Tractable Query Answering Under Probabilistic Constraints

Antoine Amarilli[1], Pierre Bourhis[2], Pierre Senellart[1,3]

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September 4th, 2014

Tractable Query Evaluation On Probabilistic Instances

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Instances and queries

- Given a relational instance (= set of facts, hypergraph)
  $I = \{R(a, b), R(b, c), S(c)\}$
- Given a conjunctive query (CQ) (existentially quantified)
  $q : \exists xy\, R(x, y) \wedge S(y)$

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$\rightarrow$ Data complexity: q is fixed

Uncertain and probabilistic instances

- Set of uncertain events

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$\rightarrow$ Query evaluation: determine the probability of q on $\widehat{I}$.

Hardness and tractability

- With arbitrary annotations
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- Our approach:
  - → Find a restriction on the instance and annotations
  - → Show that many queries are tractable in this case

Bounded treewidth

An idea from instances without probabilities...

- If an instance has low treewidth then it is almost a tree
- Assume that the instance treewidth is constant...

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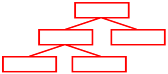
tree decomposition

$O(|I|)$ for fixed width



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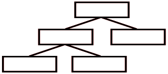

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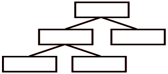
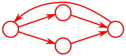


rewriting

$O(1)$ data complexity



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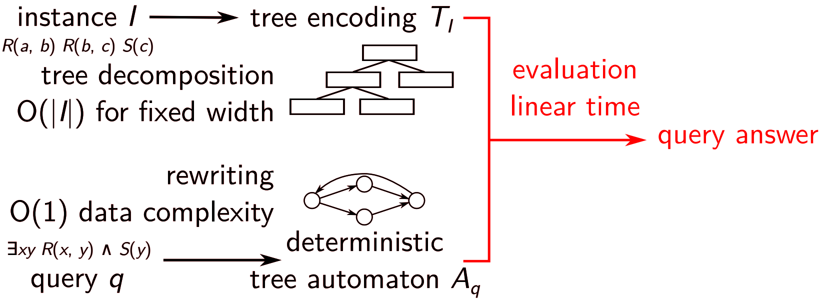

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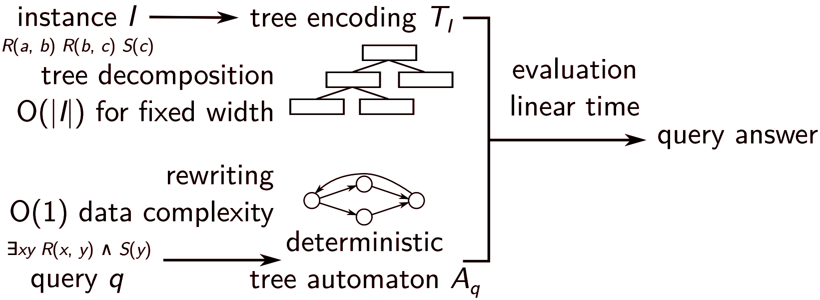

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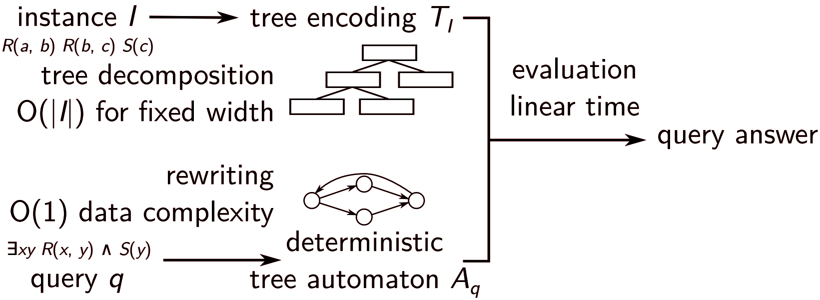

→ Linear time data complexity

Tractable inference

An idea from probabilities without instances...

- Represent a propositional formula F as a Boolean circuit
- Assume the circuit has constant treewidth

$\rightarrow$ Probability of F can be computed in linear time
(using junction tree algorithm for Bayesian networks)
(assuming constant-time arithmetic operations)

cc-tables

- Boolean circuit for the annotations

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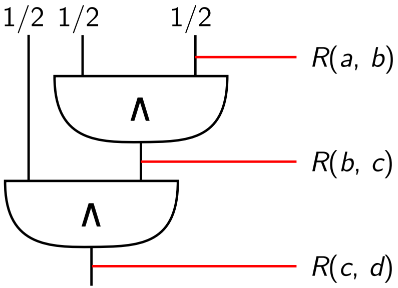

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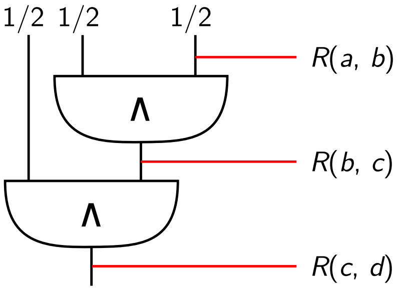

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- Instance must have low treewidth

→ Need simultaneous decomposition

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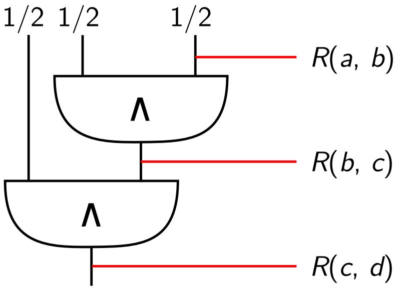

