A Dichotomy for Homomorphism-Closed Queries on Probabilistic Graphs

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Uncertain data management

In this talk, we manage data represented as a labeled graph

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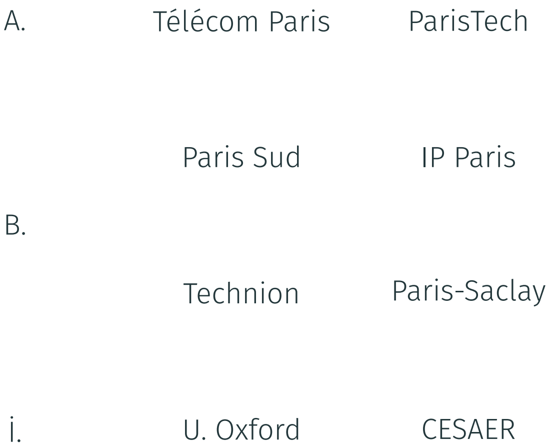

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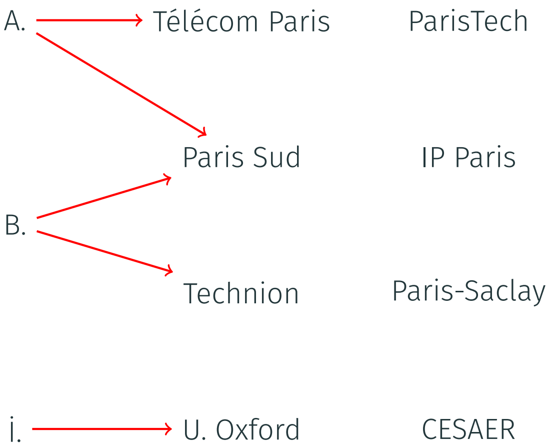

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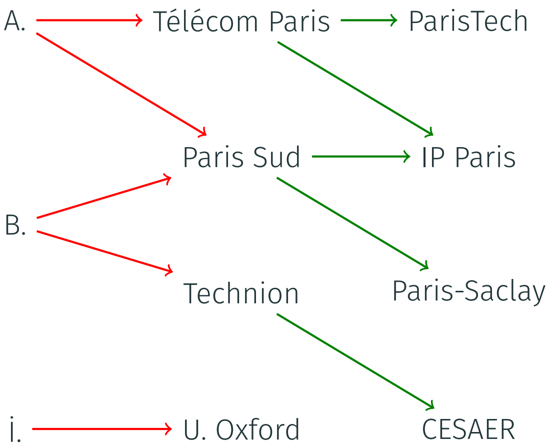

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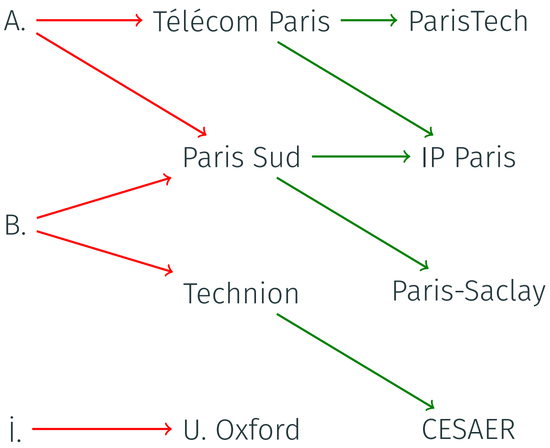
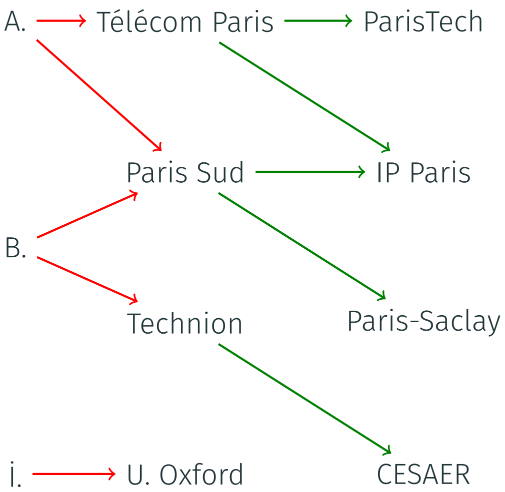


