Uniform Reliability of Self-Join-Free Conjunctive Queries

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→ Let's review the line of work on probabilistic query evaluation

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→ **Uniform reliability** amounts to a TID where **all facts have probability 1/2**

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Let **Q** *be a SJFCQ. Then:*

- *Either* **Q** *is* ***hierarchical*** *and* PQE(**Q**) *is in PTIME*
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What is this class of **hierarchical CQs**?

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For a CQ Q, write **atoms**(x) for the set of atoms where x appears

- A CQ is hierarchical if for every variables x and y
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Rest of the talk: proof sketch of this result

Reducing to R-S-T-type queries

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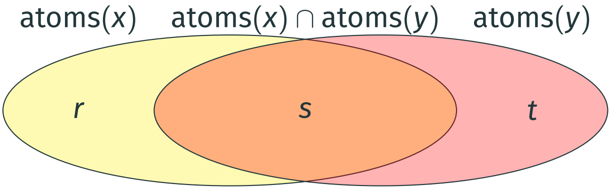

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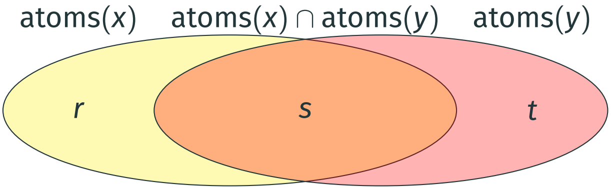

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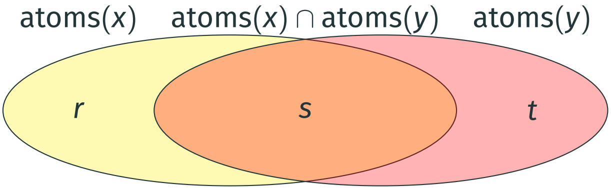

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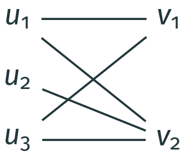
- **In this talk:** we focus for simplicity on $Q_1 : \exists x\, y\; R(x), S(x,y), T(y)$

Hard problem: counting independent sets of bipartite graphs



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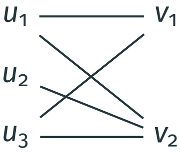

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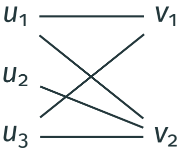

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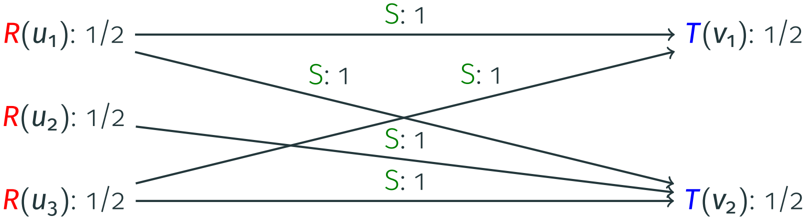

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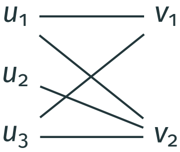

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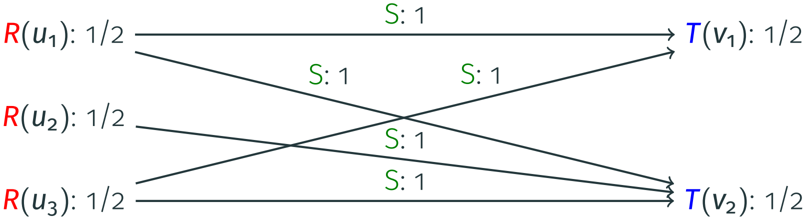

We will show how to reduce from counting independent sets to $\mathbf{UR}(Q_1)$

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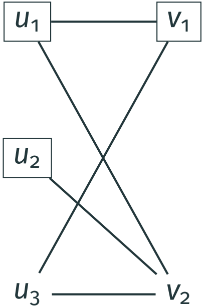

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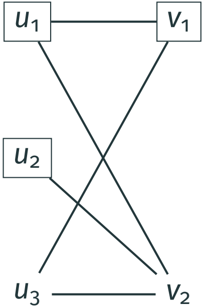

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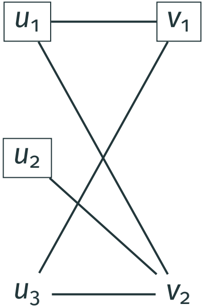

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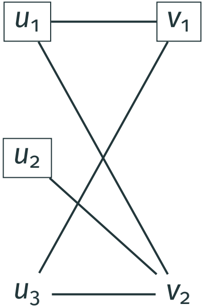

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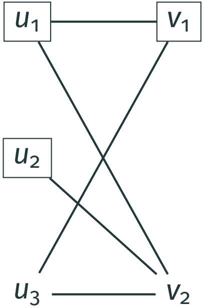

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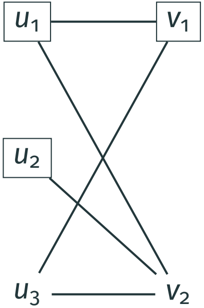

