Query Evaluation on Probabilistic Data
A Story of Dichotomies

Antoine Amarilli

April 4th, 2022

Télécom Paris

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Relational databases manage data, represented here as a labeled graph

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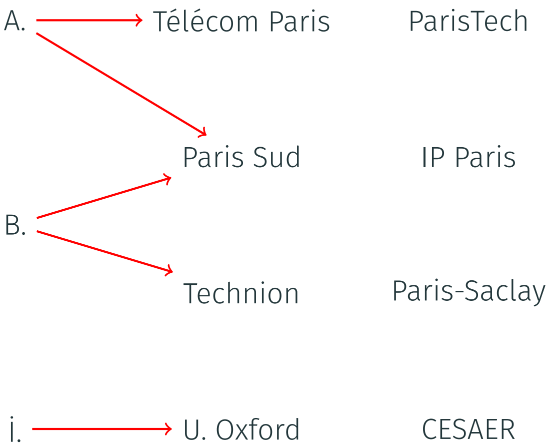

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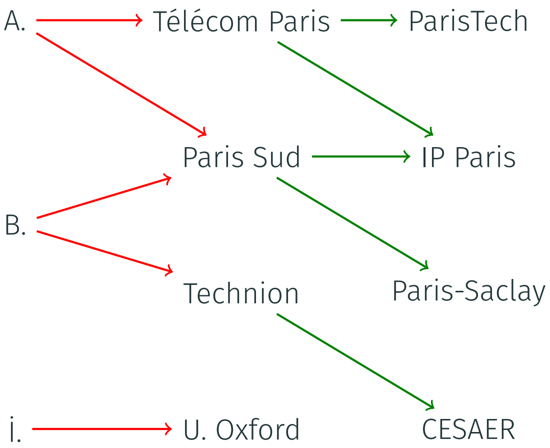

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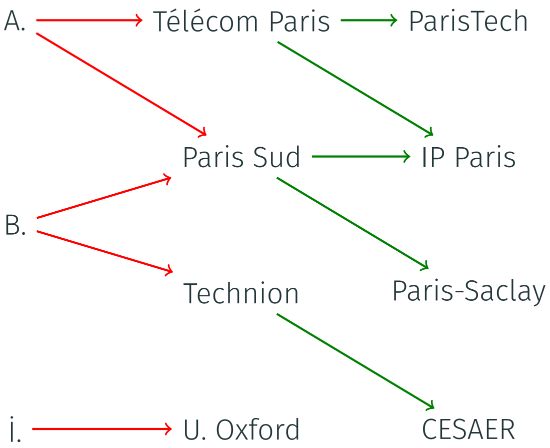
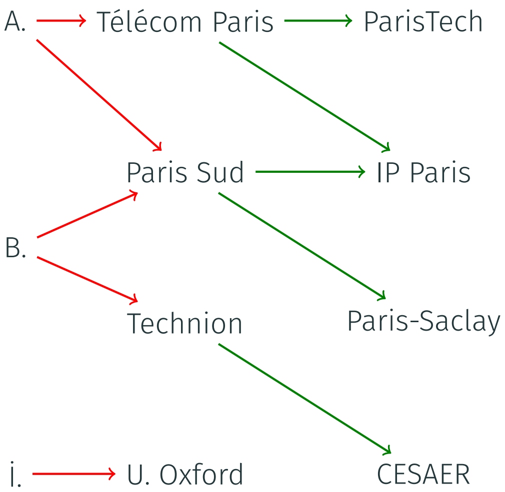


→ **Problem:** we are not **certain** about the true state of the data

Uncertain data model



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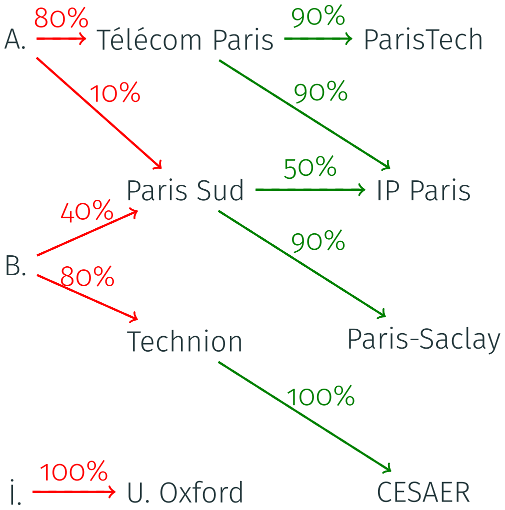

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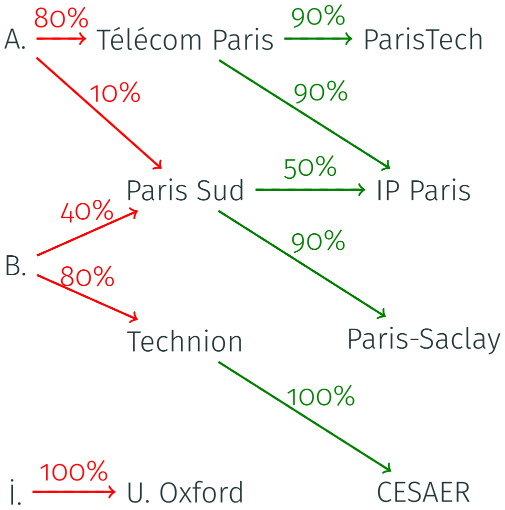

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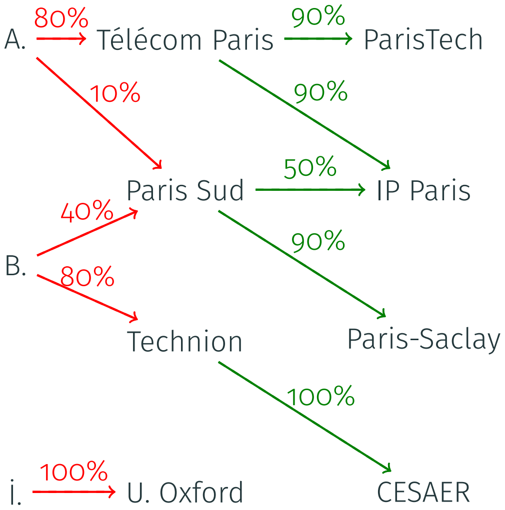

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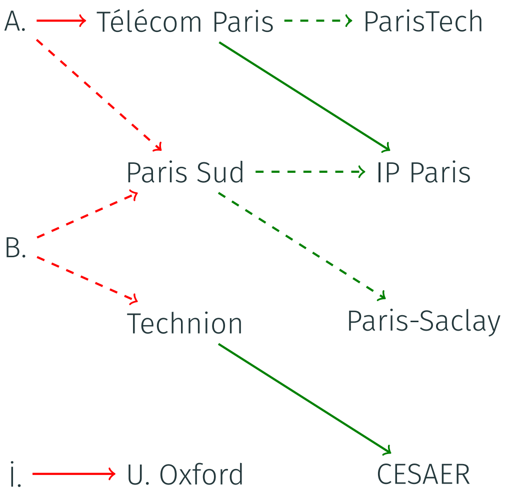

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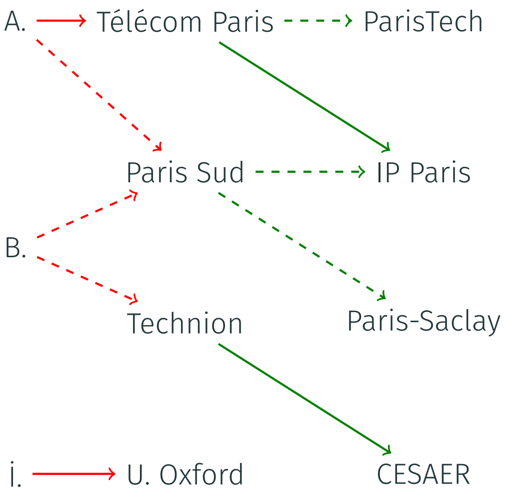