→ **Problem:** we are not **certain** about the true state of the data

Uncertain data model



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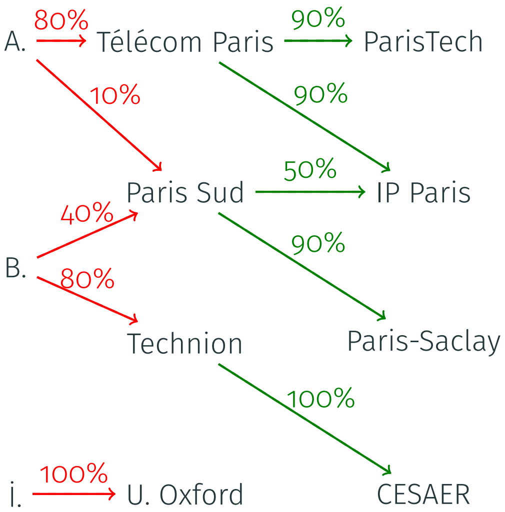

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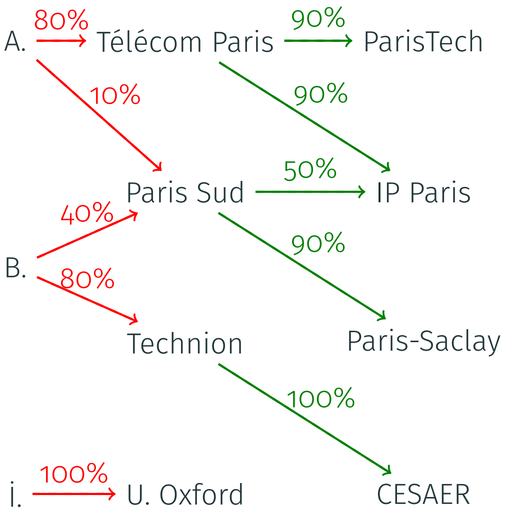

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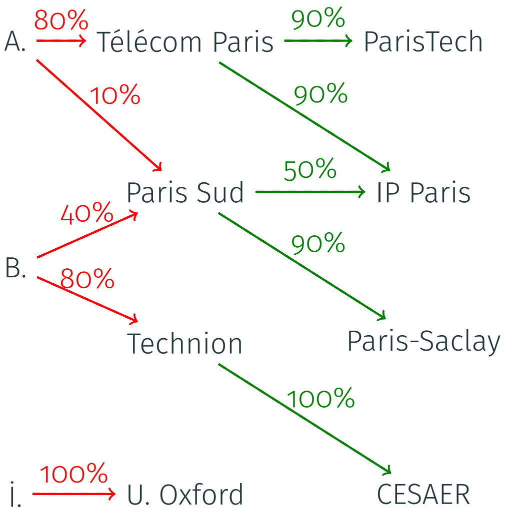

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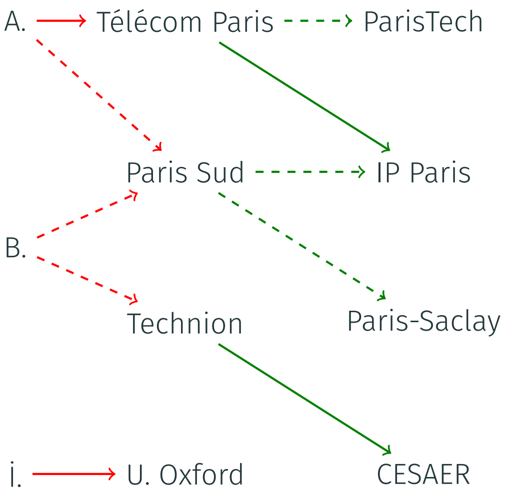

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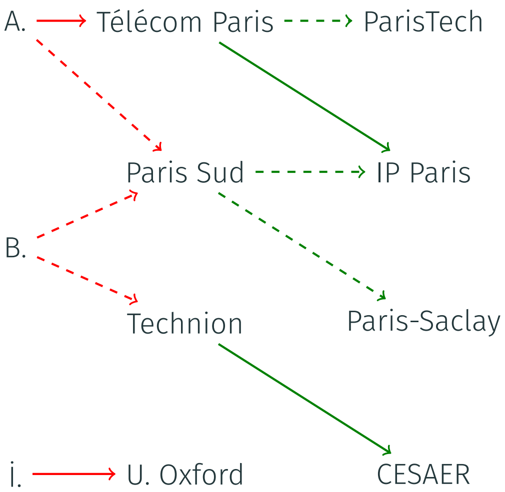

