Combining Existential Rules and Description Logics

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Open-world query answering (QA)

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- We are given:
  - Relational instance I (ground facts)
  - Logical constraints Σ
  - Boolean conjunctive query q

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Open-world query answering: – query entailment or containment

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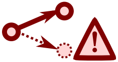
Functionality asserts $\text{Funct}(\text{Mgr}^{-})$ 	n/a

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→ QA is decidable for either language

Our problem

Can we have the best of both worlds?

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We show:

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We show:

- QA is undecidable for rich DLs and frontier-guarded rules
- QA with rich DLs is decidable for some new rule classes
- Functional dependencies can be added under some conditions

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Theorem

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- Implication of IDs and FDs is undecidable [Mitchell, 1983]
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Problem:

- Rule heads and bodies may contain cycles
- We have Funct assertions

→ We can build a grid and encode tiling problems

Undecidability of frontier-one plus DLs: proof

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- finite set of colors: ■, ■, ■

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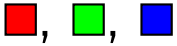
- finite set of colors: 
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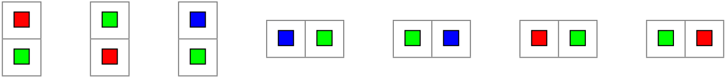

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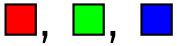
- finite set of colors: 
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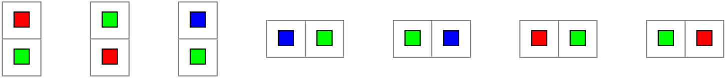

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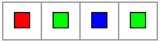
- input: initial configuration: 

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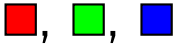
- finite set of colors: 
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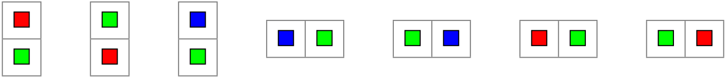

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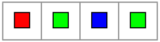
- input: initial configuration: 
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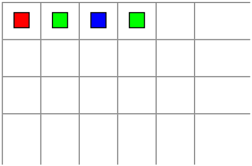

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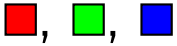
- finite set of colors: 
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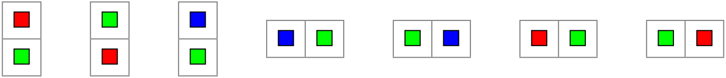

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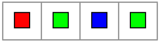
- input: initial configuration: 
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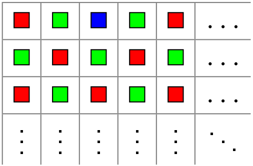

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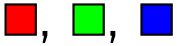
- finite set of colors: 
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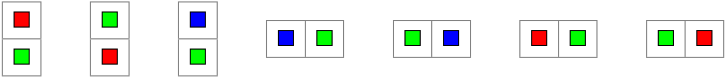

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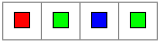
- input: initial configuration: 
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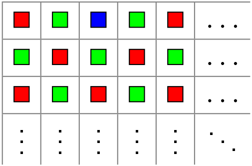

$\rightarrow$ Undecidable for some sets of colors and configurations

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$\rightarrow$ There is an extension of the instance iff there is a tiling

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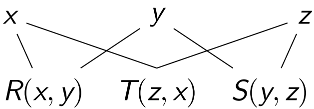

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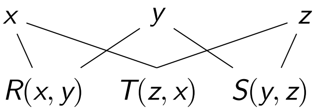

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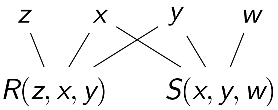

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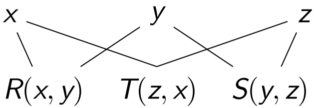

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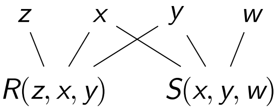

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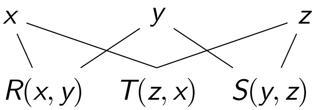

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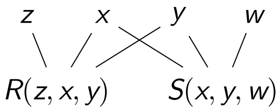

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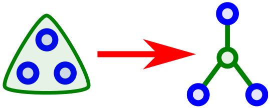

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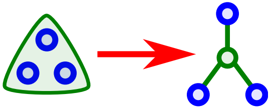

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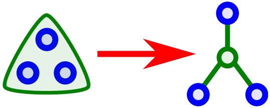

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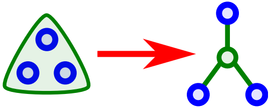

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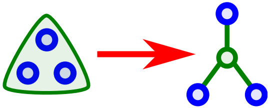

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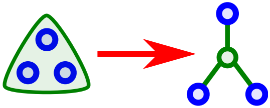

