Enumeration on Trees under Relabelings

Antoine Amarilli (LTCI, Télécom ParisTech, Université Paris-Saclay), **Pierre Bourhis** (CRIStAL, CNRS UMR 9189 & Inria Lille), **Stefan Mengel** (CNRS, CRIL UMR 8188)

Problem Description

Compute all answers of an **MSO query** on an input **tree** and maintain them efficiently under **relabeling updates**

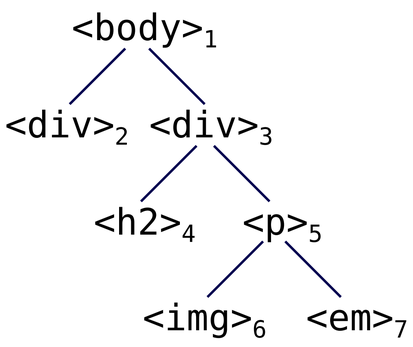
- **MSO** = **first-order logic +quantification over sets**

  Example: *find all pairs of a* ***h2 section title*** *and of an* ***image*** *located in the same section*

- **Tree** with nodes labeled in a **fixed finite alphabet**

- **Updates** are **relabelings**: change the **label** of a node

Example tree:



Example answer set: {⟨4, 6⟩}

Example updates:

- **Relabel** node 7 to <img>

New answer set: {⟨4, 6⟩, ⟨4, 7⟩}

- **Relabel** node 4 to <h3>

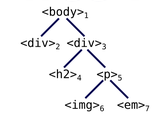
New answer set: ø

- **Complexity** is in the input tree, i.e., **data complexity**

Enumeration Algorithm with Updates

- **Phase 1:Index** the input tree T for the query Q

Tree *T*



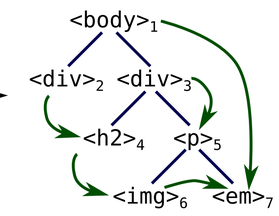
Query *Q*

$\exists s\, s \rightsquigarrow t \wedge s \rightsquigarrow i \wedge \mathrm{h2}(t) \wedge \mathrm{img}(i)$

Preprocessing

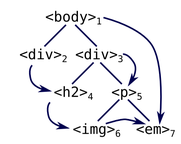
$O(f(Q) \times |T|)$

Tree +Index



- **Phase 2: Enumerate** all query answers (no duplicates)

Tree + Index



Enumeration

$O(|a|)$

New answer *a*

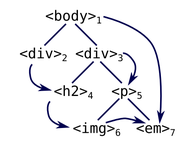
⟨4, 6⟩

New state

00010101010101011100

- **Phase 3: Update** the index when a node is relabeled

Tree + Index



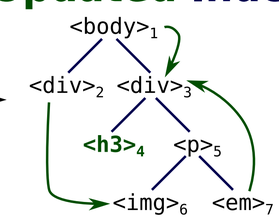
Relabeling

4 := <h3>

Update

(enumeration restarts)

Updated Tree + Updated Index



Main Result and Existing Work Comparison

Theorem (Bagan in 2006; Kazana and Segoufin in 2013):

Enumerate all **answers** to an MSO query on a tree with linear preprocessing and delay **linear** in each answer (so **constant delay** if the free variables are first-order)

Our main result:

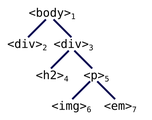
→ We can also handle **relabeling updates** in log time

Work	Delay	Updates
Bagan'06 and Kazana&Segoufin'13	O(1)	O(|T|): re-index the tree
Losemann&Martens'14	$O(\log^2 \|T\|)$	$O(\log^2 \|T\|)$
Niewerth&Segoufin'18	O(1)	O(log $\|T\|$), only on strings
Our work	O(1)	O(log $\|T\|$), only relabelings

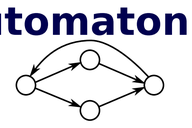
Proof Approach: Knowledge Compilation

- **Phase 1: Preprocessing:** represent all **query answers** as a **circuit** that satisfies some structural **conditions**

Tree *T*



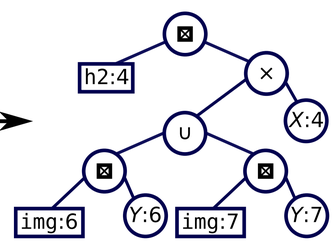
Automaton A



Preprocessing

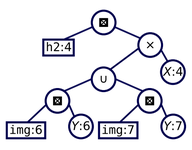
$O(f(A) \times |T|)$

Circuit



- **Phase 2: Enumeration:** general enumeration method on **any circuit** that satisfies the conditions

Circuit



Enumeration

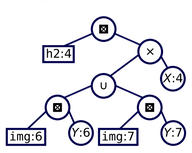
Constant delay

Answers

⟨4, 6⟩
⟨4, 7⟩
⋮

- **Phase 3: Updates:** change the value of some variables and re-evaluate parts of the circuit

Circuit



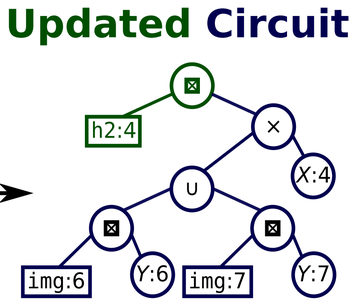
Relabeling

4 := <h3>

Update

$O(\log |T|)$

Updated Circuit



Consequences for Application-Oriented Queries

- **Group-by aggregation:** For each **group** (*page section*):
  → ***how many*** *images are in the section? (counting)*
  → *what is the* ***total size*** *of these images? (sum)*

→ Enumerate **group-value pairs**with **delay** O(log $|T|$)

- **Parameters:** find all *images* in a **user-chosen** *section*

→ Enumerate answers with **constant delay** and **change the parameters** (=relabel) in O(log $|T|$)

Proof Tool: Hybrid Circuits

- **Boolean gates:** Capture a Boolean value, depend on the **labeling**

- **Set-valued gates:** Capture a set of **answers** for each Boolean **valuation**

Variable img:6 **NOT** ¬ **AND** ∧ **OR** ∨ **Variable** *X*:4 **Union** ∪ **Product** × **Test** ⊠

- **Hybrid circuit example**

Valuation of the **Boolean gates**:
h2:4:=**1** img:6 := **1** img:7:=**0**

Captured set:
{ ⟨X:4, Y:6⟩ }