On the Complexity of Mining Itemsets from the Crowd Using Taxonomies

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Frequent itemset mining

Data mining – discovering interesting patterns in large databases

Database – a (multi)set of transactions

Transaction – a set of items (aka. an itemset)

A simple kind of pattern to identify are frequent itemsets
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\[ D = \{ \{\text{beer, diapers}\}, \{\text{beer, bread, butter}\}, \{\text{beer, bread, diapers}\}, \{\text{salad, tomato}\} \} \]

- Itemset is frequent if it occurs in \( \geq \Theta = 50\% \) of transactions
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Some databases only exist in the minds of people

Example: **popular activities in Athens:**

- $t_1$: I went to the acropolis and to the museum.
  \[ \Rightarrow \{\text{acropolis, museum}\} \]
- $t_2$: I visited Piraeus and had some ice cream.
  \[ \Rightarrow \{\text{piraeus, icecream}\} \]
- $t_3$: On Monday I attended the keynote and had coffee.
  \[ \Rightarrow \{\text{keynote, coffee}\} \]
Human knowledge mining

- Some databases only exist in the minds of people
- Example: popular activities in Athens:
  - $t_1$: I went to the acropolis and to the museum.
    $\Rightarrow \{\text{acropolis, museum}\}$
  - $t_2$: I visited Piraeus and had some ice cream.
    $\Rightarrow \{\text{piraeus, icecream}\}$
  - $t_3$: On Monday I attended the keynote and had coffee.
    $\Rightarrow \{\text{keynote, coffee}\}$

- We want frequent itemsets: frequent activity combinations
- How to retrieve this data from people?
Harvesting the data

- We **cannot** collect such data in a centralized database:
  1. It’s **impractical** to ask all users to surrender their data
     “*Everyone please tell us all you did the last three months.*”
  2. People do not **remember** the information
     “*What were you doing on August 23th, 2013?*”
Harvesting the data

- We cannot collect such data in a centralized database:
  1. It’s impractical to ask all users to surrender their data
     “Everyone please tell us all you did the last three months.”
  2. People do not remember the information
     “What were you doing on August 23th, 2013?”
- People remember summaries that we could access
  “Do you often eat ice cream when attending a keynote?”
⇒ We can just ask people if an itemset is frequent
Crowdsourcing

- **Crowdsourcing** – solving hard problems through elementary queries to a crowd of users

- **Find out if an itemset is frequent** with the crowd:
  1. **Draw** a sample of users from the crowd.  
     *(black box)*
  2. **Ask**: is this itemset frequent?  
     *"Do you often have coffee?"
  3. **Corroborate** the answers to eliminate bad answers.  
     *(black box)*
  4. **Reward** the users.  
     *(e.g., monetary incentive)*
Crowdsourcing

- **Crowdsourcing** – solving hard problems through elementary queries to a crowd of users
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  4. **Reward** the users. (e.g., monetary incentive)

⇒ The crowd is an oracle: given an itemset, say if it is frequent
Having a **taxonomy** over the items can save us work!

```
activity
  /   \
/talk  tour
  /    /
/keynote presentation  parthenon erectheion propylea
```

- \{talk, tour\} infrequent
Having a taxonomy over the items can save us work!

\[
\text{activity} \\
\text{talk} \quad \text{tour} \\
\text{keynote} \quad \text{presentation} \quad \text{parthenon} \quad \text{erectheion} \quad \text{propylea} \\
\{\text{talk, tour}\} \text{ infrequent}
\]
Taxonomies

Having a **taxonomy** over the items can save us work!