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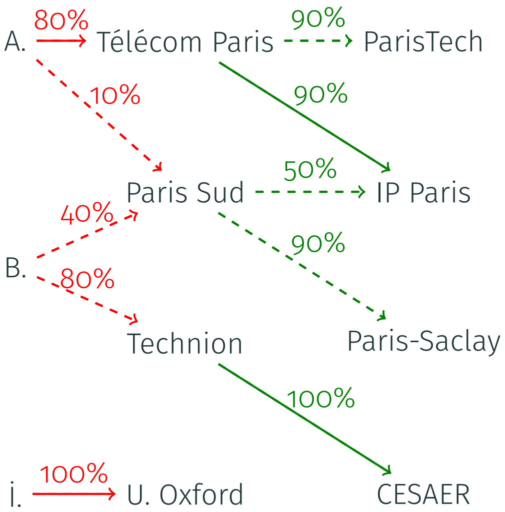

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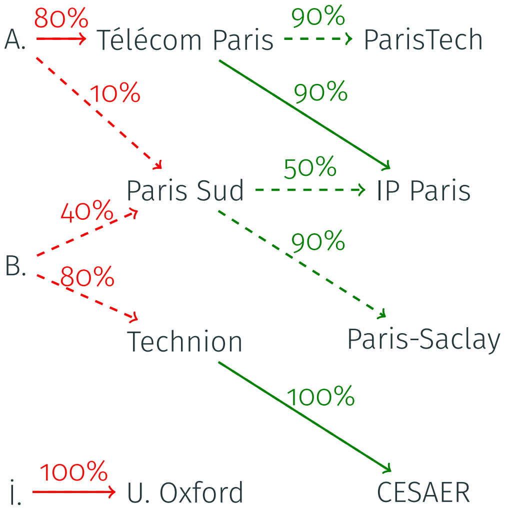

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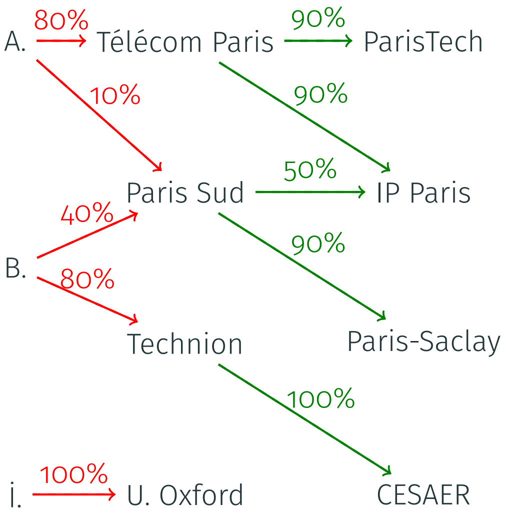

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$$\Pr(W) = \left(\prod_{F \in W} \Pr(F)\right) \times \left(\prod_{F \notin W} (1 - \Pr(F))\right)$$

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- Allows pretty wild things, e.g., “There is a path whose length is prime”

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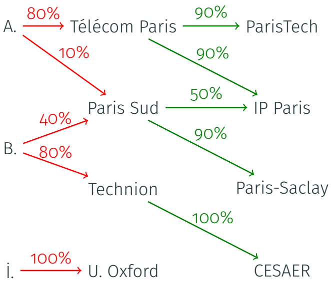

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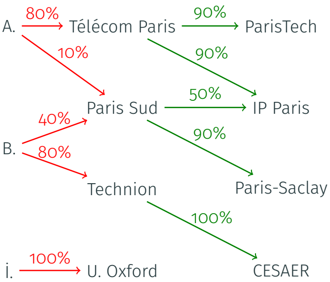

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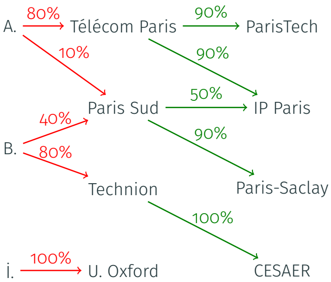

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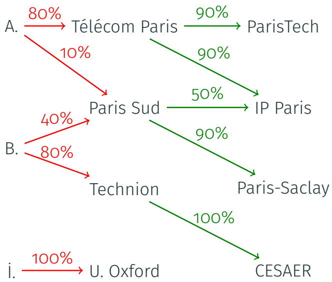

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$\rightarrow$ What is the **complexity** of the problem $\mathrm{PQE}(Q)$, depending on the query Q?