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- **Harder problem:** computing all the values:

$$X_{c,d,d',e} = \left|\{W \subseteq U \cup V \mid \mathrm{c}(W) = c \text{ and } \mathrm{d}(W) = d \text{ and } \mathrm{d}'(W) = d' \text{ and } \mathrm{e}(W) = e\}\right|$$

Idea: coding to several copies

- We want to design a reduction:
  - We reduce from (we want): given a bipartite graph G, compute the $X_{c,d,d',e}$
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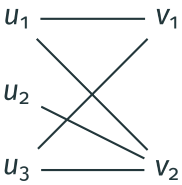
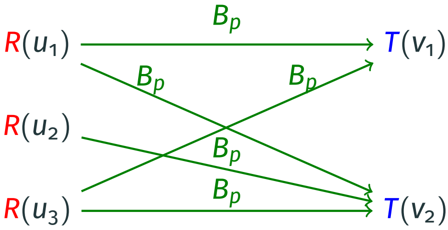
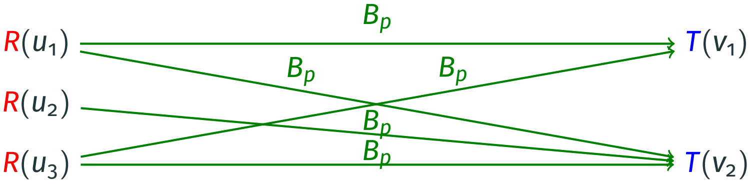
- **Idea:** code G to a **family** of instances D_p **indexed** by $p > 0$
- Fix a **box** $B_p(a, b)$ for index $p > 0$: an instance with two distinguished elements (a, b)
- **Code** G for index $p > 0$ to an instance by:
  - putting an R-fact on each U-vertex and a T-fact on each V-vertex
  - coding every edge (u, v) by a **copy of the box** $B_p(u, v)$





Getting an equation system

Take the coding of G for index p, and compute the number N_p of subinstances violating Q_1



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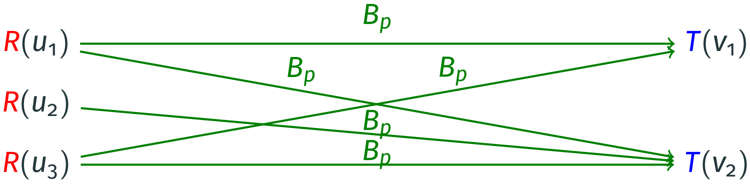

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$$N_p = \sum_{W \subseteq V} N_p^W$$

where N_p^W is the number of subinstances violating Q_1 when fixing the R-facts and T-facts to be precisely on W

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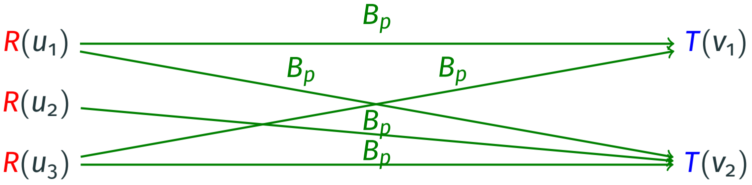

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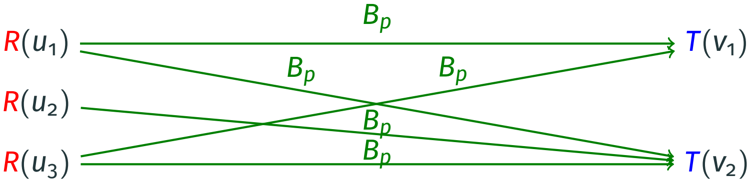

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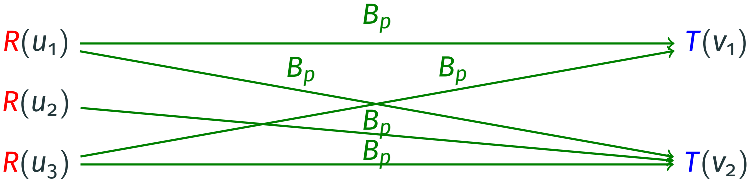

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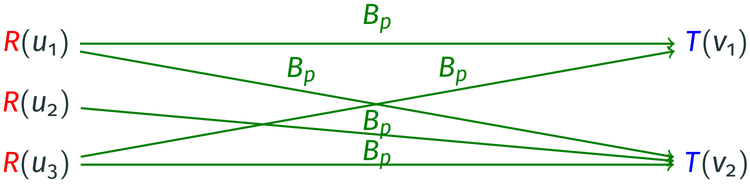

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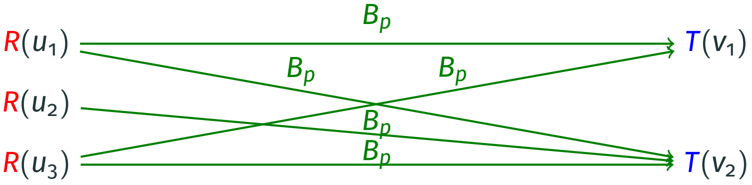

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Equation system and conclusion

We have shown the equation:

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where:

- The $X_{c,d,d',e}$ are what we **want** (to count independent sets)
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We find a box family where M is invertible, so we recover $\vec{X}$ from $\vec{N}$, showing hardness

Conclusion and future work

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Thanks for your attention!

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