Query Lineages and Knowledge Compilation

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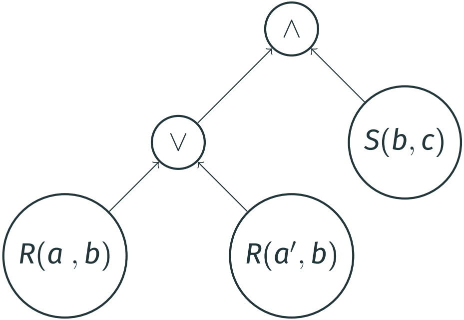

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- **Explanation:**
  - Representation of **why** the query is true
  - **What-if:** is the query still true without these facts?

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We can count the answers, enumerate them, etc.

Related work: Semiring provenance

Semiring provenance ([Green et al., 2007], PODS ToT award): annotate results of a relational algebra query with a semiring expression

A B C	
$a\ b\ c$	p
$d\ b\ e$	r
$f\ g\ e$	s

(a)

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$a\ c$	$\{p\}$
$a\ e$	$\{p, r\}$
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A C	
$a\ c$	$2p^2$
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Figure 5: Why-prov. and provenance polynomials

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- Circuit representation: more concise

Computing lineages: theory

- Unions of Conjunctive Queries (UCQ)

Theorem

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- Datalog: see [Deutch et al., 2014], PTIME

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```
a3nm=# SELECT id, name, city FROM personnel;
 id |   name   |   city
----+----------+----------
  1 | John     | New York
  2 | Paul     | New York
  3 | Dave     | Paris
  4 | Ellen    | Berlin
  5 | Magdalen | Paris
  6 | Nancy    | Paris
  7 | Susan    | Berlin
(7 rows)

a3nm=# SELECT *,formula(provenance(), 'personnel_id') FROM
(SELECT DISTINCT city FROM personnel) t;
   city   |   formula
----------+-------------
 Paris    | (3 ⊕ 5 ⊕ 6)
 Berlin   | (4 ⊕ 7)
 New York | (1 ⊕ 2)
(3 rows)
```

You can run it! https://github.com/PierreSenellart/provsql

Knowledge compilation and tractable circuit classes

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  - $\rightarrow$ Satisfiability, counting, probability computation, enumeration...

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→ Tractability: use tractable **circuit classes**

Tractable circuit classes: Theory vs practice

There is a whole zoo of tractable circuit classes...

$$
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 & \text{dec-DNNF} \arrow[rr, two heads] & & \text{d-DNNF} \arrow[rr, two heads] & & \text{DNNF} \\
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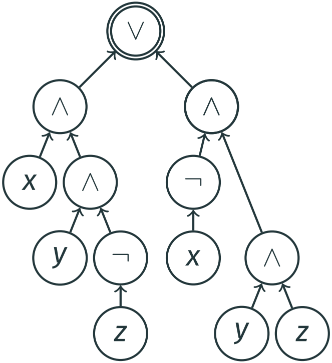
But in **practice** there are solvers for arbitrary circuits:

- **Satisfiability (SAT):** MapleSAT, Cadical, Glucose, etc.
- **Counting:** c2d, d4, dsharp, etc.

A tractable circuit class: d-SDNNF



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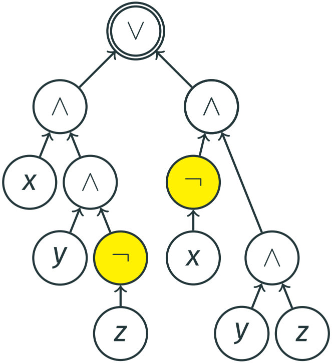

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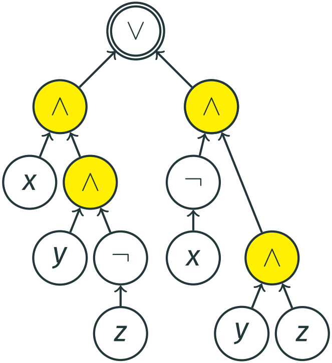

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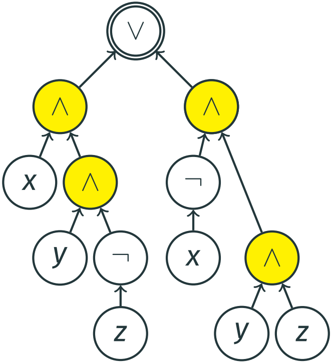

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  - → efficient satisfiability

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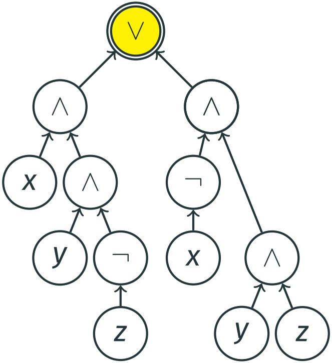

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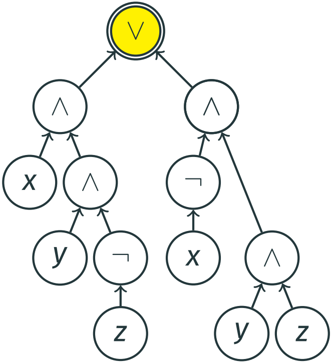
- Deterministic: inputs of $\vee$-gates are **mutually exclusive**

A tractable circuit class: d-SDNNF



- **Negation Normal Form**: negations only applied to the leaves
- **Decomposable**: inputs of $\wedge$-gates are **independent** (no variable has a path to two different inputs of the same $\wedge$-gate)
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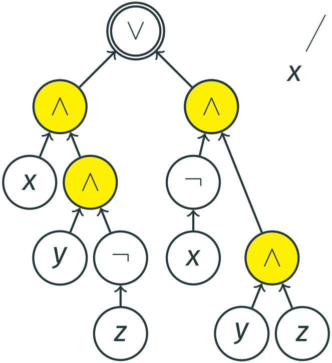

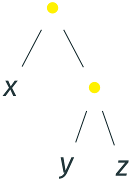

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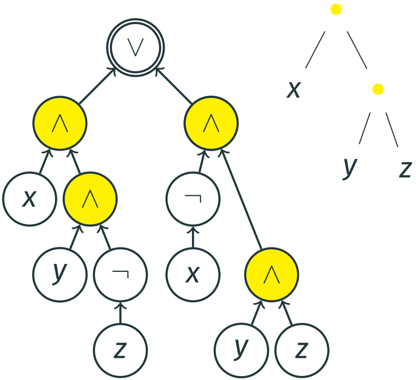

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  - $\rightarrow$ efficient **enumeration**

Computing lineages in tractable classes

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For any MSO query, given a tree (or word), we can construct a d-SDNNF representation (or OBDD) of the lineage in linear time.

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Thanks for your attention!

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To appear.