Existing results on PQE

Dichotomy on the unions of conjunctive queries (UCQs):

Theorem [Dalvi and Suciu, 2012]

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- Only exception: work on **ontology-mediated query answering** [Jung and Lutz, 2012]

Our result

We study PQE for homomorphism-closed queries and show:

Theorem

For any query Q closed under homomorphisms:

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- We do not study the **complexity of deciding which case applies**
  - Depends on how queries are represented

Proof structure

Basic idea: finding a tight pattern

The challenging part is to show:

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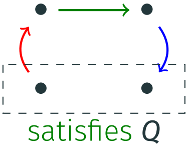

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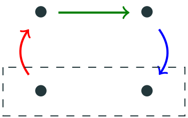

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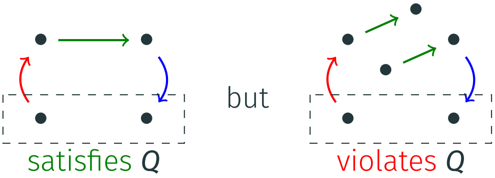

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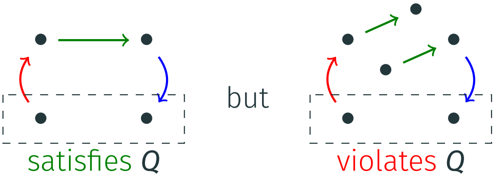

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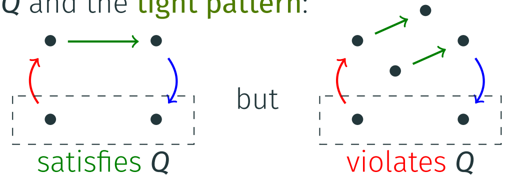

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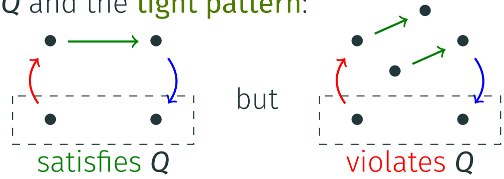
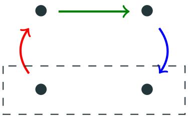


- We reduce from PQE for the **unsafe** CQ: $Q_0 : \ x \longrightarrow y \longrightarrow z \longrightarrow w$

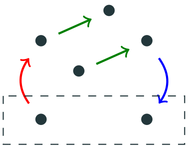
Using tight patterns to show hardness of PQE

- Fix the query Q and the tight pattern:



satisfies Q

but



violates Q

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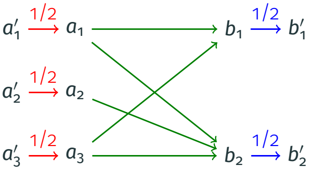
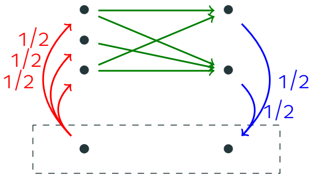


is coded as



Using tight patterns to show hardness of PQE

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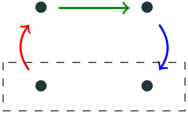
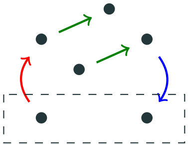
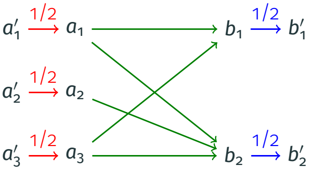


but



satisfies Q violates Q

- We reduce from PQE for the unsafe CQ: $Q_0 : x \longrightarrow y \longrightarrow z \longrightarrow w$



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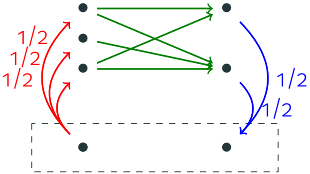

Idea: possible worlds at the left have a path that matches Q_0
iff the corresponding possible world of the TID at the right satisfies the query Q...

