)$ iff $\nu(T)$ satisfies Q

Provenance circuits on trees

Theorem

For any bottom-up tree automaton A and input tree T,
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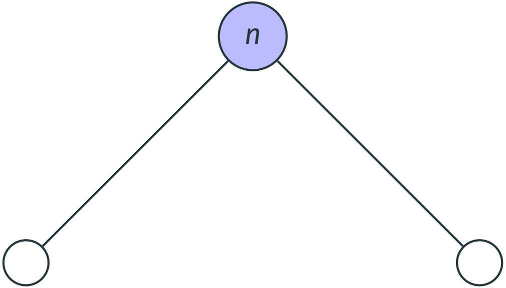

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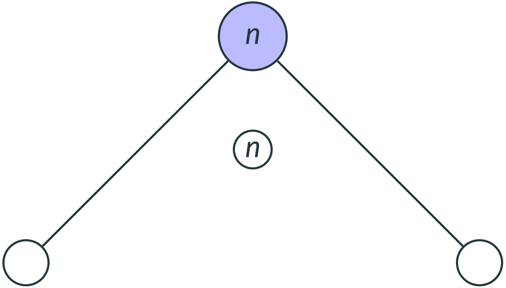

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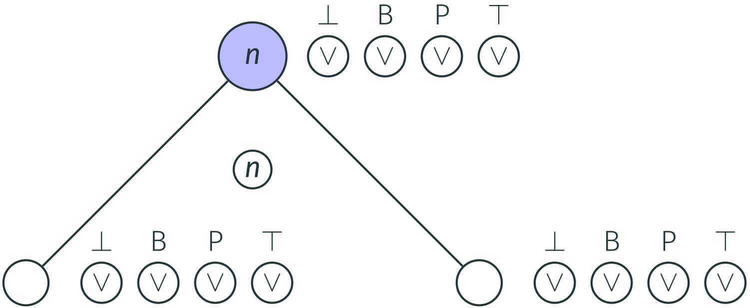

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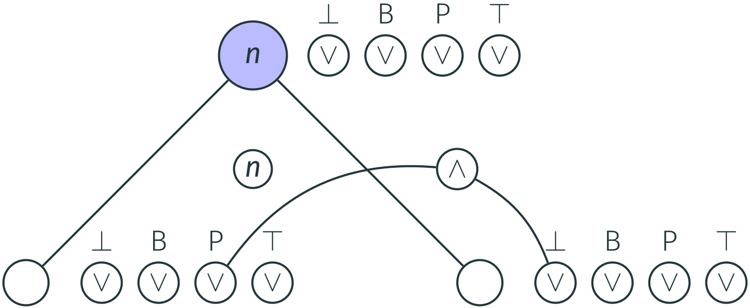

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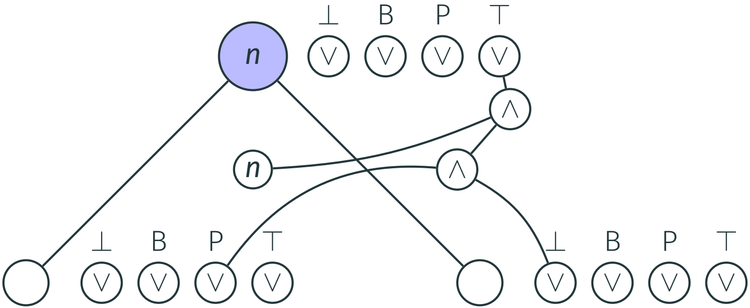

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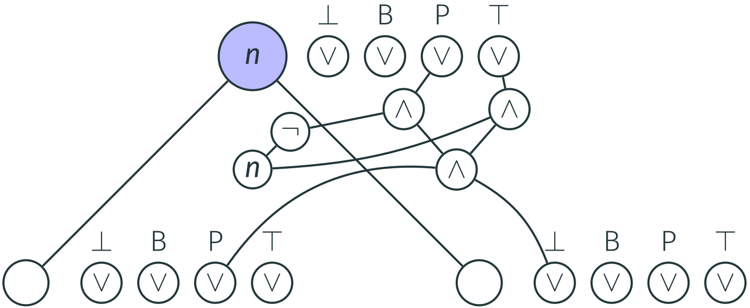

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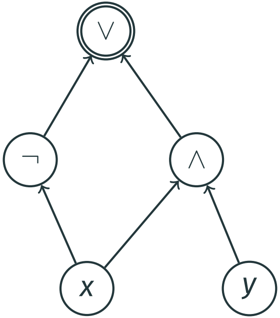
- We now have a circuit and a probability P for each variable (= tree node)



