Open-World Query Answering Under Number Restrictions

Antoine Amarilli[1,2]

[1]Télécom ParisTech, Paris, France

[2]University of Oxford, Oxford, United Kingdom

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$\Rightarrow$ Equivalently: is there a (finite) model of $I \wedge \Theta \wedge \neg q$?

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  - Inclusion Dependencies ID:
    - ⇒ ϕ is an atom, no repeated variables.
  - Unary Inclusion Dependencies UID:
    - ⇒ Only one exported variable (occurring in ϕ and A).
    - ⇒ Example: $\forall e\, b,\ \mathrm{Boss}(e, b) \Rightarrow \exists b'\ \mathrm{Boss}(b, b')$.
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- Equality-Generating Dependencies EGD: A is an equality.
  - Functional Dependencies FD:
    - ⇒ $\forall \mathbf{x}\mathbf{y}\ \left(S(\mathbf{x}) \wedge S(\mathbf{y}) \wedge \bigwedge_{l \in L} x_l = y_l\right) \Rightarrow x_r = y_r$.
  - Unary Functional Dependencies: $|L| = 1$.
    - ⇒ Example: $\forall e\, e'\, b\, b',\ \mathrm{Boss}(e, b), \mathrm{Boss}(e', b'), e = e' \Rightarrow b = b'$.
    - ⇒ Written $\mathrm{Boss}^1 \rightarrow \mathrm{Boss}^2$.
  - Key Dependencies: $\bigwedge_{r \in \mathrm{Pos}(R)} R^K \rightarrow R^r$ for some $K \subseteq \mathrm{Pos}(R)$.
  - Unary Key Dependencies: $|K| = 1$.

Logics

- Guarded Fragment GF:
  - ⇒ Contains regular atoms and equality atoms.
  - ⇒ Closed under Boolean connectives $\wedge$, $\vee$, $\neg$, etc.
  - ⇒ Quantification: given an atom $A(\mathbf{x}, \mathbf{y})$ and formula $\phi(\mathbf{x}, \mathbf{y})$ with free variables exactly as indicated:
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- Two-Variable Guarded Fragment with Counting GC^2:
  - ⇒ Quantifiers $\exists^{\leq c} y,\ A(x, y)$ and $\exists^{\geq c} y,\ A(x, y)$ with A a binary atom and $c \in \mathbb{N}$.
  - ⇒ Example: $\forall e \exists^{\leq 1} b,\ \mathrm{Boss}(e, b)$.

General Results

- Negative results:
  - $QA_\bullet(FO, CQ^-)$ is undecidable [Trakhtenbrot, 1963].
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$\Rightarrow$ Can we have both high-arity constraints and expressive low-arity constraints, including equality constraints?

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Theorem

$\text{QA}_{\bullet}(\text{UKD} \cup \text{GC}^2 \cup \text{FR1}^{\text{a}}, \text{CQ})$ *is decidable.*

Proof Idea

- Encode constraints from $UKD \cup GC^2 \cup FR1^a$ to GC^2.
- Show that QA under the original constraints is equivalent to QA for the encoded constraints (and decide it as GC^2 QA):
  - ⇒ The reification of counterexample models should be counterexample models for the encoding (easy).
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- Well-formedness constraints $\mathsf{wf}(\sigma)$ of GC^2 for the encoding:
  - ⇒ Elements are regular elements or R-facts for some $R \in \sigma_{>2}$.
  - ⇒ The R_i's connect regular elements and R-fact elements.
  - ⇒ Every fact element for R has exactly one of each R_i.
  - ⇒ The $R \in \sigma_{\leq 2}$ connect regular elements.

