Enumeration on Trees under Relabelings

Antoine Amarilli[1], Pierre Bourhis[2], Stefan Mengel[3]

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[1]Télécom ParisTech

[2]CNRS CRIStAL

[3]CNRS CRIL

Problem statement

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- Tree on a fixed alphabet
- Boolean query to test a property of the tree

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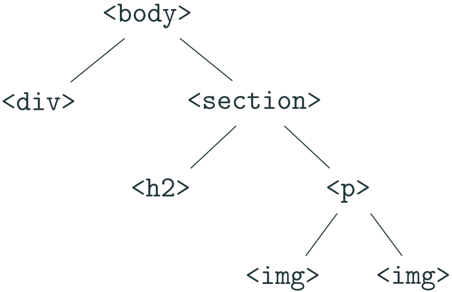

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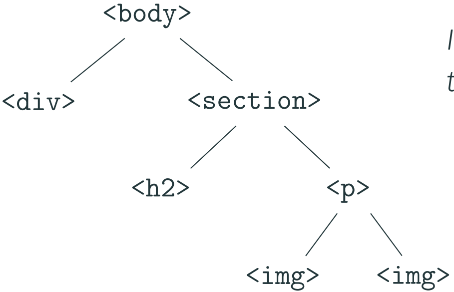

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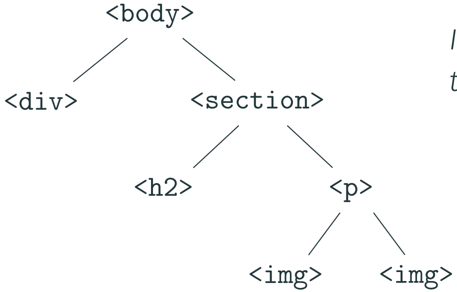

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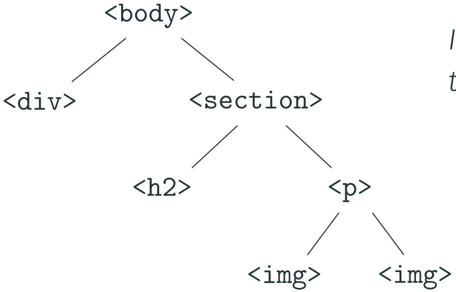

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Example answer

→**YES**

→ **Theorem:** For **monadic second-order** (MSO) queries, we can check if the query is true or not in **linear time** in the input tree

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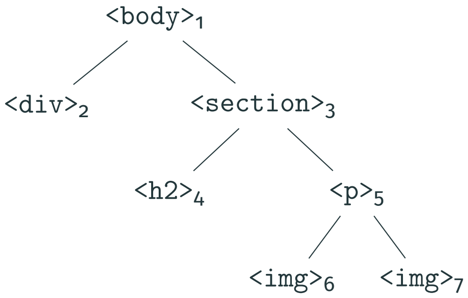

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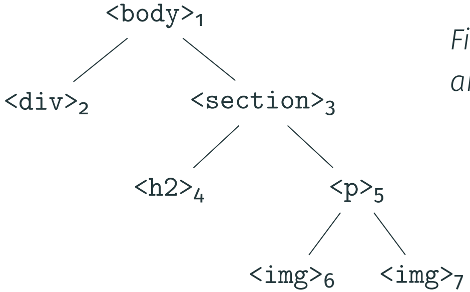

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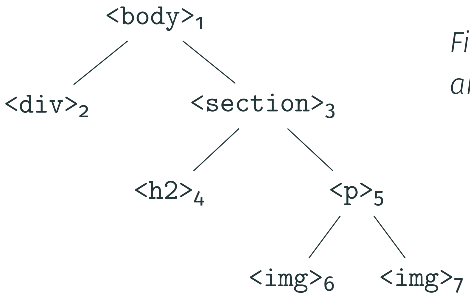

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$\rightarrow \ \{\langle 4,6\rangle,\langle 4,7\rangle\}$

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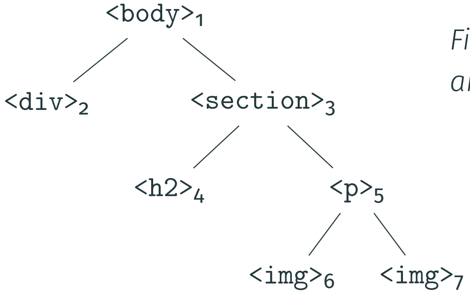

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Example answer

$\rightarrow \{\langle 4, 6\rangle, \langle 4, 7\rangle\}$

$\rightarrow$ **Corollary:** For each possible tuple, we can check in **linear time** if it is an answer to the query

Enumerating all answers

$\rightarrow$ There can be lots of answers!

- *"Find all pairs of …"*: output size can be $O(|T|^2)$

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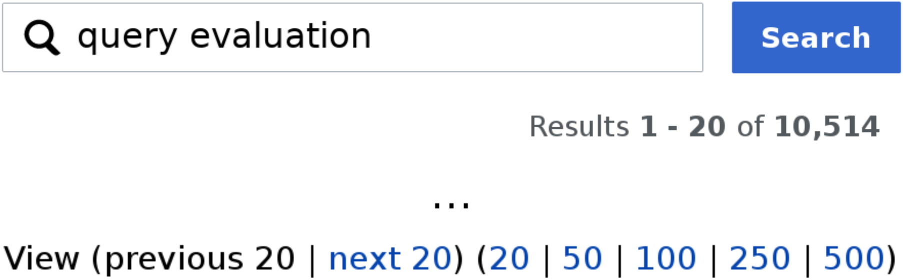

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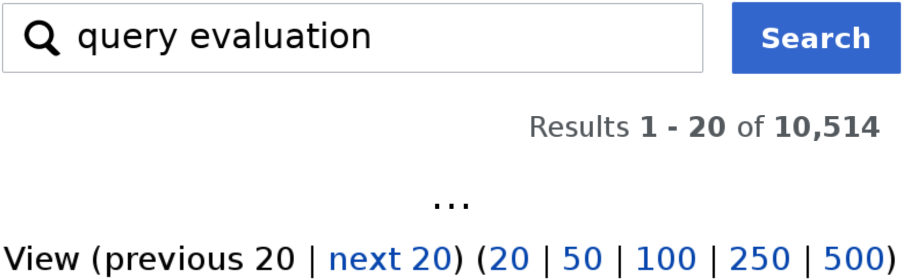
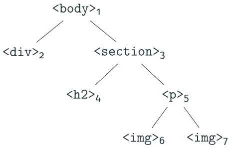


→ **Solution:** Enumerate answers **one after the other**

Enumeration

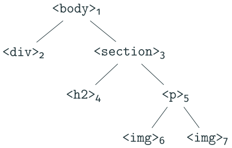


Tree

$$\exists s\ \mathtt{section}(s) \wedge$$
$$s \rightsquigarrow x \wedge s \rightsquigarrow y \wedge$$
$$\mathtt{h2}(x) \wedge \mathtt{img}(y)$$

Query

Enumeration



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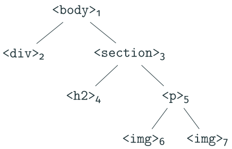

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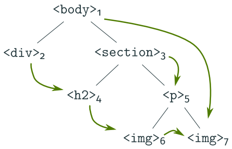
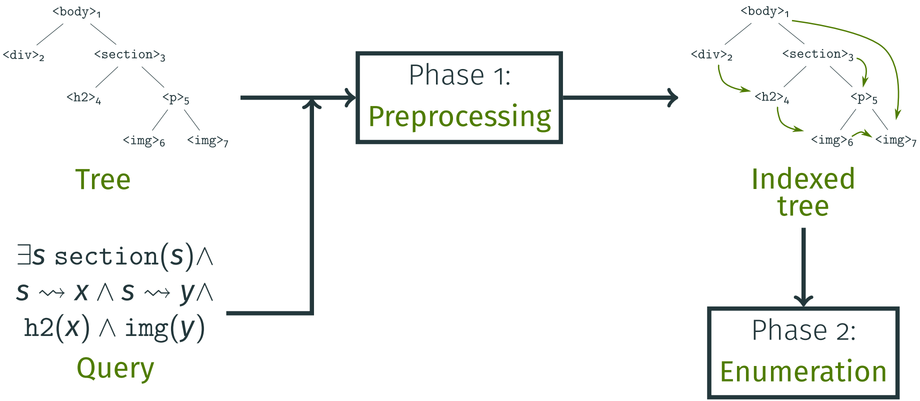


