Open-World Finite Query Answering Under Number Restrictions

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Open-world query answering

- Evaluate query q over instance I, open-world assumption:
  - The instance I is correct but incomplete
  - Consider all possible completions J satisfying constraints Σ
  - Certain answers to query q among those completions

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  - TGDs, especially inclusion dependencies (ID)

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  - Number restrictions, especially functional dependencies (FD)

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  - Number restrictions, especially functional dependencies (FD)
- Finite vs unrestricted QA

Instance:	List of employees
Constraint 1:	Each employee reviews some employee (UID)
Constraint 2:	At most one reviewer per employee (FD)
Query:	Are all employees reviewed?

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Undecidability barrier

- Entailment of IDs and FDs is undecidable [Mitchell, 1983]
- Already for binary IDs and unary FDs:
  $R[A, B] \subseteq S[C, D]$, $R[A] \rightarrow R[B]$

$\Rightarrow$ QA (finite or not) is also undecidable [Calì et al., 2003]
(Remark: this proof requires constants in the query)

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$\Rightarrow$ We can't have everything

Idea 1: Separability

- The chase for IDs: universal model
- Intuition: apply all IDs with fresh elements
- FDs are separable from IDs if they do not impact the chase
- Sufficient conditions for separability, e.g., non-conflicting:
  - $\Rightarrow$ exported positions must not be a strict superset of a key
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$\Rightarrow$ The chase is infinite in general so it doesn't work in the finite

$\Rightarrow$ Finite QA undecidable for separable IDs/FDs [Rosati, 2006] (intuition: their finite consequences may not be separable)

Idea 2: Finite controllability

- Finite controllability means that finite and infinite QA coincide
- IDs are finitely controllable [Rosati, 2006]
  - ⇒ Construction: finite chase (chase with distant reuses)
- Generalizes to the guarded fragment [Barany et al., 2010]
  - ⇒ (Guarded means that $\forall/\exists$ must be covered by an atom)
  - ⇒ Intuition: query acyclification and cycle blowup
- Generalises to IDs/FDs with foreign keys condition [Rosati, 2006]

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⇒ FDs are not expressible in the guarded fragment.

⇒ IDs/FDs are not finitely controllable!

Idea 3: Arity-two

- Finite and unrestricted QA decidable in arity-two for the two-variable guarded fragment and counting constraints [Pratt-Hartmann, 2009]
  - Intuition: again, encode the acyclic part of the query
  - Satisfiability decidable by reduction to an inequation system
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⇒ Only for arity-two signatures

⇒ No clear way to generalize to higher arity

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Our setting

- So:
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Our setting

- So:
  - Finite QA
  - TGDs and EGDs with interaction (not FC)
  - High-arity signatures
  - ⇒ Can we have all three?
- ⇒ What if we restrict the language to UIDs and FDs?
  - No direct encoding to arity-two (unlike UIDs/UKDs...)
  - UIDs are important IDs in practice
  - UIDs match the DL intuition
  - UIDs are less expressive than BIDs
  - and...

Finite closure for UIDs and FDs

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- For finite: add cycle reversal:
    - Consider only unary FDs: $R[i] \to S[j]$
    - When $R[i] \subseteq S[j]$ we have $|R[i]| \leq |S[j]|$
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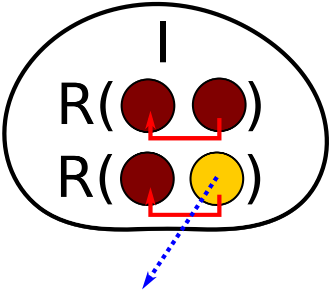
  $\Rightarrow$ In the finite, such cycles must be reversed

Finite closure example



- $R[2] \subseteq R[1]$
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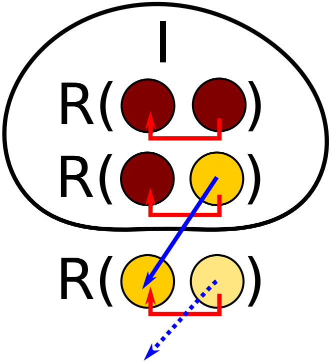

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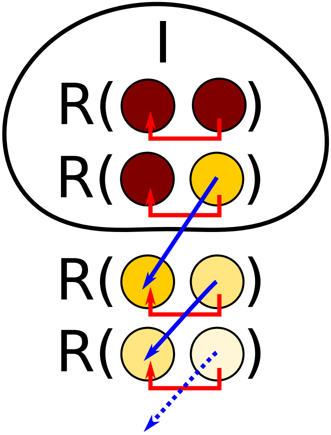