Encoding

- Encoding a key $\phi \in$ UKD to $\mathcal{R}(\phi)$:
  - $\Rightarrow$ "R^i is a key" encoded to $\forall x \exists^{\leq 1} y,\ R_i(y, x)$.
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- Encoding a high-arity constraint $\delta \in \mathrm{FR1}^{a}$ to $\mathcal{R}(\delta)$:
  - ⇒ Apply reification to the body and modify the head if $\in \sigma_{>2}$.
  - ⇒ Example:
    - $\delta : \forall xyz,\ S(y, x) \wedge R(x, x, z) \Rightarrow \exists ww',\ R(x, w, w')$
    - ⇒ $\mathcal{R}(\delta) : \forall x \big((\exists y,\ S(y, x)) \wedge (\exists f,\ R_1(f, x) \wedge R_2(f, x) \wedge (\exists z,\ R_3(f, z)))$
      $\Rightarrow \exists f, R_1(f, x)\big)$.
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- Encode the instance I to $\mathcal{R}(I)$ straightforwardly.
- Encode the query $q \in$ CQ to $\mathcal{R}(q)$ straightforwardly.
- Leave the constraints $\Theta \subseteq$ GC^2 unchanged.

Concluding the Proof

- Take an extension J of I satisfying Δ, Θ, Φ and violating q:

$\Rightarrow$ $\mathcal{R}(J)$ is an extension of $\mathcal{R}(I)$ satisfying $\mathcal{R}(\Delta)$, Θ, $\mathcal{R}(\Phi)$ and wf(σ) and violating $\mathcal{R}(q)$.

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- Conversely, take an extension of $\mathcal{R}(I)$ satisfying $\mathcal{R}(\Delta)$, Θ, $\mathcal{R}(\Phi)$ and $\mathrm{wf}(\sigma)$ and violating $\mathcal{R}(q)$.

$\Rightarrow$ Need to argue that, w.l.o.g., there are no duplicate facts (f and f' representing $R(a, b, c)$).

$\Rightarrow$ Decode an extension of I satisfying Δ, Θ, Φ and violating q.

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$\Rightarrow$ Decode an extension of I satisfying Δ, Θ, Φ and violating q.

$\Rightarrow$ Decide $\mathrm{QA}_\bullet(\mathrm{UKD} \cup \mathrm{GC}^2 \cup \mathrm{FR1}^{\mathrm{a}}, \mathrm{CQ})$ from $\mathrm{QA}_\bullet(\mathrm{GC}^2, \mathrm{CQ})$.

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The Chase and Separability

- Universal model: extension of I satisfying Θ and violating every q unless $I, \Theta \models_{\mathrm{unr}} q$.
- The chase I^Θ: infinite universal model for TGD and UCQ:
  - $\Rightarrow$ Whenever a TGD is violated, create the missing head fact.
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- $\Phi \cup \Delta \subseteq$ EGD ∪ TGD is separable if $I \models \Phi$ implies $I^{\Delta} \models \Phi$.

⇒ $\mathrm{QA_{unr}}(\mathrm{EGD} \cup (\mathrm{TGD} \cap \mathrm{GF}), \mathrm{UCQ})$ is decidable in this case:

- Check if $I \models \Phi$
- Decide $\mathrm{QA_{unr}}(\mathrm{TGD} \cap \mathrm{GF}, \mathrm{UCQ})$ problem ignoring EGDs.

⇒ $\mathrm{QA_{unr}}(\mathrm{FD} \cup \mathrm{FR1^a}, \mathrm{UCQ})$ is decidable (always separable).

Result and Intuition

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Unicity. There are no two facts $R(\mathbf{a})$ and $R(\mathbf{b})$ with $a_i = b_i$ for $R \in \sigma_{>2}$ unless both are in the instance I.

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  - $\Rightarrow$ Any FD violation for $\sigma_{>2}$ must occur in I.
  - $\Rightarrow$ FDs can be checked on I and ignored afterwards.
- Acyclicity. The Gaifman graph of $\mathcal{R}(M)$ is acyclic except for I:
  - $\Rightarrow$ $FR1 \setminus FR1^a$ dependencies can only match on I.
  - $\Rightarrow$ Convert FR1 to $FR1^a$ (enumerate matches).

$\Rightarrow$ Reduce $QA_{unr}(FD \cup GC^2 \cup FR1, CQ)$ to $QA_{unr}(GC^2 \cup FR1^a, CQ)$.

