Uniform Reliability for Unbounded Homomorphism-Closed Graph Queries

Antoine Amarilli

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Télécom Paris

Uncertain data management

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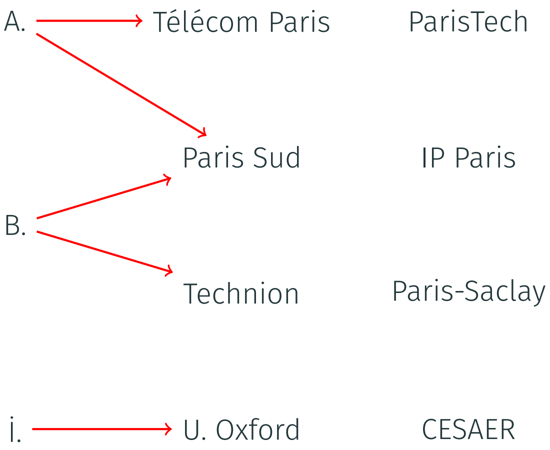

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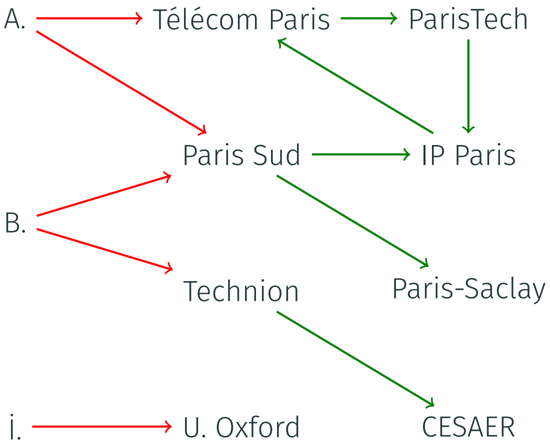

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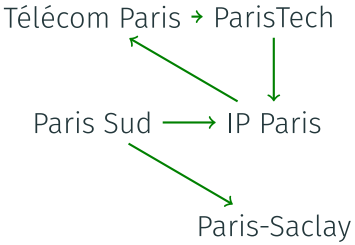

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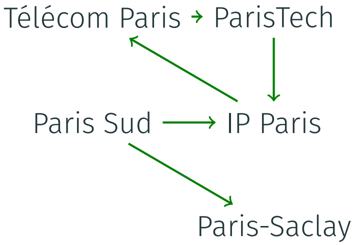

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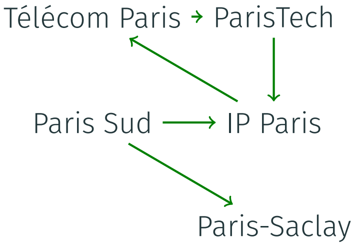

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… there are 4 that satisfy the query

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- Generalization of (two-terminal unweighted directed) network reliability [Valiant, 1979, Provan and Ball, 1983]:
    - Input: a directed graph with a source s and sink t
    - Output: the probability that there is a path from s to t if each edge can fail (independently) with probability $1/2$

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- Not supported: **inequalities**, **negation**

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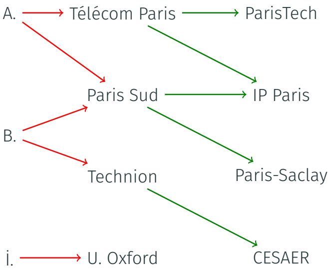

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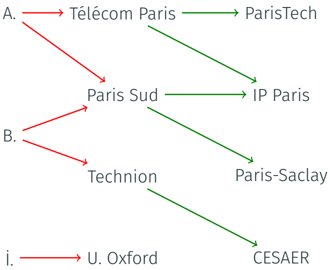

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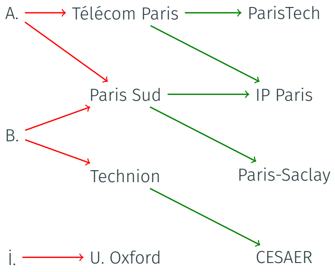

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$\rightarrow$ What is the **complexity** of the problem $\mathrm{UR}(Q)$, depending on the query Q?