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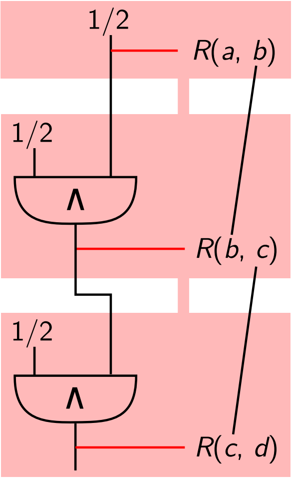
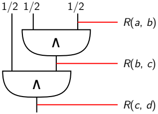


Main result

instance I



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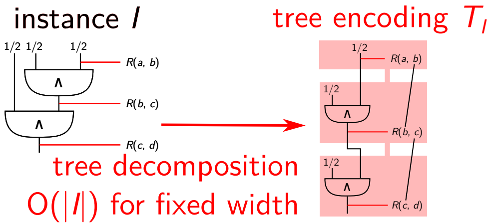
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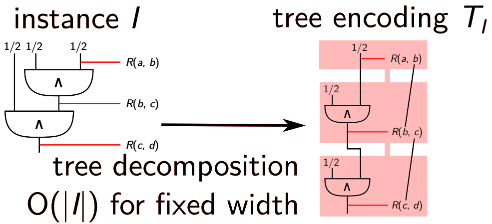

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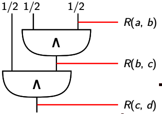

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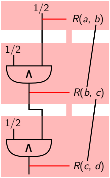

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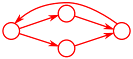

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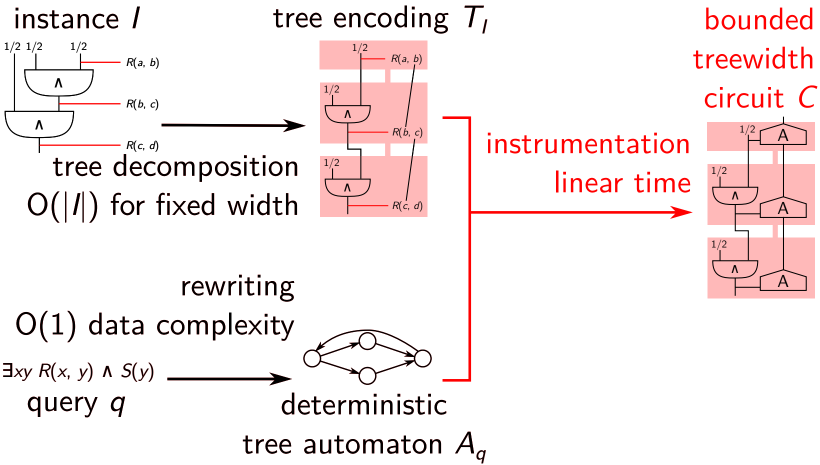

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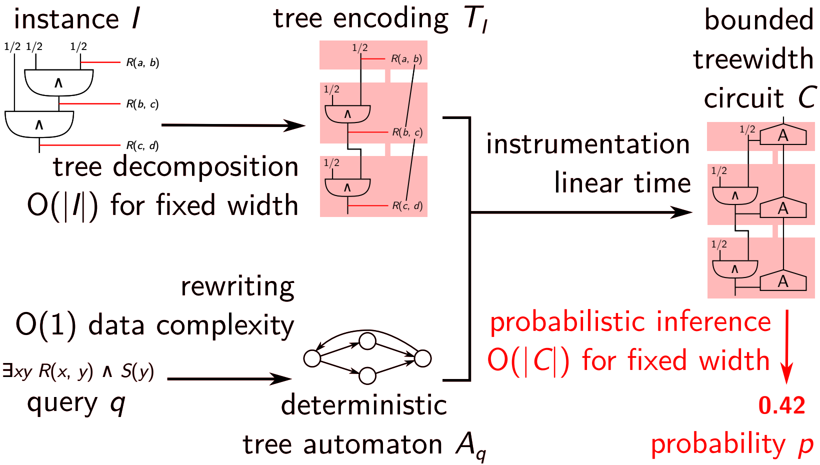
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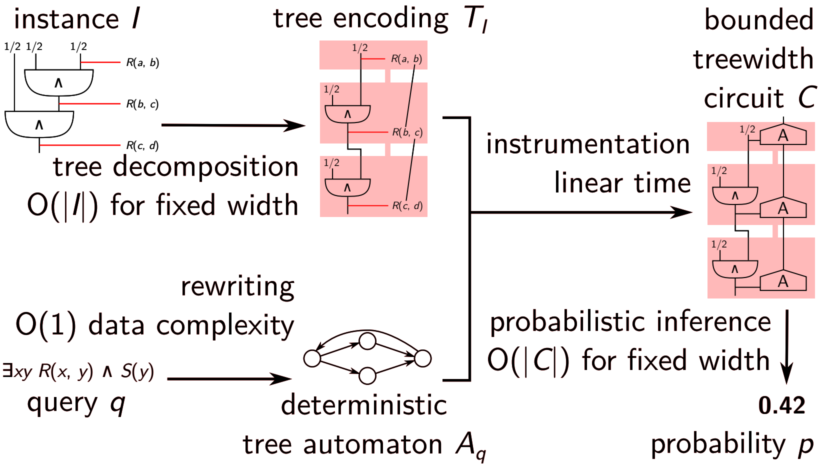
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What are bounded-treewidth circuits good for?

circuit-complexity pr.probability treewidth arithmetic-circuits

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