Unraveling the Counterexample Model

Unravelling M to a suitable M' (with mapping π'):

- Add dummy binary facts covering and connecting all elements.
- Decompose the facts in bags:
  - one bag per fact of $\sigma_{>2}$,
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  - one bag per guarded pair $\{a, b\}$ with all unary and binary facts.
- Build M' as a tree of bags by the following inductive process:

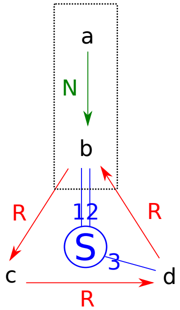
⇒ The root bag of M' is I.

⇒ The children of $t \in M'$ are, for every $a \in \mathrm{dom}(t)$:

- For every $\sigma_{\leq 2}$-bag t' of M containing $\pi'(a)$:
  An isomorphic copy of t' in M', with a and a fresh element.
- For every $R^i \in \mathrm{Pos}(\sigma_{>2})$ such that $\pi'(a)$ occurs at R^i in M,
  if a does not occur at R^i in M':
  A $\sigma_{>2}$-bag $\{R(\mathbf{b})\}$ with $\mathbf{b}$ fresh except $b_i = a$.

⇒ Do not consider in a bag the previous element used to reach it.

Example



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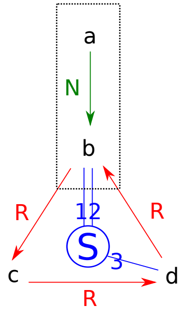

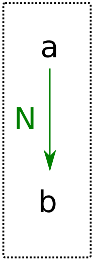

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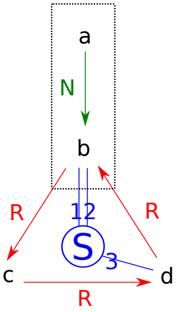

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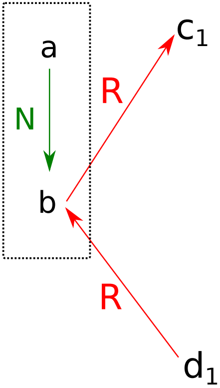

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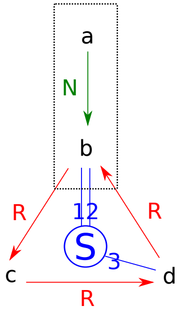

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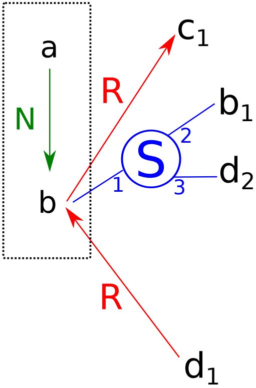

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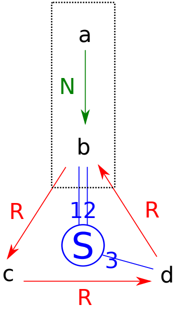

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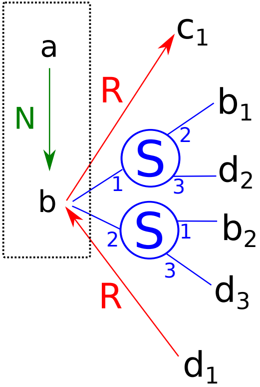

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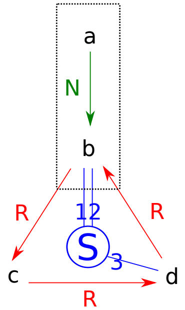

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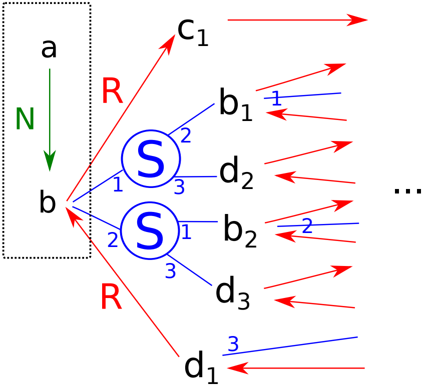

