Probabilities and Provenance on Trees and Treelike Instances

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  (= existentially quantified conjunction of atoms)
  → Example of CQ: $q : \exists x\, y\ R(x) \wedge S(x, y) \wedge T(y)$

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$\rightarrow$ Query evaluation problem for $\mathcal{Q}$ and $\mathcal{I}$:

- Fix a query $q \in \mathcal{Q}$
- Given an input instance $I \in \mathcal{I}$
- Determine whether I satisfies q (written $I \models q$)
- Complexity as a function of I, not q (= data complexity)

Query evaluation example

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- Semantics: (I, π) gives a probability distribution on $I' \subseteq I$:
  - Each fact $F \in I$ is either present or absent with probability $\pi(F)$
  - Facts are independent

Example of a probabilistic instance

S		
a	a	1
b	v	$.5$
b	w	$.2$

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Is there a smaller class $\mathcal{I}$ such that PQE is tractable for a larger $\mathcal{Q}$?

- Probabilistic XML: [Cohen et al., 2009]
  - $\mathcal{Q}$ are tree automata, $\mathcal{I}$ are trees
  - PQE is PTIME

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$\rightarrow$ Does this extend to probabilistic QE?

Our results

An instance-based dichotomy result:

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Lower bound. For any unbounded-tw family $\mathcal{I}$ and $\mathcal{Q}$ FO queries

$\rightarrow$ PQE is #P-hard under RP reductions assuming

- Signature arity is 2 (graphs)
- High-tw instances in $\mathcal{I}$ are easily constructible

Table of contents

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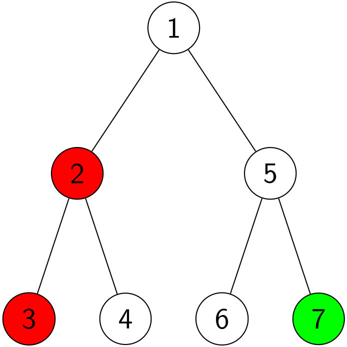
- To solve the PQE problem on treelike instances for MSO
  - First solve the problem on trees with tree automata
  - Then use the results of [Courcelle, 1990]

Uncertain trees



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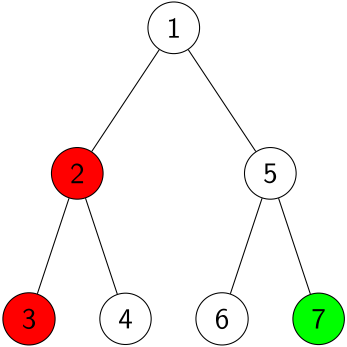

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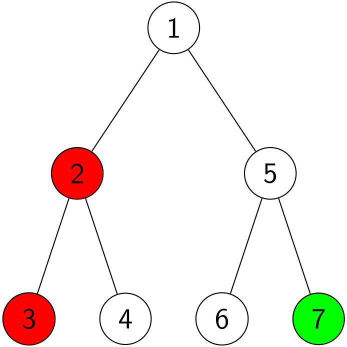

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Example tree automaton:
"Is there both a red and a green node?"

Valuation: $\{2, 3, 7\}$

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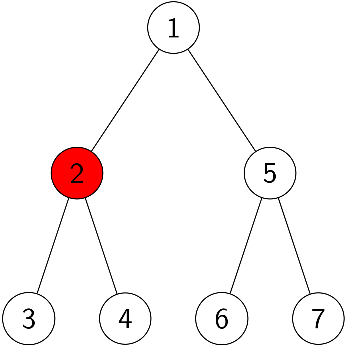

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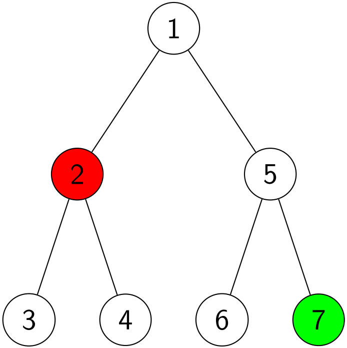