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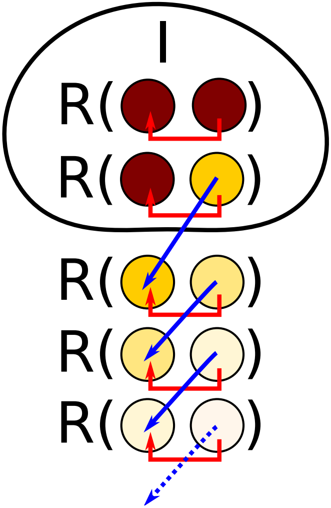

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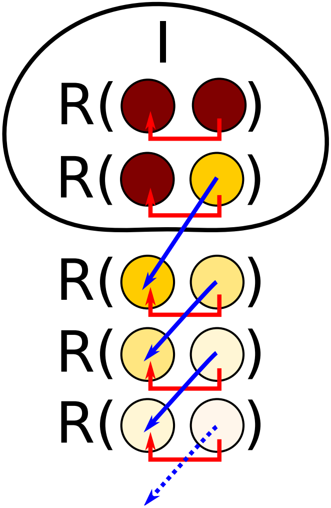

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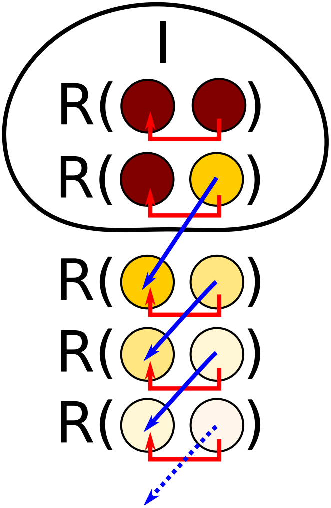

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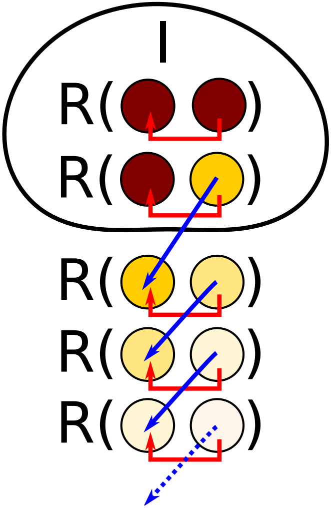

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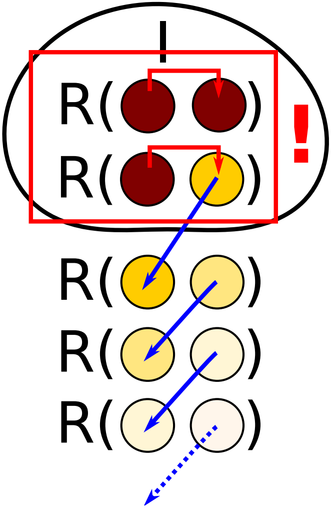

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$\Rightarrow$ No finite model!

Finite controllability up to closure

- In arity-two, UIDs/UFDs finitely controllable up to finite closure [Rosati, 2008, Ibáñez-García et al., 2014]

$\Rightarrow$ To perform finite QA on instance I, UIDs/UFDs Σ:

  - Compute Σ^* the finite closure of Σ
  - Check if I satisfies the UFDs of Σ^*
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$\Rightarrow$ Does this also hold with higher-arity relations and FDs?

The result

Theorem

UIDs and FDs, though not finitely controllable, are finitely controllable up to finite closure, on arbitrary arity signatures.

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UIDs and FDs, though not finitely controllable, are finitely controllable up to finite closure, on arbitrary arity signatures.