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  - $\Rightarrow$ Ensures that GF^2 constraints are preserved (guarded bisimilar).
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- Isomorphism between 1-neighborhoods for $\sigma_{\leq 2}$ following π'.
  - ⇒ Ensures that GF^2 constraints are preserved (guarded bisimilar).
  - ⇒ Ensures that number restrictions are preserved.
- The mapping π' is surjective for guarded pairs.
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Properties of the Construction

- Preserves the base instance *I*.
- Maps back to the original model by the homomorphism π'.
  - ⇒ Ensures that the query is still false.
- Isomorphism between 1-neighborhoods for $\sigma_{\leq 2}$ following π'.
  - ⇒ Ensures that GF^2 constraints are preserved (guarded bisimilar).
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- They do so at most once (except in the instance):
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- The model is a tree of bags.
  - ⇒ Ensures Acyclicity (and bounded treewidth).

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Finite Controllability

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  - Consider $\Theta : R^2 \to R^1, R^2 \subseteq R^1$, and $I = \{A(a), R(a,b)\}$.
  - Universal infinite chase model $A(a), R(a,b), R(b,c), \ldots$.
  - Finite model has to loop back, on a because of the FD: $A(a), R(a,b), R(b,c), \ldots, R(y,z), R(z,a)$.

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  $\Rightarrow$ For $q : R(x, y) \wedge A(y)$, we have $I, \Theta \models_{\text{fin}} q$ but $I, \Theta \not\models_{\text{unr}} q$.
- Separability not useful for finite QA (the chase is infinite):
  - Separability not closed under finite implication [Rosati, 2006].

  $\Rightarrow$ $\mathrm{QA}_{\mathrm{fin}}(\mathrm{KD} \cup \mathrm{ID}, \mathrm{CQ})$ undecidable even assuming separability.

Decidable Finite QA

- $\mathsf{QA}_{\mathsf{fin}}(\mathsf{GC}^2, \mathsf{CQ})$ not FC but decidable [Pratt-Hartmann, 2009].
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Decidable Finite QA

- $QA_{fin}(GC^2, CQ)$ not FC but decidable [Pratt-Hartmann, 2009].
  - ⇒ Only for arity-two.
- Enforce chase termination to get a finite universal model.
  - ⇒ Too restrictive.
- Restrict the language to enforce FC:
  - ⇒ $KD \cup ID$ under a foreign key condition is FC [Rosati, 2011].
  - ⇒ Also restrictive.

Result Statement

- We focus on unary IDs and (general) FDs, arbitrary arity.
- The implication problem for UIDs and FDs is decidable: PTIME finite closure construction [Cosmadakis et al., 1990].
- We show that FC holds up to finite closure:

Result Statement

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Theorem

For every $\Phi \cup \Delta \subseteq \mathsf{FD} \cup \mathsf{UID}$ *with finite closure* $\Phi^* \cup \Delta^*$, *for* $q \in \mathsf{UCQ}$ *and* I *an instance s.t.* $I \models \Phi^*$, *we have* $I, \Phi \cup \Delta \models_{\mathrm{fin}} q$ *iff* $I, \Delta^* \models_{\mathrm{unr}} q$.

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$\Rightarrow$ $\mathsf{QA}_{\mathsf{unr}}(\mathsf{FD} \cup \mathsf{UID}, \mathsf{UCQ})$ is in NP [Johnson and Klug, 1984] so $\mathsf{QA}_{\mathsf{fin}}(\mathsf{FD} \cup \mathsf{UID}, \mathsf{UCQ})$ is in NP.

Finite Chase

- The chase is a universal model but it is infinite.
- The finite chase [Rosati, 2011]: for all k, there is a finite universal model for queries of size $\leq k$.
- Reuse similar elements as nulls when chasing.

$N(a, b)$
$R(b, c)$
$R(c, d)$
$R(d, e)$
$R(e, f)$
$R(f, g)$
$R(g, h)$
$R(h, e)$

$R^2 \subseteq R^1$

Acyclic Queries

- Reuses must not make new queries true relative to the chase.
- We focus on Berge-acyclic constant-free queries of size $\leq k$.
  - The graph G of q has its atoms as vertices.
  - Two atoms are connected if they share one variable.
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Lemma

If an extension of I satisfying Δ has a homomorphism to the quotient of the chase by the k-neighborhood equivalence relation then it is universal for constant-free Berge-acyclic CQs of size $\leq k$.