Results on Uniform Reliability

Known results: SJFCQs

A **self-join-free CQ** (SJFCQ) is a CQ with no **repeated relations**,
i.e., **all colors are different**:

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The following **dichotomy** is known for a class of **hierarchical SJFCQs**
for the **probabilistic query evaluation** problem (PQE) on tuple-independent databases:

Theorem [Dalvi and Suciu, 2007]

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Theorem [A. and Kimelfeld, 2022]

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Theorem [Kenig and Suciu, 2021]

- *For any **unsafe** UCQ Q, the **#P-hardness** of PQE holds even when we only use probabilities 0, 1/2, and 1*
- *For **Type-I forbidden queries**, the UR problem is **#P-hard***

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Theorem

For any unbounded UCQ$^\infty$ Q on graphs, the uniform reliability problem for Q is #P-hard

Proof Techniques

Starting point: Hardness of a non-hierarchical query

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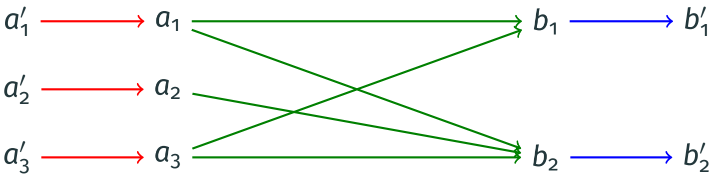

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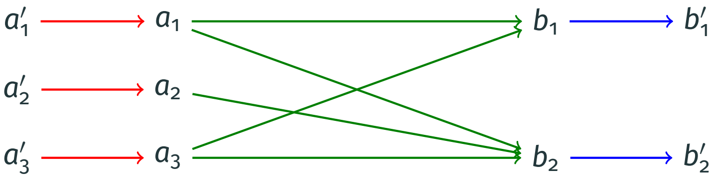

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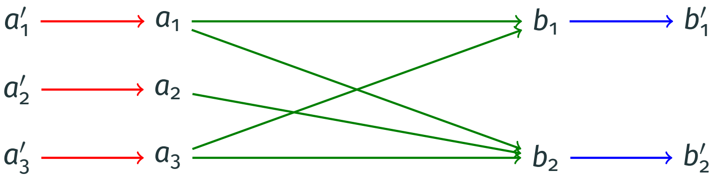

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- Otherwise, **#P-hardness** via **interpolation technique** (already in [A., Kimelfeld, 2022])

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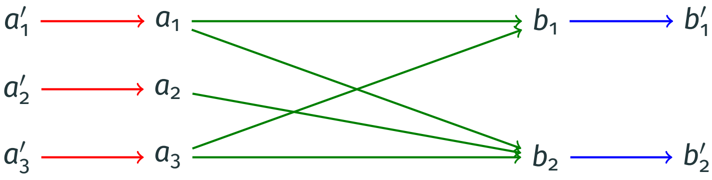
Same proof!

$$\phi : (X_1 \land Y_1) \lor (X_1 \land Y_2) \lor (X_2 \land Y_2) \lor (X_3 \land Y_1) \lor (X_3 \land Y_2)$$



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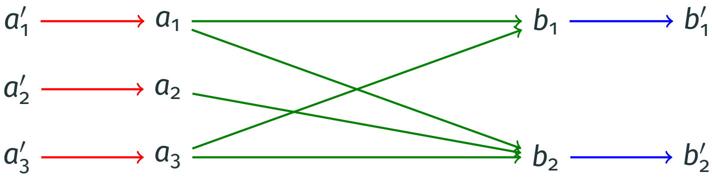

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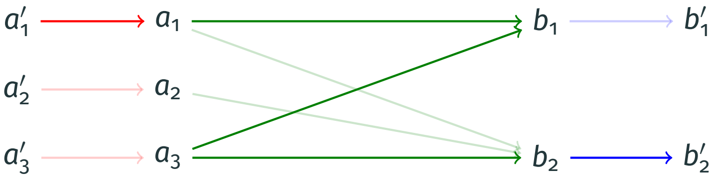

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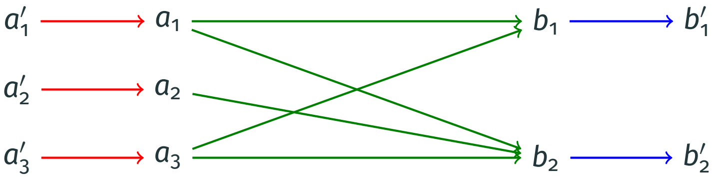

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$\rightarrow$ We call this an **iterable** unbounded query

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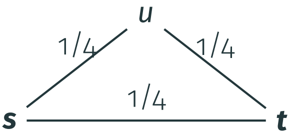

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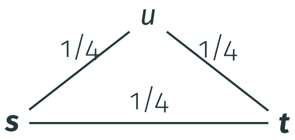

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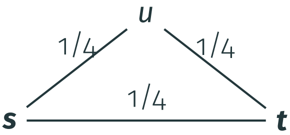

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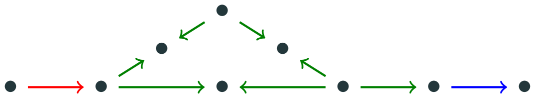

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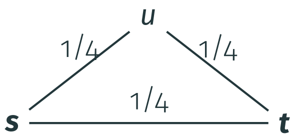