$\Rightarrow$ It suffices to show that for any k, I, and Σ^*, there is a finite completion of I by Σ^* which is universal for queries of size $\leq k$

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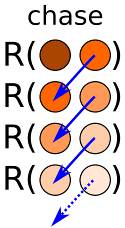
Quotienting the chase

$R[2] \subseteq R[1]$



- Consider k-neighborhood equivalence

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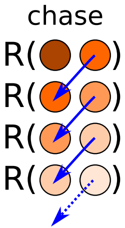

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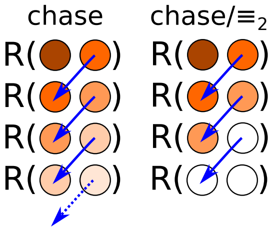

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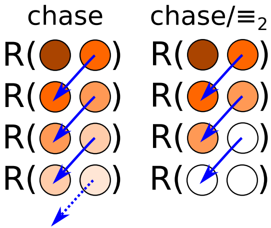

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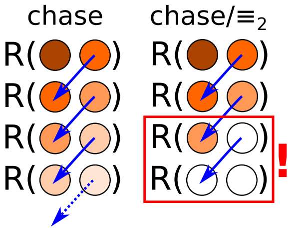

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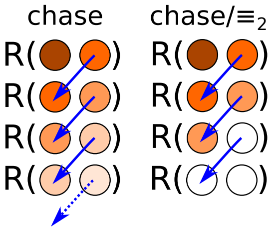

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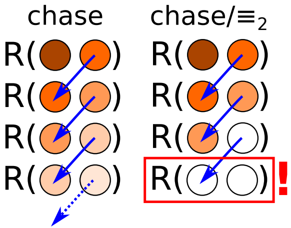

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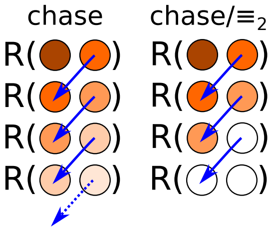

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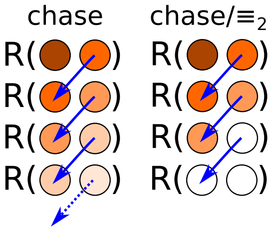

- Consider k-neighborhood equivalence
- Quotient the chase by this relation
- May violate FDs
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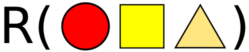
- Yet universal for $\leq k$ acyclic queries
- Keep a homomorphism to this quotient

Frugal chase steps

- Follow the chase



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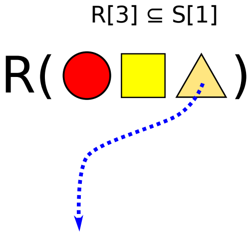

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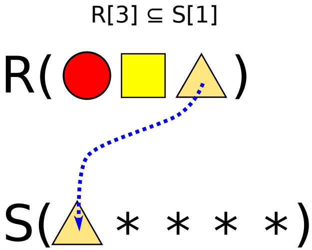

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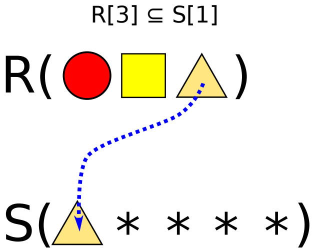

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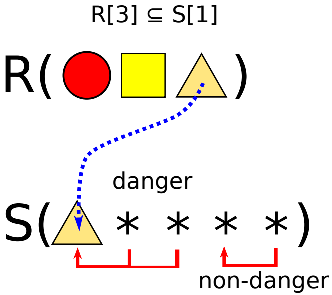

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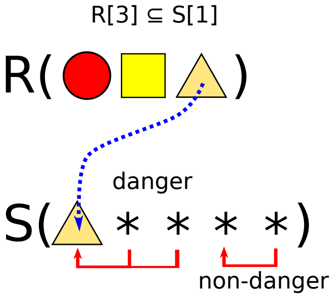

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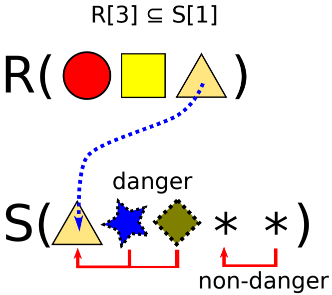

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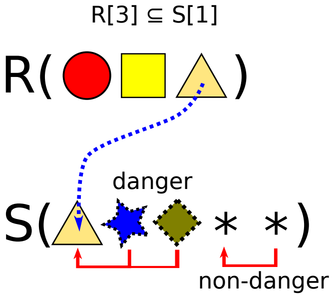

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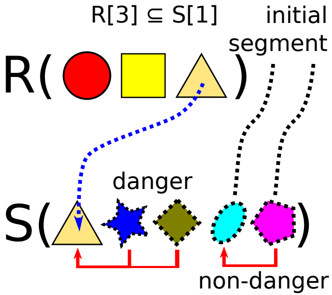