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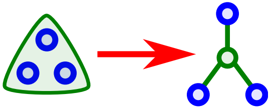

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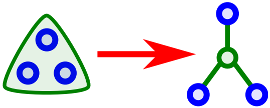

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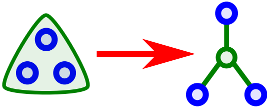

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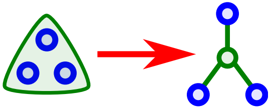

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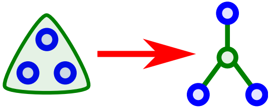

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  → Reduces to QA for GC^2: decidable [Pratt-Hartmann, 2009]

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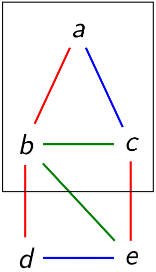
Basic idea:

- If there is a counterexample model to QA, we can unravel it
  - → It is still a counterexample
  - → It has no cycles (except in the instance part)

→ Looping rule bodies can only match on the instance part

Head-non-looping frontier-one and DLs: unraveling



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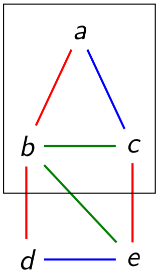

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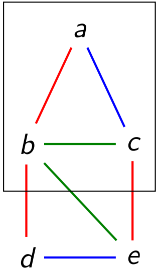

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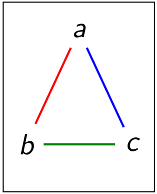

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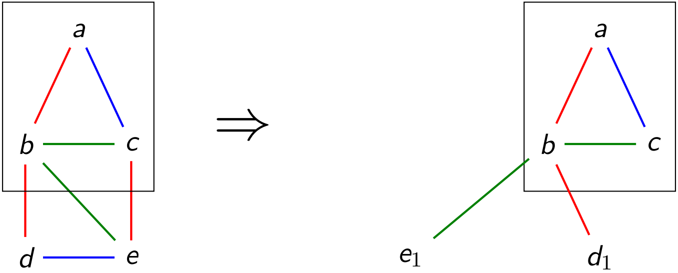

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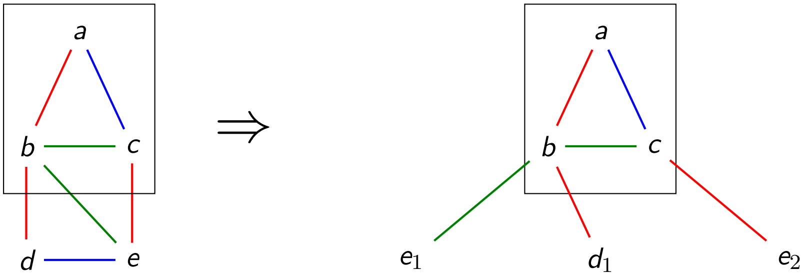

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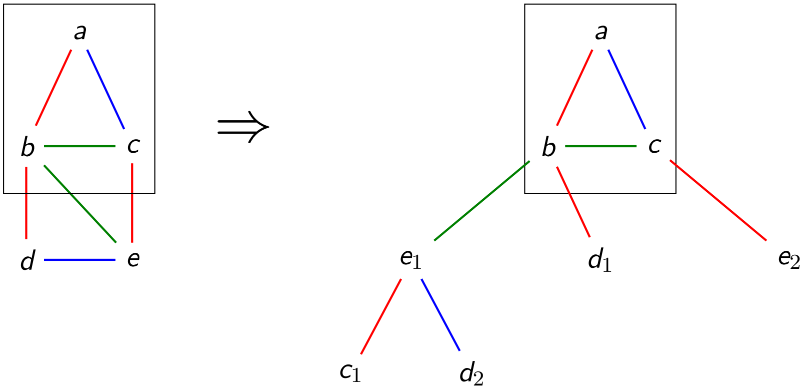

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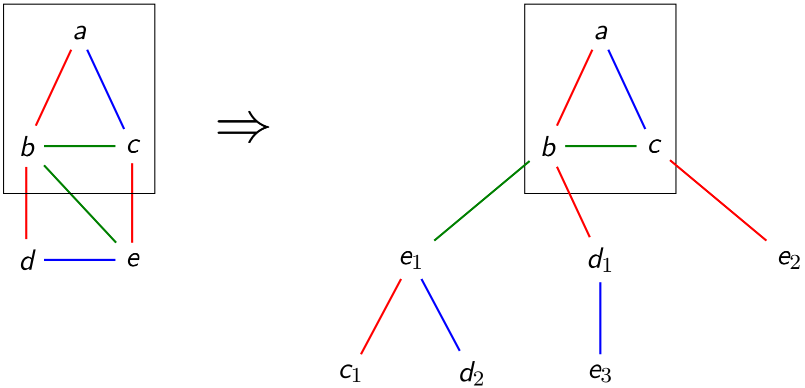

