Enumerating Pattern Matches in Texts and Trees

Antoine Amarilli[1], Pierre Bourhis[2], Stefan Mengel[3], Matthias Niewerth[4]

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[1]Télécom Paris

[2]CNRS CRIStAL

[3]CNRS CRIL

[4]Universität Bayreuth

Problem: Finding Patterns in Text

- We have a long text T:

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French national.  Appearance as of 2017.  Auth OpenPGP. OpenId.  Bitcoin.  Contact Email and XMPP
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More Résumé Location Other sites Blogging:  a3nm.net/blog Git:  a3nm.net/git ...
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  → Example: find email addresses

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  → Example: find **email addresses**
  - Write the pattern as a **regular expression**:

$P :=$ `␣ [a-z0-9.]* @ [a-z0-9.]* ␣`

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  - → Example: find **email addresses**
    - Write the pattern as a **regular expression**:

$$P := \texttt{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$

→ **How to find the pattern P efficiently in the text T?**

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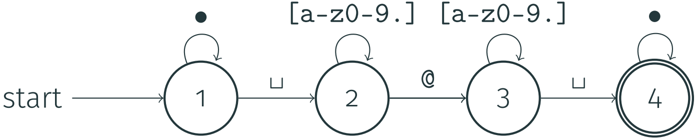

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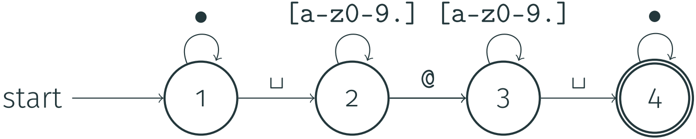

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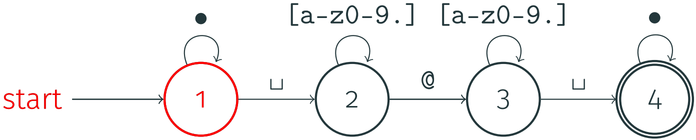

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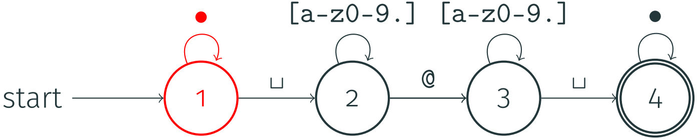

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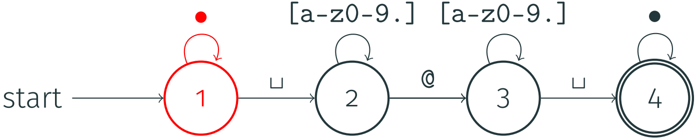

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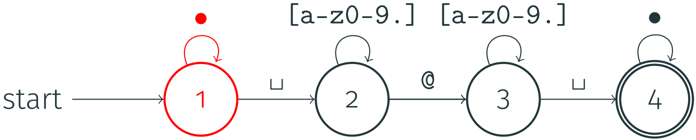

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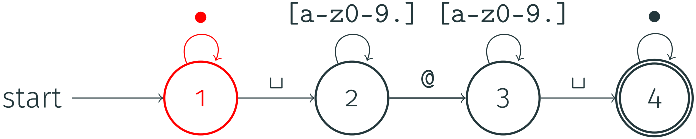

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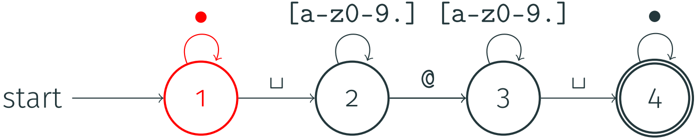

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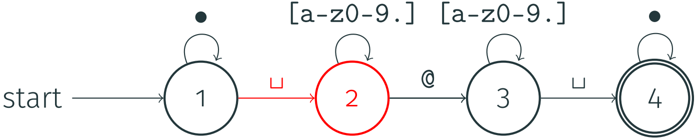

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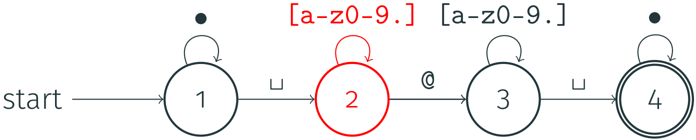

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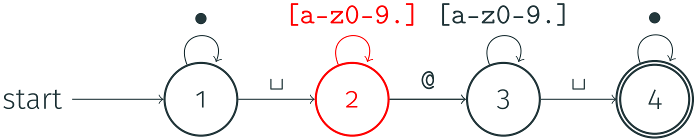

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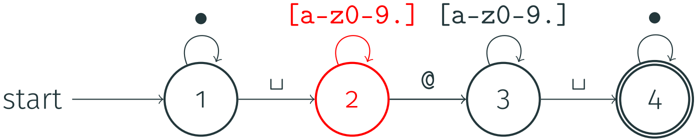

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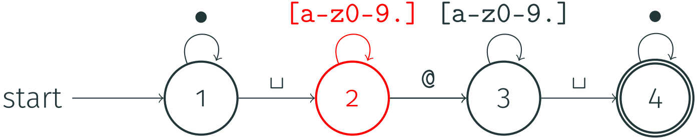

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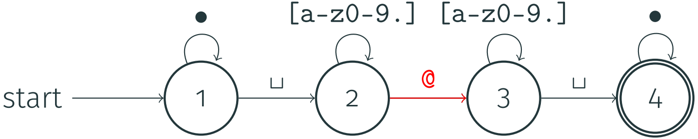

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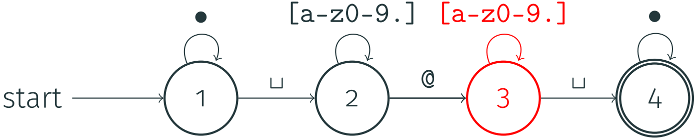

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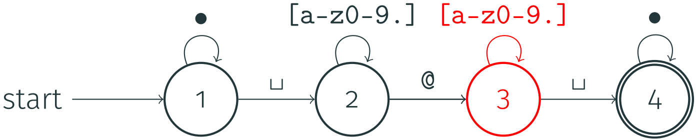

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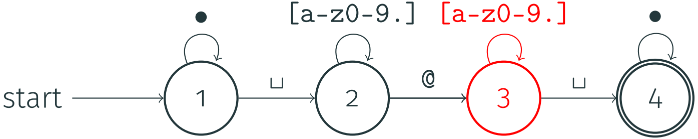

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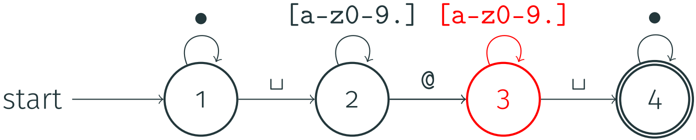

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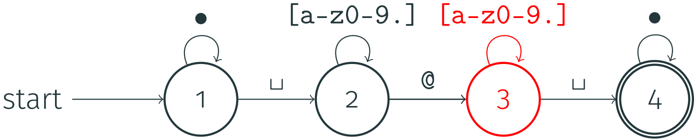

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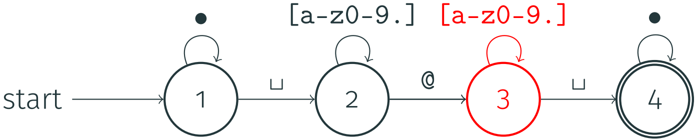

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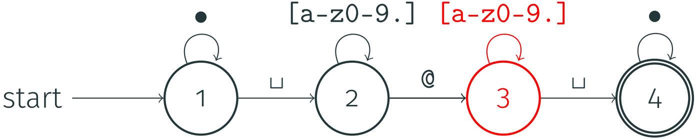

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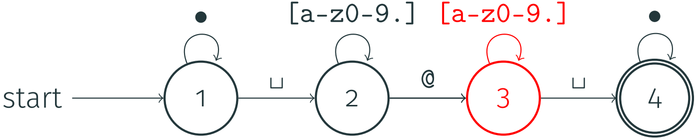

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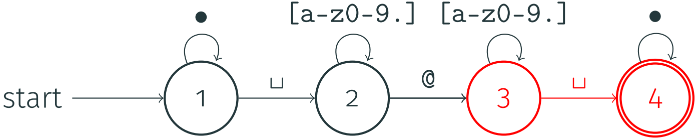

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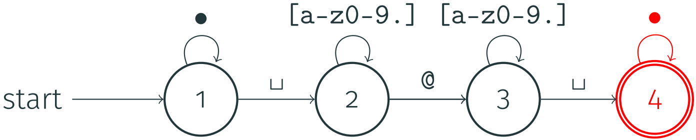

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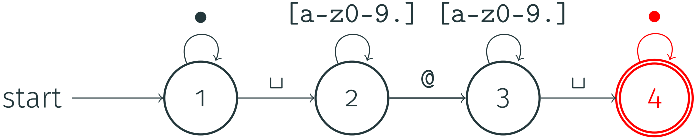

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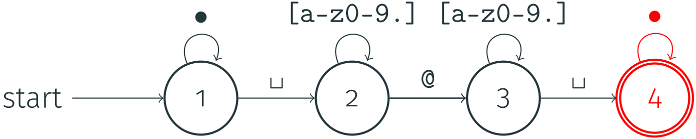

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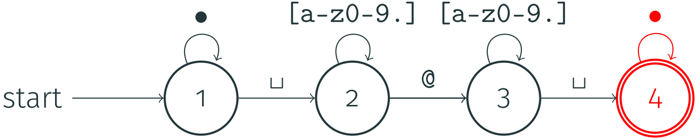

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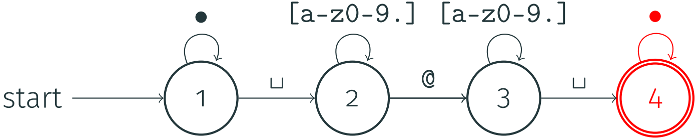

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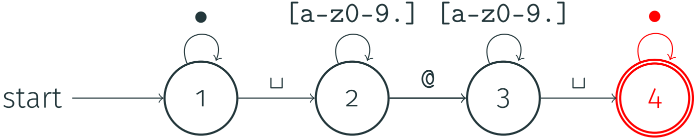