- {**talk**, **tour**} infrequent
  - Itemsets such as {**keynote**, **parthenon**} also infrequent

![Taxonomy Diagram]

- **activity**
  - **talk**
    - **keynote**
    - **presentation**
  - **tour**
    - **parthenon**
    - **erectheion**
    - **propylea**
Having a taxonomy over the items can save us work!

\[ \{ \text{talk, tour} \} \text{ infrequent} \]
\[ \Rightarrow \text{Itemsets such as } \{ \text{keynote, parthenon} \} \text{ also infrequent} \]
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activity

- talk
  - keynote
  - presentation
- tour
  - parthenon
  - ercetheion
  - propylea

- \{talk, tour\} infrequent
  - Itemsets such as \{keynote, parthenon\} also infrequent
- Without the taxonomy, we need to test all combinations!
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  - **talk**
  - **tour**
  - **keynote**
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- \{\text{talk, tour}\} infrequent
  \Rightarrow \text{Itemsets such as } \{\text{keynote, parthenon}\} \text{ also infrequent}
- Without the taxonomy, we need to test all combinations!
- Also avoids redundant itemsets like \{\text{talk, keynote}\}
Having a taxonomy over the items can save us work!

```
activity
  /   
/talk talk, tour
    /   
keynote keynote, parthenon, eractheion, propylea
```
Having a taxonomy over the items can save us work!

- \{talk, tour\} infrequent
  - Itemsets such as \{keynote, parthenon\} also infrequent
- Without the taxonomy, we need to test all combinations!
- Also avoids redundant itemsets like \{talk, keynote\}
The problem

We can now describe the problem:

- We have:
  - A known item domain $\mathcal{I}$ (set of items)
  - A known taxonomy $\Psi$ on $\mathcal{I}$ (is-a relation, partial order)
  - A crowd oracle to decide if an itemset is frequent or not

- Choose questions interactively based on past answers

$\Rightarrow$ Find out the status of all itemsets
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⇒ Find out the status of all itemsets

What is a good algorithm to solve this problem?
Cost

- How to evaluate the **performance** of a strategy to identify the frequent itemsets?

  **Crowd complexity:** The number of itemsets we ask about (monetary cost, latency...)

  **Computational complexity:** The complexity of computing the next question to ask
Cost

- How to evaluate the performance of a strategy to identify the frequent itemsets?

  **Crowd complexity:** The number of itemsets we ask about (monetary cost, latency...)

  **Computational complexity:** The complexity of computing the next question to ask

- **Tradeoff** between the two:
  - Asking *random* questions: computationally inexpensive but bad crowd complexity
  - Asking *clever* questions: optimal crowd complexity but computationally expensive
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Itemsets

- Itemsets $I(\Psi)$ – the sets of pairwise incomparable items
Itemsets

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  - $\{\text{icdt, parthenon}\}$ is an itemset
Itemsets

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- **Order** over itemsets:
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    - $\{parthenon\}$ also frequent
  - $\{parthenon\}$ frequent
    - $\{tour\}$ also frequent
Itemset taxonomy example

Taxonomy $\Psi$

- activity
  - icdt
  - tour
  - parthenon
  - piraeus

Itemset taxonomy $I(\Psi)$

- nil
- activity
  - icdt
  - tour
  - parthenon
  - piraeus
  - icdt
  - parthenon
  - piraeus
  - icdt
  - parthenon
  - piraeus
  - icdt
  - parthenon
  - piraeus
Itemset taxonomy example

Taxonomy $\Psi$

activity
   /    
icdt   tour
  /       /
parthenon piraeus

Itemset taxonomy $I(\Psi)$

nil
   /   
activity
   /   
icdt   tour

parthenon piraeus
   /   
icdt   piraeus
   /   
parthenon piraeus
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- **activity**
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Itemset taxonomy $I(\Psi)$

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    - icdt
    - tour
      - icdt
      - parthenon
      - piraeus
    - icdt
    - parthenon
    - piraeus
    - icdt
    - parthenon
    - piraeus
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Taxonomy $\Psi$

- activity
  - icdt
  - tour
  - parthenon
  - piraeus

Itemset taxonomy $\mathcal{I}(\Psi)$

- nil
- activity
  - icdt
  - tour
  - parthenon
  - piraeus
  - icdt
  - parthenon
  - piraeus
  - icdt
  - parthenon
  - piraeus
  - icdt
  - parthenon
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Itemset taxonomy example

