Top-k Querying of Incomplete Data under Order Constraints

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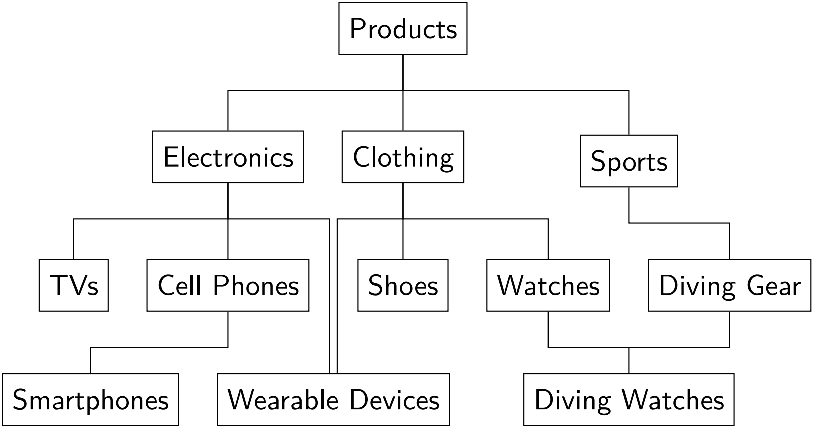
Introduction

Taxonomy of items for a store



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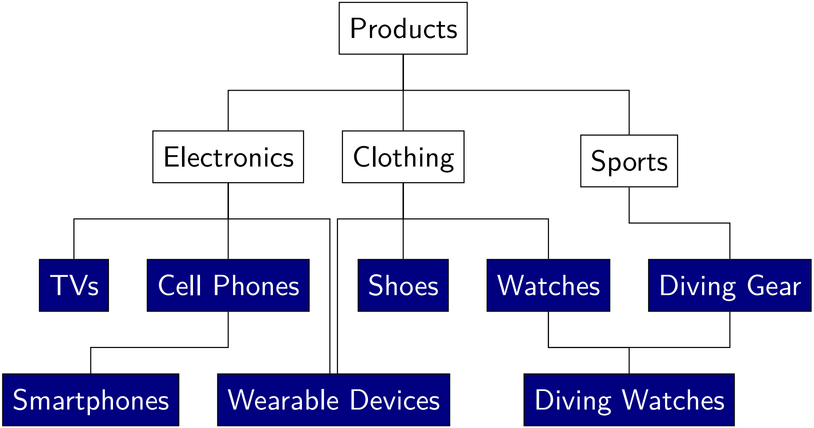

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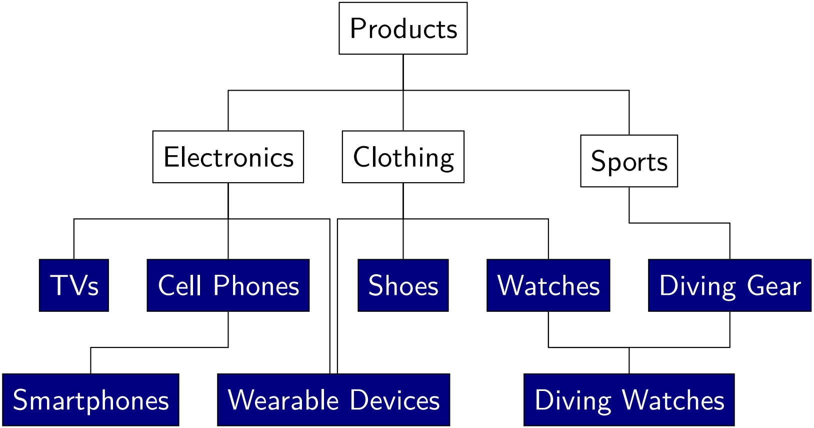