Indexed tree

Enumeration



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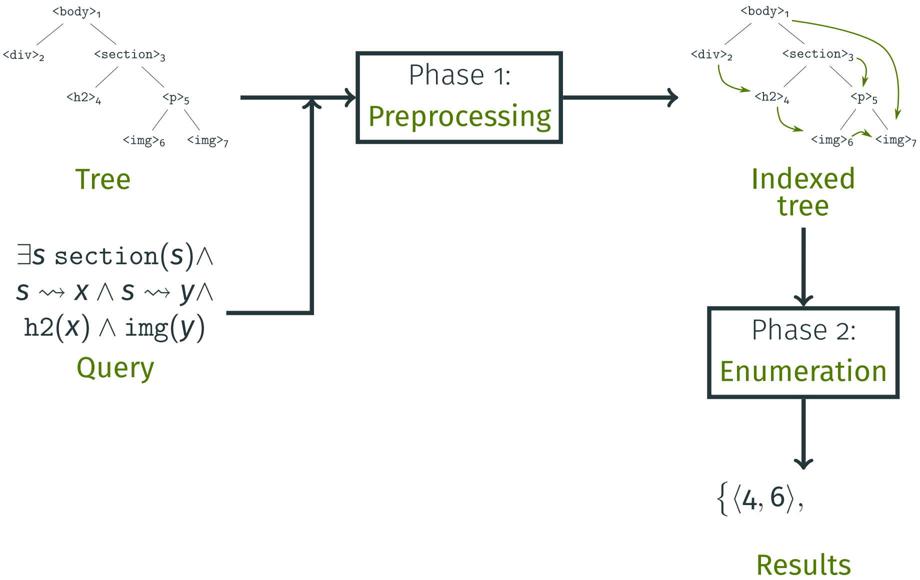

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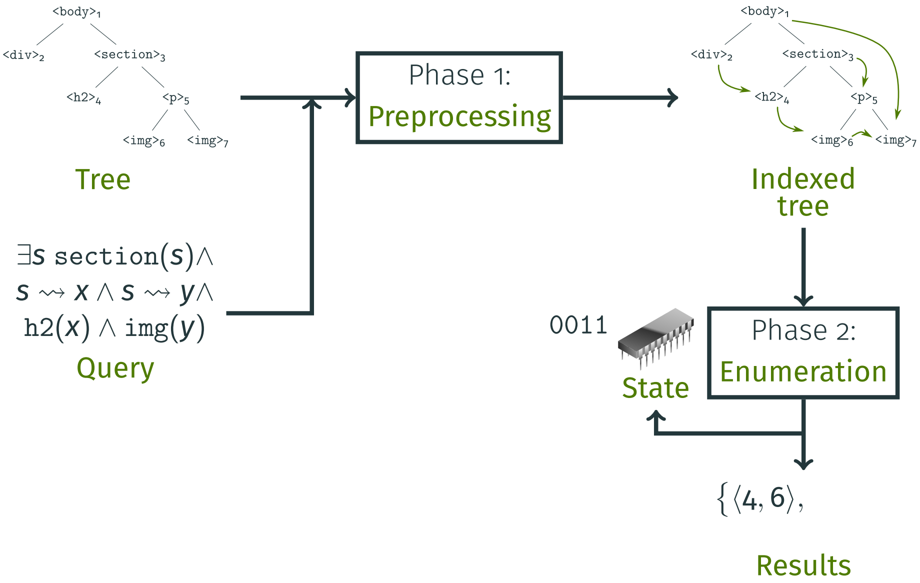

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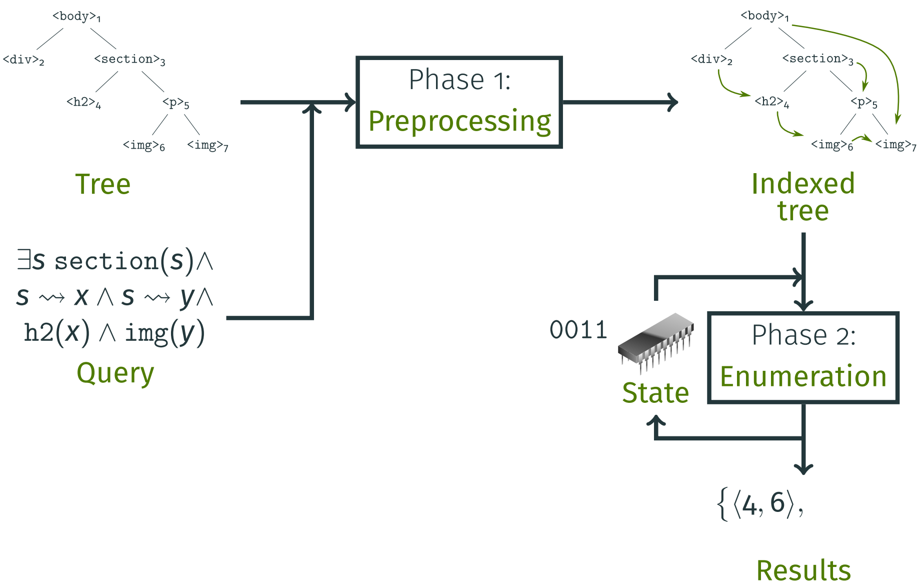

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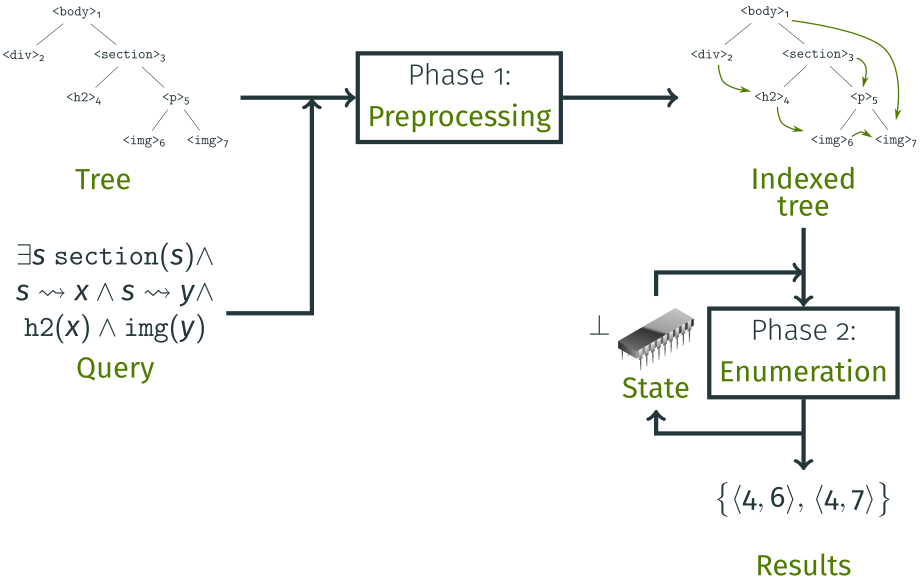

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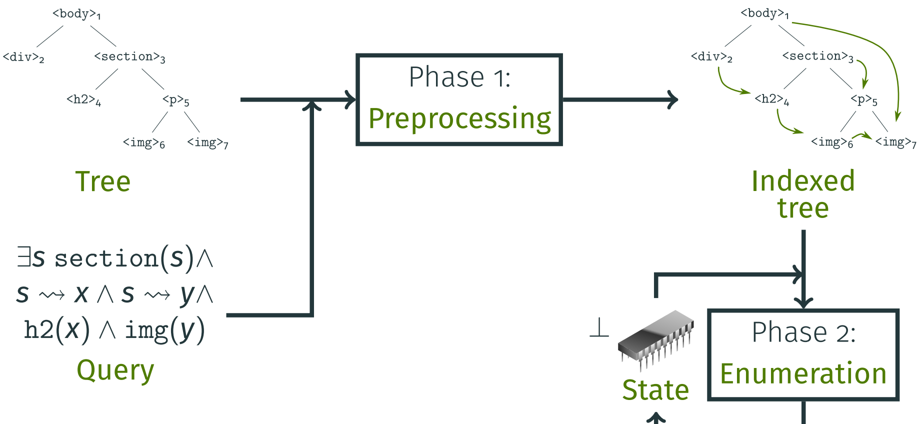