**Taxonomy \( \Psi \)**

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  - tour
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**Itemset taxonomy \( I(\Psi) \)**

- nil
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    - icdt
    - tour
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    - piraeus
  - icdt
    - parthenon
    - piraeus
  - icdt
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Taxonomy $\Psi$

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Itemset taxonomy $I(\Psi)$

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    - parthenon
    - piraeus
    - icdt
    - parthenon
    - piraeus
    - icdt
    - parthenon
    - piraeus
    - icdt
    - parthenon
    - piraeus
    - icdt
    - parthenon
    - piraeus
Solutions

```
nil
  /|
activity
  |  
  icdt  tour
  /|
icdt tour
  |  |
  icdt tour parthenon piraeus
  |
  icdt parthenon
  |
  icdt parthenon piraeus
```

- Is \{\textit{parthenon}, \textit{piraeus}\} frequent? => Yes!
- Is \{\textit{parthenon}\} frequent? => Yes!
- Is \{\textit{icdt}, \textit{parthenon}\} frequent? => Yes!
- Is \{\textit{icdt}, \textit{piraeus}\} frequent? => No!
- Is \{\textit{icdt}\} frequent? => Yes!
- Is \{\textit{icdt}, \textit{parthenon}, \textit{piraeus}\} frequent? => No!

"Being frequent" is a monotone predicate over \(\text{I(Ψ)}\).
“Being frequent” is a monotone predicate over $l(\Psi)$
Solutions

“Being frequent” is a monotone predicate over $I(\Psi)$.
“Being frequent” is a monotone predicate over I(Ψ)

Ask questions:
“Being frequent” is a monotone predicate over $I(\Psi)$.

Ask questions:

- Is \{piraeus\} frequent?
- Is \{parthenon\} frequent?
- Is \{icdt, parthenon\} frequent?
- Is \{icdt, piraeus\} frequent?
- Is \{parthenon\} frequent?
- Is \{icdt\} frequent?
- Is \{parthenon, piraeus\} frequent?
- Is \{icdt\} frequent?
- Is \{parthenon\} frequent?
- Is \{icdt\} frequent?
Solutions

• “Being frequent” is a monotone predicate over $\mathcal{I}(\Psi)$

• Ask questions:

• Is \{piraeus\} frequent?
  ⇒ Yes!

• Is \{parthenon, piraeus\} frequent?
  ⇒ No!

• Is \{parthenon\} frequent?
  ⇒ Yes!

• Is \{icdt, parthenon\} frequent?
  ⇒ Yes!

• Is \{icdt, piraeus\} frequent?
  ⇒ No!

• Is \{icdt\} frequent?
  ⇒ Yes!

• Is \{tour\} frequent?
  ⇒ Yes!

• Is \{activity\} frequent?
  ⇒ Yes!

• Is \{nil\} frequent?
  ⇒ Yes!
Solutions

- “Being frequent” is a monotone predicate over $l(\Psi)$
- Ask questions:
  - Is \{piraeus\} frequent?
    \[\Rightarrow \text{Yes!}\]
  - Is \{parthenon, piraeus\} frequent?
    \[\Rightarrow \text{No!}\]

Tree diagram with nodes representing different activities and places, illustrating the frequent sets.
“Being frequent” is a monotone predicate over $I(\Psi)$

- Ask questions:
  - Is \{piraeus\} frequent?
    - Yes!
  - \{parthenon, piraeus\}?
    - Yes!
  - \{parthenon\}?
    - Yes!
  - \{icdt, parthenon\}?
    - Yes!
  - \{icdt, piraeus\}?
    - No!
  - \{icdt\}?
    - Yes!
  - \{activity\}?
    - Yes!
Solutions

- “Being frequent” is a monotone predicate over \( I(\Psi) \)
- Ask questions:
  - Is \( \{\text{piraeus}\} \) frequent?
    \( \Rightarrow \) Yes!
  - \( \{\text{parthenon, piraeus}\}? \)
    \( \Rightarrow \) No!
  - \( \{\text{parthenon}\}? \)
Solutions

- "Being frequent" is a monotone predicate over I(Ψ)
- Ask questions:
  - Is \{piraeus\} frequent?  
    \[\Rightarrow\] Yes!
  - \{parthenon, piraeus\}?  
    \[\Rightarrow\] No!
  - \{parthenon\}?  
    \[\Rightarrow\] Yes!
Solutions

“Being frequent” is a monotone predicate over $I(\Psi)$

Ask questions:

- Is \{piraeus\} frequent?  
  \Rightarrow Yes!