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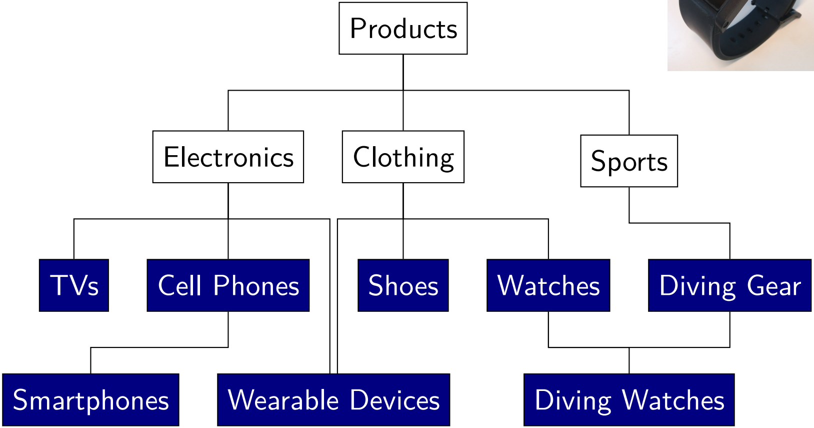

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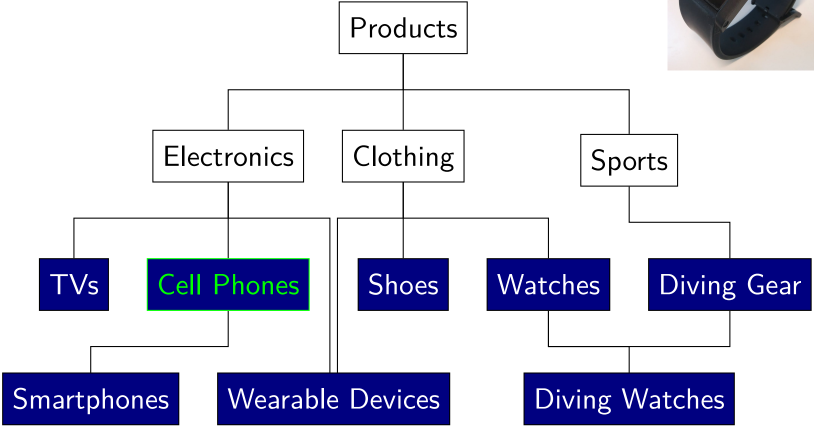

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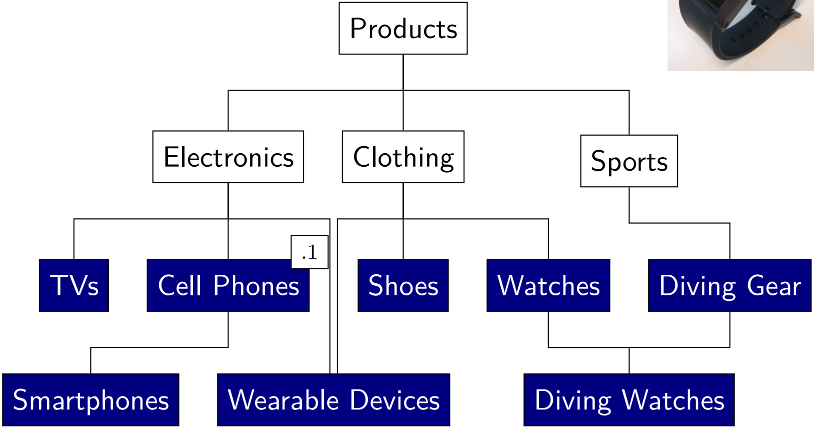

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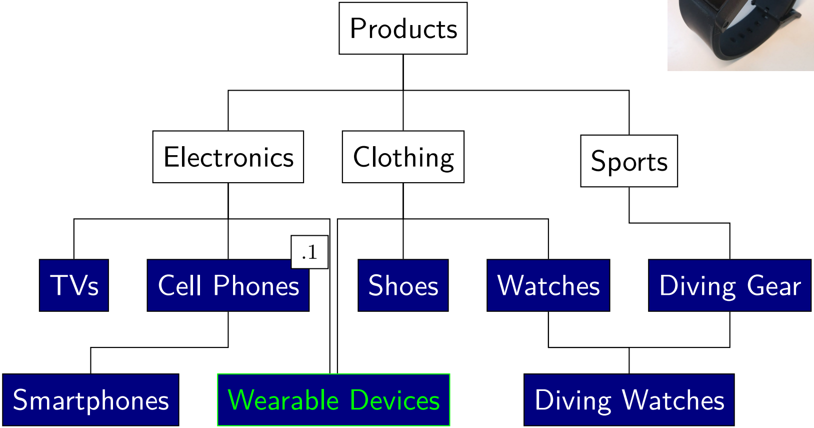