Finite Chase and FDs

- The dangerous positions of R^i are the $R^j \in \mathrm{Pos}(R) \setminus \{R^i\}$ such that the FD $R^j \to R^i$ holds.
- At non-dangerous positions, reusing elements cannot violate unary FDs.
- At dangerous positions, we cannot reuse elements!

$N(a, b)$
$R(b, c)$
$R(c, d)$
$R(d, \mathbf{e})$
$R(e, f)$
$R(f, g)$
$R(g, h)$
$R(h, \mathbf{e})$

$R^2 \subseteq R^1$
$R^2 \to R^1$

Finite Chase and FDs and Closure

- Finite closure [Cosmadakis et al., 1990]:
  - Whenever $R^i \subseteq S^j$ holds then $\langle R^i \rangle \leq \langle S^j \rangle$.
  - Whenever $S^i \rightarrow S^j$ holds then $\langle S^j \rangle \leq \langle S^i \rangle$.
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  - Inequality chains imply the reverse inequalities in the finite.
  - Add the reverse dependencies for such invertible cycles.

$\Rightarrow$ When we create a chain with no possiblity to reuse, the reverse dependencies must hold.

$\Rightarrow$ Intuitively: glue both chains together.

$N(a, b)$
$R(b, c)$
$R(c, d)$ $R(z, b)$
$R(d, e)$ $R(y, z)$
$R(e, f)$ $R(x, y)$
$R(f, g)$ $R(w, x)$
$R(g, w)$

$R^2 \subseteq R^1$
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$\mathbf{R^2 \subseteq R^1}$
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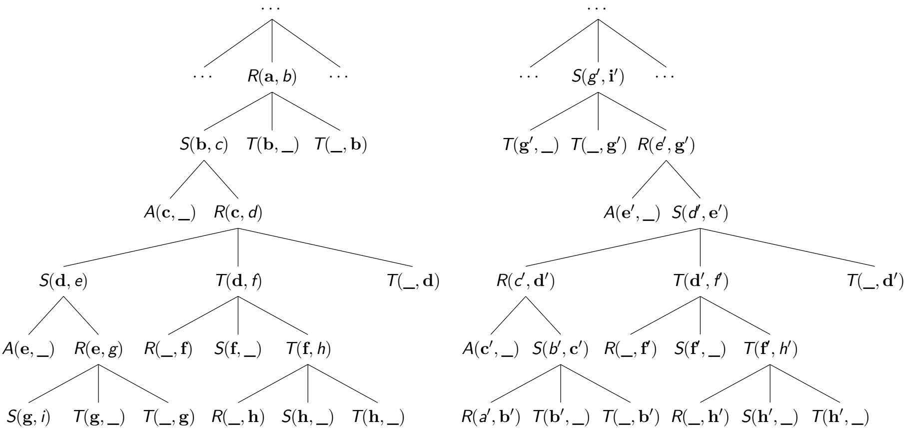
Locality Result



Lemma

After chasing by k consecutive reversible UIDs, elements at positions connected by UIDs have the same k-neighborhood.

General Scheme

- Start with the instance I.
- Chase by the IDs.
- Reuse elements at non-dangerous positions.
- Connect together elements at dangerous positions.
  - ⇒ Use the previous lemma to justify they can be paired.

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- Start with the instance I.
- Chase by the IDs.
- Reuse elements at non-dangerous positions.
- Connect together elements at dangerous positions.
  - ⇒ Use the previous lemma to justify they can be paired.
- Connect elements within an invertible cycle:
  - ⇒ We say that $(R^i \subseteq S^j) \rightarrowtail (S^p \subseteq T^q)$ if $S^p \rightarrow S^j$.
  - ⇒ An invertible path is a cycle of $\rightarrowtail$.
  - ⇒ Chase by the ID of SCCs of $\rightarrowtail$ in topological order.