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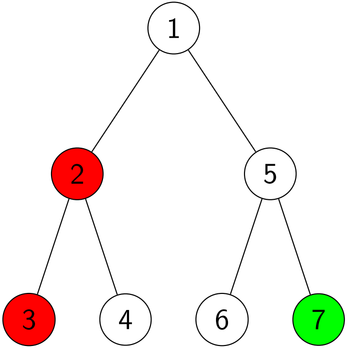
Provenance formulae and circuits on trees



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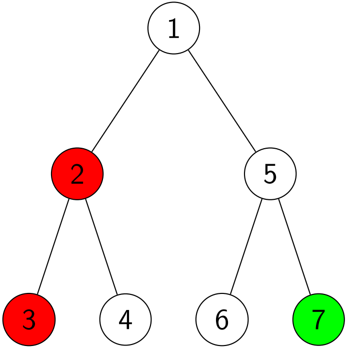

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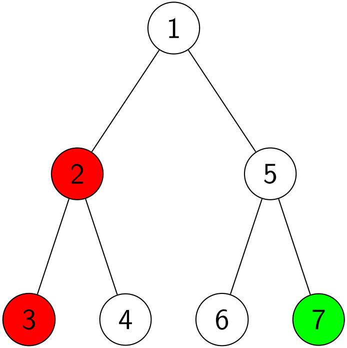

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- Provenance circuit of A on T [Deutch et al., 2014]
  - Boolean circuit C
  - with input gates g_2, g_3, g_7

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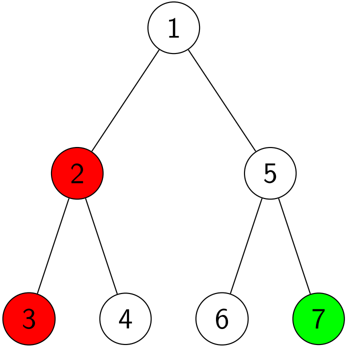
Example



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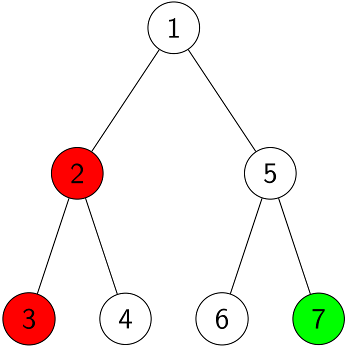

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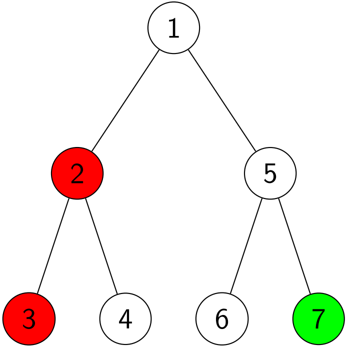

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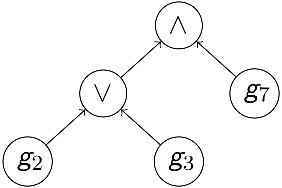

Our main result on trees

Theorem

For any bottom-up (nondet) tree automaton A and input tree T, we can build a provenance circuit of A on T in linear time in A and T.

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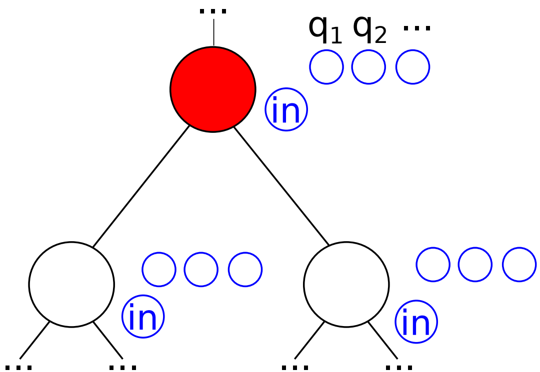