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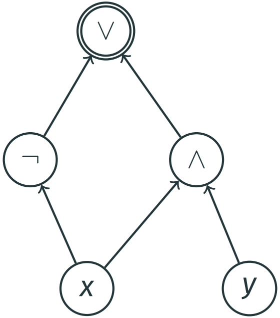

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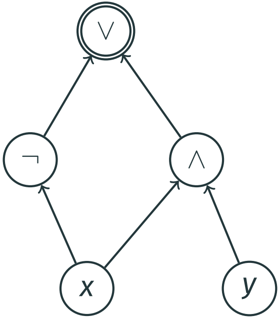

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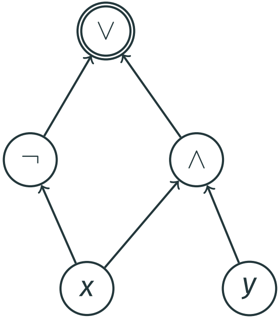

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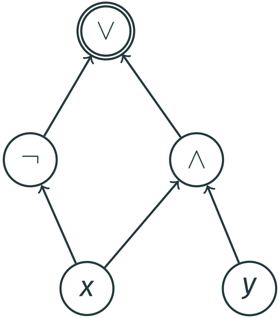

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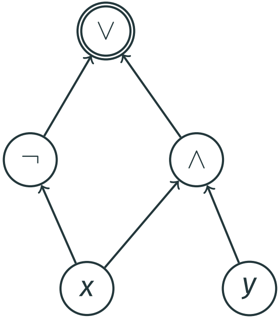

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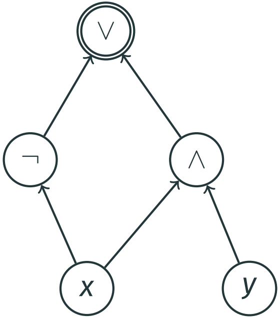

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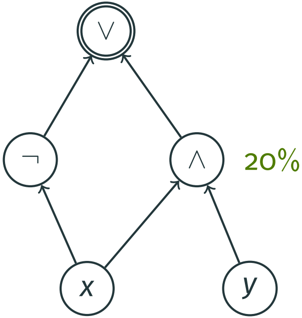

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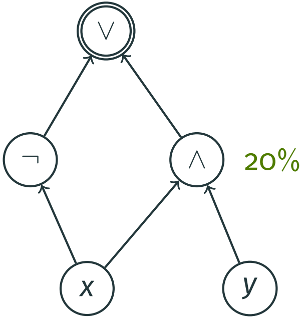

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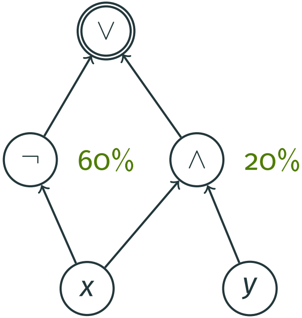

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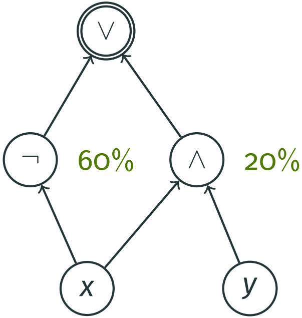

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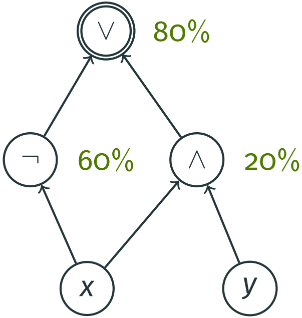

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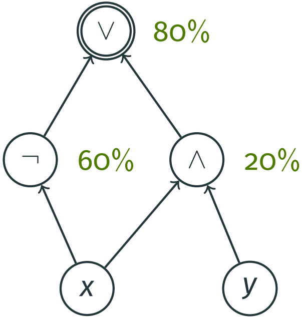

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$\rightarrow$ The circuit that we constructed falls in a restricted class satisfying such conditions

d-DNNFs

Lemma

For unambiguous automata, the provenance circuit that we compute is a d-DNNF

d-DNNF requirements

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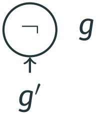