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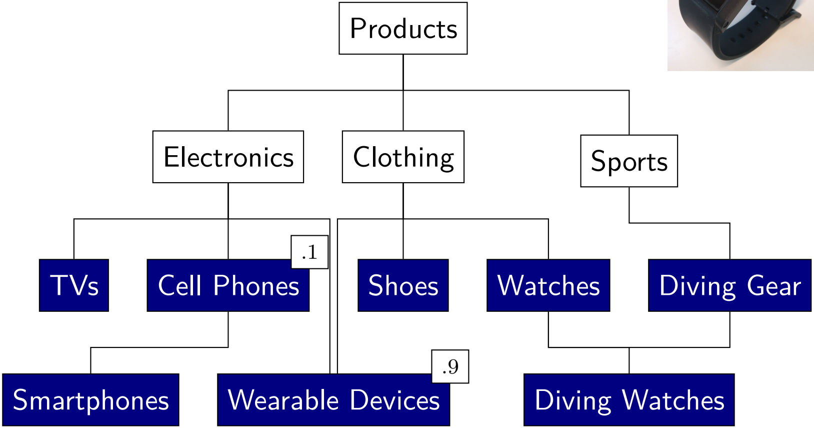

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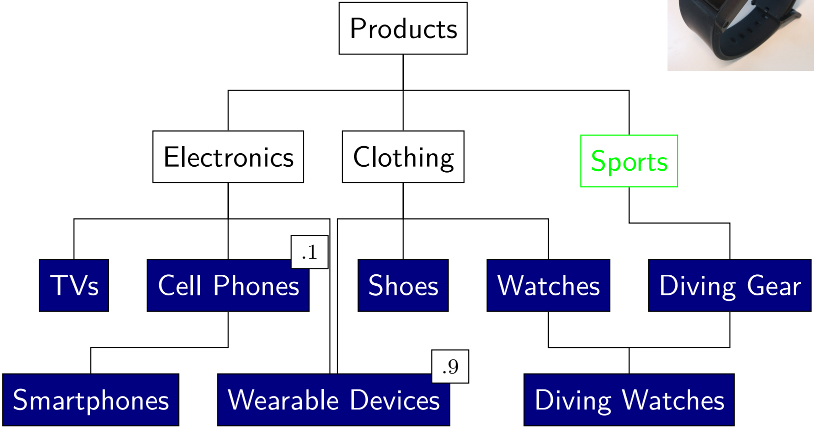

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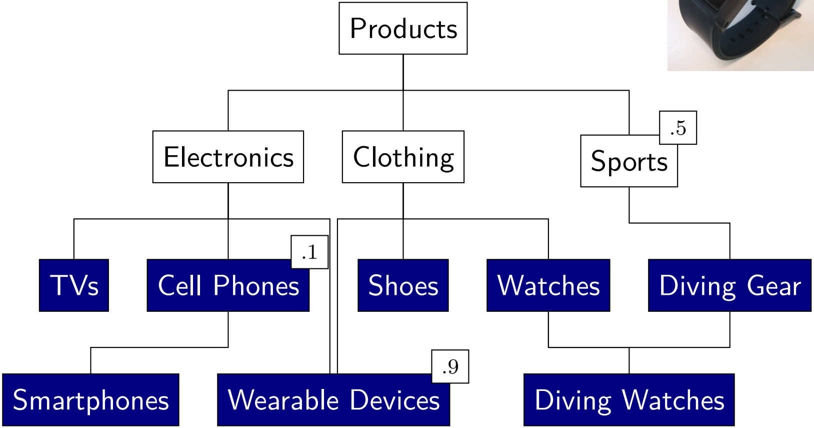