Using tight patterns to show hardness of PQE

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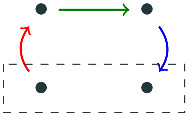
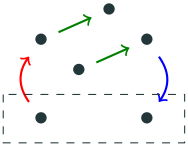


satisfies Q

but



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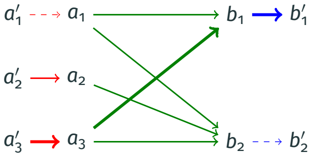

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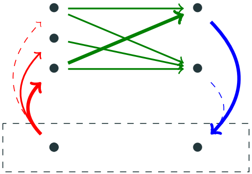

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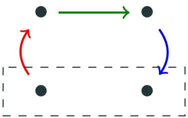

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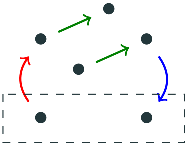

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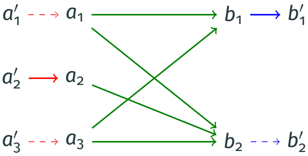
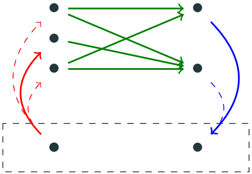


is coded as



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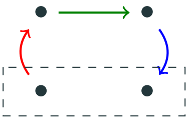

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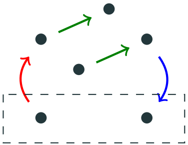

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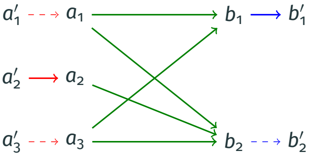

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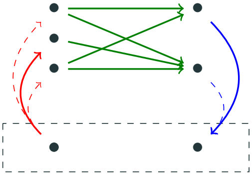

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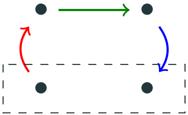
Using tight patterns to show hardness of PQE

- Fix the query Q and the **tight pattern**:



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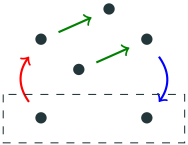
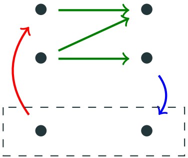


AND



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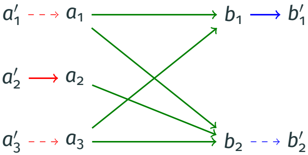

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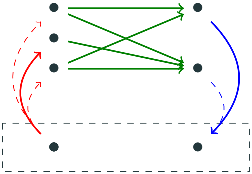

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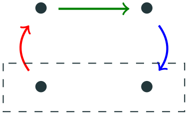
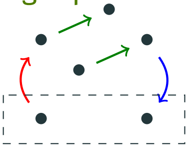
Saving the proof

We know that we have a **tight pattern**:



satisfies Q

but



violates Q

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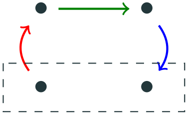

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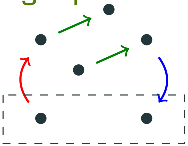

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Consider its **iterates**

Saving the proof

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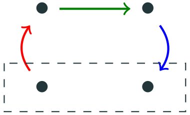

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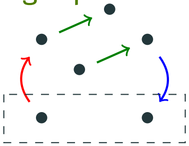
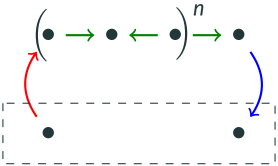


violates Q

Consider its iterates for each $n \in \mathbb{N}$:



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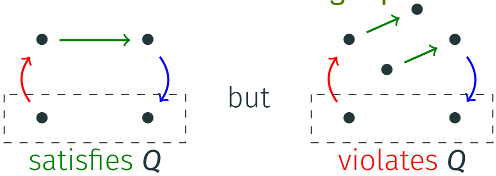

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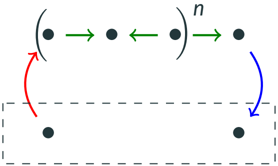

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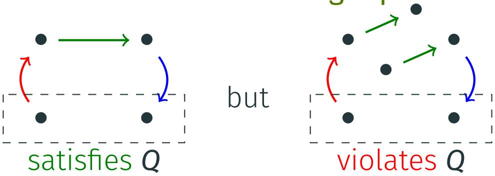

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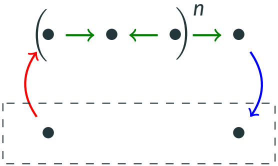
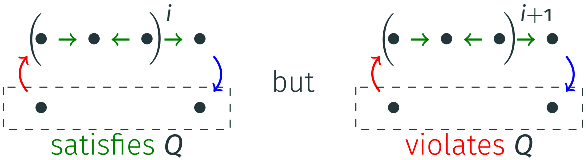