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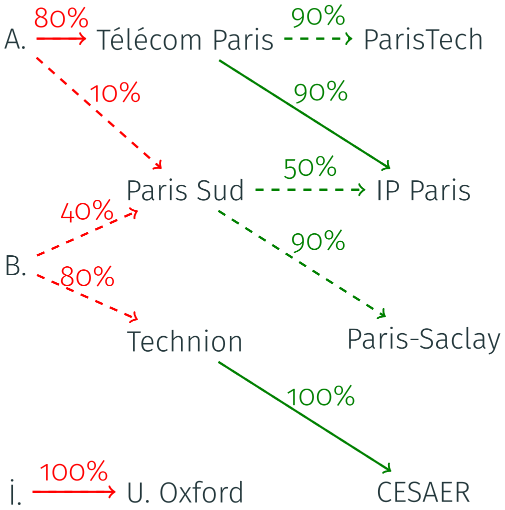

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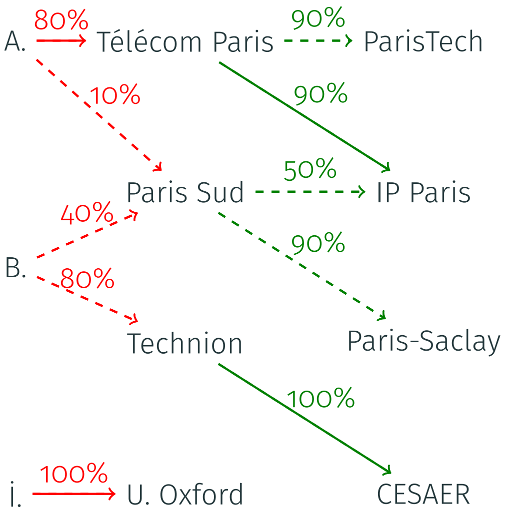

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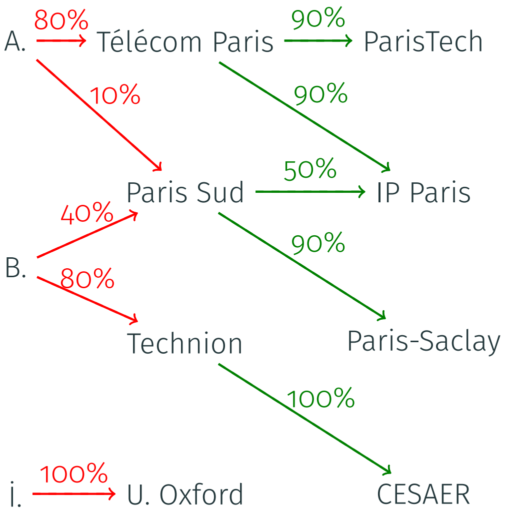

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$$\Pr(W) = \left(\prod_{F \in W} \Pr(F)\right) \times \left(\prod_{F \notin W} (1 - \Pr(F))\right)$$

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  - Formally: a **finite disjunction** of CQs

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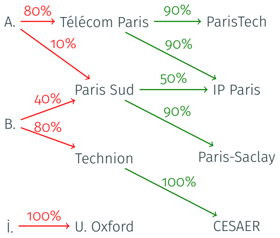

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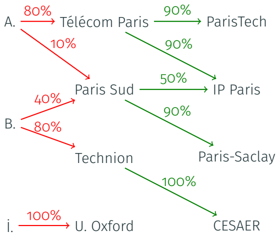

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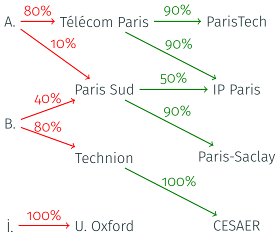

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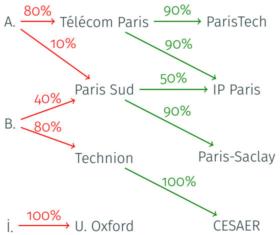

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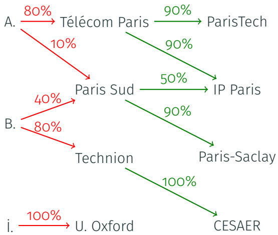

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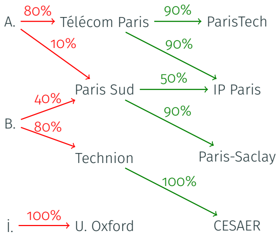

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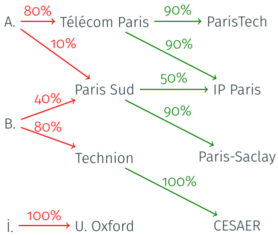

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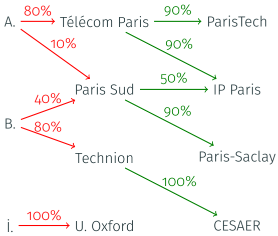

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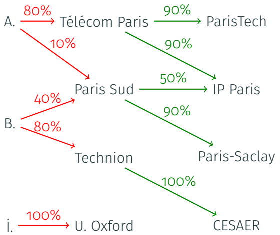

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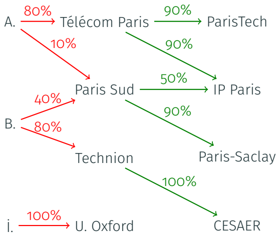

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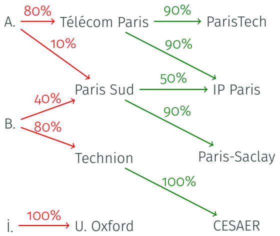

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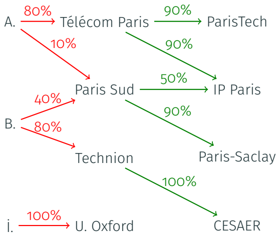

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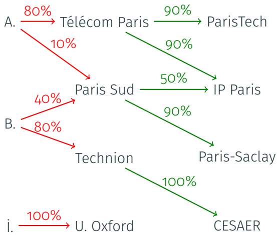

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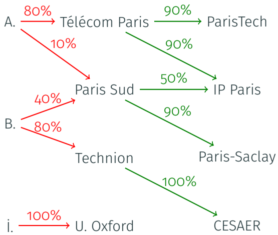

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Research goal: Understanding the complexity of PQE

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- **Restricted instances:** $\mathrm{PQE}(Q)$ for MSO queries...
  - Is **in PTIME** if the input data is restricted to have **bounded treewidth**
  - Is **intractable otherwise** under some assumptions

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  - $\mathrm{PQE}(Q)$ is **#P-hard** for all homomorphism-closed queries not equivalent to a safe UCQ
- **Restricted instances:** $\mathrm{PQE}(Q)$ for MSO queries...
  - Is **in PTIME** if the input data is restricted to have **bounded treewidth**
  - Is **intractable otherwise** under some assumptions
- **Restricted instances:** if all probabilities are **50%** then the complexity is the same

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  $\rightarrow$ e.g., single-atom CQs

  $\rightarrow$ e.g., $x \longrightarrow y \longrightarrow z$

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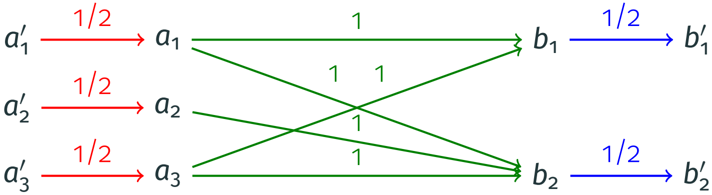

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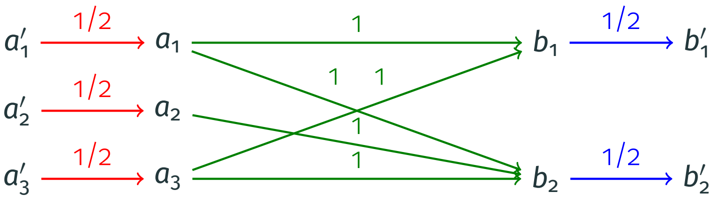

Idea: Satisfying valuations of ϕ correspond to **possible worlds** with a **match** of Q

The “small” Dalvi and Suciu dichotomy

- Self-join-free CQ: only one edge of each color (no repeated color)

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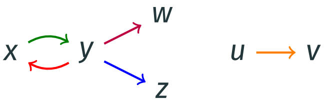

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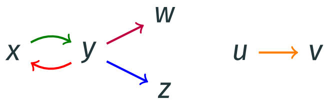

- The dichotomy generalizes to **higher-arity data** (hierarchical queries)

Proving the small dichotomy (upper bound)

$$\begin{tikzcd} & & w \\ x \arrow[r, bend left] & y \arrow[l, bend left] \arrow[ur] \arrow[dr] & \\ & & z \end{tikzcd} \qquad u \longrightarrow v$$

How to solve $\mathrm{PQE}(Q)$ for Q a self-join-free star?