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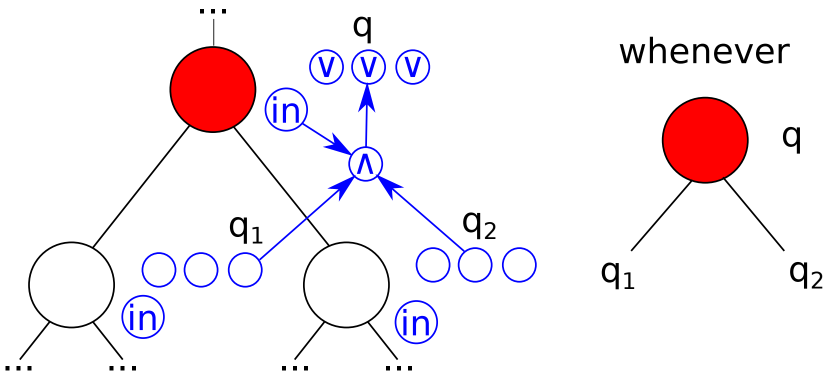

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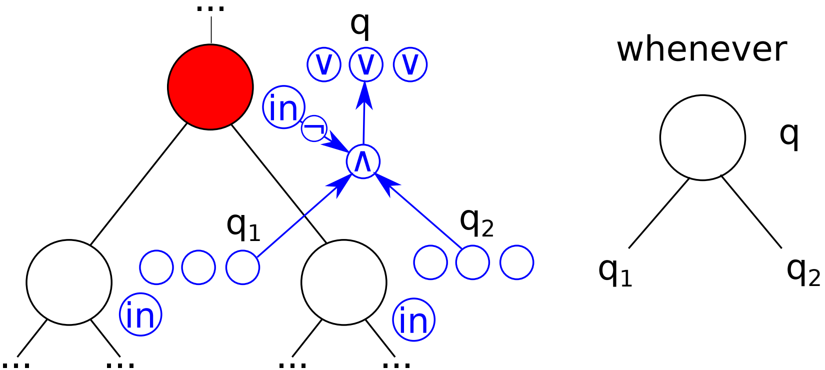

Treelike instances

- Treelike instance I
- Tree encoding: tree E on fixed alphabet, represents I
- MSO query on I translates to

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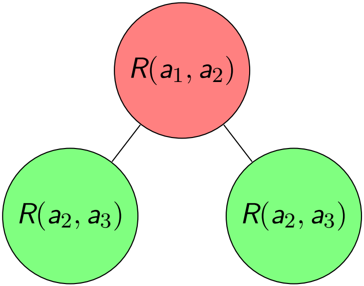

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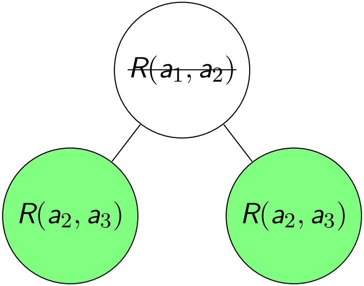

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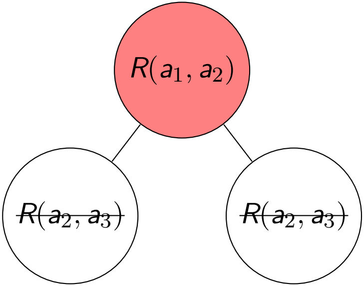

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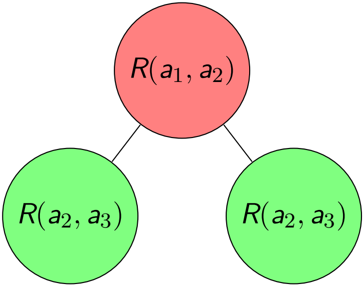

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Corollary

Probabilistic query evaluation of MSO queries on treelike instances is in linear time up to arithmetic operations.