Case 1: some iterate **violates** the query:



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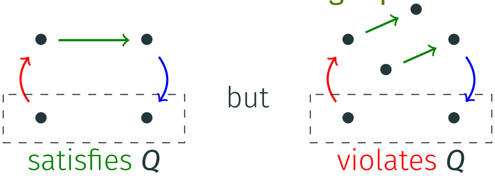

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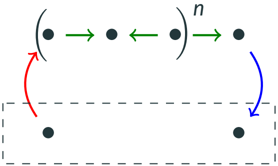

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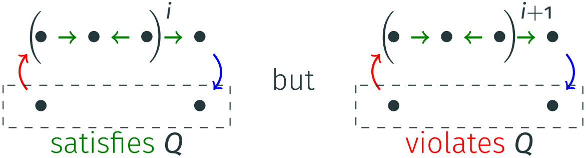

$\rightarrow$ Reduce from $\mathrm{PQE}(Q_0)$ as we explained

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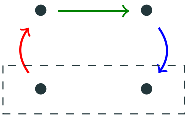
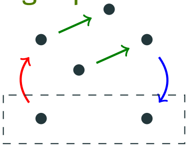


satisfies Q

but



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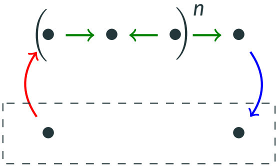

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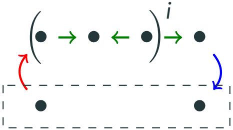


satisfies Q

but

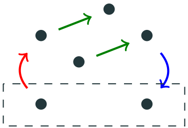
violates Q

$\rightarrow$ Reduce from $\mathrm{PQE}(Q_0)$ as we explained

Case 2: all iterates **satisfy** the query:

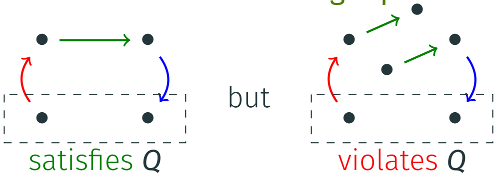
satisfies Q for all $n \in \mathbb{N}$

but



violates Q

We know that we have a **tight pattern**:



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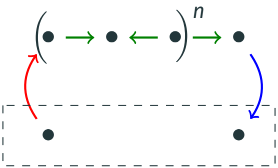

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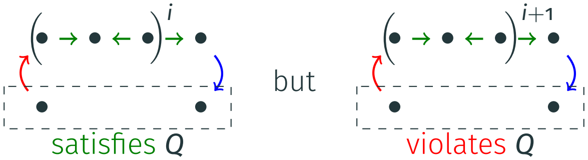

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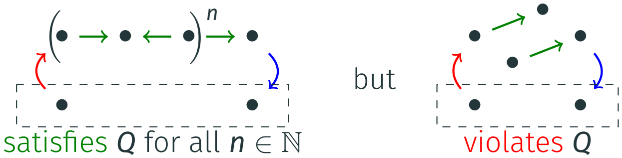


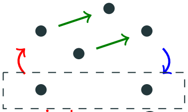
$\rightarrow$ Call this an **iterable pattern**

Using iterable patterns to show hardness of PQE

We have an iterable pattern:

satisfies Q for all $n \in \mathbb{N}$

but



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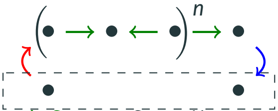

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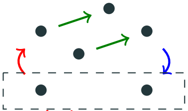

violates Q

Idea: reduce from the **#P-hard** problem **source-to-target connectivity**:

- Input: **undirected graph** with a **source** s and **target** t, all edges have probability $1/2$
- Output: what is the **probability** that the source and target are **connected**?

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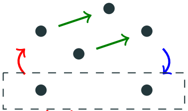

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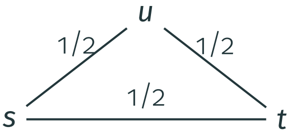

Using iterable patterns to show hardness of PQE

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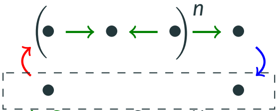

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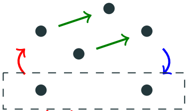

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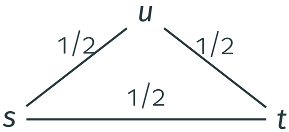

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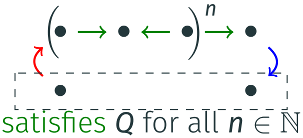