Theorem (Bagan'06; Kazana & Segoufin'13)

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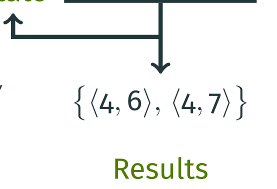

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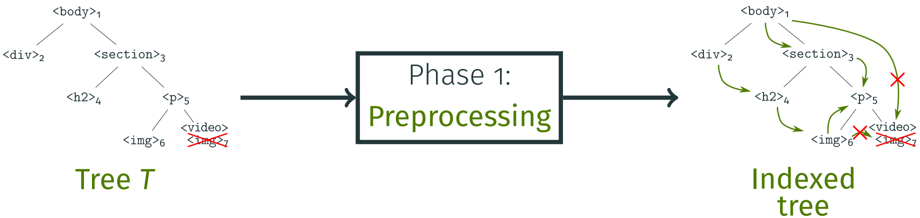

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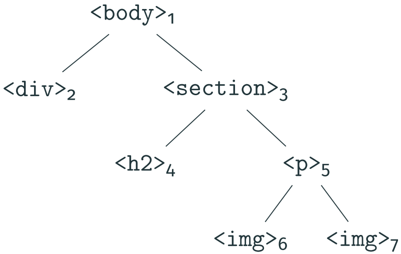

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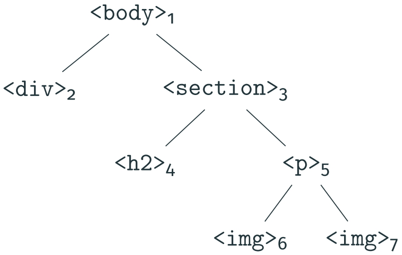

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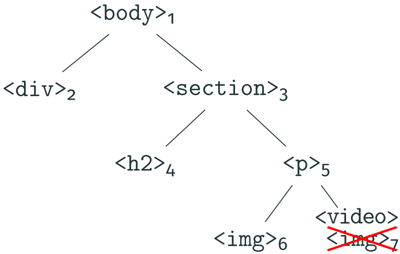

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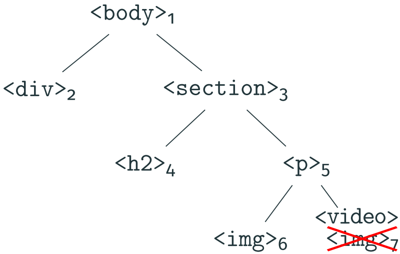

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- **Example:** relabel node `7` to `<video>`
- The tree's **structure** never changes

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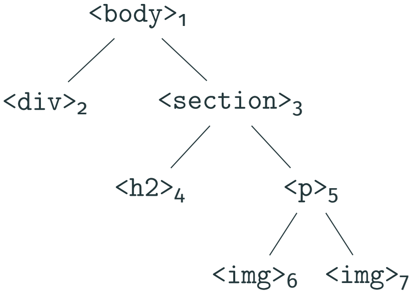
  - Example: *“Find all images in a* ***user-selected*** *section”*



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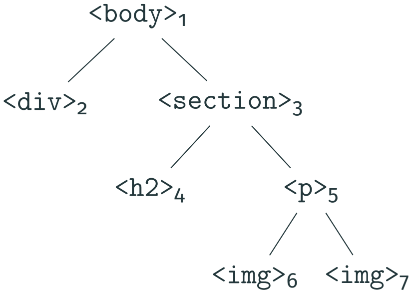

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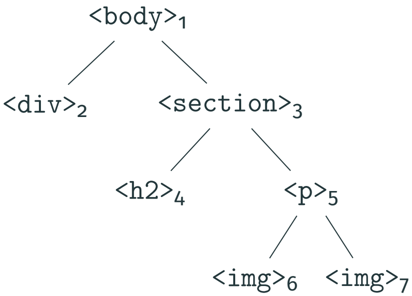

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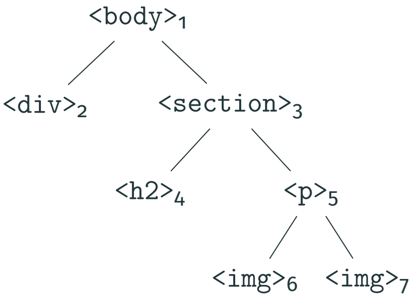

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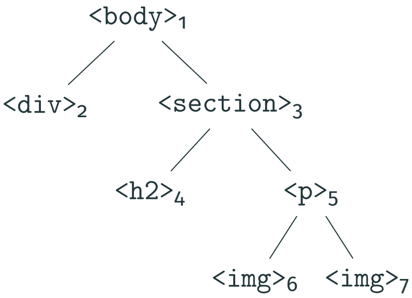

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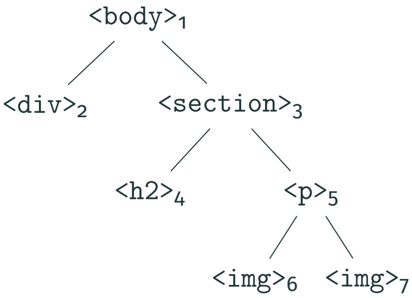

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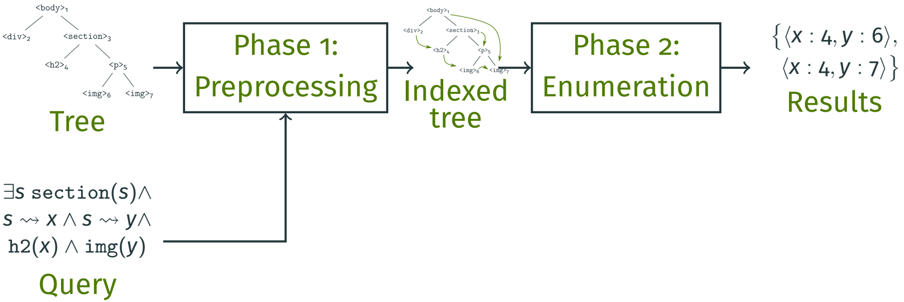
$\rightarrow$ Consequences for group-by, aggregation, parameterized queries

Proof techniques

Knowledge compilation

To make the proof modular, we follow knowledge compilation:



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To make the proof **modular**, we follow **knowledge compilation**:

- **Preprocessing:** Compute a **circuit representation** of the answers
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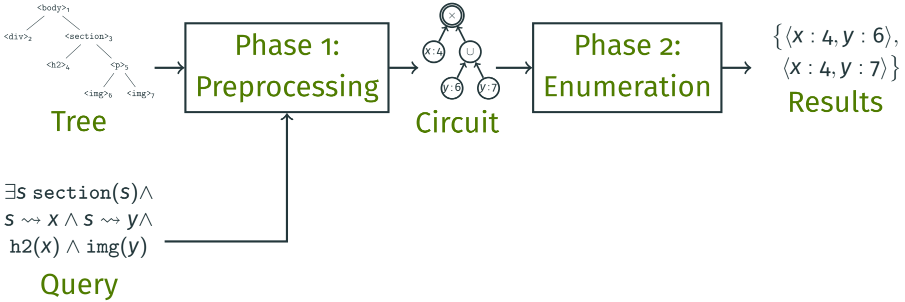

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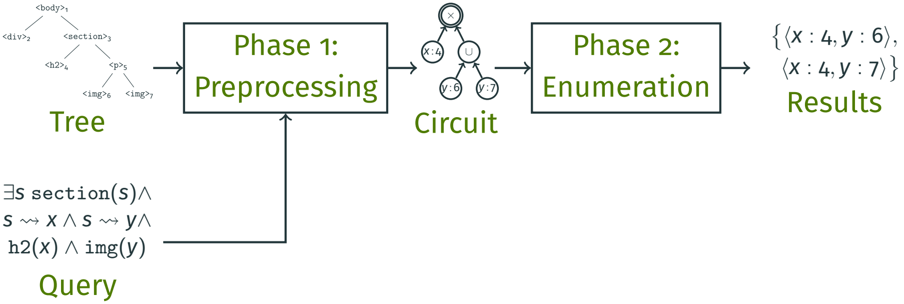