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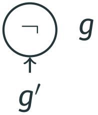

$$P(g) := 1 - P(g')$$

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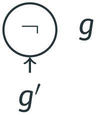

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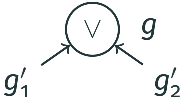

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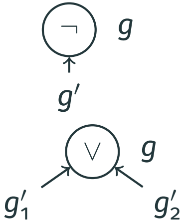

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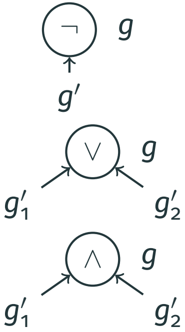
... make probability computation **easy**!



$$P(g) := 1 - P(g')$$

$$P(g) := P(g'_1) + P(g'_2)$$

d-DNNFs

Lemma

*For **unambiguous automata**, the **provenance circuit** that we compute is a **d-DNNF***

d-DNNF requirements

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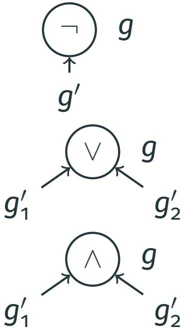

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For unambiguous automata, the provenance circuit that we compute is a d-DNNF

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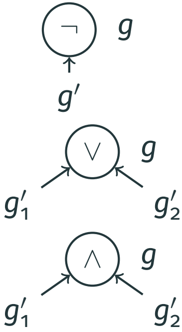

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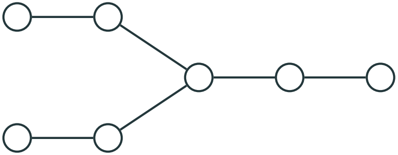
$\rightarrow$ Connections to other circuit classes in the field of **knowledge compilation**

Treewidth

We have shown tractability of PQE on **trees**; let us extend to **bounded treewidth**

Treewidth by example:



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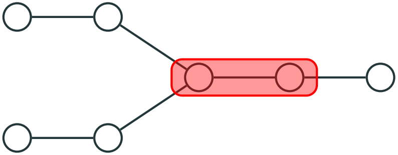

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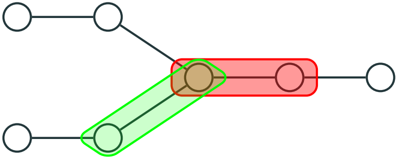

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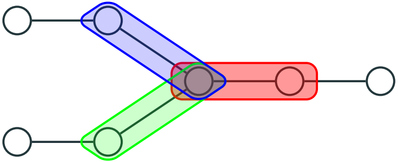

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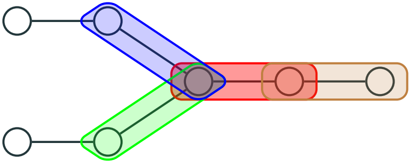

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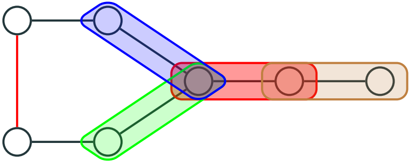

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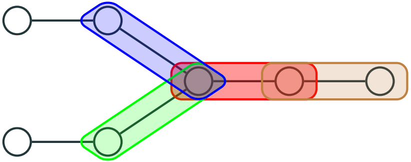

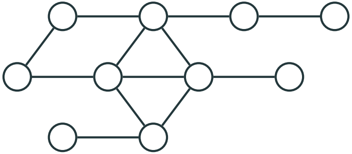

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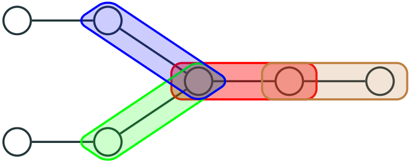

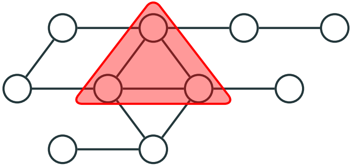

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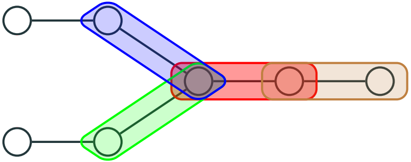

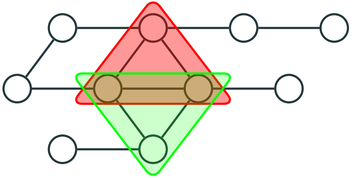

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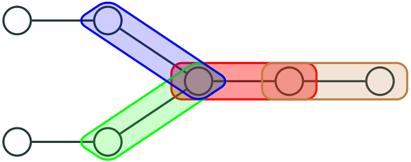