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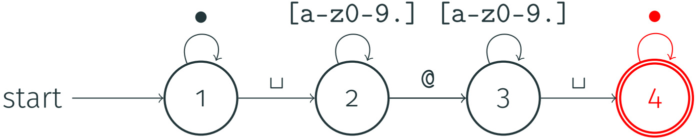

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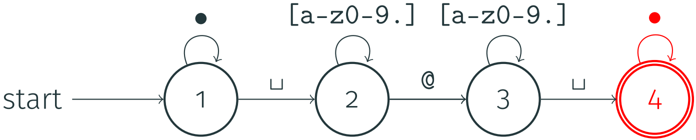

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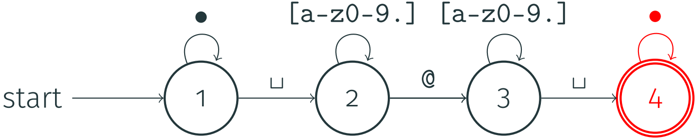

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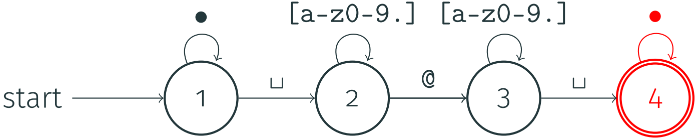

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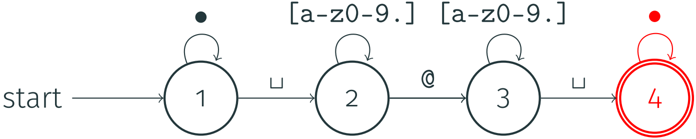

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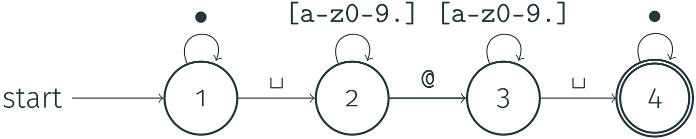

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- The **complexity** is $O(|A| \times |T|)$, i.e., **linear** in T and **polynomial** in P

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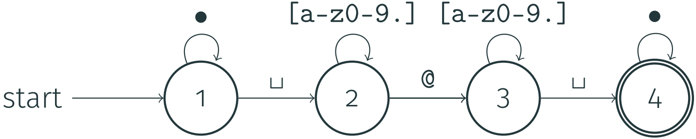
$$P := \sqcup\ \texttt{[a-z0-9.]}^{*}\ \texttt{@}\ \texttt{[a-z0-9.]}^{*}\ \sqcup$$



- Then, evaluate the automaton on the text T

Email␣a3nm@a3nm.net␣Affiliation

- The complexity is $O(|A| \times |T|)$, i.e., linear in T and polynomial in P
  → This is very efficient in T and reasonably efficient in P

Actual Problem: Extracting all Patterns

- This only tests if the pattern occurs in the text!
  - → ``YES''

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  → ‘‘YES’’

- Goal: find all substrings in the text *T* which match the pattern *P*

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0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
E	m	a	i	l	␣	a	3	n	m	@	a	3	n	m	.	n	e	t	␣	A	f	f	i	l	i	a	t	i	o	n

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→ **One match:** $[5, 20\rangle$

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Formal Problem Statement

- Problem description:

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    - Input:
        - A text *T*

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    - A pattern *P* given as a regular expression

$$P := \textvisiblespace\ \texttt{[a-z0-9.]}^*\ \texttt{@}\ \texttt{[a-z0-9.]}^*\ \textvisiblespace$$

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  - Output: the list of substrings of *T* that match *P*:

$$[186, 200\rangle, \quad [483, 500\rangle, \quad \ldots$$

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Measuring the Complexity

- **Naive algorithm:** Run the automaton *A* on **each substring** of *T*

```
    l   o   l
```

Measuring the Complexity

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```
[⟩ l   o   l
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Measuring the Complexity

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```
    l   o   l [⟩
```

Measuring the Complexity

- **Naive algorithm:** Run the automaton A on **each substring** of T

```
    l   o   l
```

→ **Complexity** is $O(|T|^2 \times |A| \times |T|)$

Measuring the Complexity

- **Naive algorithm:** Run the automaton A on **each substring** of T

```
    l   o   l
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Measuring the Complexity

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- **Problem:** We may need to output $\Omega(|T|^2)$ matching substrings:
  - Consider the **text** T:

```
aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa
```

Measuring the Complexity

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```
    l   o   l
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```
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```

  - Consider the **pattern** $P :=$ `a`*

Measuring the Complexity

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  `l o l`

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- **Problem:** We may need to output $\Omega(|T|^2)$ matching substrings:
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    `aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa`

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Measuring the Complexity

- **Naive algorithm:** Run the automaton A on **each substring** of T

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```
aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa
```

  - Consider the **pattern** $P :=$ `a`*
  - The **number of matches** is $\Omega(|T|^2)$

→ We need a **different way** to measure complexity

Enumeration Algorithms

Idea: In real life, we do not want to compute all the matches
we just need to be able to enumerate matches quickly

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how to find patterns

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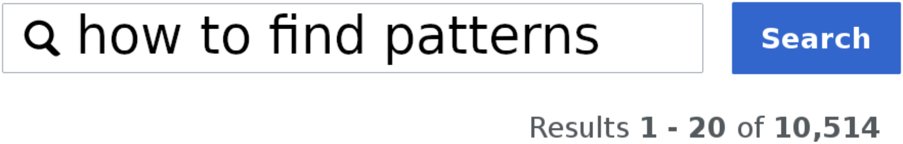

how to find patterns

Results **1 - 20** of **10,514**

Enumeration Algorithms

Idea: In real life, we do not want to compute all the matches
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...

Enumeration Algorithms

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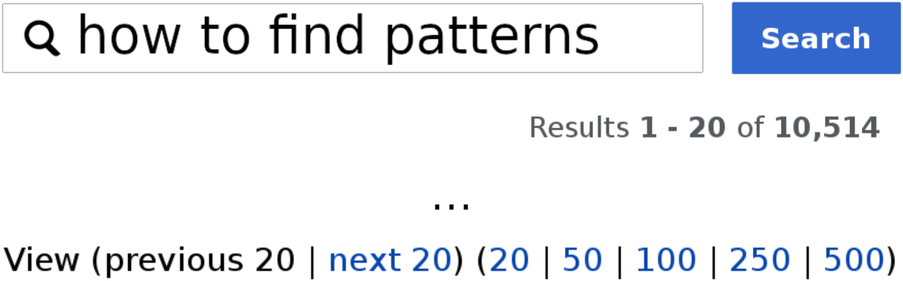

how to find patterns

Results **1 - 20** of **10,514**

...

View (previous 20 | next 20) (20 | 50 | 100 | 250 | 500)

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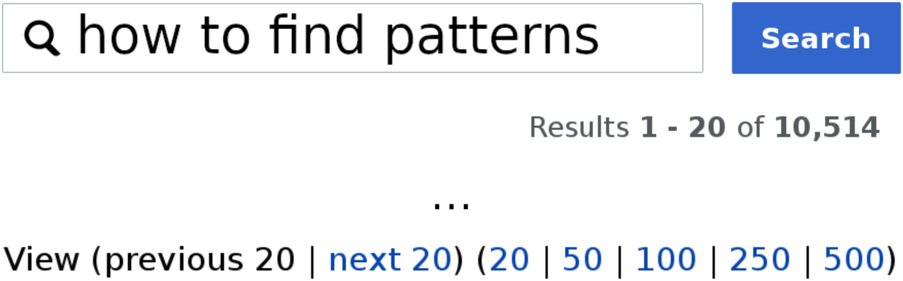

how to find patterns

Results **1 - 20** of **10,514**

...

View (previous 20 | next 20) (20 | 50 | 100 | 250 | 500)

→ Formalization: **enumeration algorithms**

Formalizing Enumeration Algorithms

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```
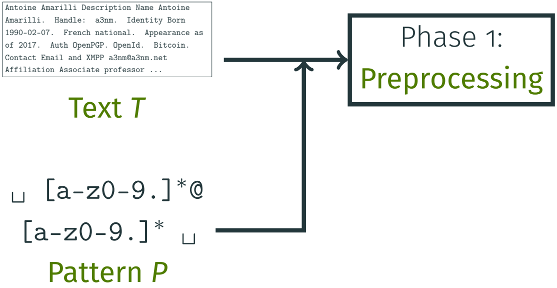

Text *T*