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  $\rightarrow$ The provenance for $\mathbb{N}[X]$ can be specialized to any $K[X]$
- Captures many useful semirings:
  - counting the number of matches of a query
  - computing the security level of a query result
  - computing the cost of a query result

$\mathbb{N}[X]$-provenance example

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How is $\mathbb{N}[X]$ more expressive than $\mathrm{PosBool}[X]$?

→ Coefficients: counting multiple matches

→ Exponents: using facts multiple times

Capturing $\mathbb{N}[X]$-provenance

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$\rightarrow$ What fails for MSO and Datalog?

- Unbounded maximal multiplicity of fact uses

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More expressive formalism: Block-Independent Disjoint instances:

name	**city**	**iso**	p
pods	san francisco	us	0.8
pods	los angeles	us	0.2
icalp	rome	it	0.1
icalp	florence	it	0.9

pc-tables

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date	teacher	room	
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11	John	C47	$\neg x_2 \land \neg x_1$
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x_1 John gets sick

$\rightarrow$ Probability 0.1

x_2 Room C017 is available

$\rightarrow$ Probability 0.2

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Probabilistic query evaluation of MSO queries on treelike pc-tables is in linear time up to arithmetic operations.

"Tree-like" refers to the underlying instance, adding facts to represent variable occurrences and co-occurrences

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- Class $\mathcal{I}$ of unbounded-treewidth instances, query q in class $\mathcal{Q}$.
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  → Otherwise instances of treewidth k in $\mathcal{I}$ could be very large... see [Makowsky and Marino, 2003]

Our lower bound result

Theorem

There is a first-order query q such that for any unbounded-tw, tw-constructible, arity-2 instance family $\mathcal{I}$, probabilistic query eval for q on $\mathcal{I}$ is #P-hard under RP reductions.

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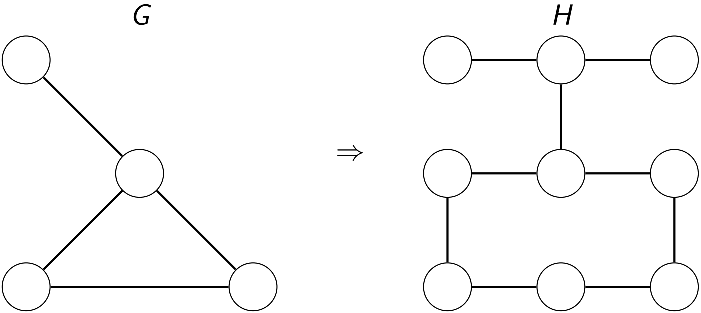

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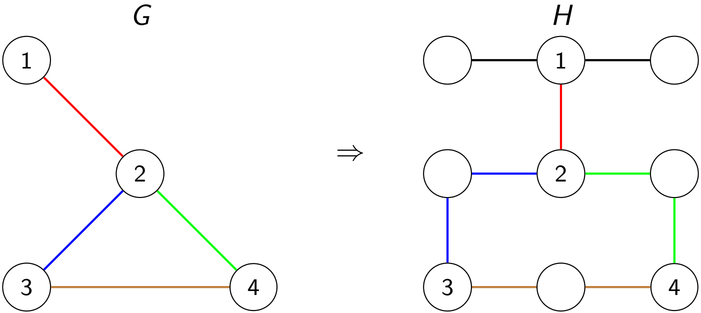

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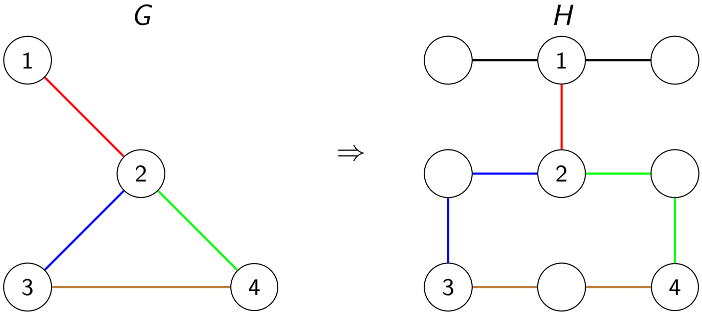