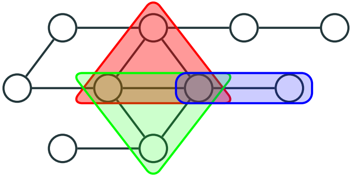

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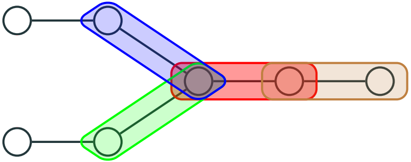

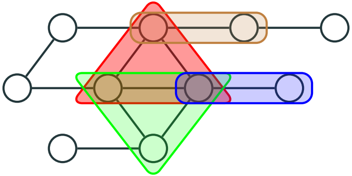

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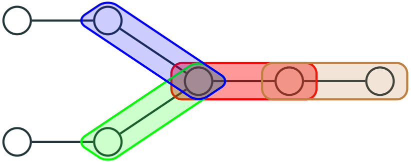

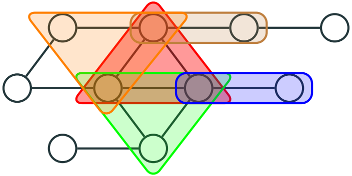

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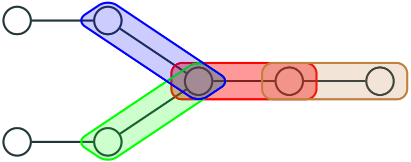

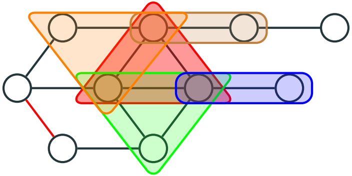

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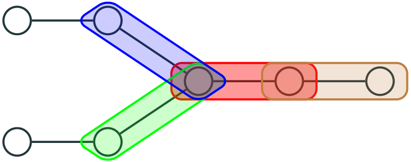

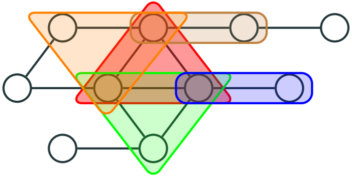

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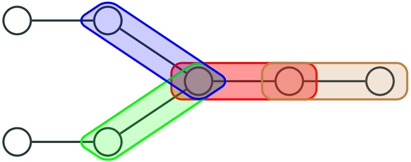

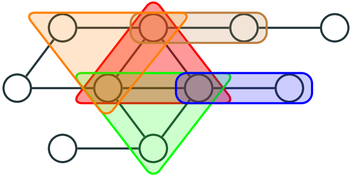

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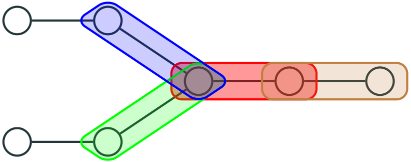

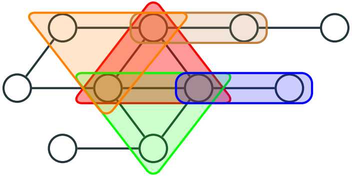

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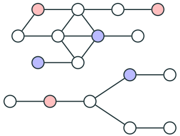
$\rightarrow$ **Treelike**: the **treewidth** is bounded by a **constant**

Courcelle's theorem and extension to PQE

Treelike **data**



MSO query

$$\exists x\, y \; P_{\bigcirc}(x) \wedge P_{\bigcirc}(y)$$

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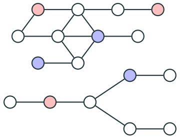

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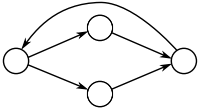

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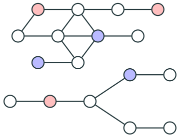

linear

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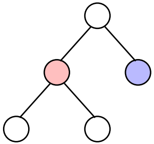

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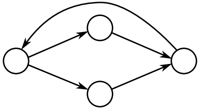