```
␣ [a-z0-9.]*@
[a-z0-9.]* ␣
```

Pattern *P*

Formalizing Enumeration Algorithms



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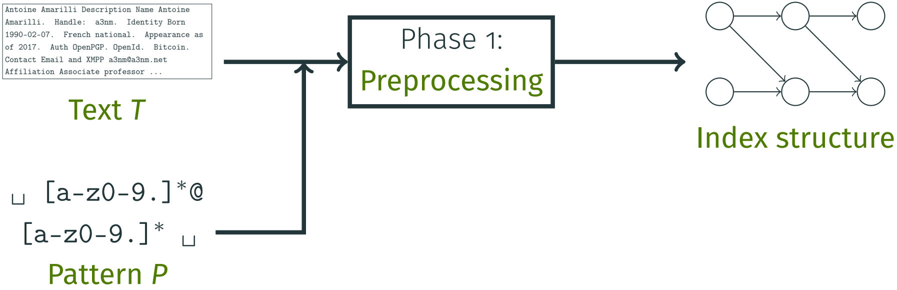
Phase 1: Preprocessing

Formalizing Enumeration Algorithms



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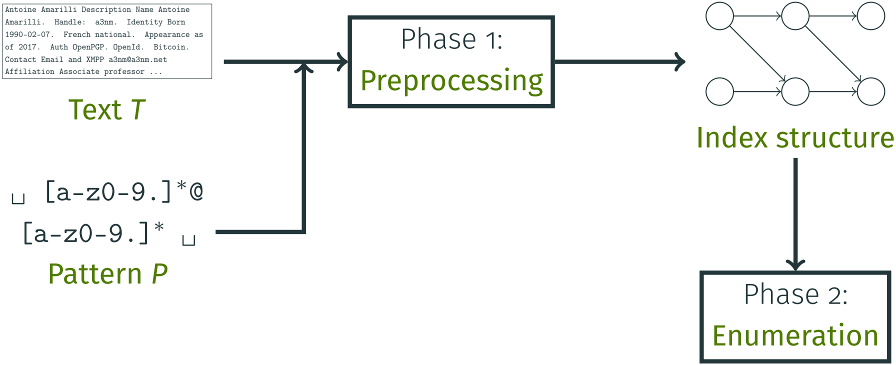
Phase 1: Preprocessing

Index structure

Formalizing Enumeration Algorithms



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Text *T*

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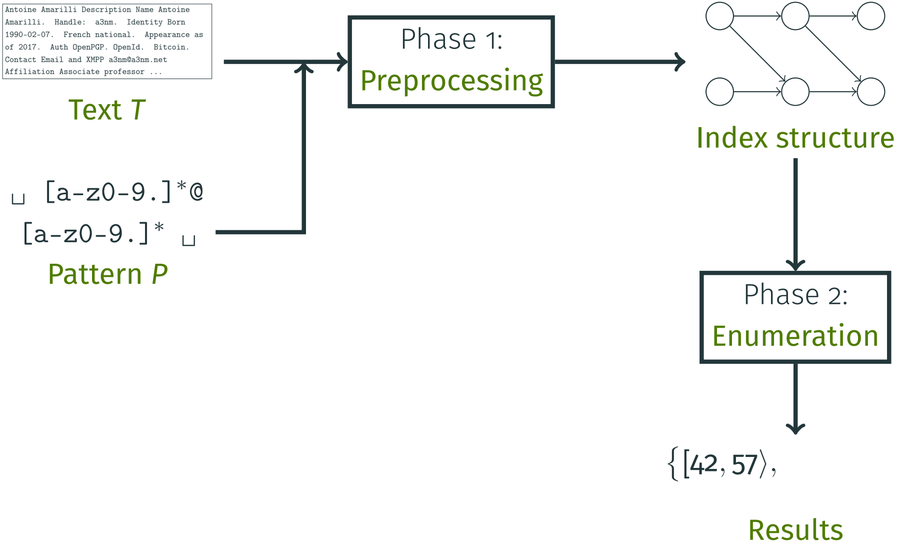
Phase 1: Preprocessing

Index structure

Phase 2: Enumeration

Formalizing Enumeration Algorithms



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Text T

`␣[a-z0-9.]*@[a-z0-9.]*␣`

Pattern P

Phase 1: Preprocessing

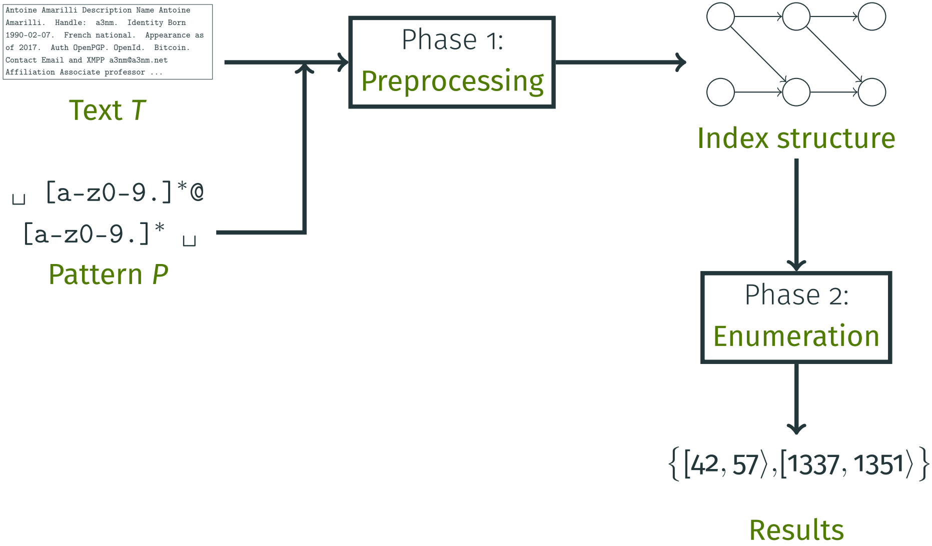
Index structure

Phase 2: Enumeration

$\{[42, 57\rangle,$

Results

Formalizing Enumeration Algorithms



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```

Text *T*

`␣ [a-z0-9.]*@ [a-z0-9.]* ␣`

Pattern *P*

Phase 1:

Preprocessing

Index structure

Phase 2:

Enumeration

$\{[42,57\rangle,[1337,1351\rangle\}$

Results

Formalizing Enumeration Algorithms

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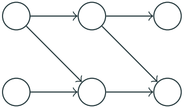

Index structure

Phase 2:
Enumeration

$\{[42,57\rangle,[1337,1351\rangle\}$

Results

Two ways to measure performance:

- Total time for phase 1
- Delay between two results in phase 2

... as a function of the text and pattern

Complexity of Enumeration Algorithms

- Recall the inputs to our problem:
  - A text *T*

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$$P := \texttt{␣ [a-z0-9.]* @ [a-z0-9.]* ␣}$$

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- What is the **delay** of the **naive algorithm**?

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→ it is the **maximal time** to find the next **matching substring**

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→ i.e. $O(|T|^2 \times |A|)$, e.g., if only the **beginning** and **end** match

Complexity of Enumeration Algorithms

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```

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→ it is the **maximal time** to find the next **matching substring**
→ i.e. $O(|T|^2 \times |A|)$, e.g., if only the **beginning** and **end** match

→ Can we do **better**?

Results for Enumerating Pattern Matches

- Existing work has shown the best possible bounds:

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Theorem [Florenzano et al., 2018]

We can enumerate all matches of a pattern P on a text T with:

- *Preprocessing linear in T*
- *Delay constant (independent from T)*

Results for Enumerating Pattern Matches

- Existing work has shown the best possible bounds in T:

Theorem [Florenzano et al., 2018]

We can enumerate all matches of a pattern P on a text T with:

- *Preprocessing linear in T and exponential in P*
- *Delay constant (independent from T) and exponential in P*

→ Problem: They only measure the complexity as a function of T!

Results for Enumerating Pattern Matches

- Existing work has shown the best possible bounds in T:

Theorem [Florenzano et al., 2018]

We can enumerate all matches of a pattern P *on a text* T *with:*

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- *Delay constant (independent from* T*) and exponential in* P

→ Problem: They only measure the complexity as a function of T!

- Our contribution is:

Results for Enumerating Pattern Matches

- Existing work has shown the best possible bounds in T:

Theorem [Florenzano et al., 2018]