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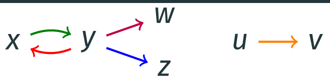

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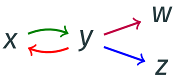

- We consider each connected component separately

$\rightarrow$ **Independent conjunction** over the connected components

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How to solve $\mathrm{PQE}(\boldsymbol{Q})$ for $\boldsymbol{Q}$ a self-join-free star?

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- For every match, we consider every **other variable** separately

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- We consider every value for the **other variable**

$\rightarrow$ **Independent disjunction** over the possible assignments

$\rightarrow$ **Independent conjunction** over the facts

Proving the small dichotomy (lower bound)

Every non-star self-join-free CQ contains a pattern essentially like:

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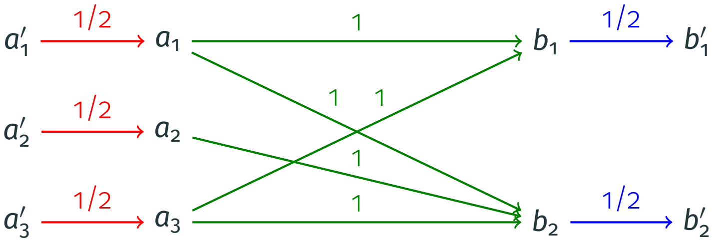
We can use this to reduce from #SAT like before:



The "big" Dalvi and Suciu dichotomy

Full dichotomy on the **unions of conjunctive queries** (UCQs):

Theorem (Dalvi and Suciu 2012)

Let Q be a UCQ:

- *If Q is handled by a complicated algorithm $\mathrm{PQE}(Q)$ is in **PTIME***
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This result is **far more complicated** (but still generalizes to higher arity)

- **Upper bound:**
  - an algorithm generalizing the previous case with **inclusion-exclusion**
  - many **unpleasant details** (e.g., a ranking transformation)
- **Lower bound:** hardness proof on minimal cases where the algorithm does not work

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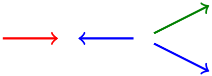
We study the case of **queries closed under homomorphisms**

Homomorphism-closed queries

- A homomorphism from a graph G to a graph G' maps the vertices of G to those of G' while preserving the edges



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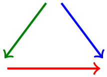

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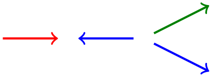
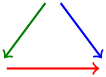
 has a homomorphism to 

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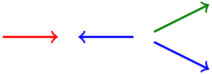
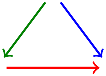
 has a homomorphism to 

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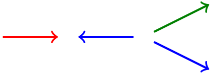
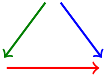
 has a homomorphism to 

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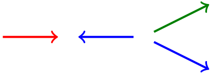

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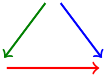

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- Queries with **negations** or **inequalities** are not homomorphism-closed
- Homomorphism-closed queries can equivalently be seen as **infinite unions of CQs** (corresponding to their models)

Our result

We show:

Theorem (Amarilli and Ceylan 2020)

For any query Q closed under homomorphisms:

- *Either* Q *is equivalent to a tractable UCQ and* $\mathrm{PQE}(Q)$ *is in PTIME*
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The challenging part is to show:

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For any query Q closed under homomorphisms and unbounded, $\mathrm{PQE}(Q)$ is #P-hard

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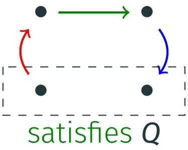

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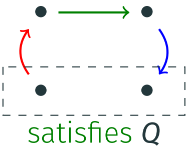

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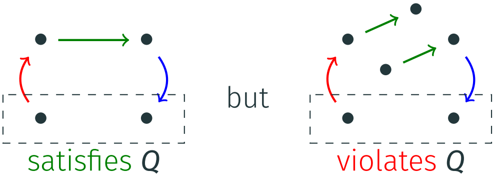

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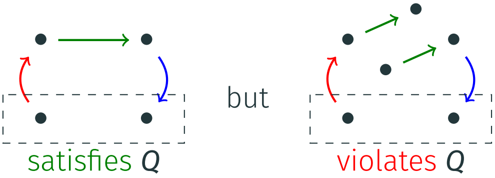

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Using tight patterns to show hardness of PQE

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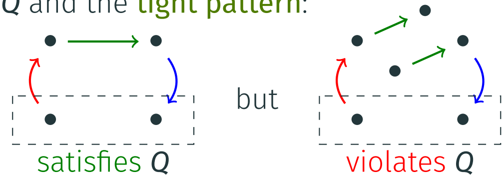

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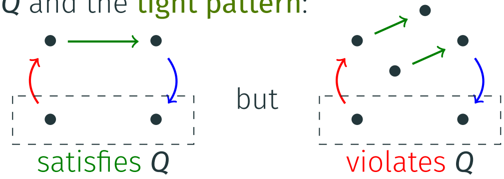

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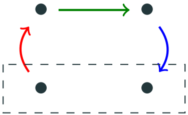

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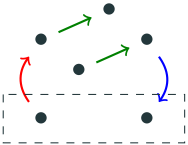

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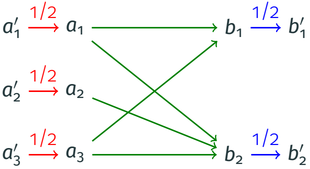

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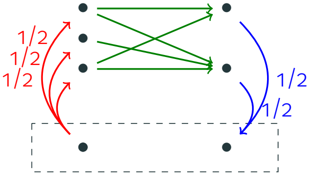

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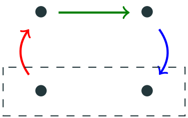

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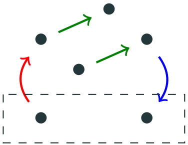

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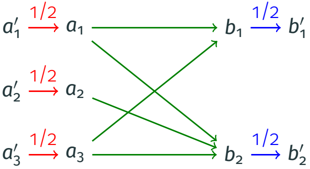

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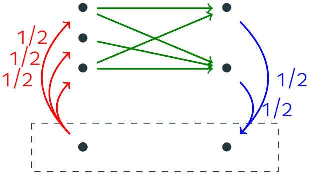

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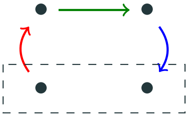

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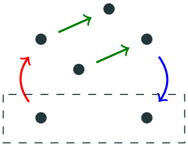

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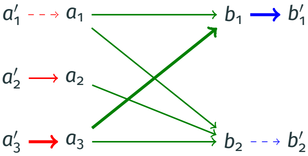

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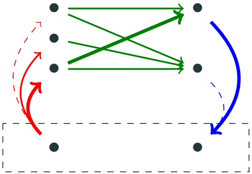

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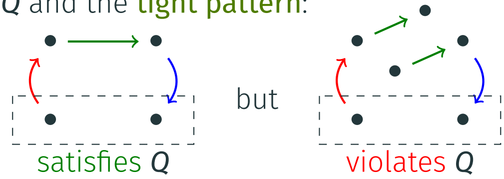

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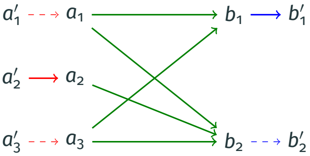

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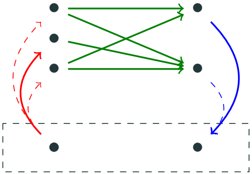

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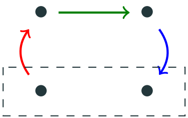

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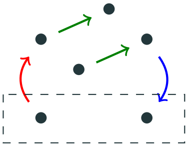