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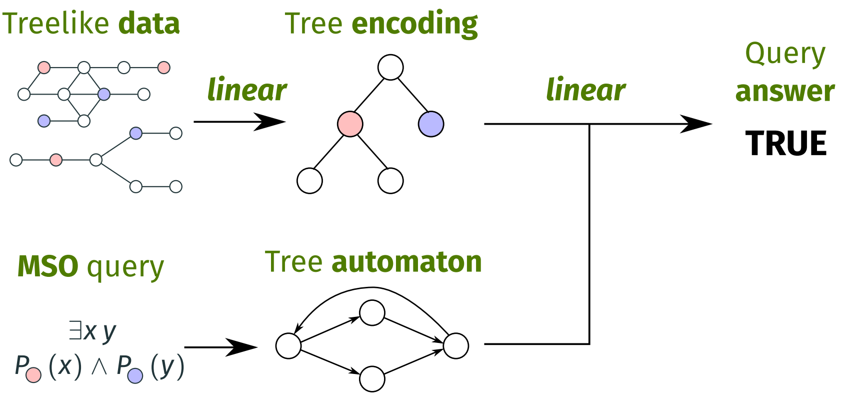

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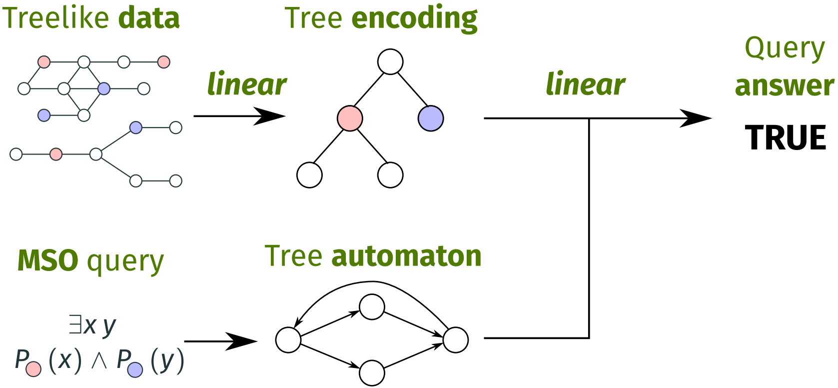
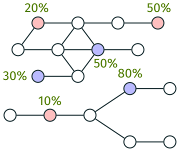


Theorem (Courcelle 1990)

For any fixed Boolean MSO query Q *and* $k \in \mathbb{N}$*, given a database* D *of treewidth* $\leq k$*, we can compute in linear time in* D *whether* D *satisfies* Q

Courcelle's theorem and extension to PQE

Probabilistic
treelike **data**



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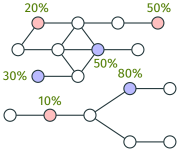

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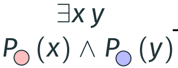

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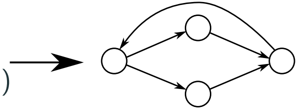

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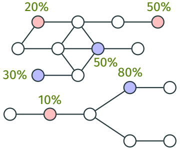

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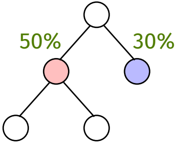

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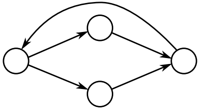

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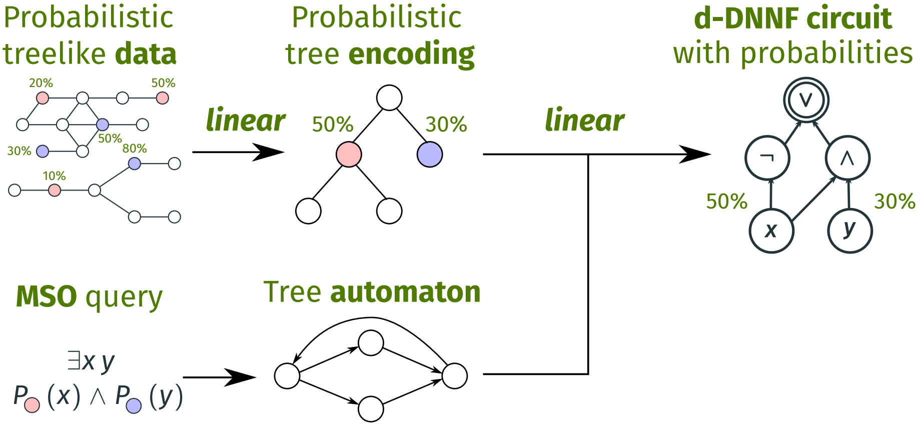