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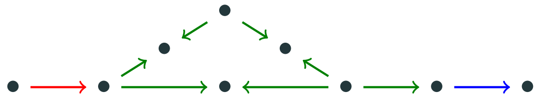

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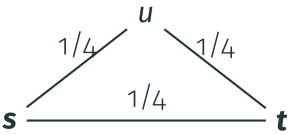

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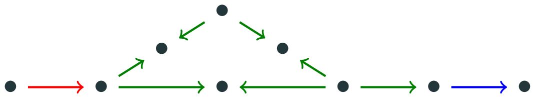

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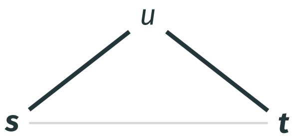

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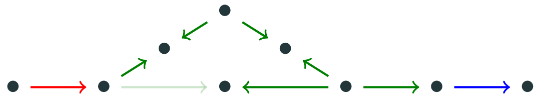

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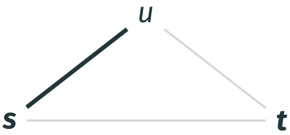

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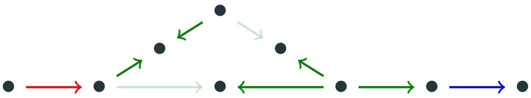

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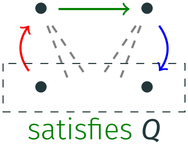

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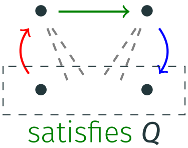

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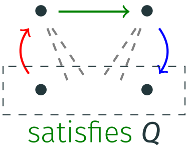

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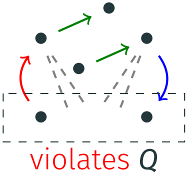

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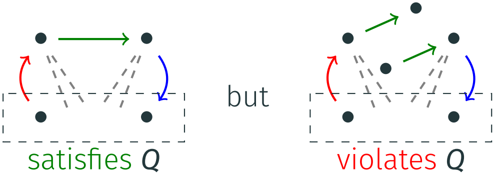

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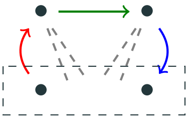

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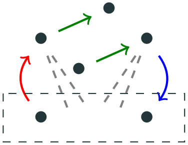

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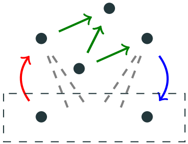

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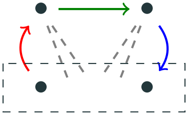

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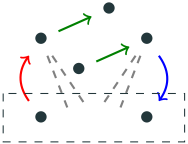

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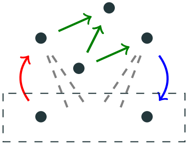

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$\rightarrow$ What **breaks down** in the unweighted case?

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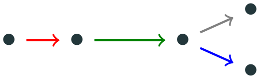

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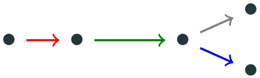

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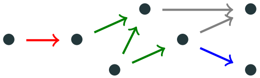

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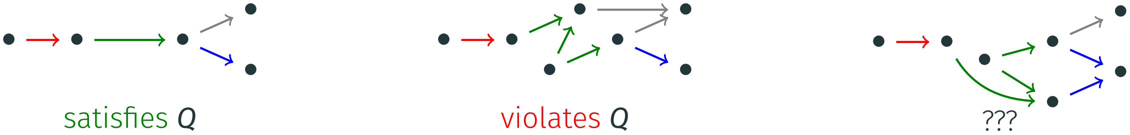

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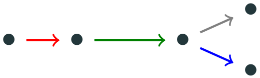

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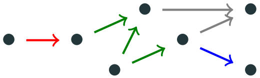

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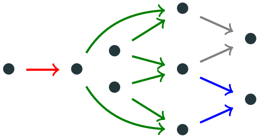

???

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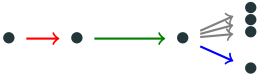

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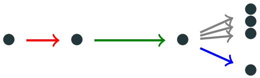

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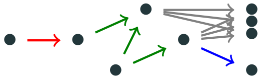

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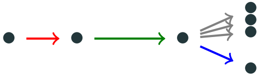

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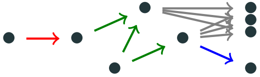

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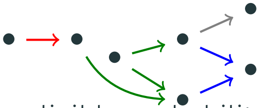

negligible probability

But serious **limitations**:

- Only applicable in the **non-iterable** case
  (query matches must have constant size to make the probability negligible)
- Only works with **binary facts**!

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Conclusion and Future Work

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Thanks for your attention!

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