We can enumerate all matches of a pattern P *on a text* T *with:*

- *Preprocessing linear in* T *and exponential in* P
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Theorem

We can enumerate all matches of a pattern P *on a text* T *with:*

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Automaton Formalism

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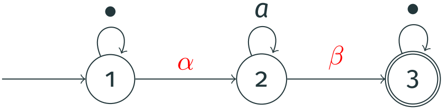

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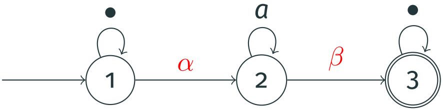

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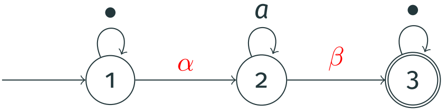

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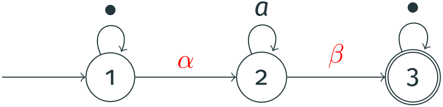

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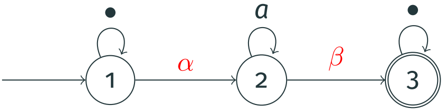

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- **Challenge:** Because of nondeterminism we can have many different runs of A producing the same tuple!

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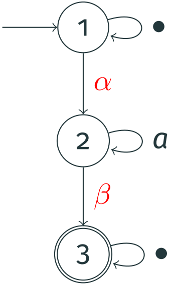

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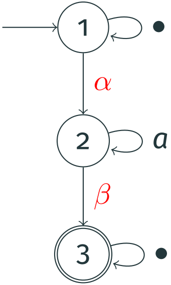

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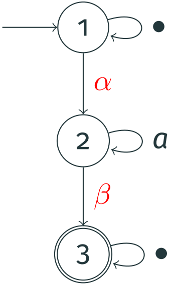

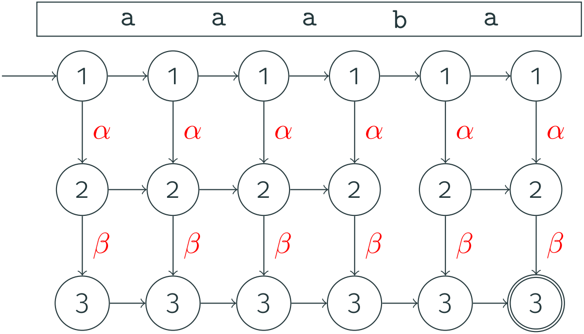

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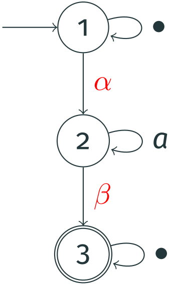

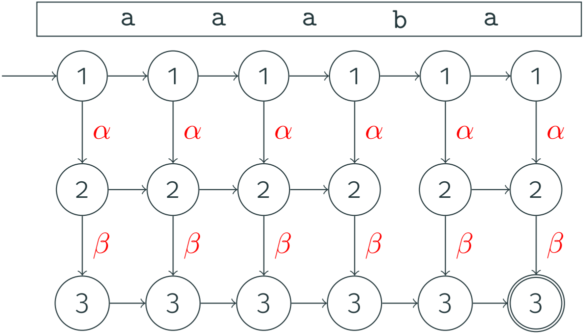

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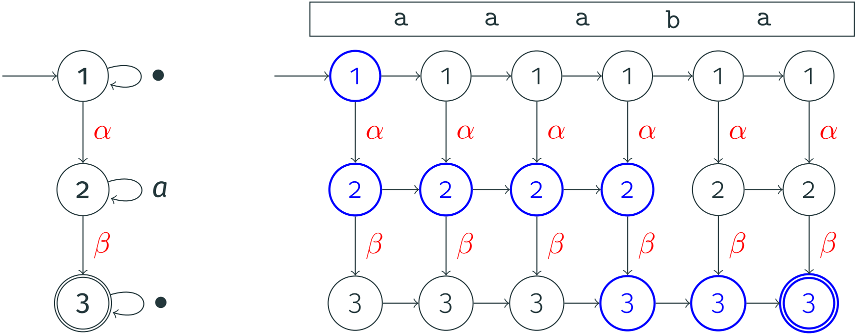

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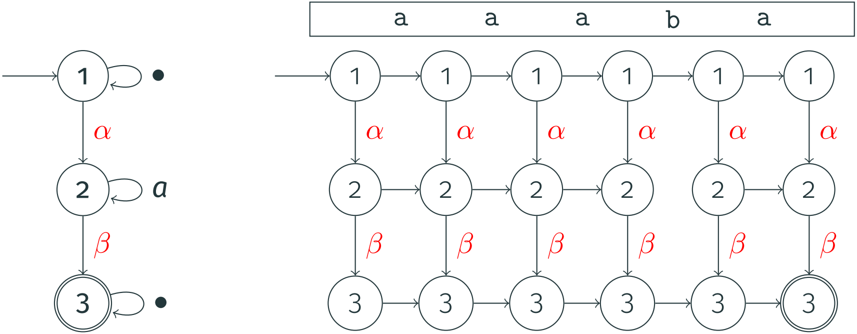

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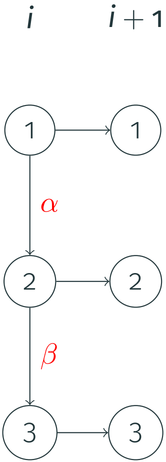
→ **Challenge:** Enumerate paths but avoid **duplicate matches** and do not **waste time** to ensure constant delay

Proof idea: on-the-fly computation to avoid duplicates



- We are at a position i and set of states in blue

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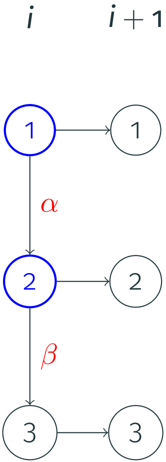

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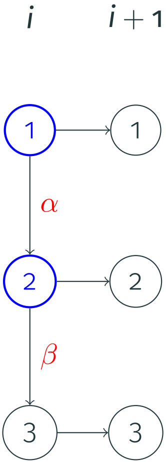

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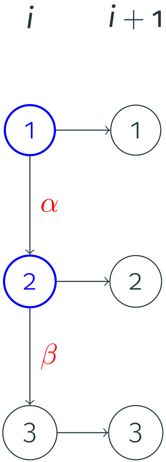

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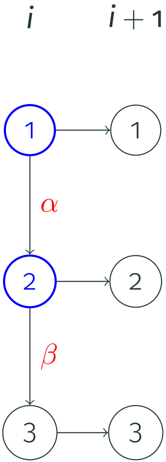

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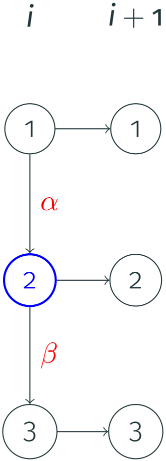

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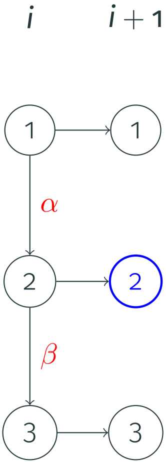

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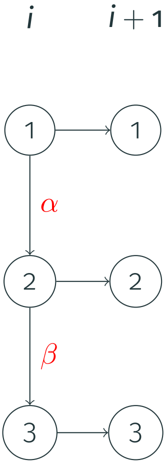

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→ We must have **preprocessed** the DAG to make sure that we can always finish the run

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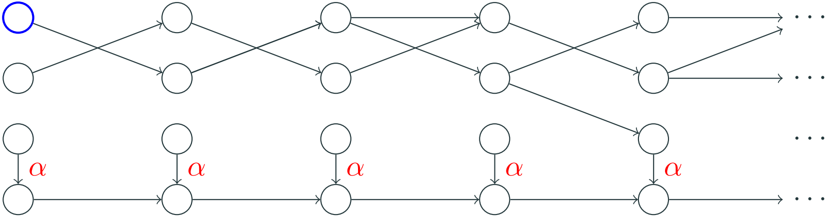

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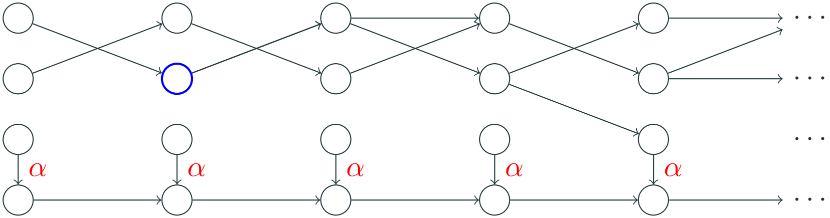

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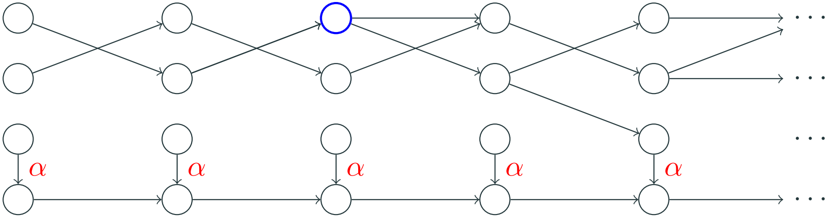

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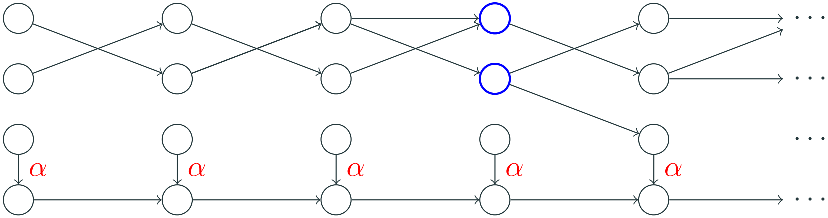

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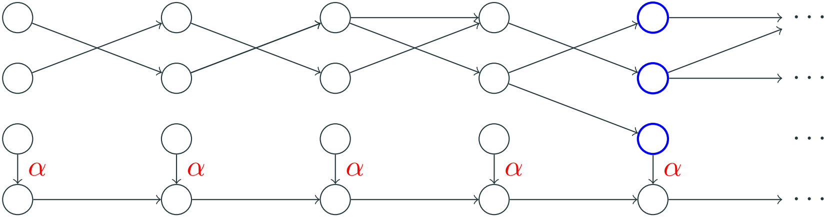

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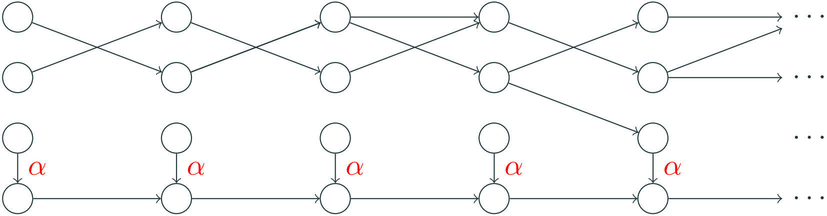

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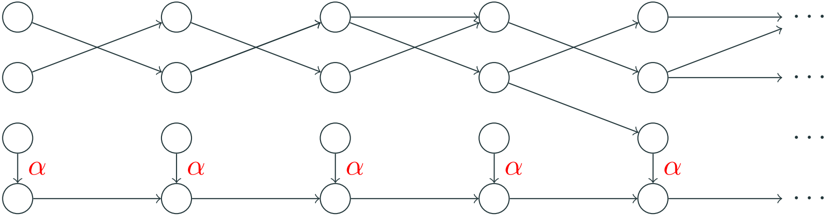

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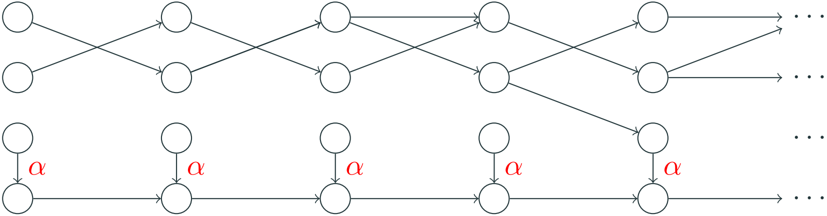

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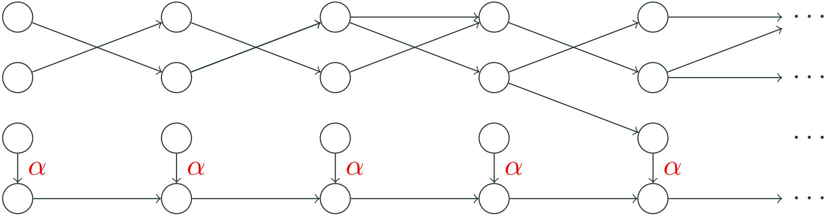