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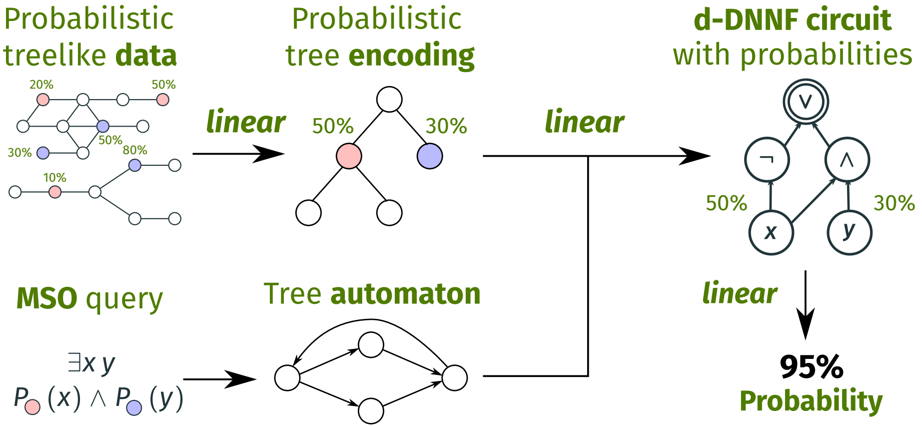

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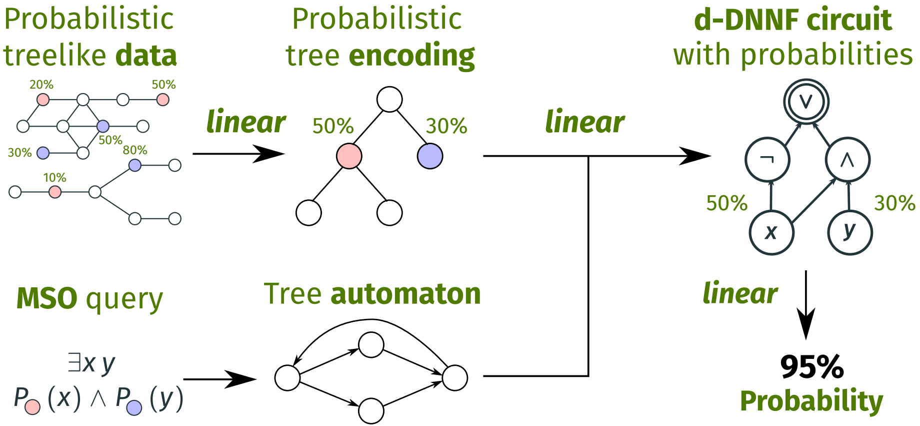

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Why is this a dichotomy? Where's the lower bound?

Theorem (Amarilli, Bourhis, and Senellart 2016)

*For any set of edge colors, there is a **first-order** query **Q** such that for any constructible **unbounded-treewidth** family $\mathcal{I}$ of probabilistic graphs, the PQE problem for **Q** and $\mathcal{I}$ is **#P-hard** under RP reductions*

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→ Proof idea: **extract wall graphs as topological minors** (Chekuri and Chuzhoy 2014) and adapt a technique of Ganian, Hlineny, Langer, Obdrzalek, Rossmanith, and

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We study to **self-join-free CQs** and extend the "small" Dalvi and Suciu dichotomy to SC:

Theorem (Amarilli and Kimelfeld 2020)

Let Q *be a self-join-free CQ:*

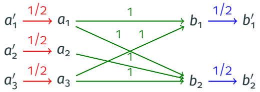
- *If* Q *is a* ***star****, then* $\mathrm{PQE}(Q)$ *is in* ***PTIME***
- *Otherwise, even* $\mathrm{SC}(Q)$ *is* ***#P-hard***

$\rightarrow$ This also extends **beyond arity two** (hierarchical queries)

Proof technique

Hard part: show hardness for (variants of) the query Q: $x \longrightarrow y \longrightarrow z \longrightarrow w$

We reduce from $\mathrm{PQE}(Q)$, on **probabilistic graphs** G of the following form:



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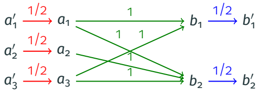

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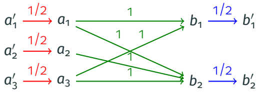

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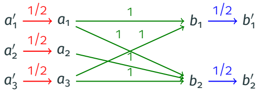

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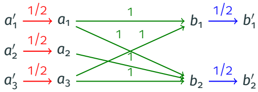

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- Show **invertibility** of this matrix to recover the X_i from the N_i

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Thanks for your attention!

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