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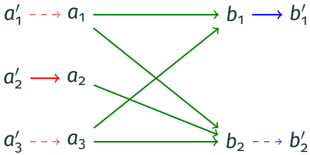

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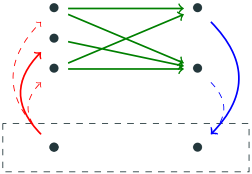

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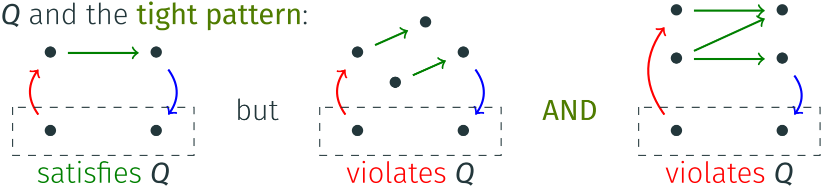

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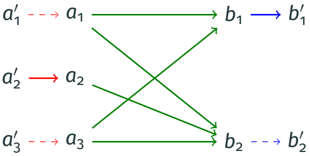

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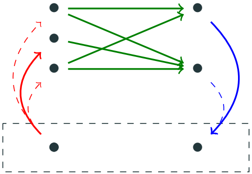

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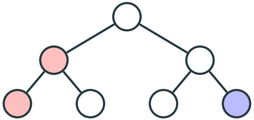
Database: a **tree** T where nodes have a color from an alphabet



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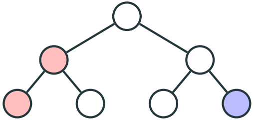

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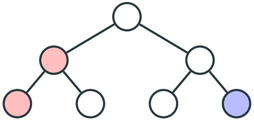

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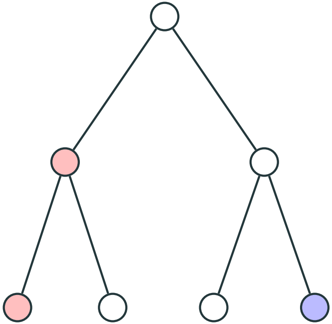

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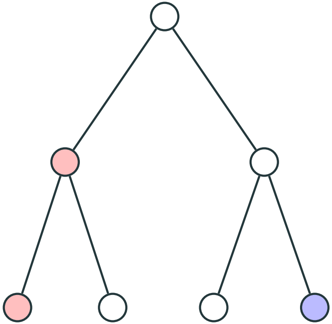

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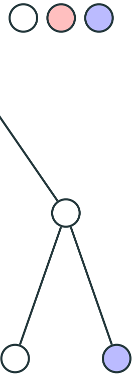

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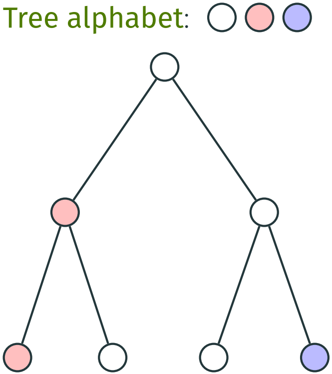

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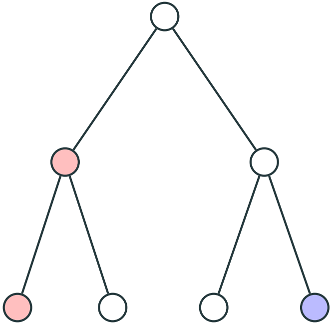

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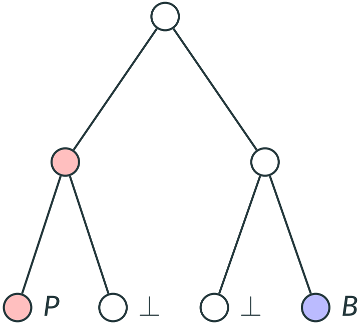

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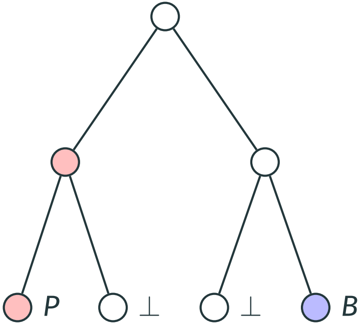

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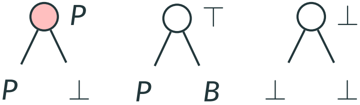

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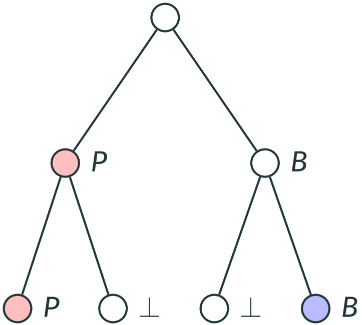

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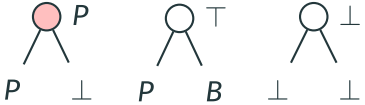

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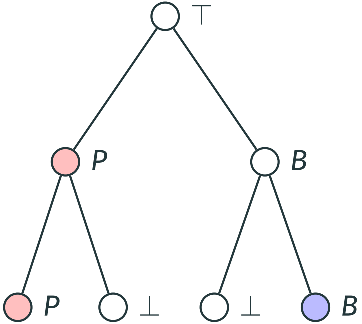

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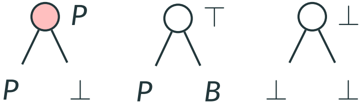

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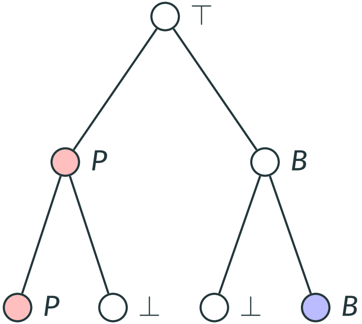

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Theorem (Thatcher and Wright 1968)

MSO and tree automata have the same expressive power on trees

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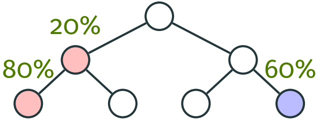

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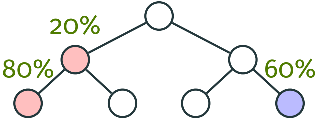

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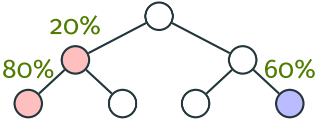

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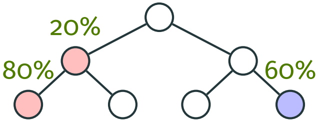

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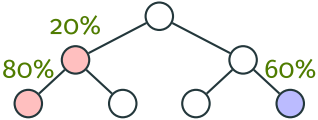


Query Q: in monadic second-order logic (MSO)

$$\exists x\, y\; P_{\circ}(x) \land P_{\circ}(y)$$

Result: **probability** that the probabilistic tree T satisfies the query Q

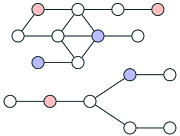
Theorem

For any fixed ***MSO query*** Q*, the problem* $\mathrm{PQE}(Q)$ *on trees is in* ***PTIME***

We can extend this to **treelike data**, i.e., data of bounded **treewidth**...