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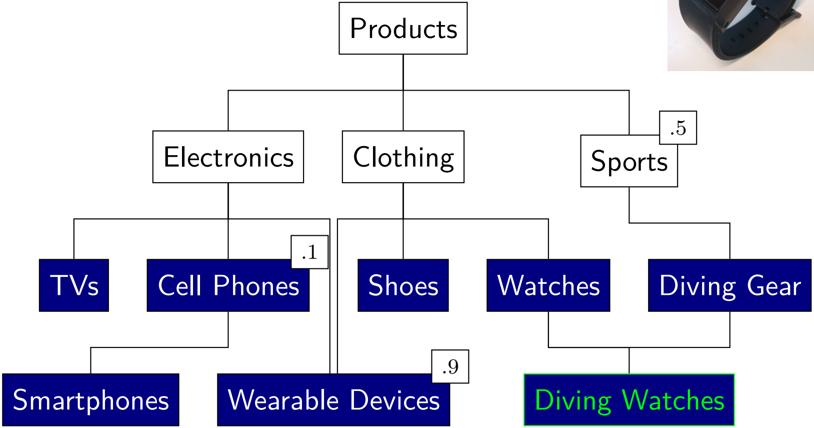

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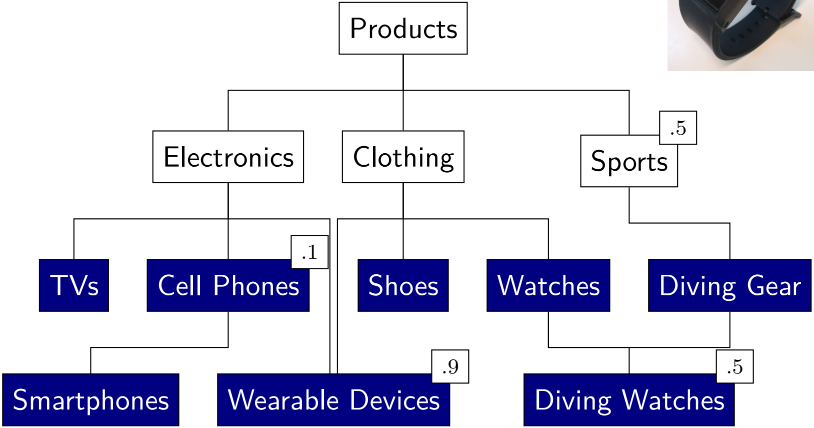

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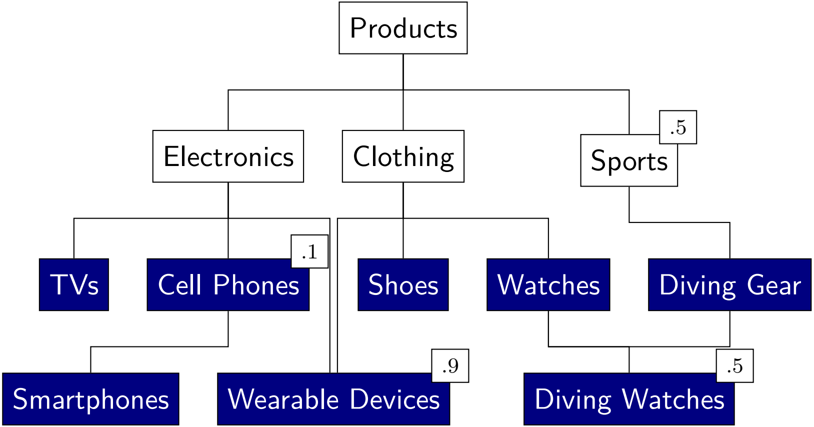

Monotonicity: compatibility increases as we go up.

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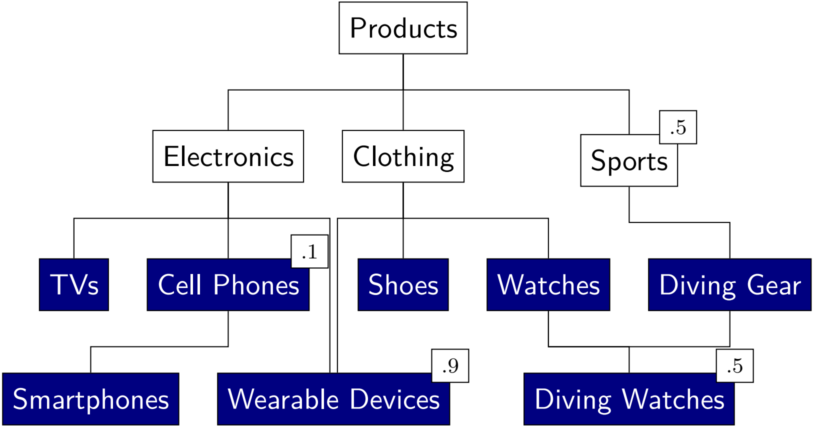