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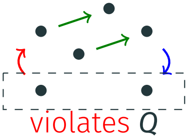

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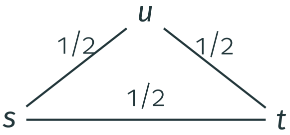

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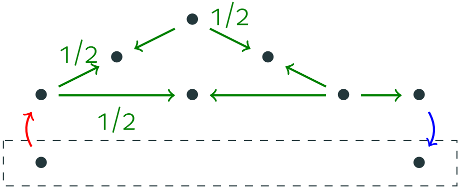

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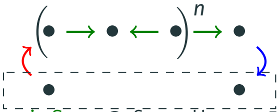

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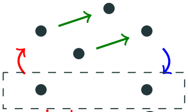

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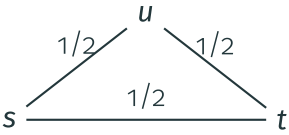

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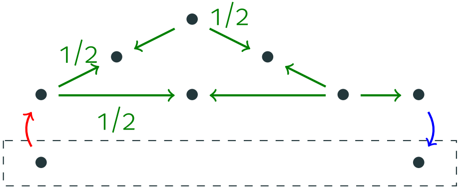

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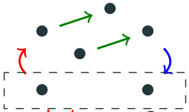

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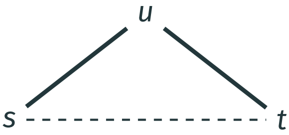

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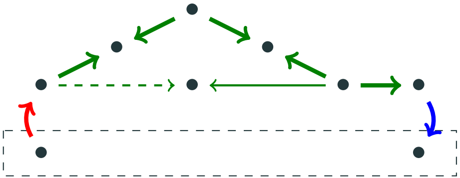

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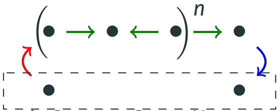

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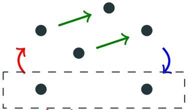

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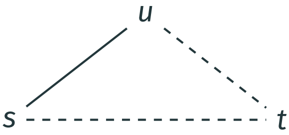

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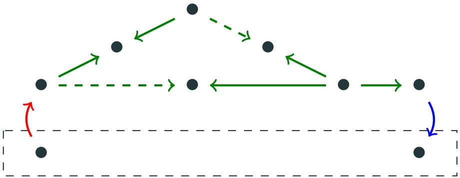

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Thanks for your attention!

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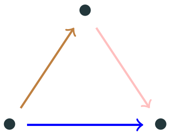

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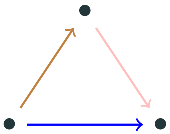

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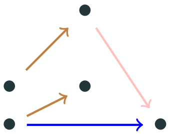

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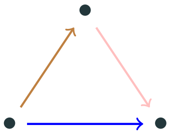

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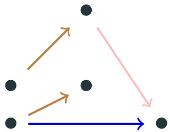

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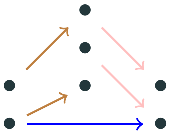

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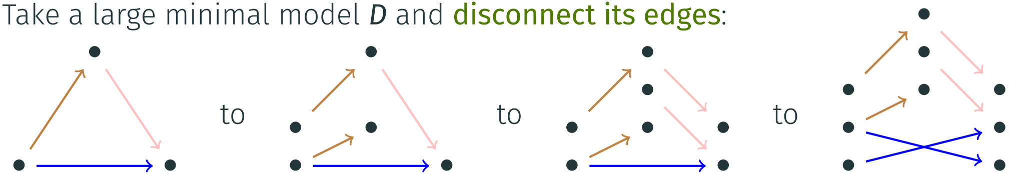

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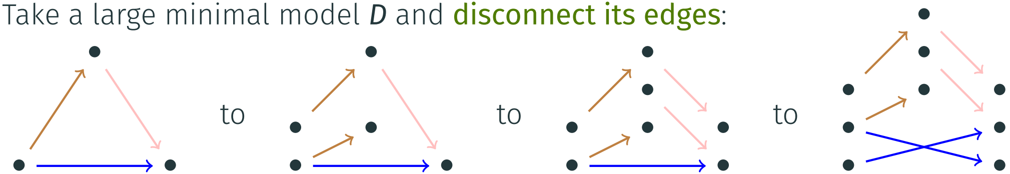