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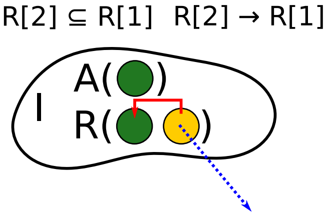

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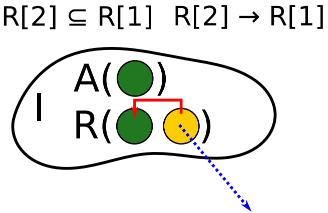

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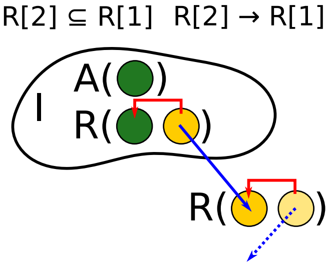

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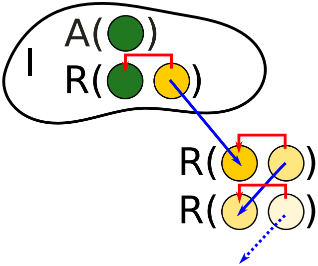

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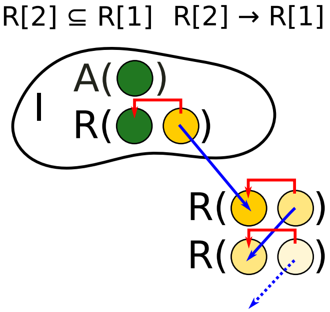

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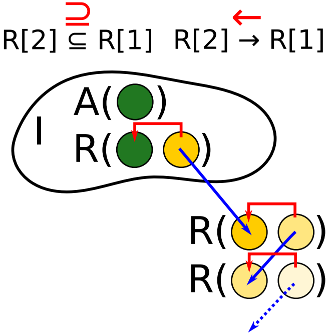

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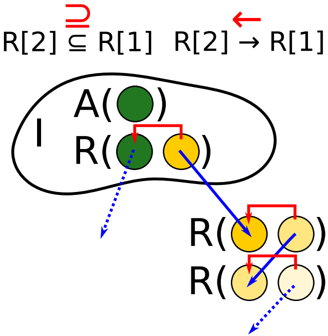

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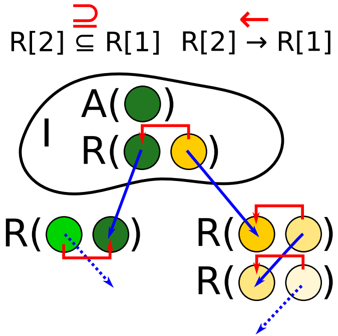

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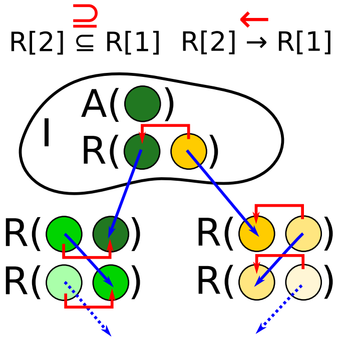

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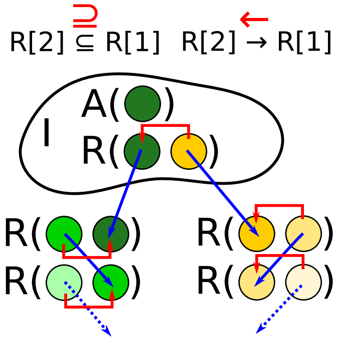

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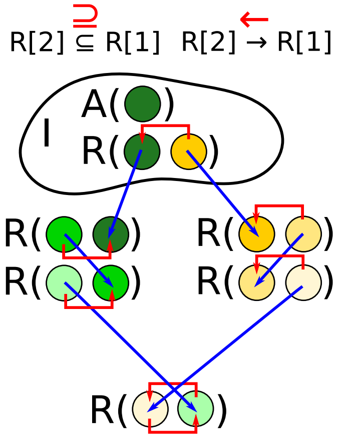

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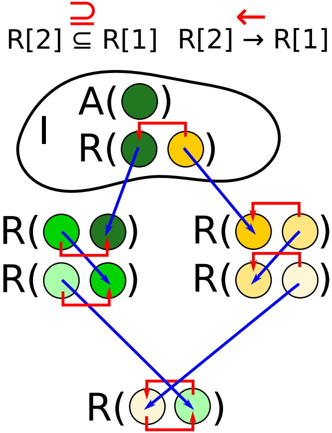