First present approach **without relabelings** (as in our ICALP'17 paper) then extend the approach to **support relabelings**

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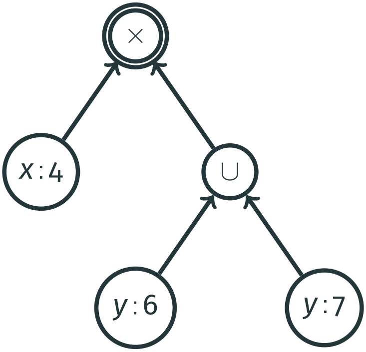

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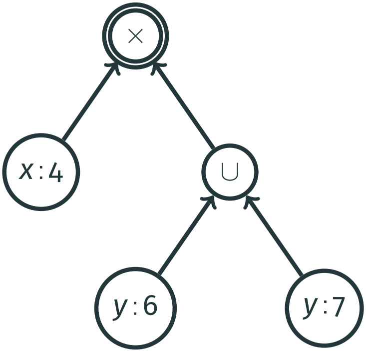

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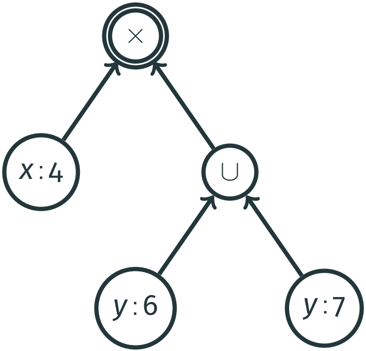

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- Union gate $\cup$:

  $\rightarrow$ union of sets of tuples

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A **set circuit** represents a **set of answers** to a query $Q(x, y)$

- **Singleton** $x:6 \to$ *"the free variable x is mapped to node 6"*
- **Tuple** $\langle x:4, y:6\rangle$: tuple of singletons
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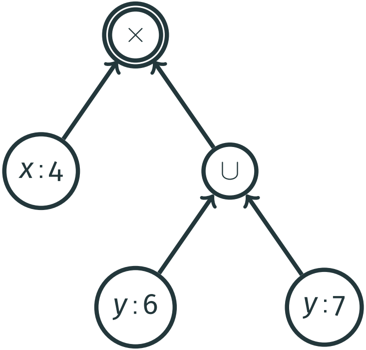

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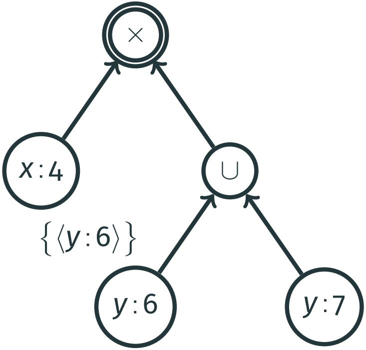

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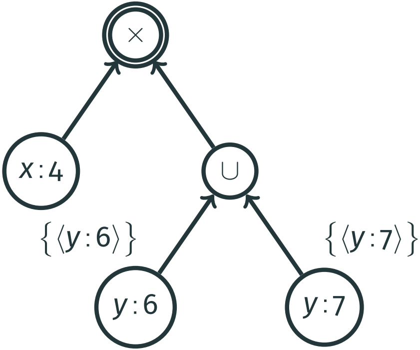

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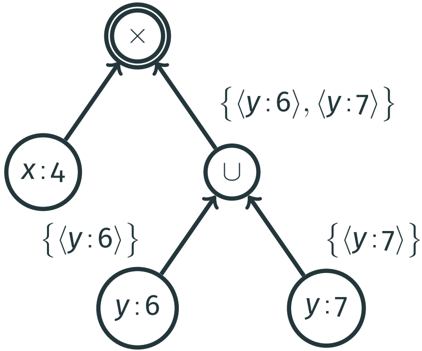

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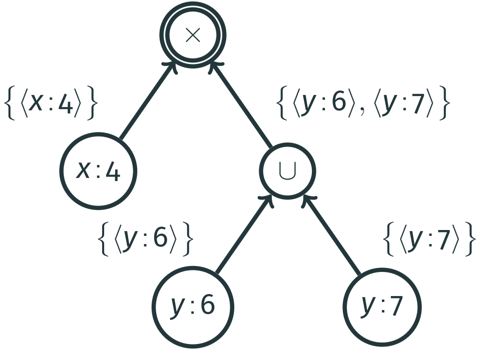

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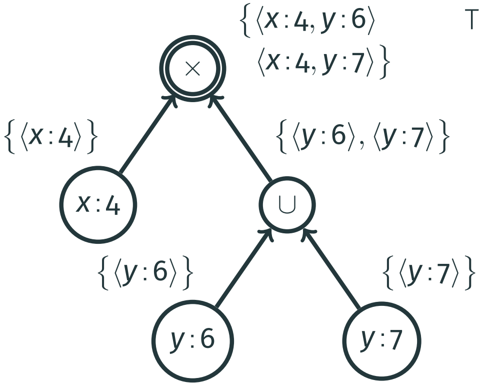

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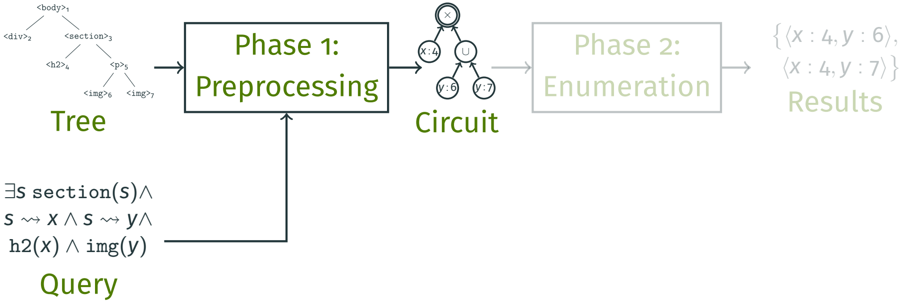
Preprocessing: Set circuit construction



Theorem

For any MSO query $Q(x_1, \ldots, x_k)$, given a tree T, we can build in $O(T)$ a set circuit capturing exactly the set of answers of Q on T:
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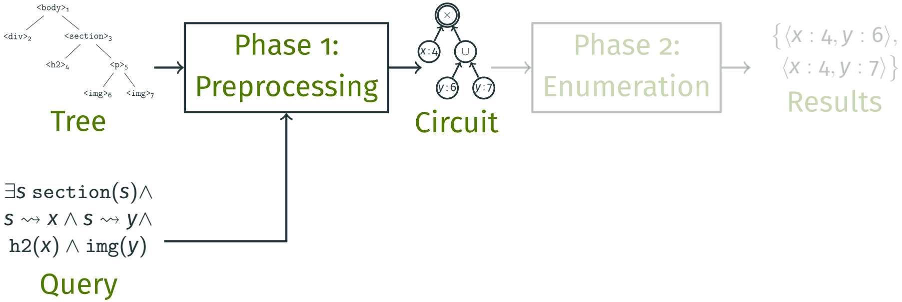

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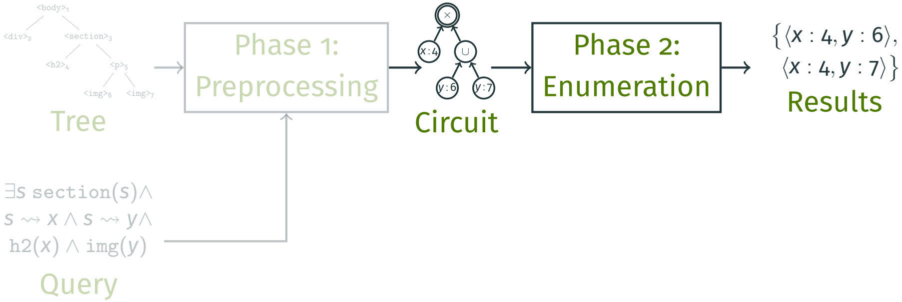
- **Proof idea:** Translate query to **bottom-up tree automaton** and build a **provenance circuit** following the structure of the tree

Enumeration on set circuits



Theorem

Given a set circuit satisfying some conditions, we can enumerate all tuples that it captures with linear preprocessing and constant delay