Courcelle's theorem and extension to PQE

Treelike **data**



MSO query

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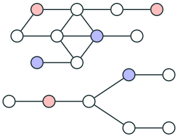

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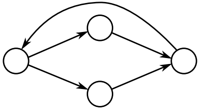

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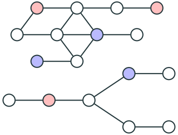
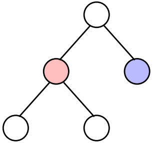


linear

Tree **encoding**



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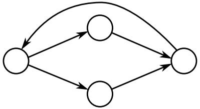

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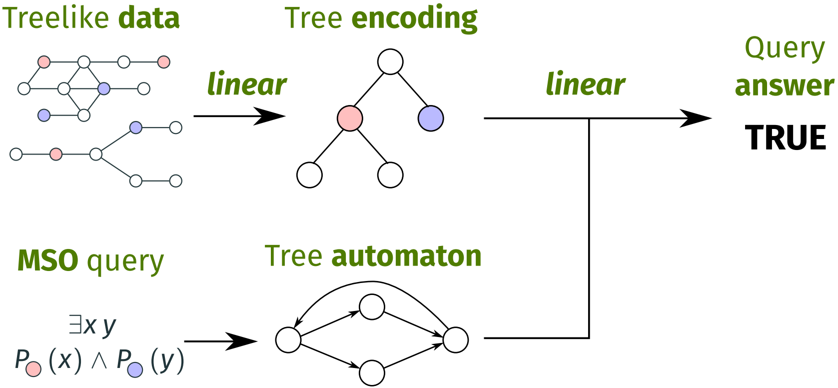

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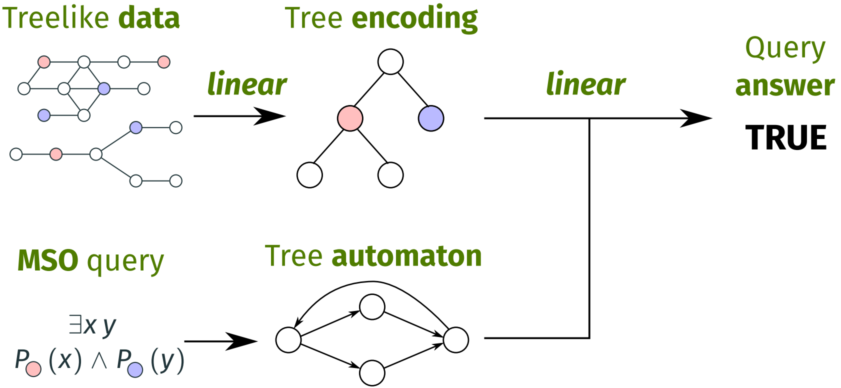
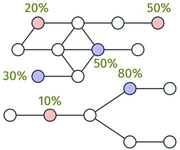


Theorem (Courcelle 1990)

For any fixed Boolean MSO query $\boldsymbol{Q}$ *and* $\boldsymbol{k} \in \mathbb{N}$*, given a database* $\boldsymbol{D}$ *of treewidth* $\leq \boldsymbol{k}$*, we can compute in linear time in* $\boldsymbol{D}$ *whether* $\boldsymbol{D}$ *satisfies* $\boldsymbol{Q}$

Courcelle's theorem and extension to PQE

Probabilistic treelike **data**



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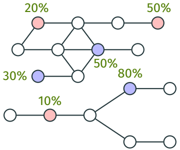

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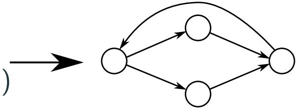

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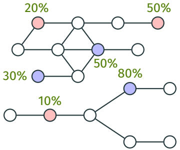

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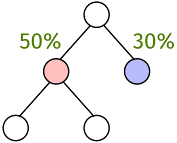

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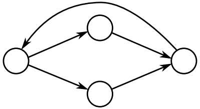

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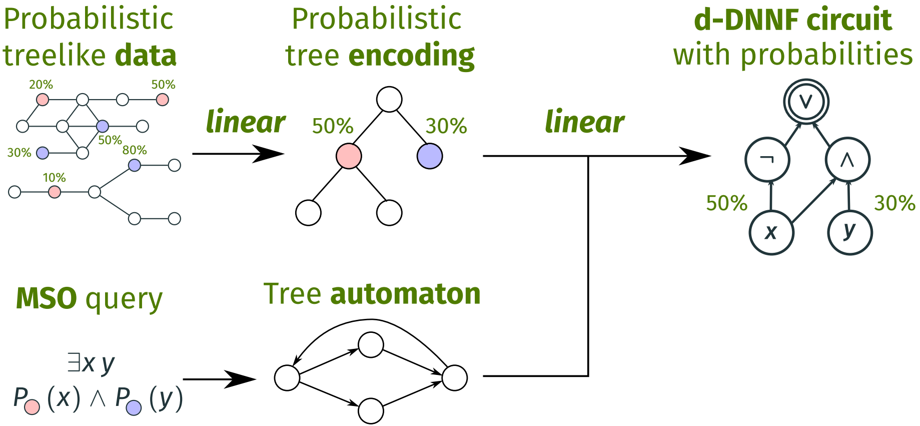

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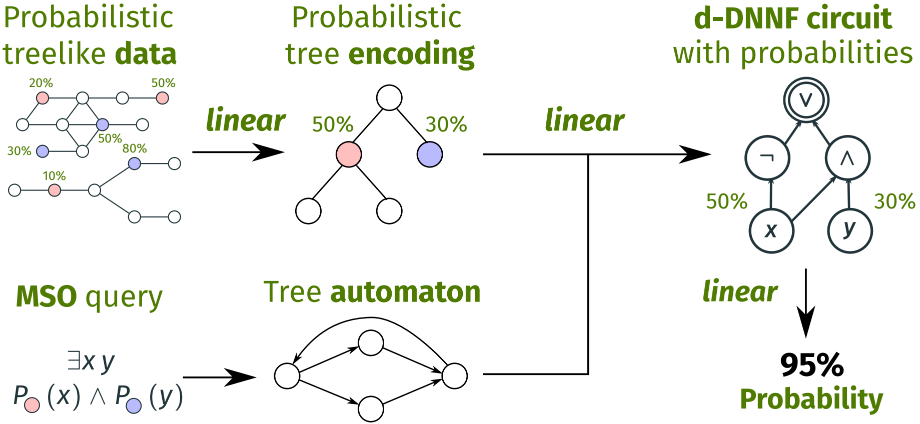

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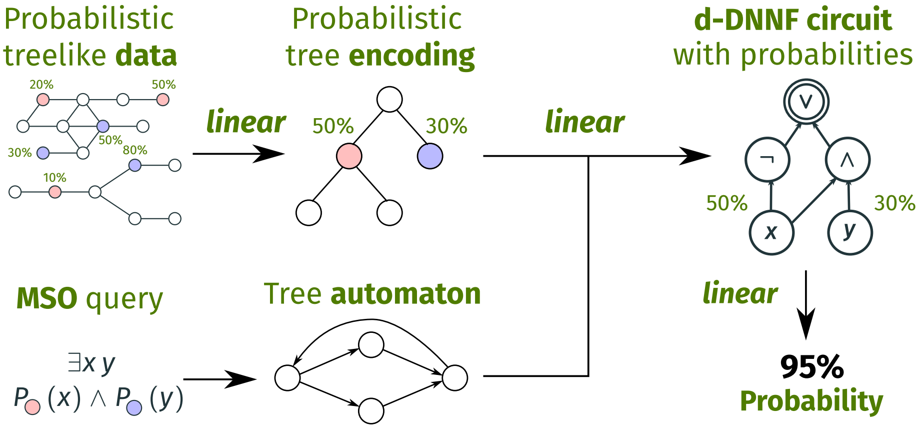

Theorem (Amarilli, Bourhis, and Senellart 2015; Amarilli, Bourhis, and Senellart 2016)

For any fixed Boolean MSO query Q *and* $k \in \mathbb{N}$*, given a database* D *of treewidth* $\leq k$*, we can solve the PQE problem in linear time (assuming constant-time arithmetics)*

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Problem statement

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We limit to **self-join-free CQs** and extend the "small" Dalvi and Suciu dichotomy to SC:

Theorem (Amarilli and Kimelfeld 2020)

Let Q be a self-join-free CQ:

- *If Q is a **star**, then $\mathrm{PQE}(Q)$ is in **PTIME***
- *Otherwise, even $\mathrm{SC}(Q)$ is **#P-hard***

$\rightarrow$ This also extends **beyond arity two** (hierarchical queries)

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Conclusion and open problems

We have seen:

- PQE is **#P-hard** for all homomorphism-closed queries except safe UCQs
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Thanks for your attention!

Amarilli, Antoine, Pierre Bourhis, and Pierre Senellart (2015). “Provenance Circuits for Trees and Treelike Instances”. In: *ICALP*.

— (2016). “Tractable Lineages on Treelike Instances: Limits and Extensions”. In: *PODS*.