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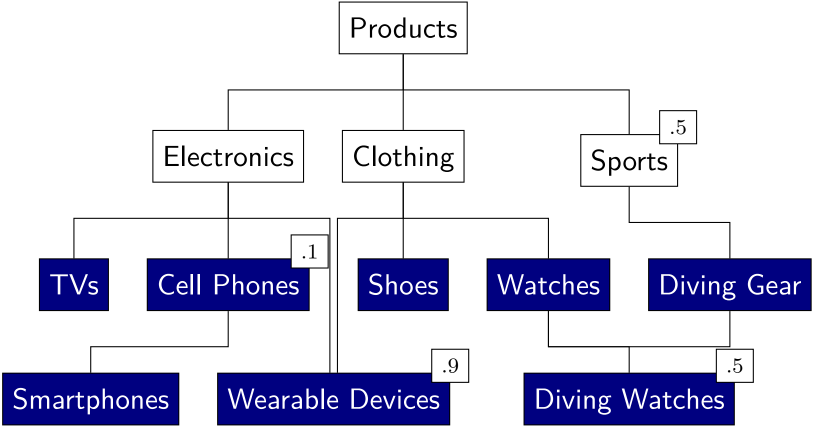

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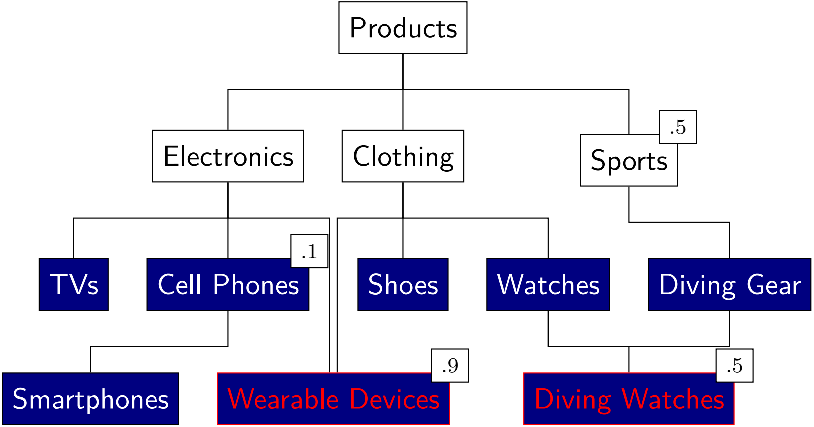

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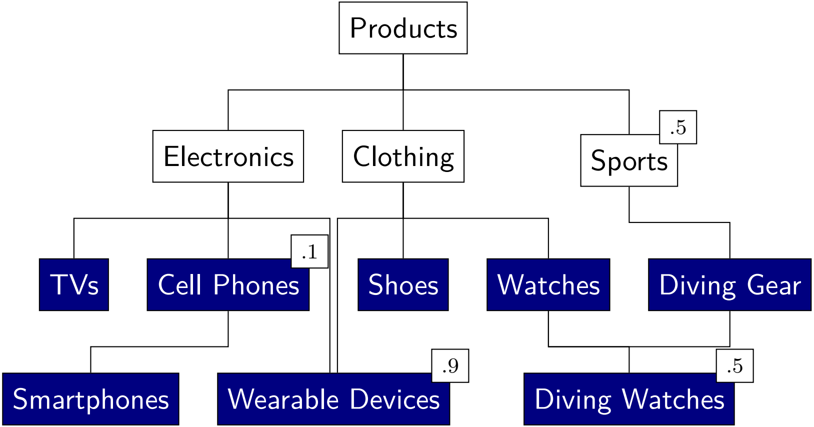