- \{parthenon, piraeus\}?  
  \Rightarrow No!

- \{parthenon\}?  
  \Rightarrow Yes!

- \{icdt, parthenon\}?
“Being frequent” is a monotone predicate over $I(\Psi)$

Ask questions:
- Is \{piraeus\} frequent?  
  \[ \Rightarrow \text{Yes!} \]
- \{parthenon, piraeus\}?  
  \[ \Rightarrow \text{No!} \]
- \{parthenon\}?  
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Ask questions:
- Is \{piraeus\} frequent?
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- \{parthenon, piraeus\}?
  $\Rightarrow$ No!
- \{parthenon\}?
  $\Rightarrow$ Yes!
- \{icdt, parthenon\}?
  $\Rightarrow$ Yes!
- \{icdt, piraeus\}?
Solutions

“Being frequent” is a monotone predicate over $I(\Psi)$

- Ask questions:
  - Is \{piraeus\} frequent?
    - $\Rightarrow$ Yes!
  - \{parthenon, piraeus\}?
    - $\Rightarrow$ No!
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    - $\Rightarrow$ Yes!
  - \{icdt, parthenon\}?
    - $\Rightarrow$ Yes!
  - \{icdt, piraeus\}?
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“Being frequent” is a monotone predicate over $I(\Psi)$

Ask questions:

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- $\{\text{icdt, parthenon}\}$?
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- $\{\text{icdt, piraeus}\}$?
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Crowd complexity lower bound

- How many **questions** do we need to ask?
- Each query yields **one bit** of information
Crowd complexity lower bound

- How many **questions** do we need to ask?
- Each query yields **one bit** of information
- **Information-theoretic lower bound**: at least $\Omega(\log N)$ queries, with $N$ the number of solutions

$$N = \Omega \left( 2^{|I(\Psi)|} \right) \text{ and } |I(\Psi)| = \Omega \left( 2^{|\Psi|} \right)$$

- **W.r.t. the original taxonomy** $\Psi$, $\Omega \left( 2^{\text{width}(\Psi)} / \sqrt{\text{width}[\Psi]} \right)$
### Crowd complexity upper bound

<table>
<thead>
<tr>
<th>Itemset</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>nil</td>
<td>6/7</td>
</tr>
<tr>
<td>a1</td>
<td>5/7</td>
</tr>
<tr>
<td>a2</td>
<td>4/7</td>
</tr>
<tr>
<td>a3</td>
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</tr>
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- Query itemsets that are frequent in **about half** of the solutions
**Crowd complexity upper bound**

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Crowd complexity upper bound

- Query itemsets that are frequent in about half of the solutions
- Itemset split: min of proportion where frequent and proportion where infrequent
Crowd complexity upper bound

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- Query itemsets that are frequent in about half of the solutions
- Itemset split: min of proportion where frequent and proportion where infrequent
- Existing result from order theory
  [Linial and Saks, 1985]: there is a constant $\delta_0 \approx 1/5$ such that some itemset achieves a split $\geq \delta_0$
  $\Rightarrow$ The previous bound is tight: we need $\Theta(\log N)$ queries
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Maximal frequent itemsets

- Complexity with respect to the output size
- Output representation: Maximal frequent itemsets (MFI)
- Minimal infrequent itemset (MII)
Maximal frequent itemsets

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- Complexity with respect to the output size
- Output representation: Maximal frequent itemsets (MFI)
- Minimal infrequent itemset (MII)
- Must query all MFIs and MIIs
- Solutions with few MFIs/MIIs should be easier to find
MFI/MII upper bound