Amarilli, Antoine and Ismail Ilkan Ceylan (2020). “A Dichotomy for Homomorphism-Closed Queries on Probabilistic Graphs”. In: *ICDT*.

Amarilli, Antoine and Benny Kimelfeld (2020). “Uniform Reliability of Self-Join-Free Conjunctive Queries”. Preprint: https://arxiv.org/abs/1908.07093.

Chekuri, Chandra and Julia Chuzhoy (2014). “Polynomial bounds for the grid-minor theorem”. In: *STOC*. DOI: 10.1145/2591796.2591813. URL: http://doi.acm.org/10.1145/2591796.2591813.

Courcelle, Bruno (1990). “The Monadic Second-Order Logic of Graphs. I. Recognizable Sets of Finite Graphs”. In: *Inf. Comput.* 85.1. DOI: `10.1016/0890-5401(90)90043-H`. URL: `http://dx.doi.org/10.1016/0890-5401(90)90043-H`.

Dalvi, Nilesh and Dan Suciu (2007). “The dichotomy of conjunctive queries on probabilistic structures”. In: *Proc. PODS*.

— (2012). “The dichotomy of probabilistic inference for unions of conjunctive queries”. In: *J. ACM* 59.6.

Fink, Robert and Dan Olteanu (2016). “Dichotomies for queries with negation in probabilistic databases”. In: 41.1, 4:1–4:47.

Ganian, Robert, Petr Hlineny, Alexander Langer, Jan Obdrzalek, Peter Rossmanith, and Somnath Sikdar (2014). “Lower bounds on the complexity of MSO1 model-checking”. In: *JCSS* 1.80.

Jung, Jean Christoph and Carsten Lutz (2012). "Ontology-based access to probabilistic data with OWL QL". In: *Proceedings of the 11th International Conference on The Semantic Web - Volume Part I*, pp. 182–197.

Thatcher, James W. and Jesse B. Wright (1968). "Generalized Finite Automata Theory with an Application to a Decision Problem of Second-Order Logic". In: *Mathematical systems theory* 2.1, pp. 57–81.

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- Unbounded queries have **arbitrarily large** minimal models
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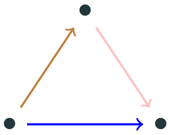

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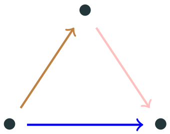

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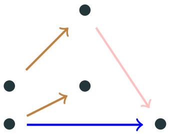

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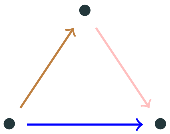

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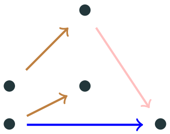

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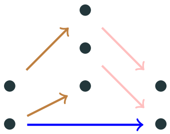

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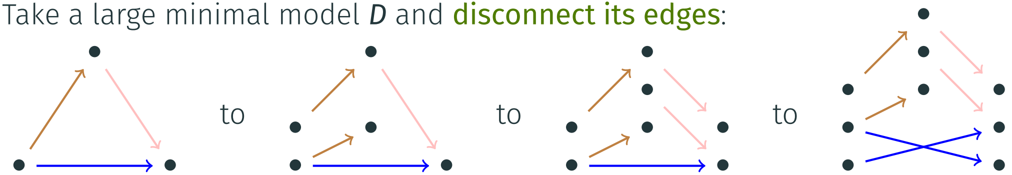

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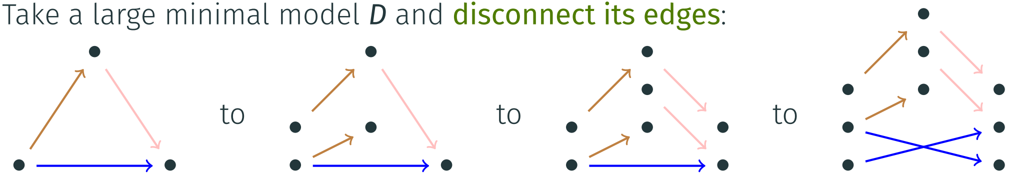

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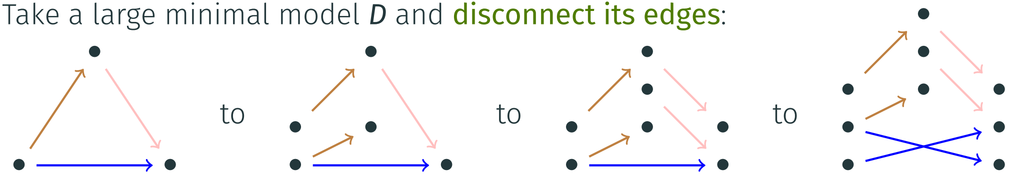

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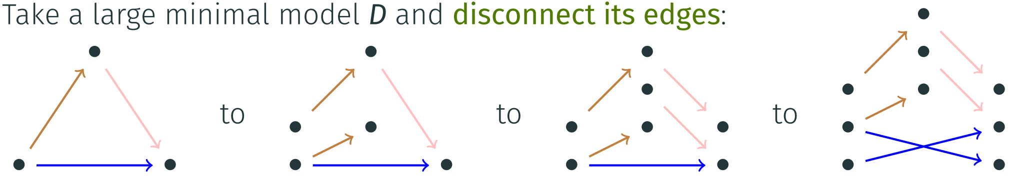

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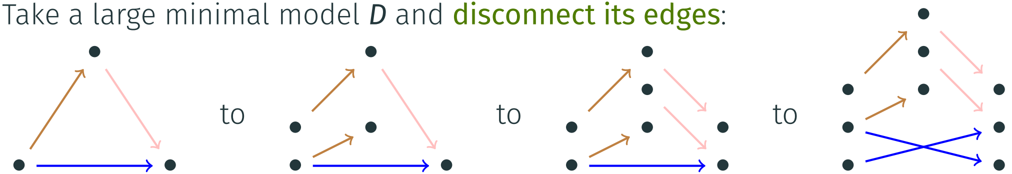

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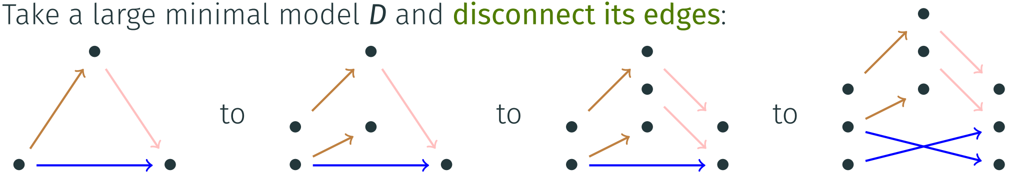

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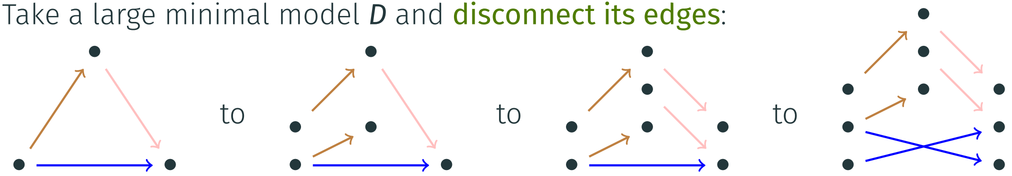

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  - This contradicts the **minimality** of the large D

Rescuing the proof

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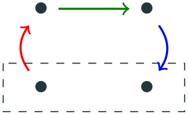

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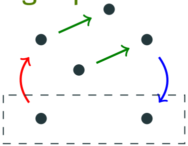

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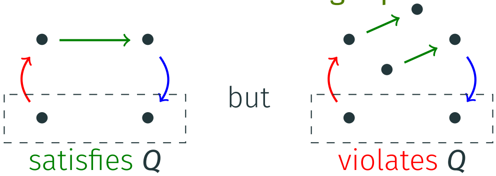

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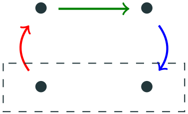

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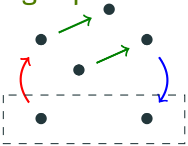