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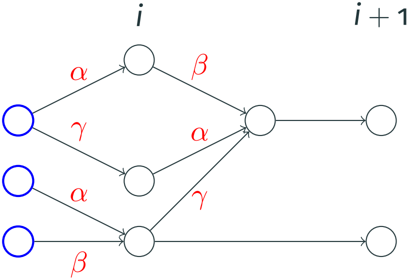

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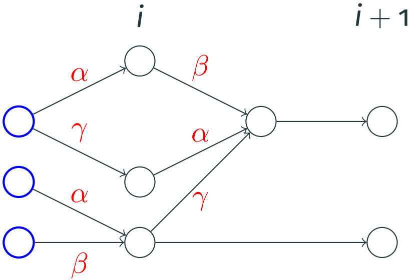

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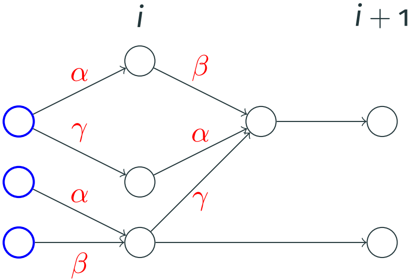

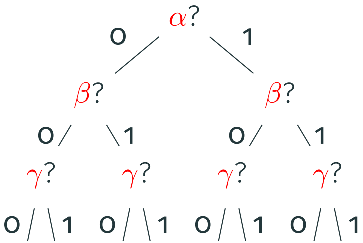

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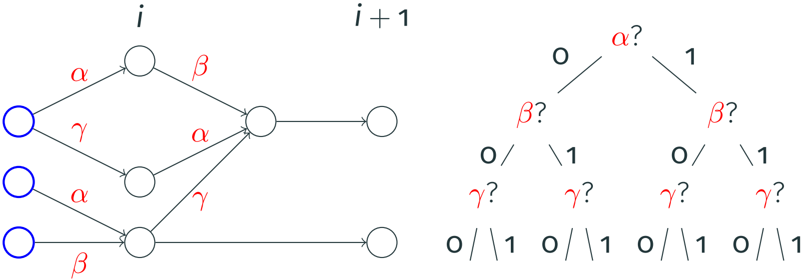

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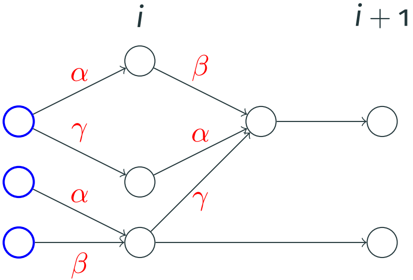

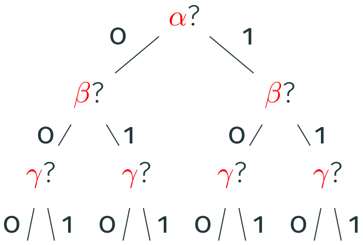

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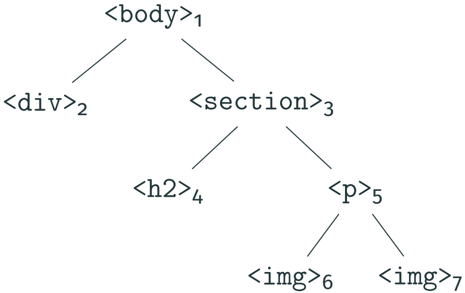
Extension: From Text to Trees

Pattern Matching on Trees

- The data T is no longer text but is now a tree:



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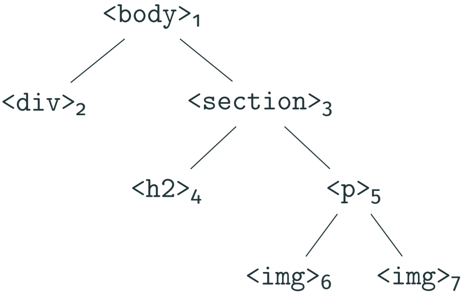

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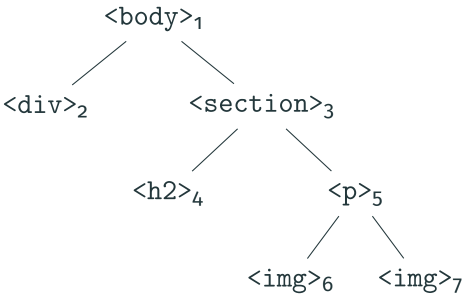

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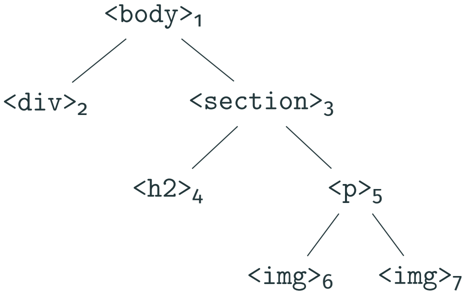

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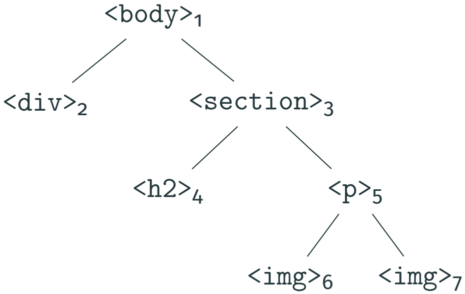

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Theorem [Amarilli et al., 2019]

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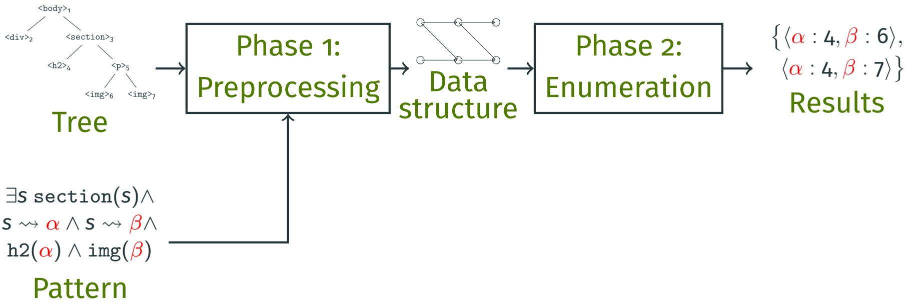
Proof Idea for Trees: Structure

Similar structure to the previous proof, but with a **circuit**:



Tree

Phase 1: Preprocessing

Data structure

Phase 2: Enumeration

$\{\langle \alpha : 4, \beta : 6 \rangle,$
$\langle \alpha : 4, \beta : 7 \rangle\}$

Results

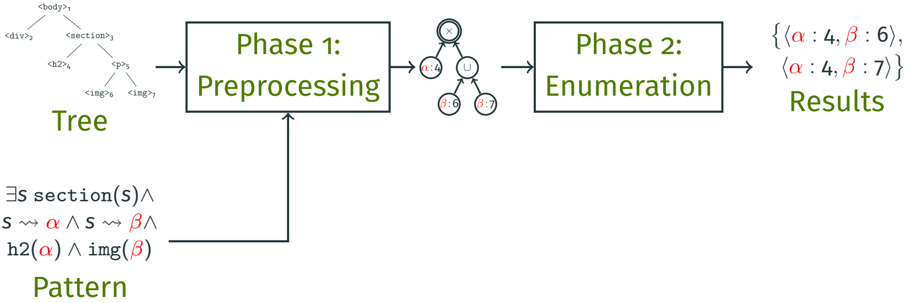
$\exists s\ \texttt{section}(s) \wedge$
$s \rightsquigarrow \alpha \wedge s \rightsquigarrow \beta \wedge$
$\texttt{h2}(\alpha) \wedge \texttt{img}(\beta)$

Pattern

Proof Idea for Trees: Structure

Similar structure to the previous proof, but with a circuit:

- Preprocessing: Compute a circuit representation of the answers
- Enumeration: Apply a generic algorithm on the circuit



Tree

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$\{\langle \alpha : 4, \beta : 6\rangle, \langle \alpha : 4, \beta : 7\rangle\}$

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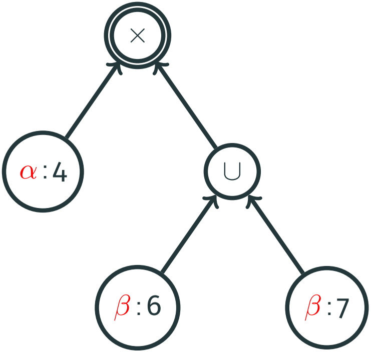

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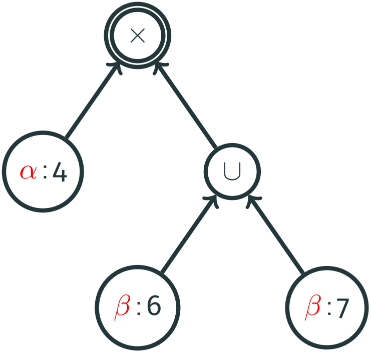

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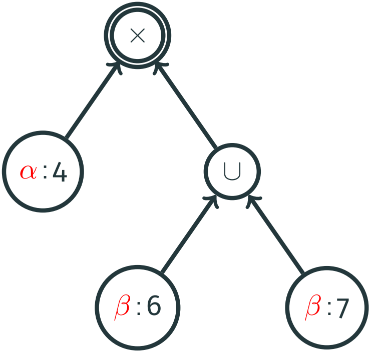

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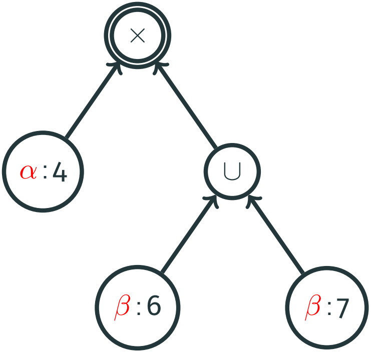

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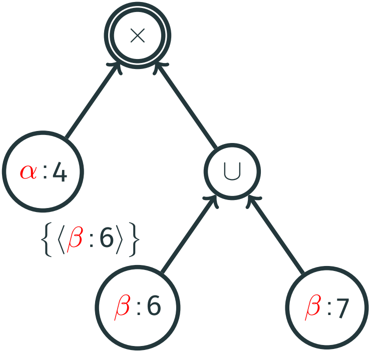

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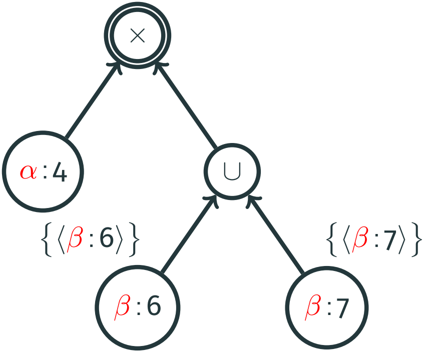
×
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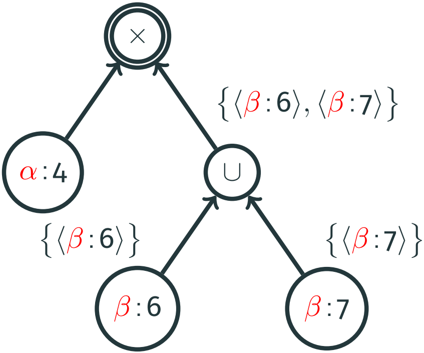

$\times$
$\{\langle \beta : 6 \rangle, \langle \beta : 7 \rangle\}$
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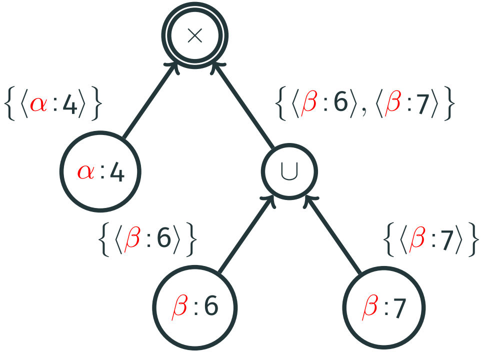

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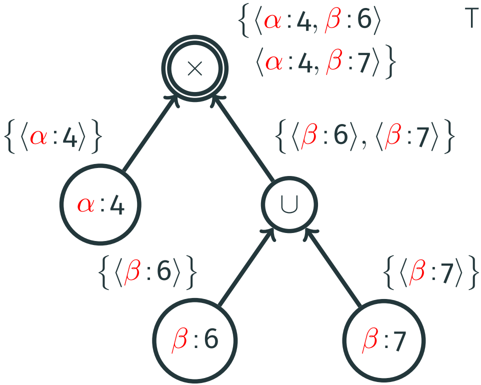

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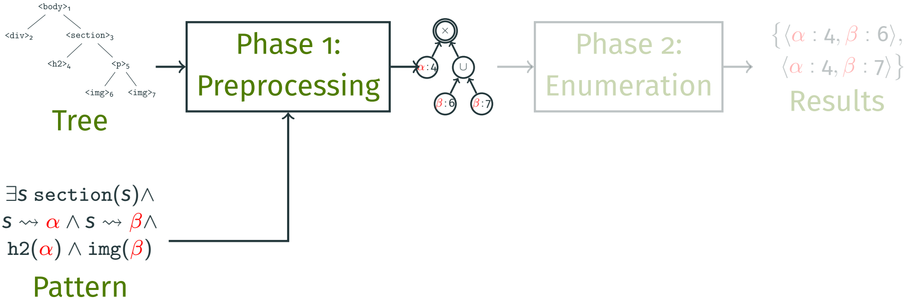


Tree

Phase 1: Preprocessing

Phase 2: Enumeration

$\{\langle \alpha : 4, \beta : 6\rangle, \langle \alpha : 4, \beta : 7\rangle\}$

Results

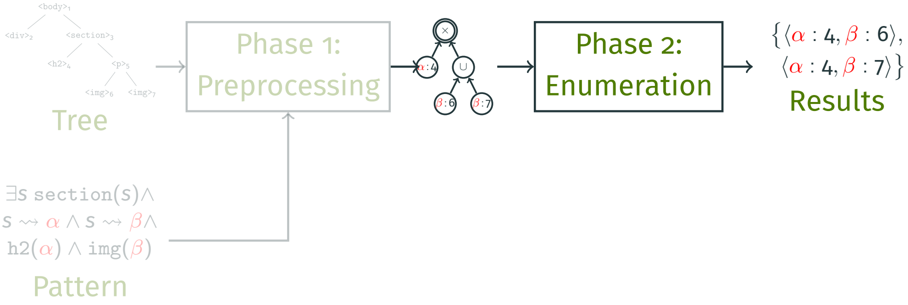
$\exists s\ \mathtt{section}(s) \land s \rightsquigarrow \alpha \land s \rightsquigarrow \beta \land \mathtt{h2}(\alpha) \land \mathtt{img}(\beta)$

Pattern

Theorem