Higher-Arity FDs

- Non-dangerous positions defined w.r.t. unary FDs.
- The non-unary FDs are not considered in the finite closure.
- Reusing the same patterns may violate higher-arity FDs:
  - ⇒ Must make many patterns out of limited reusable elements.
  - ⇒ Ex: $R(x_1, a_1, b_1)$, $R(x_2, a_2, b_2)$, $R(x_3, a_1, b_2)$, $R(x_4, a_2, b_1)$.
  - ⇒ If $R^2 \rightarrow R^3$ then the non-dangerous positions have a unary key so higher-arity FDs are subsumed by UFDs.

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⇒ We need to justify that we can make many patterns out of a limited number of elements to reuse.

- ⇒ Formally: from N elements, for any K, make NK patterns (unless there is a unary key preventing this).

Dense Models

The possibility to find such patterns is a consequence of:

Lemma

For any FDs Φ over R, there exists $D \leq |R|$ such that either R has a unary key, or there exists a finite model of Φ with $O(N)$ elements and $O(N^{D/(D-1)})$ facts.

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For any FDs Φ over R, there exists $D \leq |R|$ such that either R has a unary key, or there exists a finite model of Φ with $O(N)$ elements and $O(N^{D/(D-1)})$ facts.

- First, collapse any UFD cycles of R.
- Then, consider the UFD "roots" T of R (there are ≥ 2) such that $\forall t \in T, \nexists s \in \mathrm{Pos}(R), s \rightarrow t$, and reduce to the case:
  - the attributes of R are the non-empty parts of T.
  - the roots that determine $X \in \mathrm{Pos}(R)$ are exactly those of X.
  - the non-unary FDs are as pessimistic as possible.
- Finally, construct the desired model on this relation.

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- Build the product of the model with a finite acyclic group:
  - Let $L(M) = \{l_i^F \mid F \in M, 1 \leq i \leq |F|\}$.
  - Let G be a k-acyclic group generated by $L(M)$.
  - For $F = R(\mathbf{a}) \in M, g \in G$, create $R((a_1, gl_1^F), \ldots, (a_{|R|}, gl_{|R|}^F))$.
  - Ex: $M = \{R(a,a)\}, M' = \{R((a,e),(a,g)), R((a,g),(a,e))\}$.

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  - Ex: $M = \{R(a, a)\}, M' = \{R((a, e), (a, g)), R((a, g), (a, e))\}$.
- Properties:
  - ⇒ Can be adjusted to preserve the instance as-is.
  - ⇒ Preserves unary overlaps so preserves UIDs.
  - ⇒ Homomorphism back to M so no new queries are true.
  - ⇒ Cycles in M' of size $\leq k$ must take one edge back-and-forth.
  - ⇒ This may violate FDs!

Expanding Cycles With FDs

- Our models have a homomorphism h to $I^{\Theta}/{\equiv_k}$.
- Overlaps are between facts with the same h-image.
- Adjust the product $M \times G$ with $L(I^{\Theta}/{\equiv_k})$ not $L(M)$:
  - ⇒ If $F = R(a, b, c)$ and $F' = R(a, b, d)$ then $h(F) = h(F')$ and the FD $R^1 \to R^2$ cannot be violated.
  - ⇒ Any cycles in $M \times G$ are mapped by the homomorphism $(x, g) \mapsto (h(x), g)$ to cycles in the "regular" product $I^{\Theta}/{\equiv_k} \times G$.
  - ⇒ In other words:
    - M satisfies the right dependencies (including FDs),
    - $I^{\Theta}/{\equiv_k} \times G$ satisfies the right queries,
    - $M \times G$ satisfies both.
- More work required to preserve the instance.

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Summary

We have shown the decidability of:

- $\mathsf{QA}_\bullet(\mathsf{UKD} \cup \mathrm{GC}^2 \cup \mathsf{FR1}^{\mathsf{a}}, \mathsf{CQ})$
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Further work:

- Derive upper and lower complexity bounds.
- For unrestricted QA:
  - ⇒ Find a more homogeneous fragment than $GF^2 \cup FR1$.
  - ⇒ Must limit the interaction with FD and number restrictions.
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Thanks for your attention!

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