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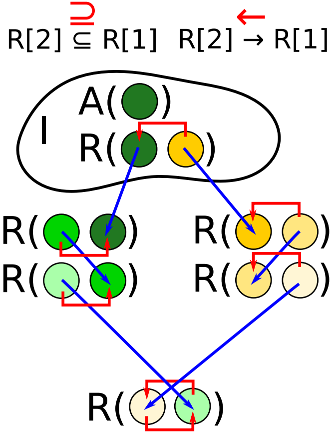

Blueprint

- Infinite functional paths...
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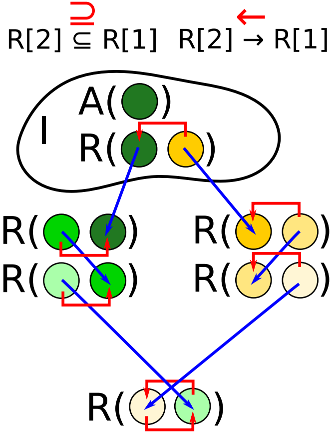
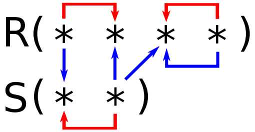
- Connect it back (match elements)
- More complex if many positions of many relations are involved...
- Uses cardinality along cycles...
- ... but also initial chasing to force "generic neighborhoods"



Dependency graph

- Build a DAG on the dependency cycles



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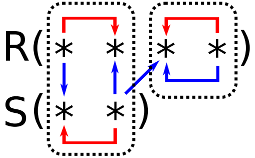

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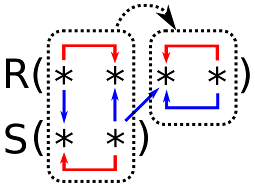

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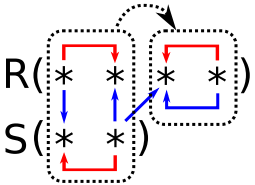

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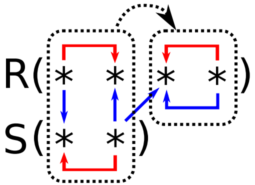

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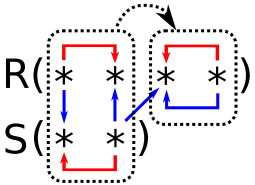

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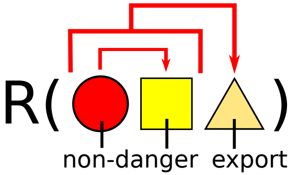
$\Rightarrow$ Finite extension that satisfies UIDs/UFDs with a homomorphism to the quotient

Higher-arity FDs

- Ignored so far
- May only be triggered at non-dangerous reuses
- Idea: if non-dangerous but dangerous for higher-arity FD then no unary key



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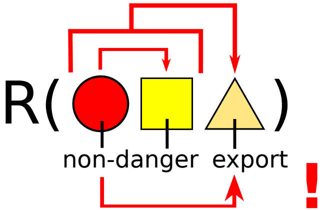

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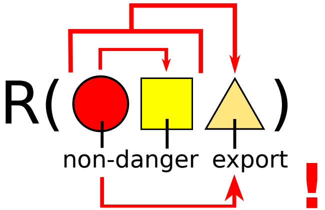
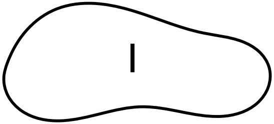


- Idea:
  - create many reuse candidates
  - combine them in different patterns

⇒ Lemma: if no UKD then $O(n^{>1})$ patterns for $O(n)$ elements

Higher-arity FDs



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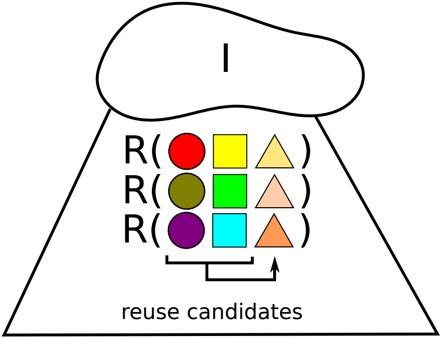