*For any **tree automaton** A with capture variables $\alpha_1, \ldots, \alpha_k$, given a **tree** T, we can build in $O(|T| \times |A|)$ a **set circuit** capturing exactly the set of tuples $\{\langle \alpha_1 : n_1, \ldots, \alpha_k : n_k\rangle\}$ in the output of A on T*

Proof Idea for Trees: Results


<body>$_1$
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Tree
$\exists s$ section$(s) \wedge$
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Results

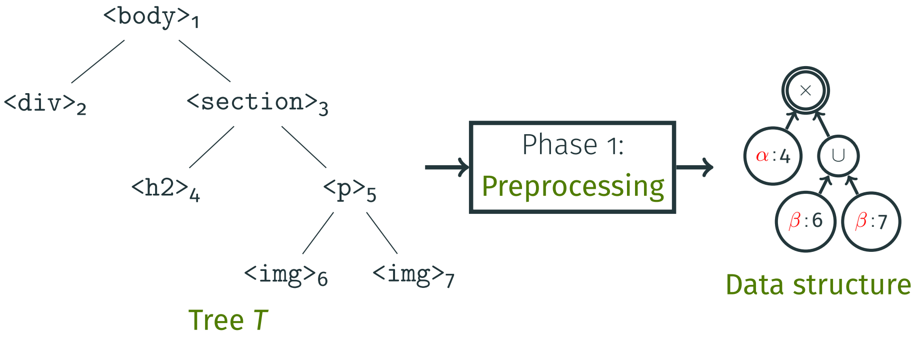
Theorem

*Given a set circuit **satisfying some conditions**, we can enumerate all tuples that it captures with linear preprocessing and constant delay*

E.g., for $\{\langle\alpha : 4, \beta : 6\rangle, \langle\alpha : 4, \beta : 7\rangle\}$: enumerate $\langle\alpha : 4, \beta : 6\rangle$ then $\langle\alpha : 4, \beta : 7\rangle$

Extension: Supporting Updates

Updates



- The input data can be **modified** after the preprocessing

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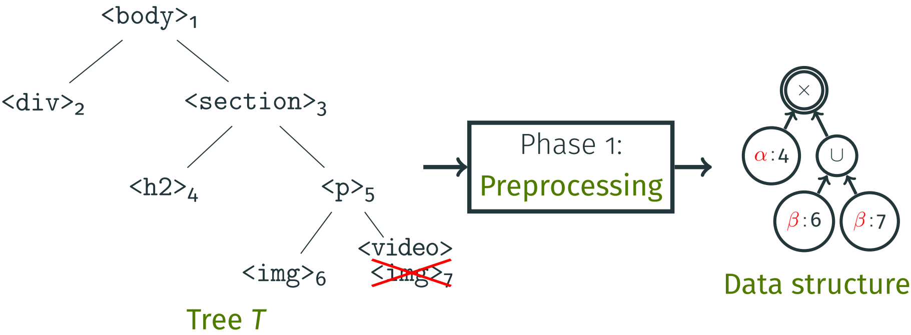

Tree T

Phase 1:
Preprocessing

Data structure

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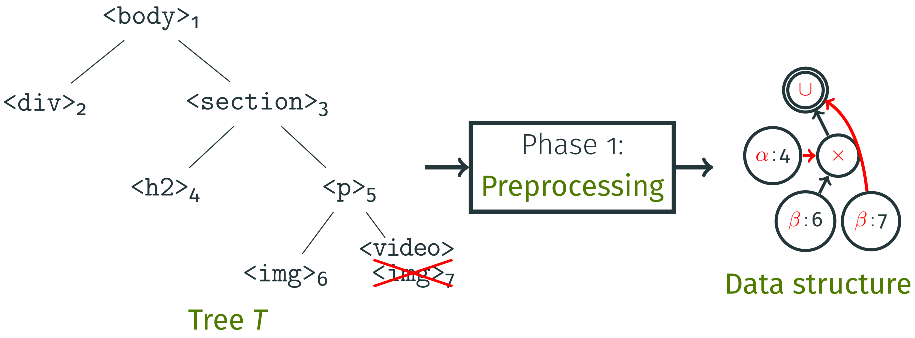

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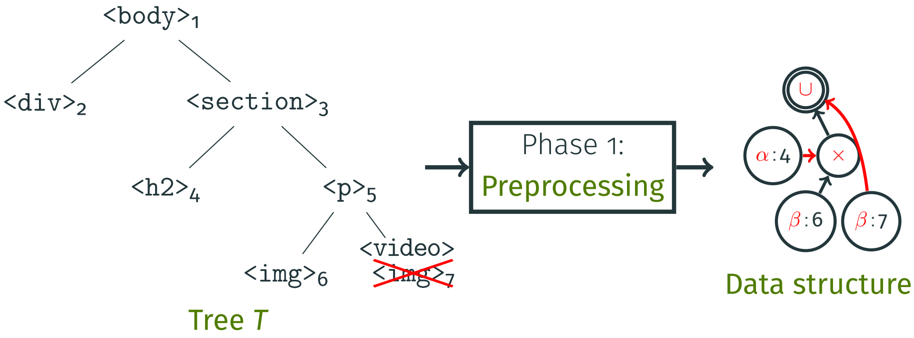

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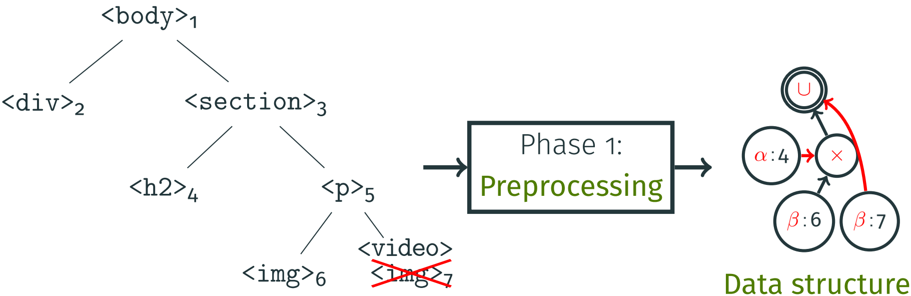

Tree T

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→ Can we **do better**?

Results on dynamic trees

All these results are on data complexity in T (for a fixed pattern):

Work	Data	Preproc.	Delay	Updates
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Extension: Connection to Circuits

Connections with Boolean circuits

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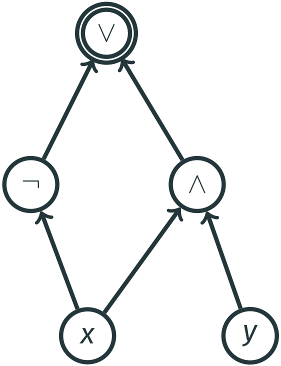
→ Task: Given a Boolean circuit, how to efficiently enumerate its satisfying valuations?

Boolean circuits



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- **Output** gate: ◎
- **Variable** gates: x
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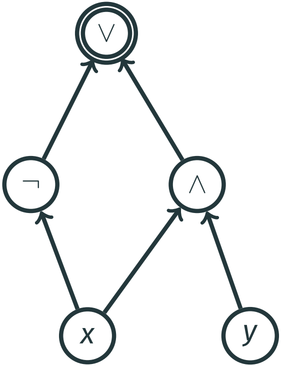

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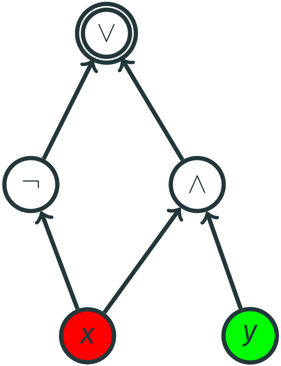

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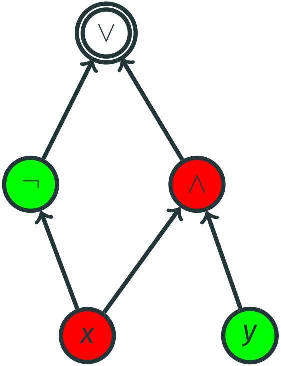

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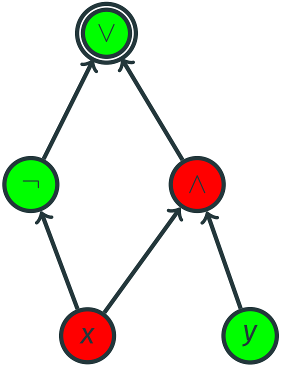

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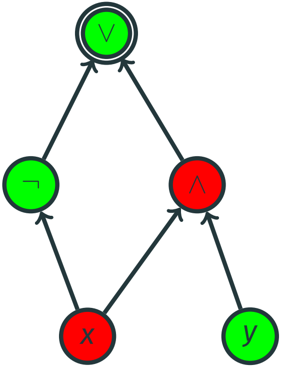

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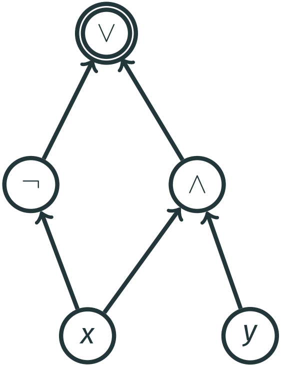

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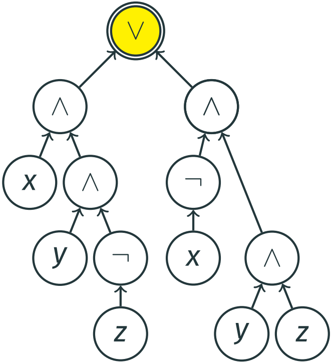
Our task: Enumerate all **satisfying assignments** of an input circuit

Circuit restrictions

d-DNNF:

- $\vee$ are all **deterministic**:

The inputs are **mutually exclusive** (= no valuation ν makes two inputs simultaneously evaluate to 1)



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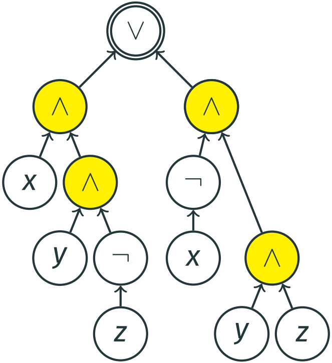

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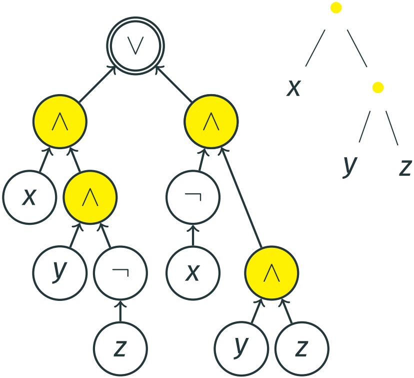
v-tree: ∧-gates follow a **tree** on the variables



Main results

Theorem

*Given a **d-DNNF circuit** C with a **v-tree** T, we can enumerate its **satisfying assignments** with preprocessing **linear in** $|C| + |T|$ and delay **linear in each assignment***

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Also: restrict to assignments of **constant size** $k \in \mathbb{N}$ (at most k variables are set to 1):

Theorem

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Main results

Theorem

Given a d-DNNF circuit C with a v-tree T, we can enumerate its satisfying assignments with preprocessing linear in $|C| + |T|$ and delay linear in each assignment