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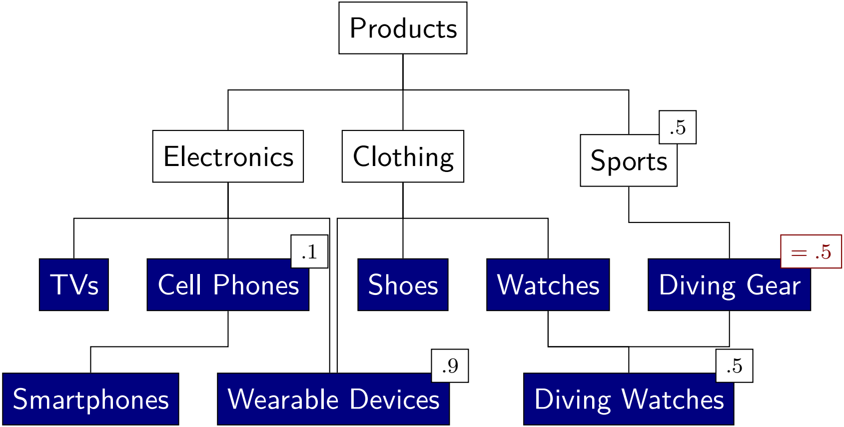

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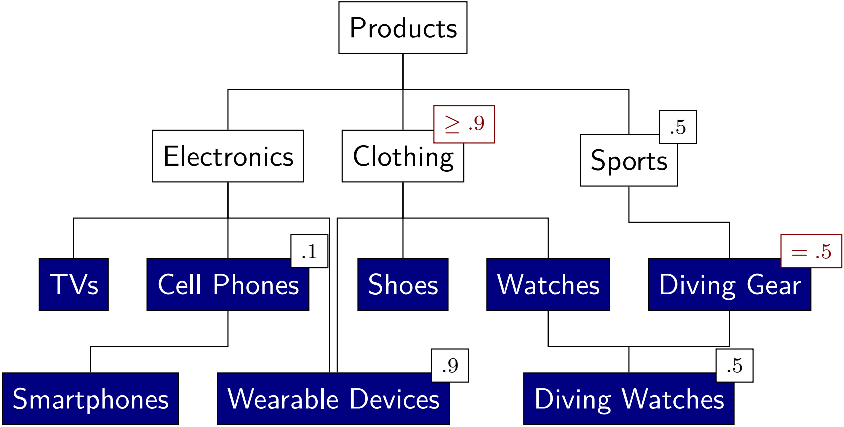

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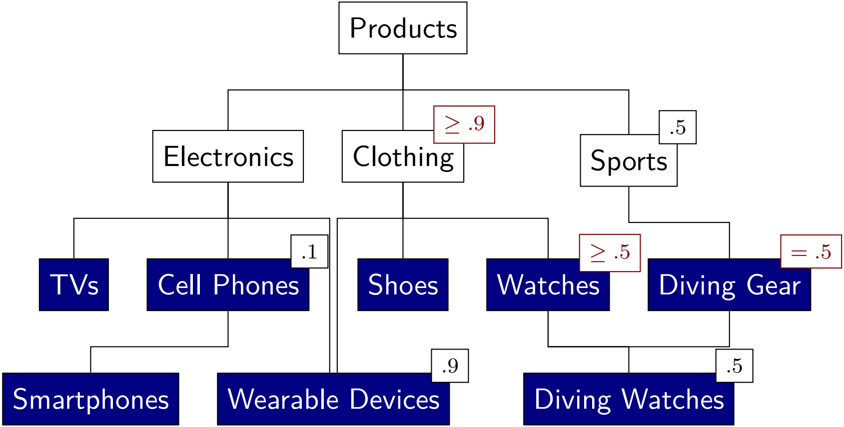

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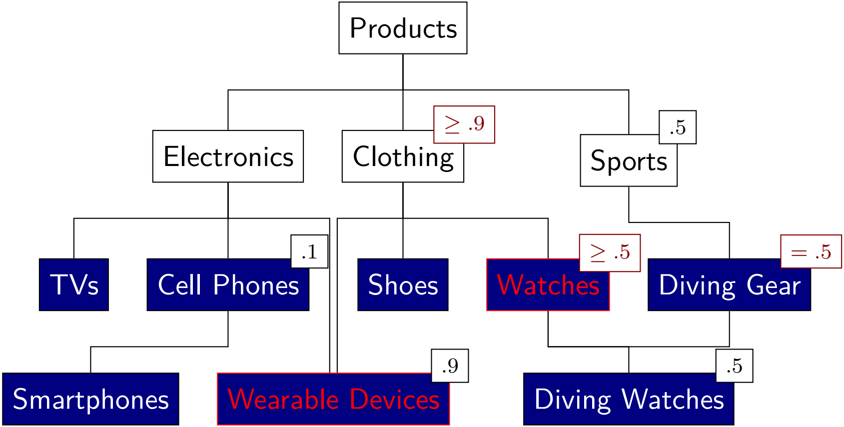