E.g., for $\{\langle x:4, y:6\rangle, \langle x:4, y:7\rangle\}$: enumerate $\langle x:4, y:6\rangle$, then $\langle x:4, y:7\rangle$

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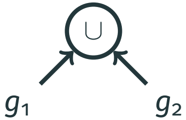

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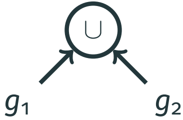

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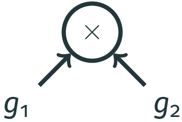
$\times$-gate



Lexicographic product:

for every t_1 in $T(g_1)$:
    for every t_2 in $T(g_2)$:
        output $t_1 + t_2$

Circuit conditions

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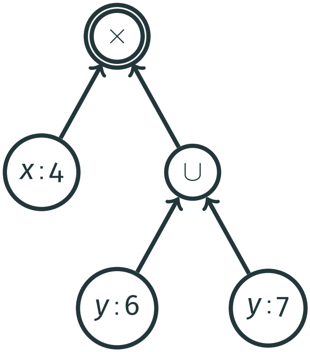

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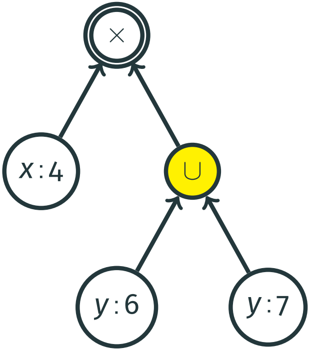

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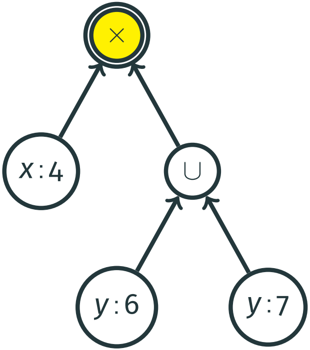

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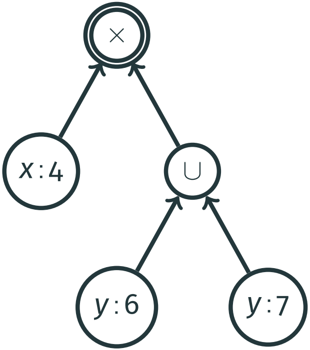

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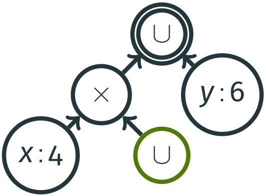
- Also an additional **upwards-determinism** condition

Enumeration subtleties



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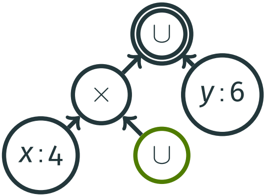

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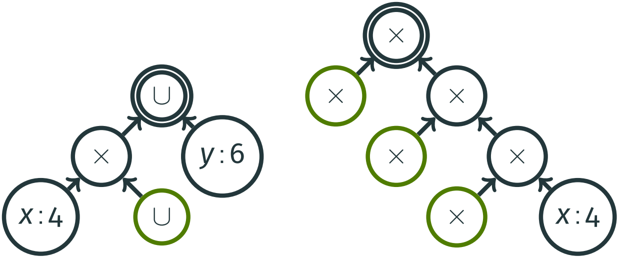

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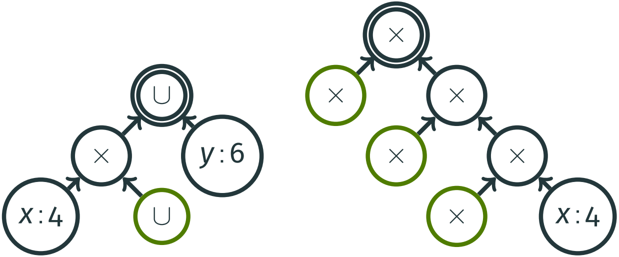

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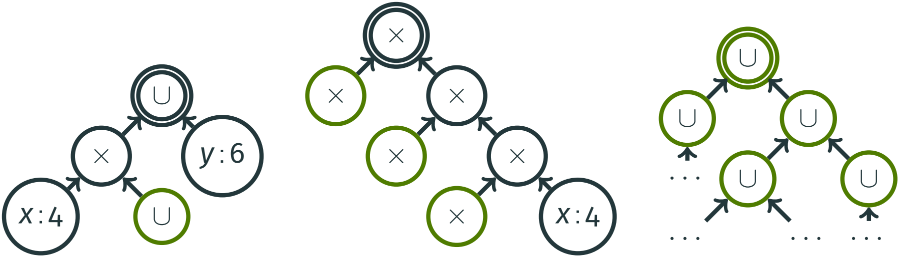

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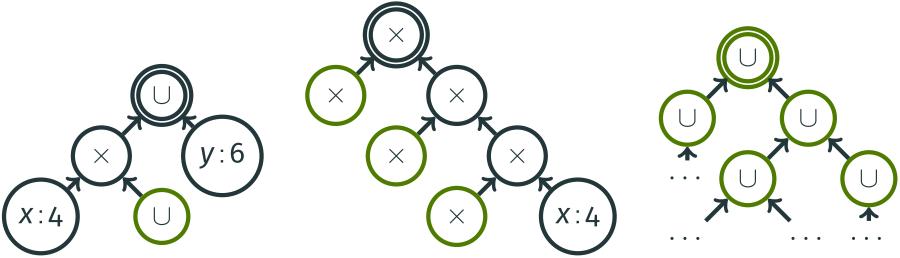

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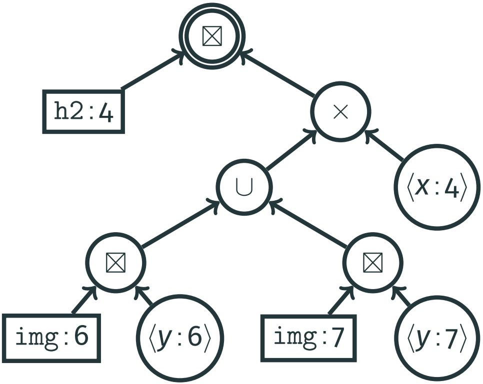
  - $\rightarrow$ Precompute a **reachability index** (uses **upwards-determinism**)

Hybrid circuits to support relabelings

To support relabelings, we use hybrid circuits that have:

- Boolean gates that depend only on the labeling
- Set gates that capture a set of tuples for each labeling



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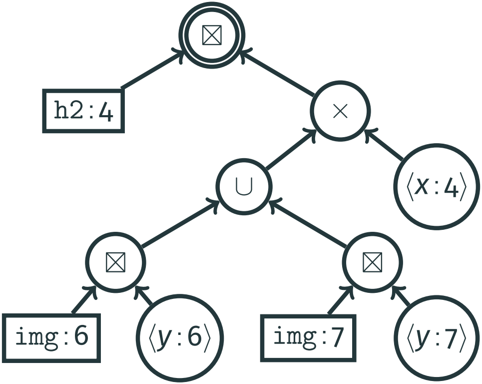

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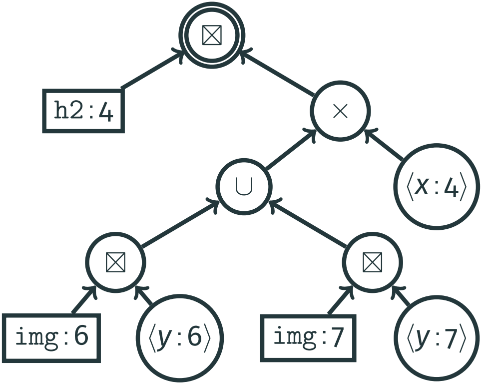

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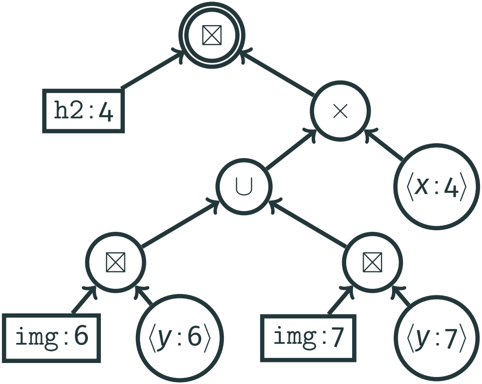