- Explicit algorithm to find each MFI/MII in \( \leq |I| \) queries
- Example:
MFI/MII upper bound

- Explicit algorithm to find each MFI/MII in $\leq |I|$ queries
- Example:
  - Pick an itemset:
MFI/MII upper bound

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- Example:
  - Pick an itemset: \{tour\}
MFI/MII upper bound

- Explicit algorithm to find each MFI/MII in \( \leq |\mathcal{I}| \) queries
- Example:
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  - Specialize it...
**MFI/MII upper bound**

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Explicit algorithm to find each MFI/MII in $\leq |I|$ queries

Example:
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At most \(|\mathcal{I}|\) specializations
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- Example:
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- At most \(|\mathcal{I}|\) specializations

\[ \Rightarrow \text{Complexity: } O(|\mathcal{I}| \cdot (|\text{MFI}| + |\text{MII}|)) \]
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Output computational complexity lower bound

- Previous algorithm assumes $|I(\Psi)|$ is materialized
- Do we need to?
Output computational complexity lower bound

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- Decide if finished: do the MFIs/MIIs cover all itemsets?
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Previous algorithm assumes $|I(\Psi)|$ is materialized.
Do we need to?
Decide if finished: do the MFIs/MIIs cover all itemsets?
This is EQ-hard, for problem EQ [Bioch and Ibaraki, 1995] (exact complexity open)
Computational complexity lower bound

- Find an unclassified itemset of \( I(\Psi) \) frequent for **about half** of the possible solutions.
- We can count the possible solutions (exponential in \(|I(\Psi)|\))
- A solution is an “itemset” of \( I(\Psi) \), an antichain, and counting the antichains of \( I(\Psi) \) is \( \text{FP}^\#P \)-complete.

\[\Rightarrow\] Finding the best-split element in \( I(\Psi) \) is \( \text{FP}^\#P \)-hard in \(|I(\Psi)|\)?
Computational complexity lower bound

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- A solution is an “itemset” of $I(\Psi)$, an antichain, and counting the antichains of $I(\Psi)$ is FP$\#^P$-complete.

$\Rightarrow$ Finding the best-split element in $I(\Psi)$ is FP$\#^P$-hard in $|I(\Psi)|$?

- **Problem:** $I(\Psi)$ is not a general DAG, so we only show hardness in $|\Psi|$ for restricted (fixed-size) itemsets
- **Intuition:** count antichains by comparing to a known poset; use a best-split oracle to compare; perform a binary search
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Summary and further work

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- Balance **crowd complexity** and **computational complexity**
- Function of the **input taxonomy size** or the **output size**
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    \(k\) itemsets (work in progress)

Thanks for your attention!

Greedy algorithms

- Querying an element of the chain may remove $< 1/2$ possible solutions
- Querying the isolated element $b$ will remove exactly $1/2$ solution
- However, querying $b$ classifies far less itemsets
  ⇒ Classifying many itemsets isn’t the same as eliminating many solutions

Finding the greedy-best-split item is FP$^P$-hard
Restricted itemsets

- Asking about large itemsets is irrelevant.

  “Do you often go cycling and running while drinking coffee and having lunch with orange juice on alternate Wednesdays?”

- If the itemset size is bounded by a constant, $I(\Psi)$ is tractable

$\Rightarrow$ The crowd complexity $\Theta(\log |S(\Psi)|)$ is tractable too
Chain partitioning

- Optimal strategy for chain taxonomies: binary search
- We can determine a chain decomposition of the itemset taxonomy and perform binary searches on the chains
- Optimal crowd complexity for a chain, performance in general is unclear
- Computational complexity is polynomial in the size of $I(\Psi)$ (which is still exponential in $\Psi$)
To describe the solution, we need the MFIs or the MIIs. However, we need to query both the MFIs and the MIIs to identify the result uniquely: $\Omega(|MFI| + |MII|)$ queries. We can have $|MFI| = \Omega(2^{|MII|})$ and vice-versa. This bound is not tight (e.g., chain).