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Problem statement

- Taxonomy:
  - Partial order, i.e., directed acyclic graph
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- Compatibility values:
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$\rightarrow$ How to complete the missing values?

$\rightarrow$ What are the top-k and their expected values?

$\rightarrow$ What is our confidence in the answer?

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Admissible polytope

- Each unknown value has one variable
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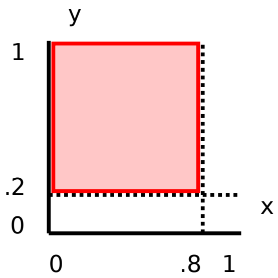

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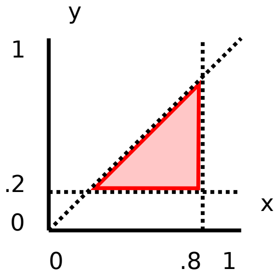

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- Consider the admissible polytope
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(Possible extensions: variance, marginal distribution...)

Easy case: total order

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- (For marginal distribution: order statistics, Beta distribution)

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→ Gives the average for the actual taxonomy!

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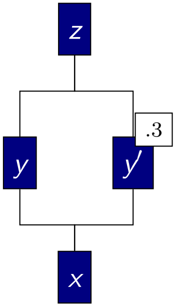

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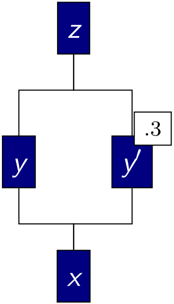

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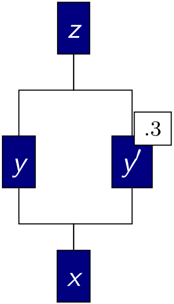

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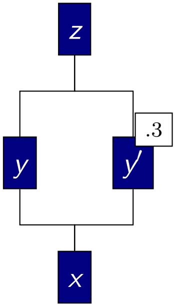

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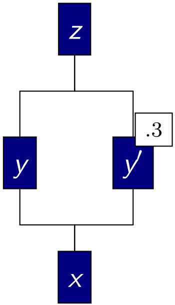

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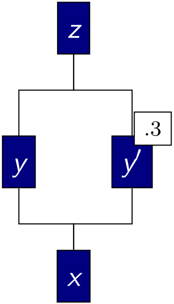

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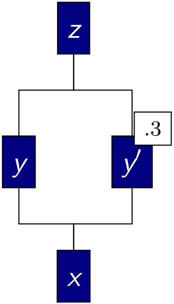

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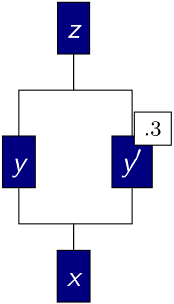

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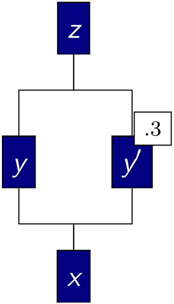

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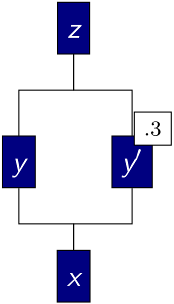

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→ Final result: y has expected value $.43$

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→ Can we show hardness of our problems?

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→ $\mathrm{FP}^{\#\mathrm{P}}$-membership of our problems

- → Non-trivial as polytope volume computation is not in $\mathrm{FP}^{\#\mathrm{P}}$!

Tractable cases

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- Are there tractable subcases?
- Common situation: taxonomy is a tree

$\rightarrow$ PTIME expected value computation
(Compute the marginal distributions as piecewise polynomials)

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Thanks for your attention!

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