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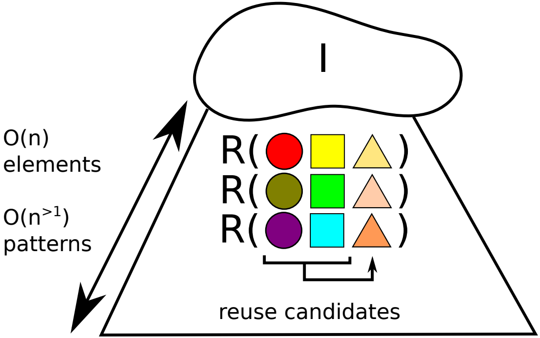

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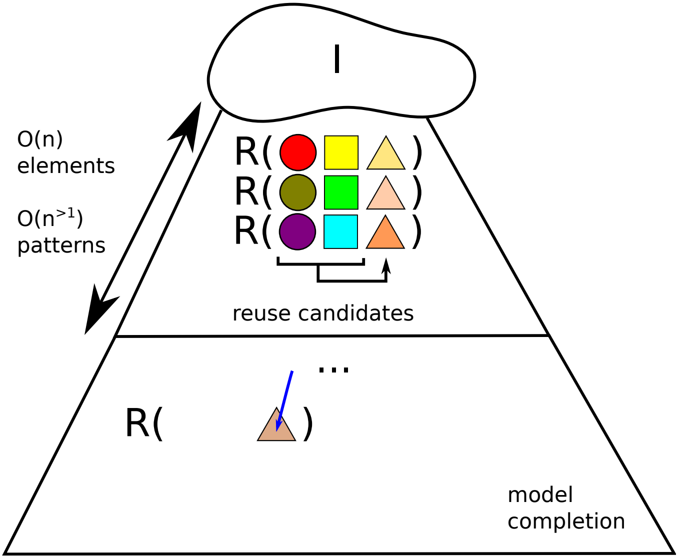

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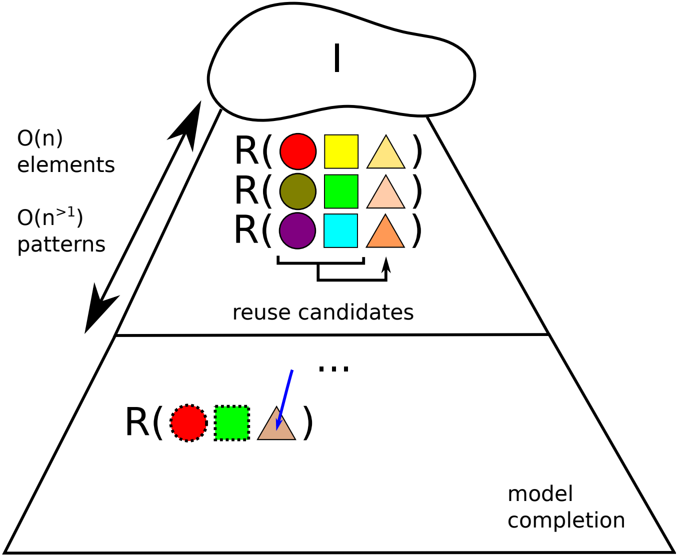

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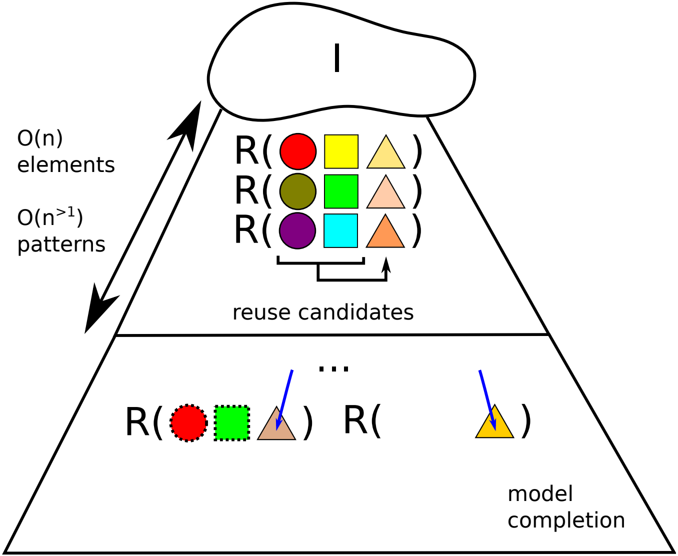