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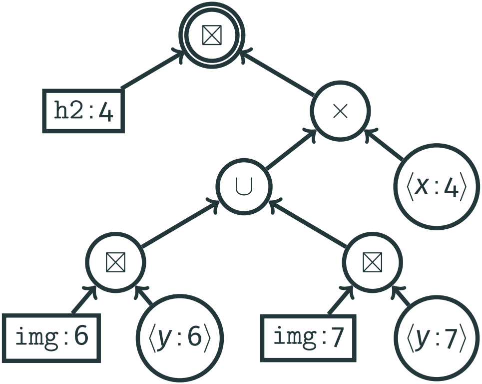

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One new **set-valued gate**:

- $\boxtimes$: **Test gate**
  → $\emptyset$ if Boolean input is false
  → like the set input otherwise

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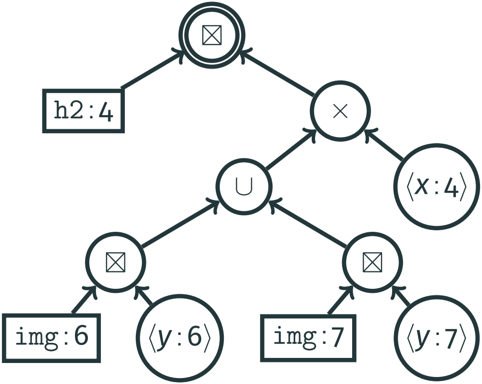

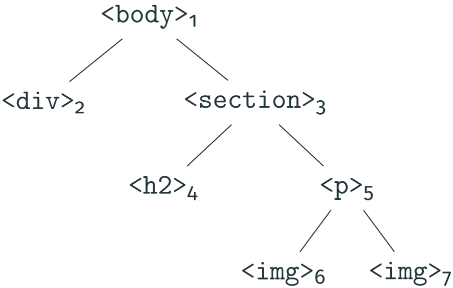

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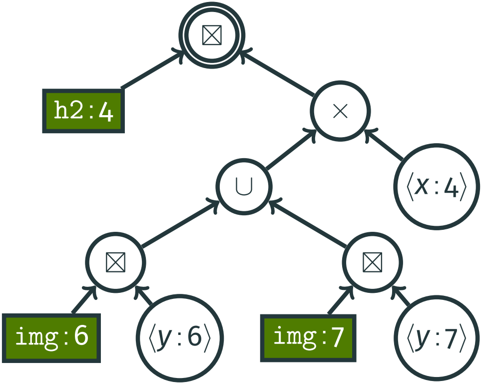

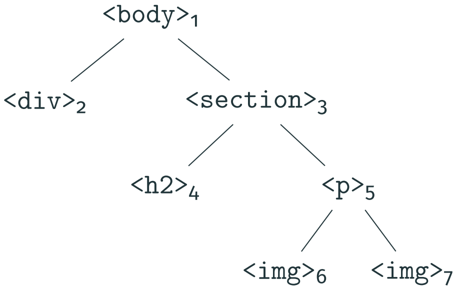

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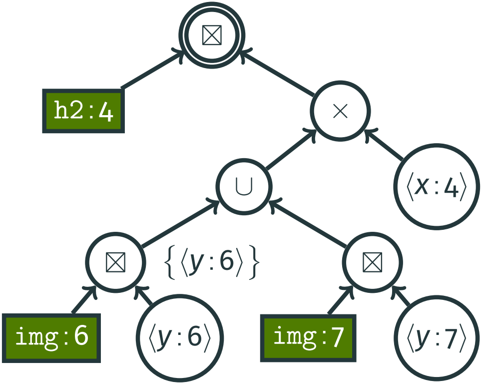

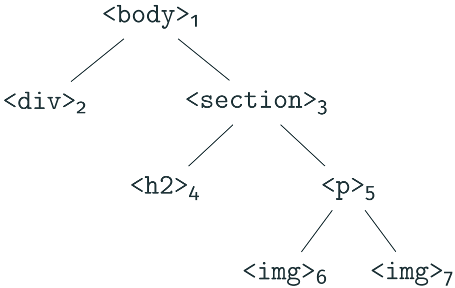

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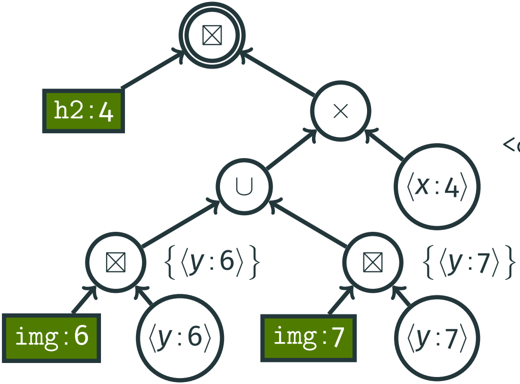

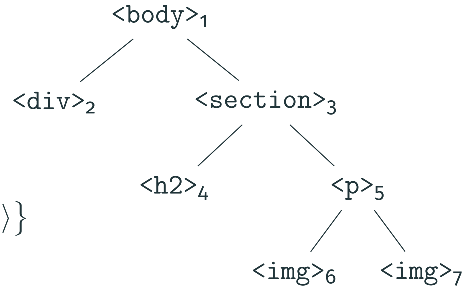

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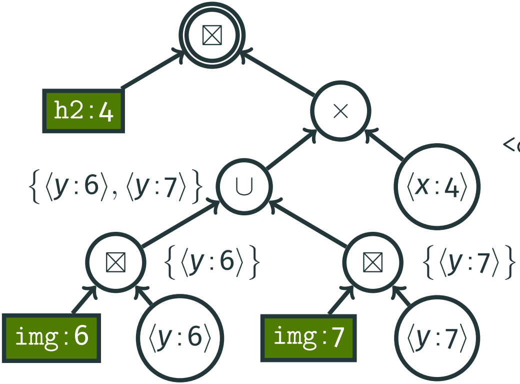

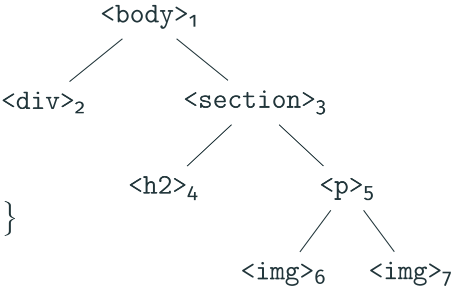

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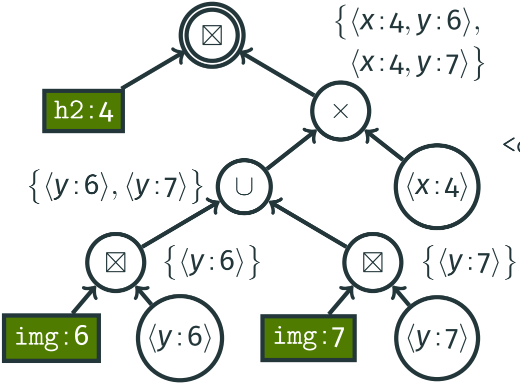

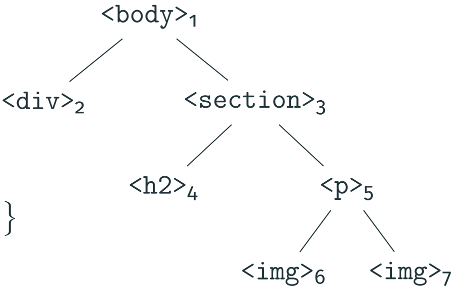

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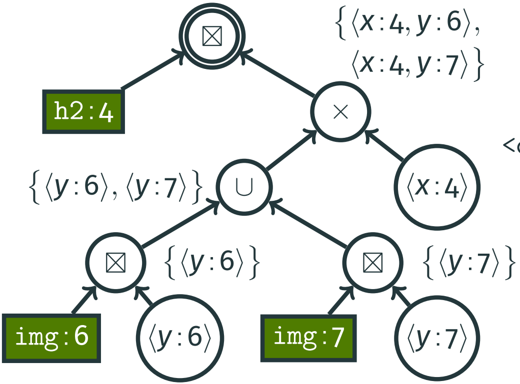