Also: restrict to assignments of constant size $k \in \mathbb{N}$ (at most k variables are set to 1):

Theorem

Given a d-DNNF circuit C with a v-tree T, we can enumerate its satisfying assignments of size $\leq k$ with preprocessing linear in $|C| + |T|$ and constant delay

Subtleties: Must complete to a set circuit; memory usage problems

Summary and Future Work

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- **Problem:** given a text T and a pattern P, enumerate efficiently all matches of P on T

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Extensions:

- Enumeration on trees rather than words
- Handling updates to the underlying data
- Enumerating satisfying valuations of a circuit

Ongoing research and future work

With P. Bourhis, R. Dupré, M. Niewerth, S. Mengel:

Efficient implementation of the approach

`https://github.com/PoDMR/enum-spanner-rs`

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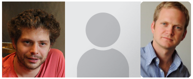
With L. Jachiet, M. Muñoz, C. Riveros:

Can we enumerate for **context-free languages**?



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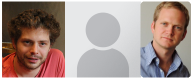

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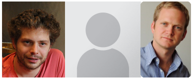

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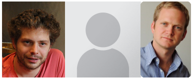

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How to enumerate **maximal matches** of a pattern?

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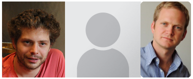

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Thanks for your attention!

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To appear.

Proof idea for trees: set circuit construction (details)

- **Automaton:** *“Select all node pairs* (α, β)*”*
- **States:** $\{\emptyset, \alpha, \beta, \alpha\beta\}$

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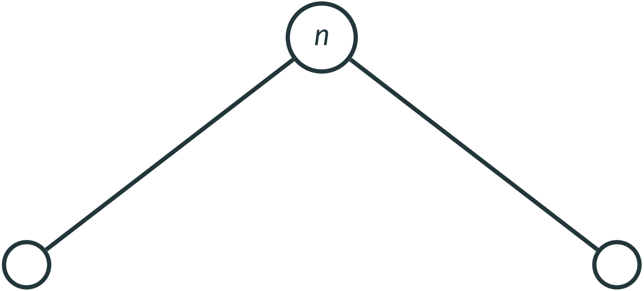

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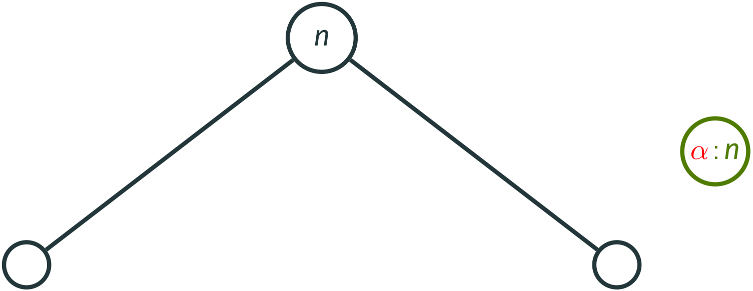

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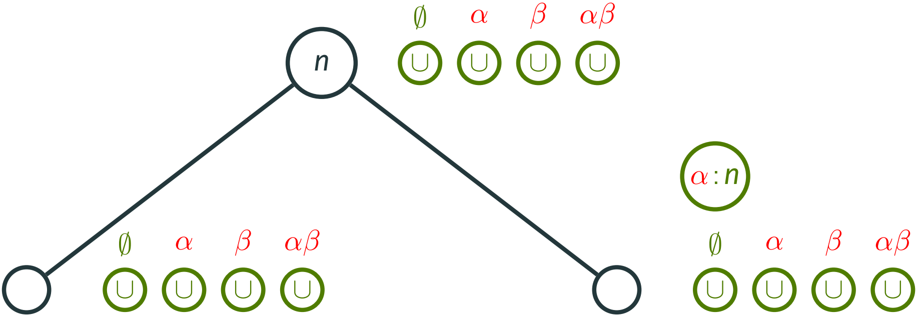

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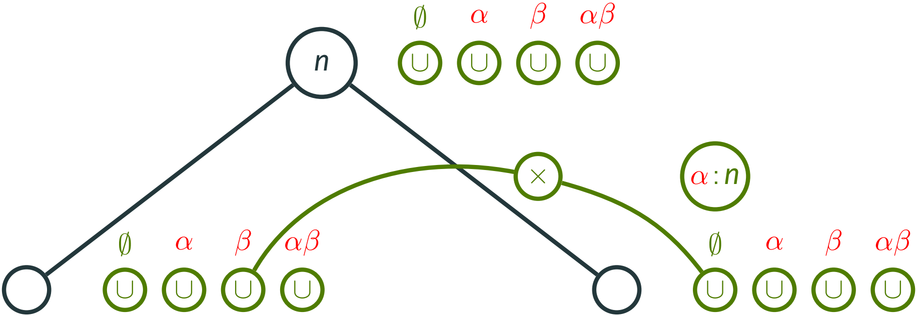

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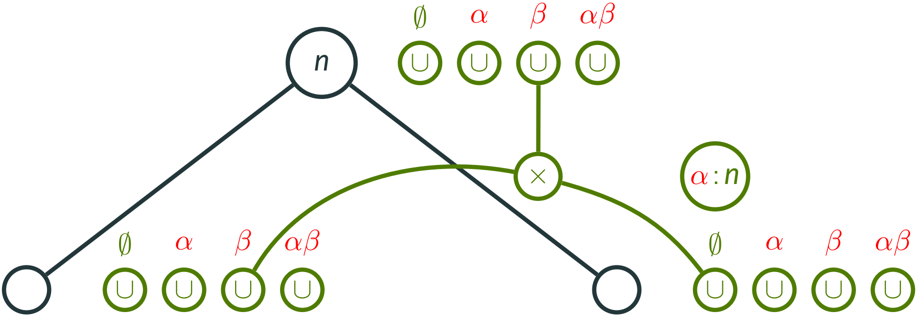

n
$\emptyset$ α β $\alpha\beta$
$\cup$ $\cup$ $\cup$ $\cup$
$\times$
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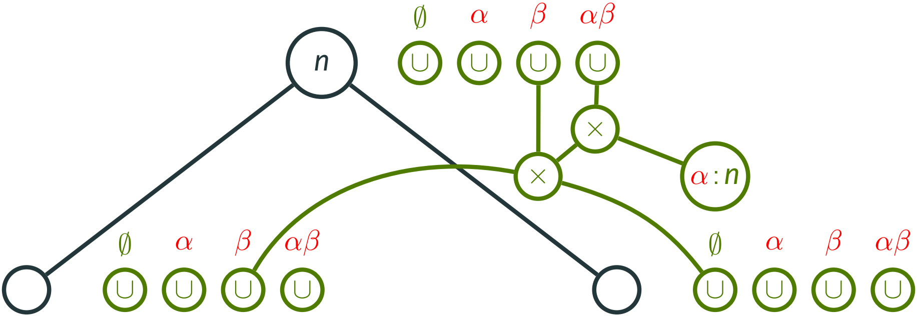

Proof idea for trees: general enumeration approach

$\rightarrow$ Enumerate the set $T(g)$ captured by each gate g

$\rightarrow$ Do it by top-down induction on the circuit

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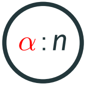
Base case: variable 

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Base case: variable $\alpha : n$: enumerate $\langle \alpha : n \rangle$ and stop

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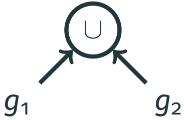
∪-gate



Concatenation: enumerate $T(g_1)$
and then enumerate $T(g_2)$

Proof idea for trees: general enumeration approach

→ Enumerate the set $T(g)$ captured by each gate g

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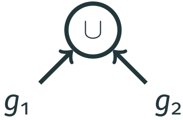
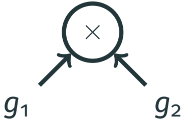
Base case: variable : enumerate $\langle \alpha : n \rangle$ and stop

$\cup$-gate



Concatenation: enumerate $T(g_1)$ and then enumerate $T(g_2)$

$\times$-gate

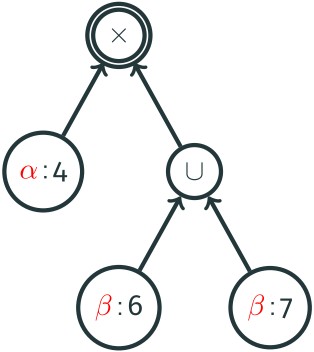


Lexicographic product:

```
for every $t_1$ in $T(g_1)$:
    for every $t_2$ in $T(g_2)$:
        output $t_1 + t_2$
```

Proof idea for trees: circuit conditions

Enumeration relies on some **conditions** on the input circuit (d-DNNF):



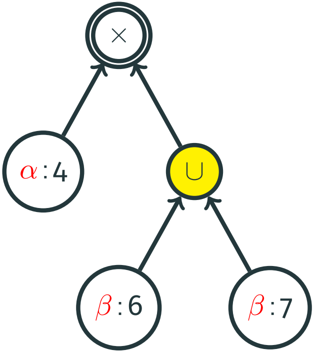
Proof idea for trees: circuit conditions

Enumeration relies on some conditions on the input circuit (d-DNNF):

- $\cup$ are all deterministic:

For any two inputs g_1 and g_2 of a $\cup$-gate, the captured sets $T(g_1)$ and $T(g_2)$ are disjoint (they have no tuple in common)

→ Avoids duplicate tuples



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- $\times$ are all **decomposable**:

For any two inputs g_1 and g_2 of a $\times$-gate, no variable has a path to both g_1 and g_2

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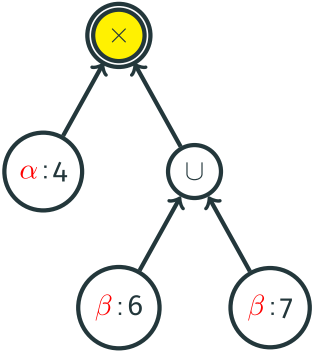

Proof idea for trees: circuit conditions

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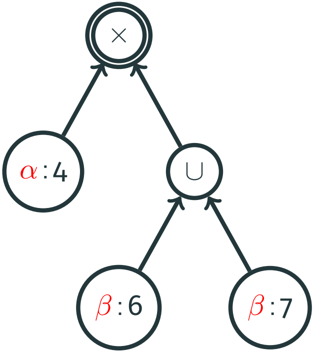

- × are all decomposable:

For any two inputs g_1 and g_2 of a ×-gate, no variable has a path to both g_1 and g_2

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- Also an additional upwards-determinism condition

Proof idea for trees: circuit conditions

Enumeration relies on some **conditions** on the input circuit (d-DNNF):

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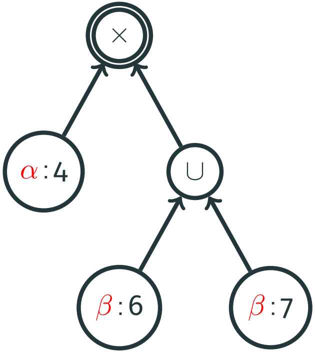
→ Avoids **duplicate tuples**

- × are all **decomposable**:

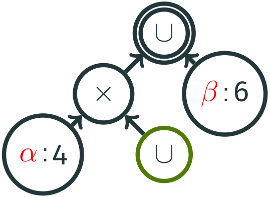
For any two inputs g_1 and g_2 of a ×-gate, no variable has a path to both g_1 and g_2

→ Avoids **duplicate singletons**

- Also an additional **upwards-determinism** condition
- Our circuit satisfies these thanks to **automaton determinism**



Proof idea for trees: enumeration subtleties



- We must not waste time in gates capturing $\emptyset$

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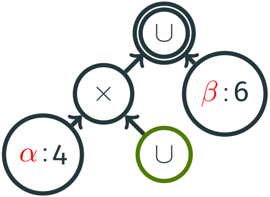

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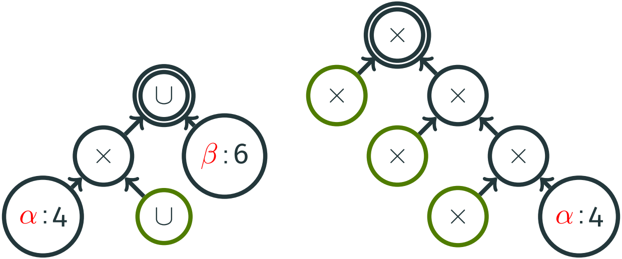

- We must not waste time in gates capturing $\emptyset$

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- We must not waste time because of gates capturing $\{\langle\rangle\}$

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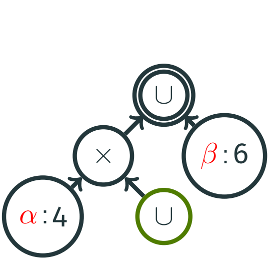

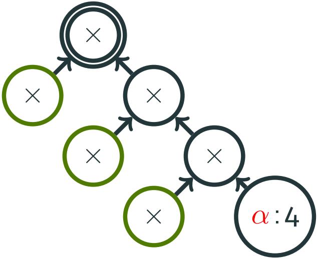

- We must not waste time in gates capturing $\emptyset$
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  - $\rightarrow$ **Homogenization** to set them aside

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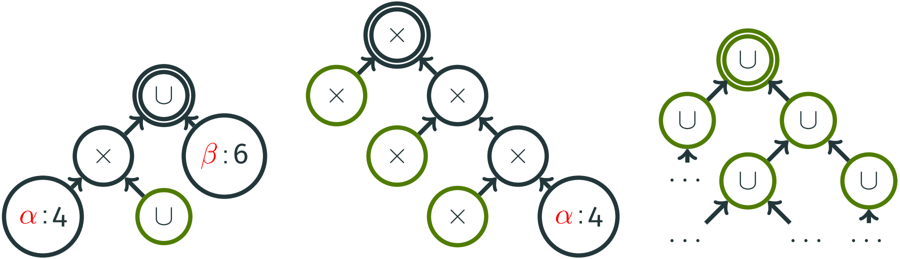

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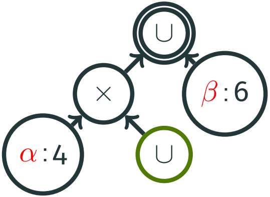

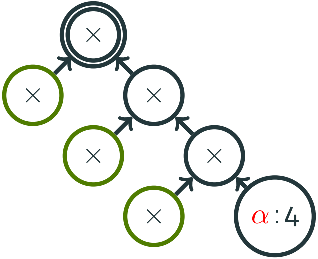

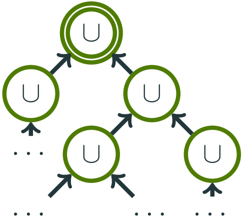

- We must not waste time in gates capturing $\emptyset$
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- We must not waste time because of gates capturing $\{\langle\rangle\}$
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- We must not waste time in **hierarchies of $\cup$-gates**
  - → Precompute a **reachability index** (uses **upwards-determinism**)