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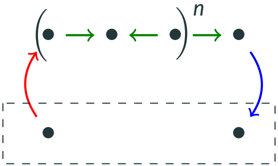

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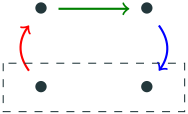

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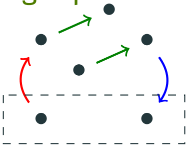

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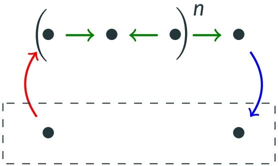

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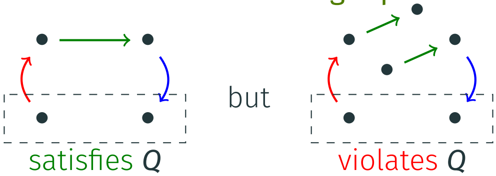

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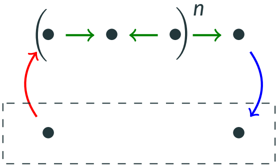

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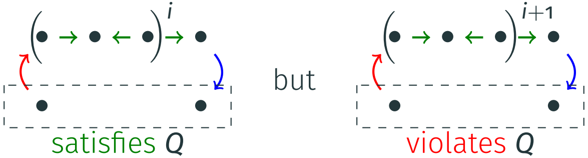

Rescuing the proof

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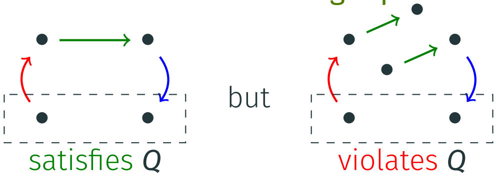

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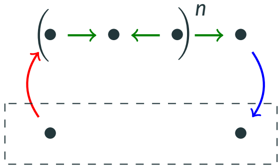

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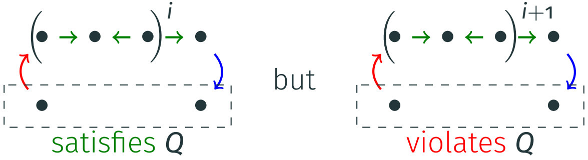

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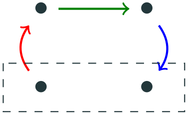

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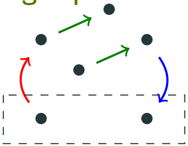

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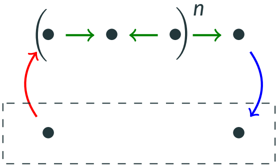

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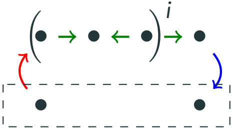

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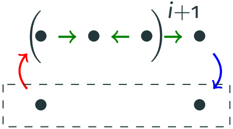

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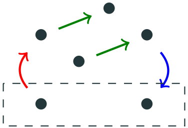
Case 2: all iterates **satisfy** the query:

satisfies Q for all $n \in \mathbb{N}$

but



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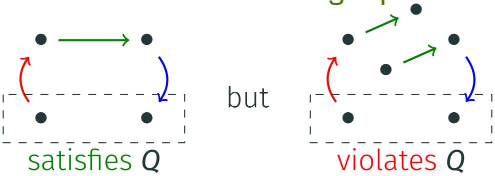

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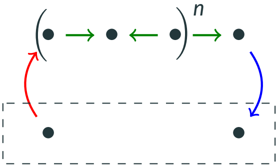

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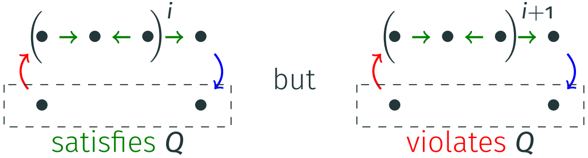

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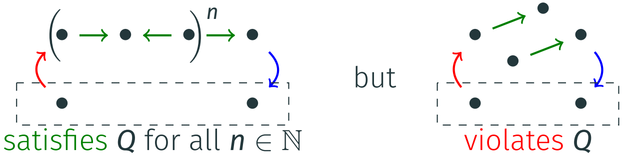
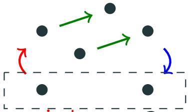


$\rightarrow$ Call this an **iterable pattern**

Using iterable patterns to show hardness of PQE

We have an **iterable pattern**:  but 

satisfies Q for all $n \in \mathbb{N}$ | violates Q

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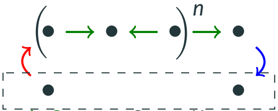

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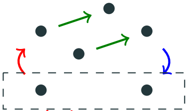

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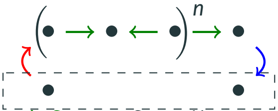

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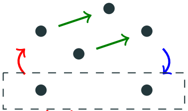

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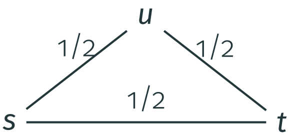

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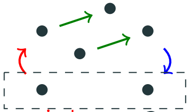

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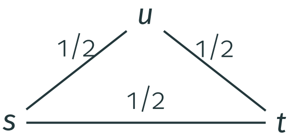

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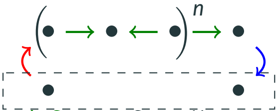

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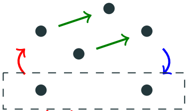

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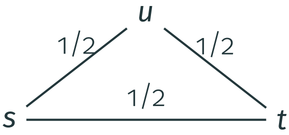

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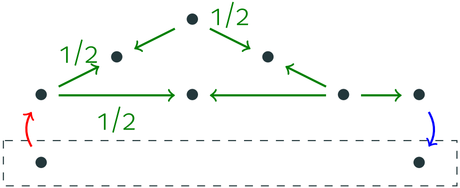

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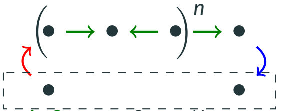

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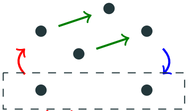

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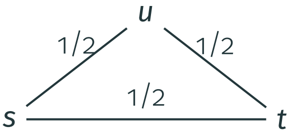

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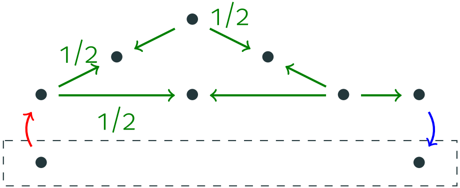

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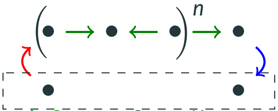

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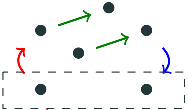

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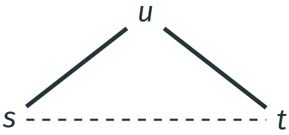

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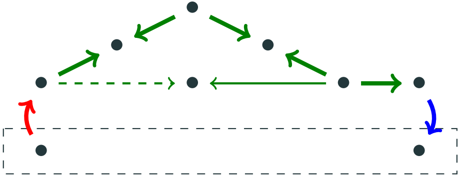

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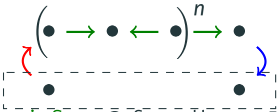

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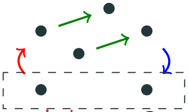

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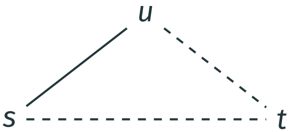

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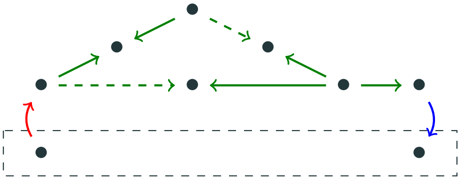

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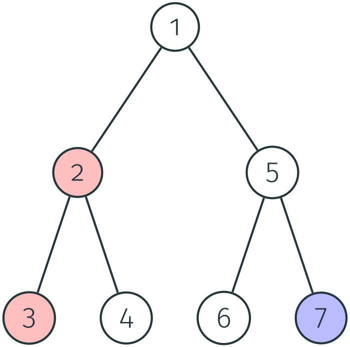
Uncertain trees: capturing how the query result depends on the choices