- Map vertices to vertices
- Map edges to vertex-disjoint paths

Topological minor extraction results

Theorem ([Robertson and Seymour, 1986])

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More recently:

Theorem ([Chekuri and Chuzhoy, 2014])

There is a certain constant $c \in \mathbb{N}$ such that
for any planar graph G of degree ≤ 3,
for any graph H of treewidth $\geq |G|^c$,
G is a topological minor of H and
we can embed G in H (with high probability) in PTIME in $|H|$.

Intuition for our result: reduction

- Choose a problem from which to reduce:
  - Must be #P-hard on planar degree-3 graphs
  - Must be encodable to an FO query q (more later)

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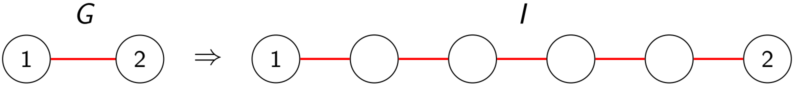
- Construct a probability valuation π of I such that:
  - Unneccessary edges of I are removed
  - Probability eval for q gives the answer to the hard problem

Technical issue



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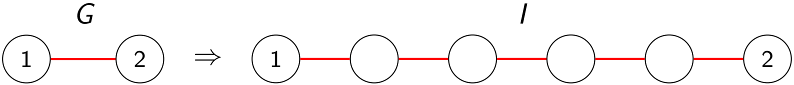

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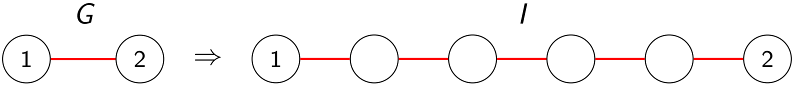

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→ For non-probabilistic evaluation, using FO does not work [Frick and Grohe, 2001]

→ Lower bounds for non-probabilistic evaluation are for MSO [Ganian et al., 2014]

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→ This UCQ with inequalities is hard in a weaker sense
(no polynomial-size OBDD representations of provenance)

→ We don't know whether it's #P-hard (because of subdivisions)

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Lower. PQE for FO on any tw-constructible, arity-2, unbounded-tw instance family is #P-hard under RP reductions

→ Bounded treewidth is the right notion for tractability of PQE?

Future work (upper bound)

Two promising directions:

- Restricting both instances and queries
    - Hard query on unbounded-treewidth instances may be easy!
    - Query-specific tree decomposition or instance simplification?
    - Tractability criterion based on the instance and query?
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  - Tractability criterion based on the instance and query?
  - Understand the connection to the query-based dichotomy?

- Combined complexity: tractability in the query and data
  - Cost in the MSO query is nonelementary in general
  - Lower for some query languages? (... on some instances?)
  - Monadic Datalog approaches? [Gottlob et al., 2010]

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Thanks for your attention!

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Encoding treelike instances [Chaudhuri and Vardi, 1992]

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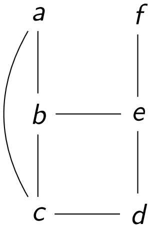

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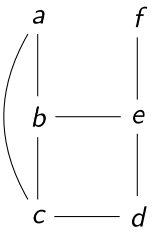

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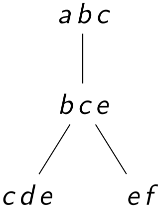

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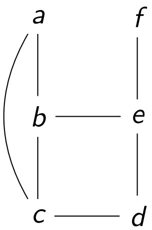

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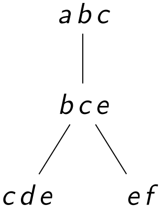

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