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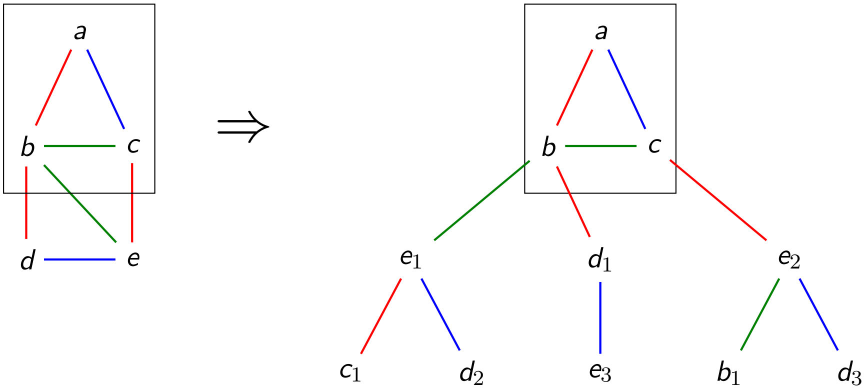

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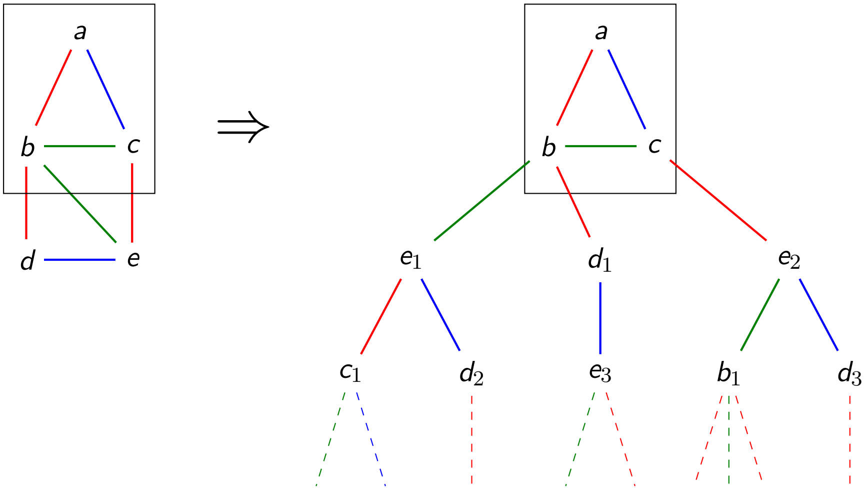

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- We have functional dependencies Funct(R) on binary relations
- Could we also allow FDs on higher-arity relations?
  Ex.: Talk[$speaker, session$] determines Talk[$title$]

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Proof ideas:

- Reduce from implication of unary FDs and frontier-2 IDs
- Leverage variable reuse and FDs to export two variables: to encode the ID $R[1,2] \subseteq R[3,4]$ with the FD $R[1] \rightarrow R[2]$, write $R(x, y, z, w) \Rightarrow R(x, y', x, y')$: we must have $y = y'$

$\rightarrow$ We need an additional restriction for decidability

Non-conflicting rules and FDs [Calì et al., 2012]

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Examples: for the FD $R[1] \to R[3]$:

- $T(x) \Rightarrow R(y, y, x)$ is... non-conflicting
- $T(x, y) \Rightarrow R(x, y, z)$ is... conflicting (superset)
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Non-conflicting rules and FDs [Calì et al., 2012]

Consider QA under single-head rules Σ and FDs Φ

- Σ and Φ are separable if $\mathrm{QA}(\Sigma, \Phi) \Leftrightarrow \mathrm{QA}(\Sigma)$ when $I \models \Phi$
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Decidability for non-conflicting FDs

We know from [Calì et al., 2012]:

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We show:

Theorem

QA is decidable for:

- *Rich DL constraints (with Funct)*
- *Single-head (hence, head-non-looping) frontier-one rules*
- *Non-conflicting FDs (on higher-arity predicates)*

Decidability for non-conflicting FDs: proof ideas

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- if $S' = S$ for S' an FD determiner
  → copy only one such fact, distinguish its other elements
  (no equality between them is required by the constraints)

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- What about QA on finite models?
- Could we have an expressive frontier-one language? (FDs, disjunctions... like DLs but higher-arity)

Related things I work on

- Adding transitive and order relations to existential rules[1]
  - → QA for frontier-guarded is decidable with transitive relations
  - → Also for order relations (with atom-covered requirement)

[1] With Michael Benedikt, ongoing work

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Thanks for your attention!

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