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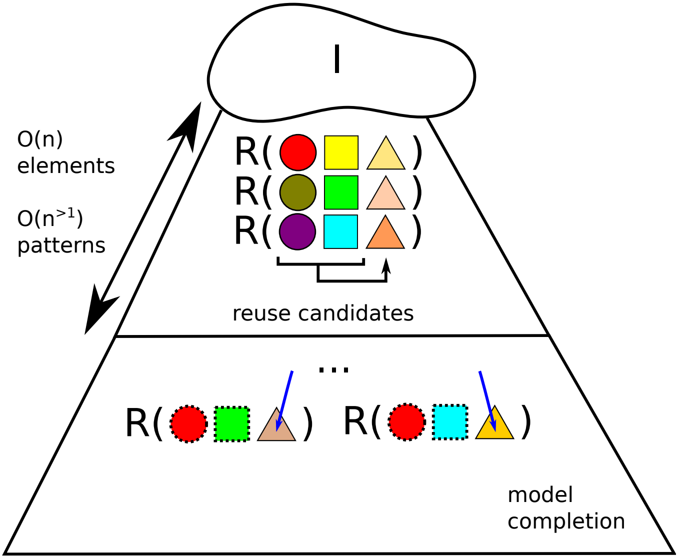

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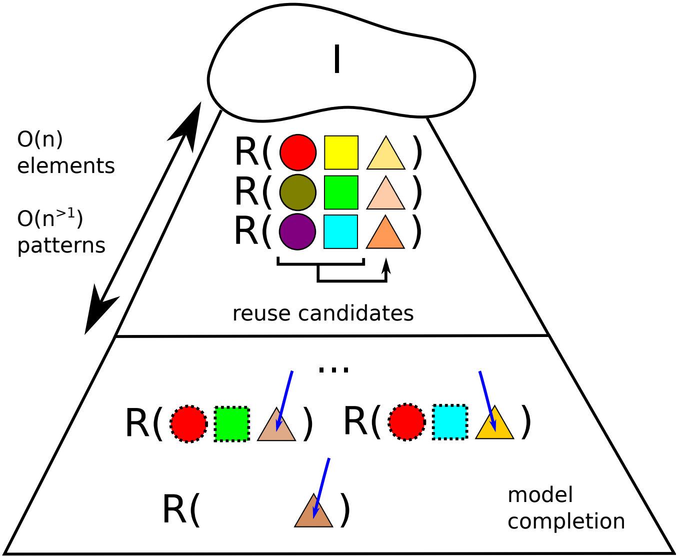

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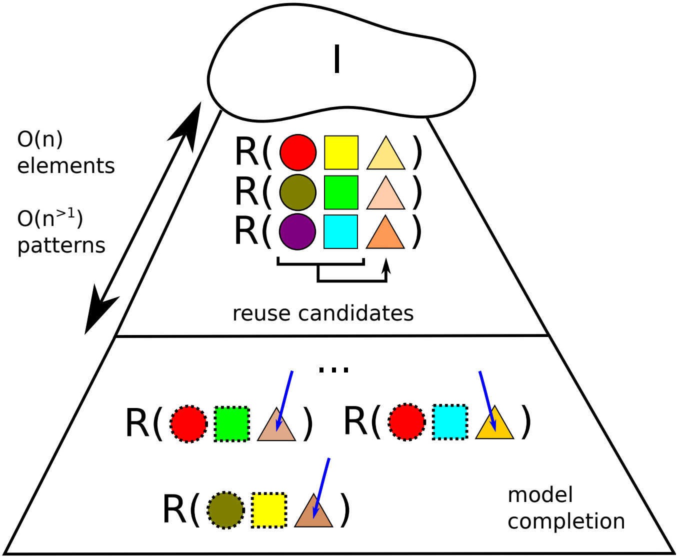

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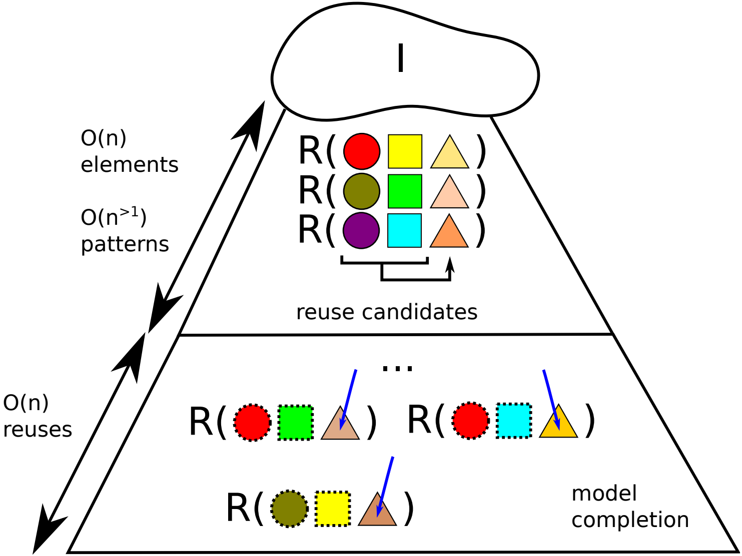