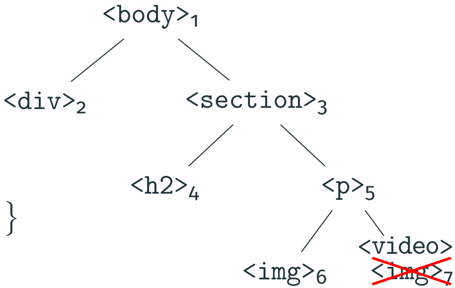

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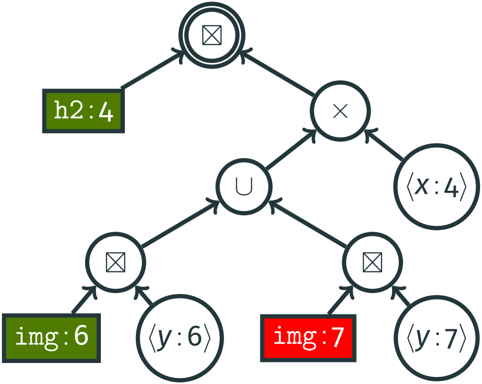

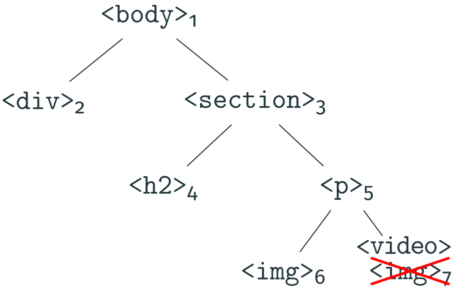

Hybrid circuit semantics

- For every labeling, the hybrid circuit captures a set of tuples
  - $\rightarrow$ Here, the captured set is $\{\langle x{:}4, y{:}6\rangle, \langle x{:}4, y{:}7\rangle\}$
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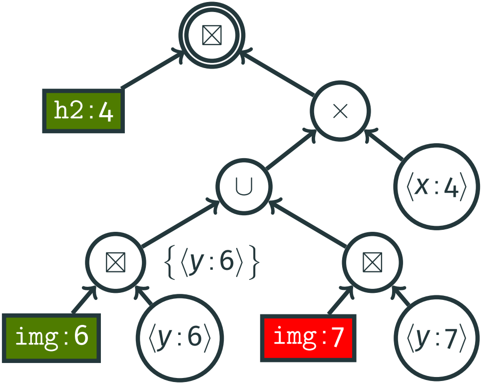

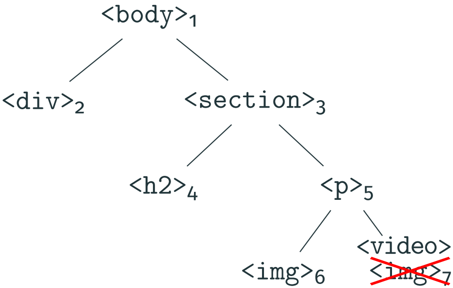

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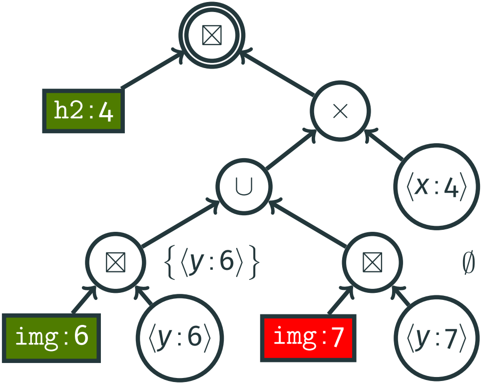

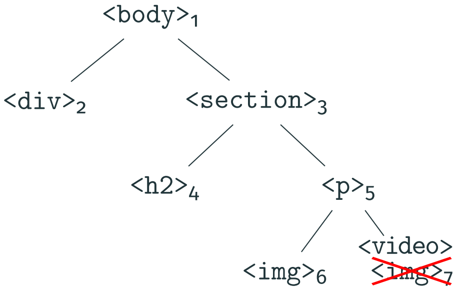

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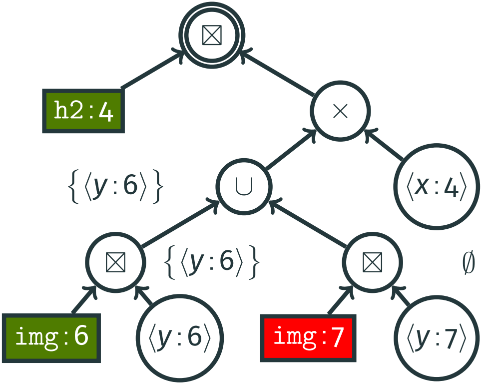

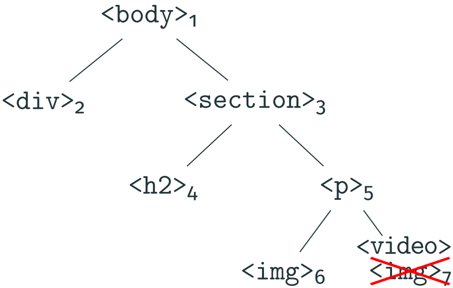

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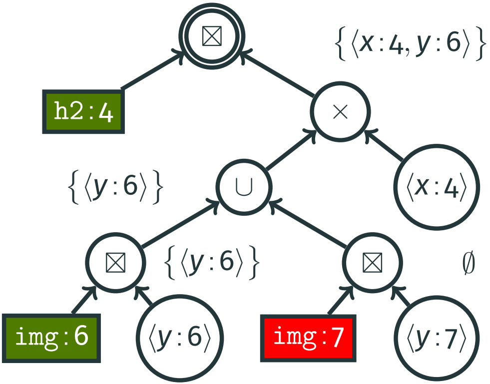

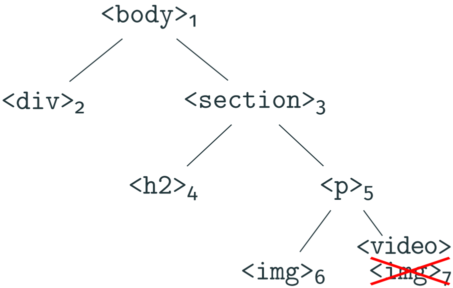

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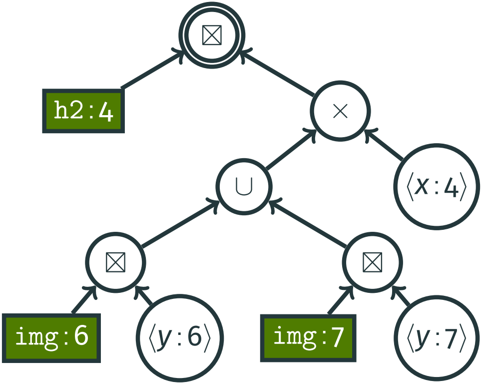

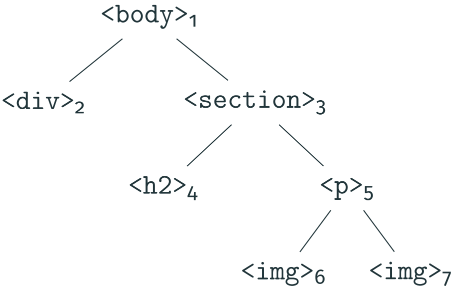

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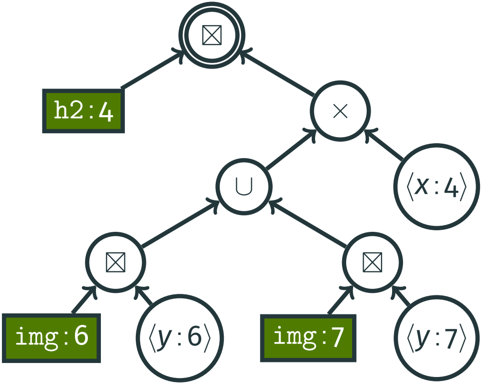