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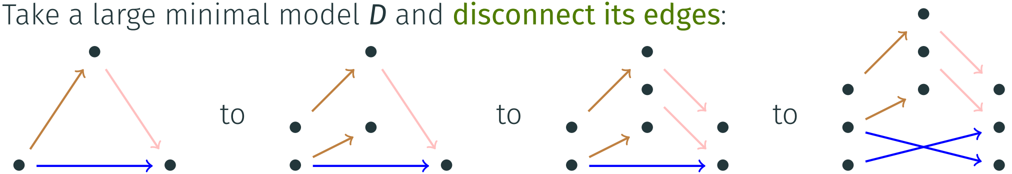

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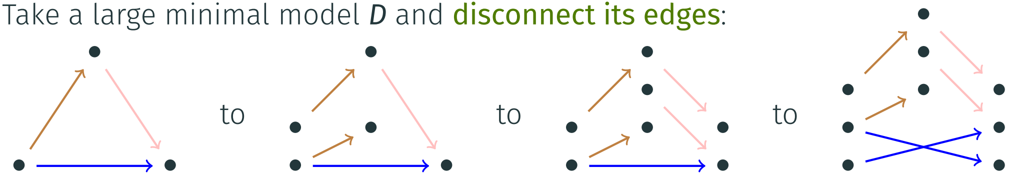

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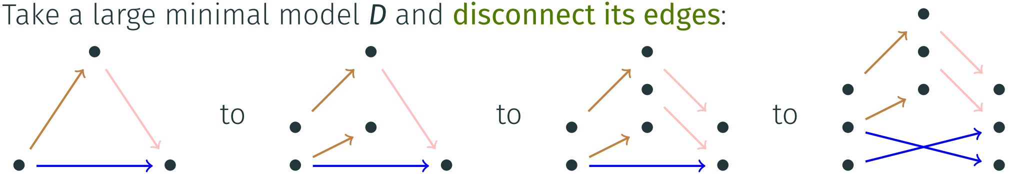

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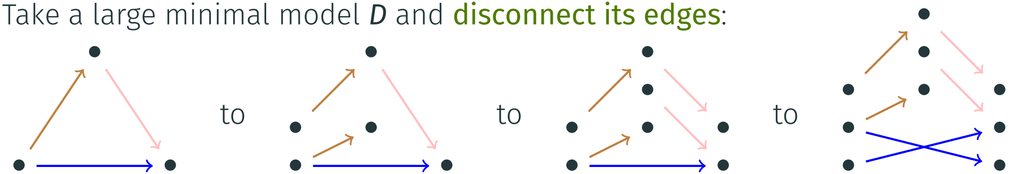

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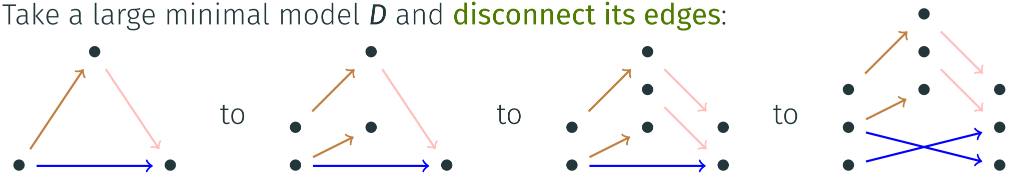

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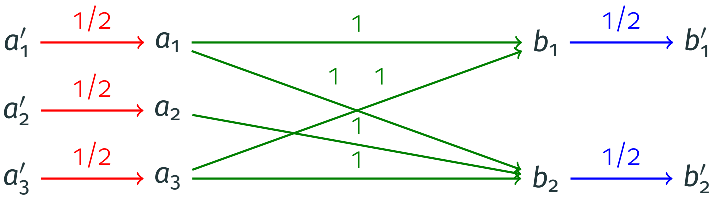

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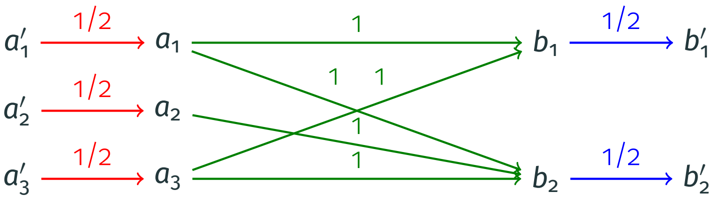

Idea: Satisfying valuations of ϕ correspond to **possible worlds** with a **match** of Q

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