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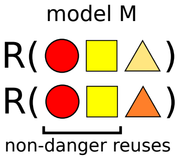

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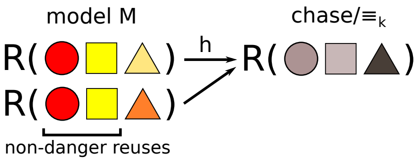

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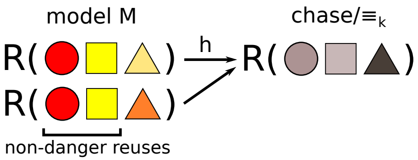


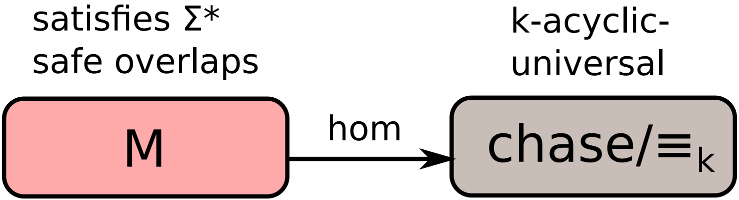
⇒ Blow up cycles, but not those cycles

Blowing up cycles via the product

k-acyclic-universal

$\boxed{\text{chase}/\equiv_k}$

Blowing up cycles via the product



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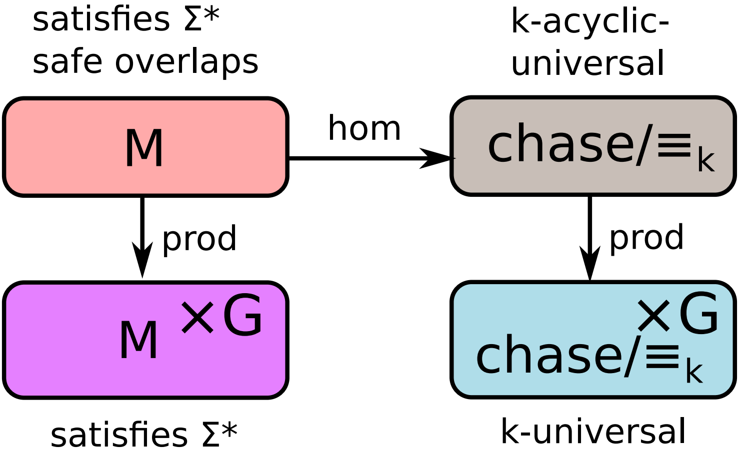

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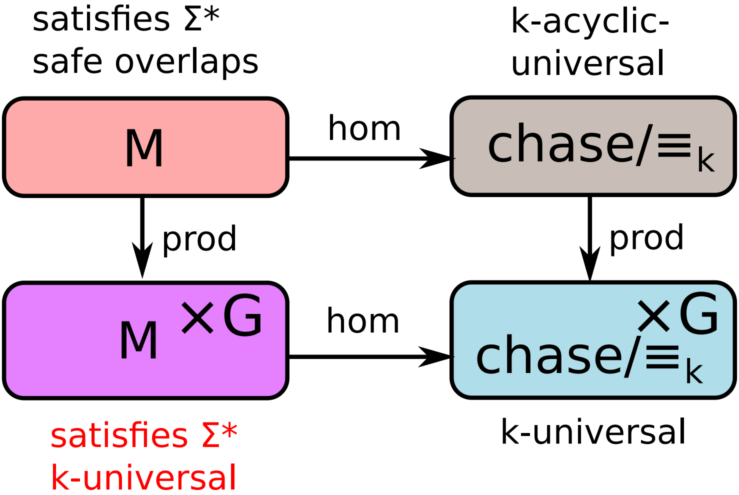

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Thanks for your attention!

References I

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