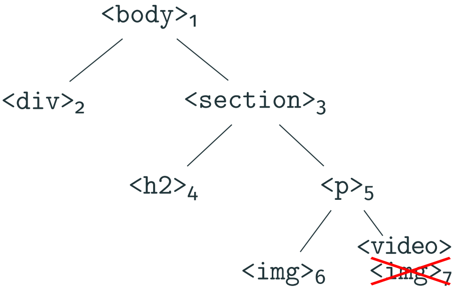

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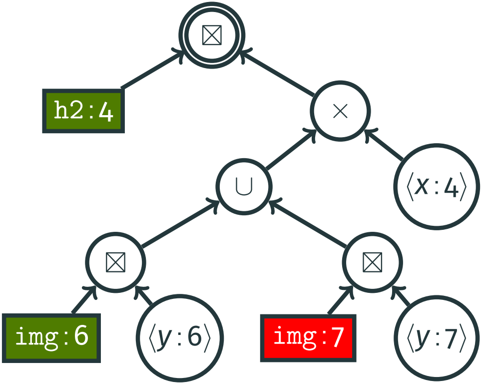

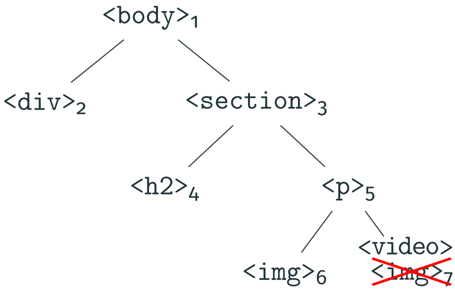

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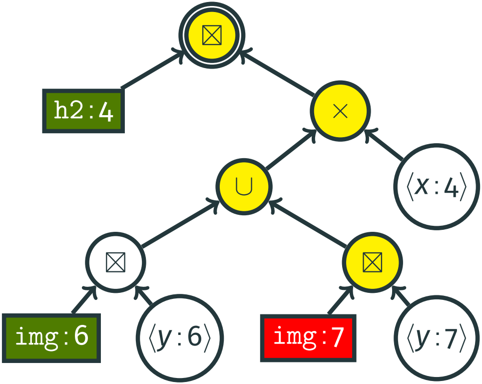

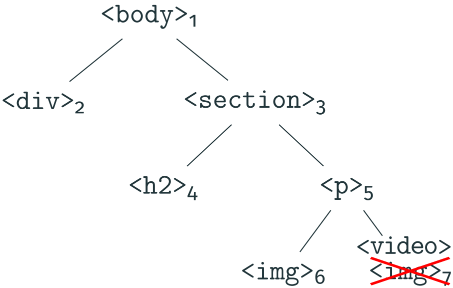

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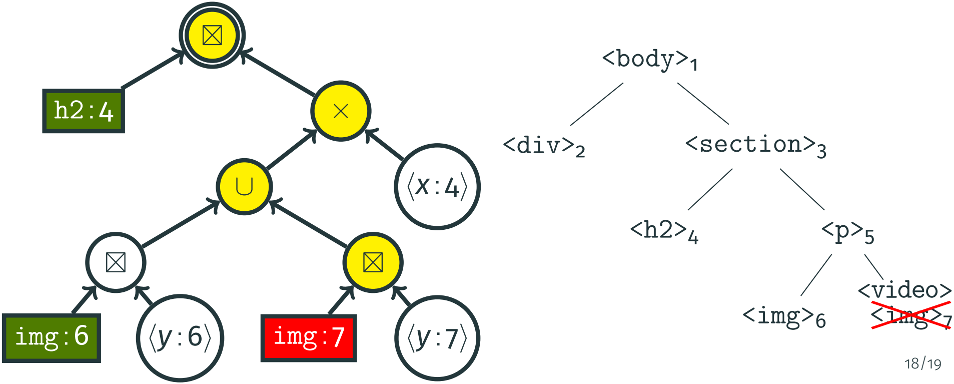
$\rightarrow$ **Balancing lemma:** Rewrite the **input tree** to make it **balanced**



Conclusion

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Thanks for your attention!

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To appear.

Set circuit construction

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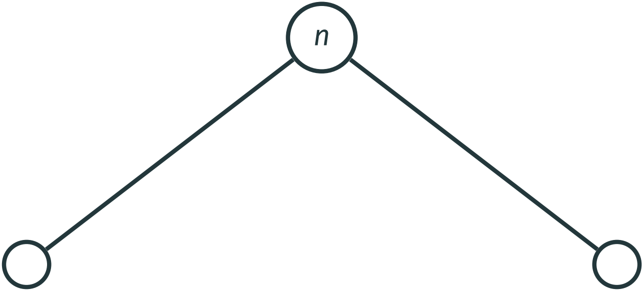

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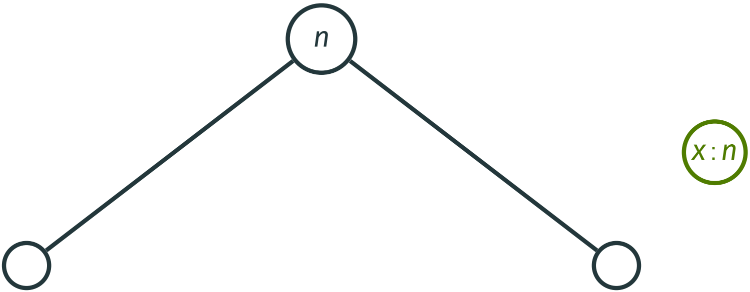

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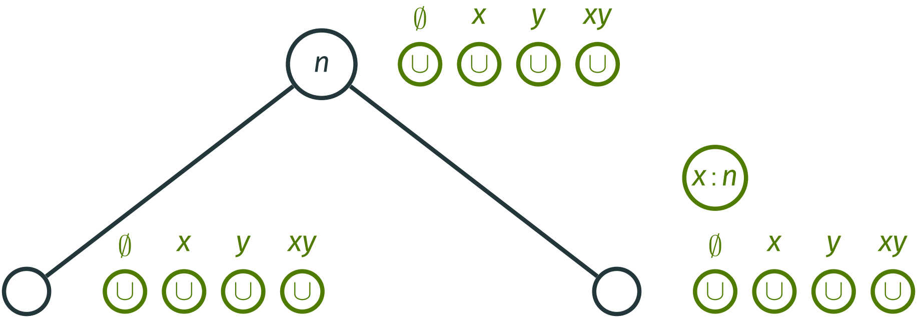

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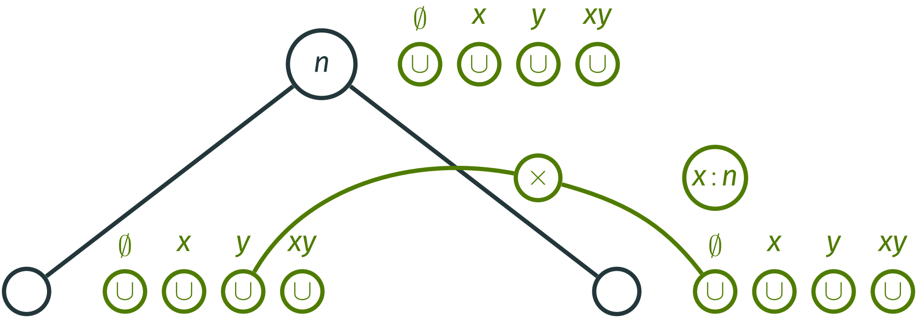

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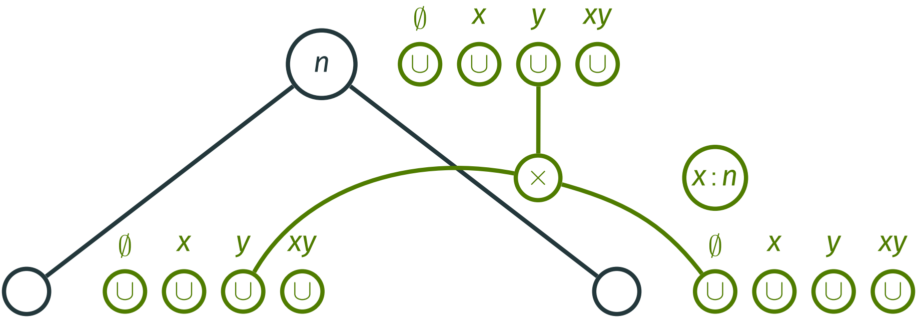

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