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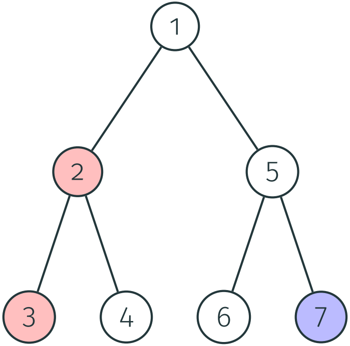

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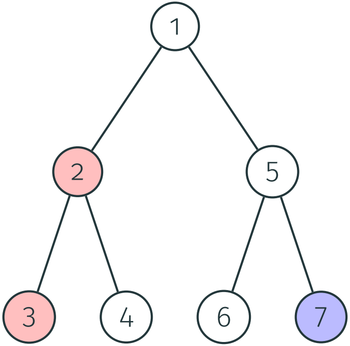

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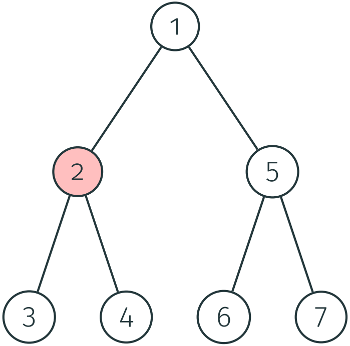

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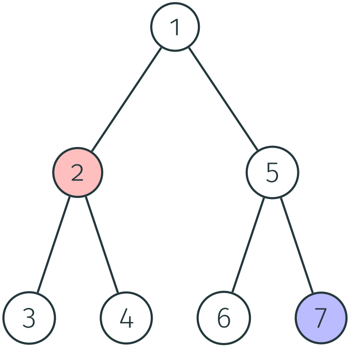

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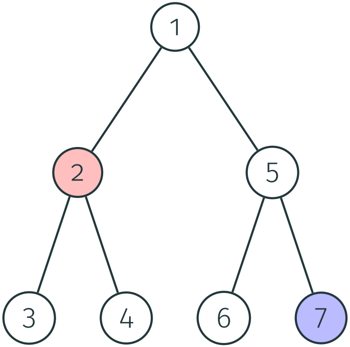

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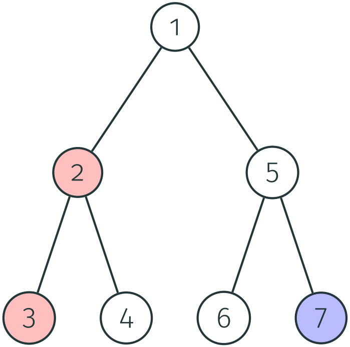

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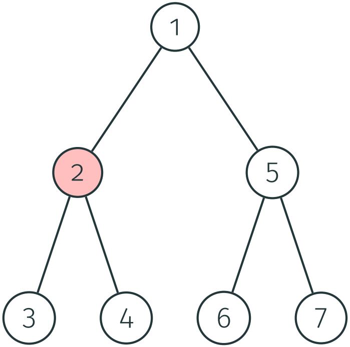

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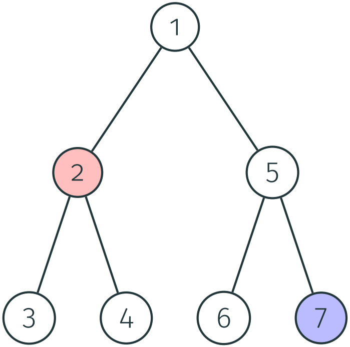

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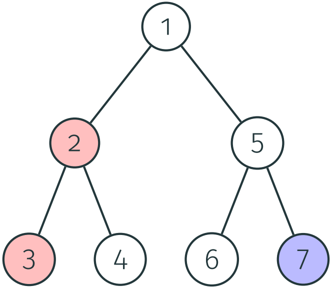

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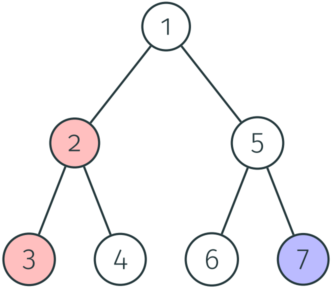

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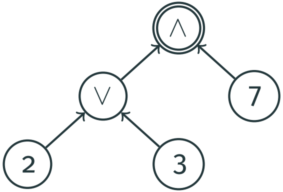

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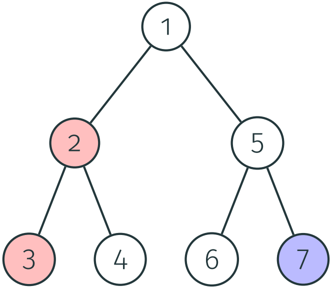

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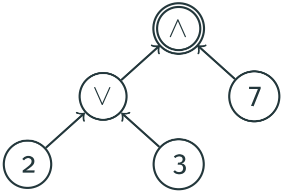

Formal definition of provenance circuits:

- Boolean query Q, uncertain tree T, circuit C
- **Variable gates** of C: nodes of T
- **Condition:** Let ν be a valuation of T, then $\nu(C)$ iff $\nu(T)$ satisfies Q

Provenance circuits on trees

Theorem

For any bottom-up tree automaton A and input tree T,
we can build a Boolean provenance circuit of A on T in $O(|A| \times |T|)$

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- **States:** $\{\bot, B, P, \top\}$
- **Final:** $\{\top\}$
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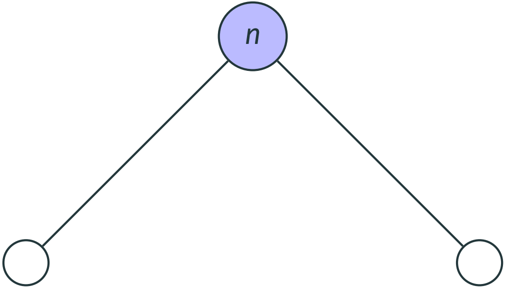

Provenance circuits on trees

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For any bottom-up tree automaton A and input tree T, we can build a Boolean provenance circuit of A on T in $O(|A| \times |T|)$

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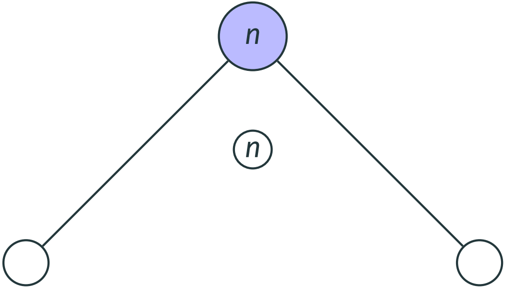

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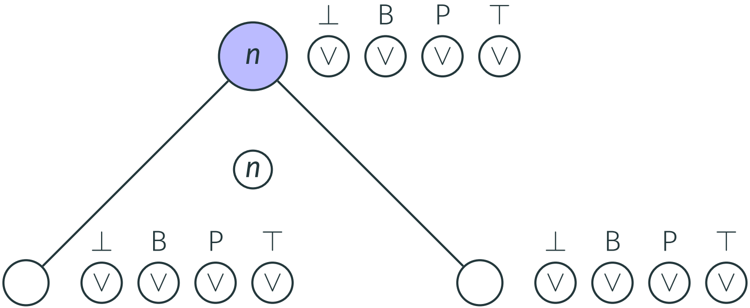

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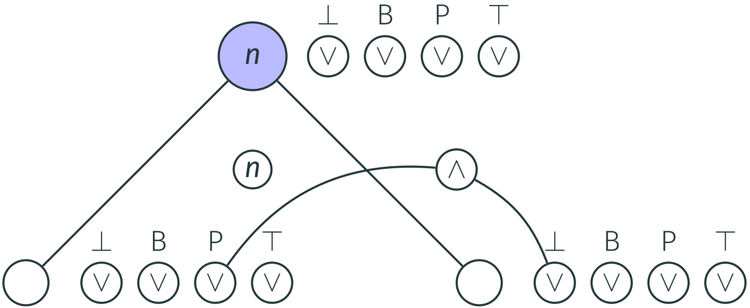

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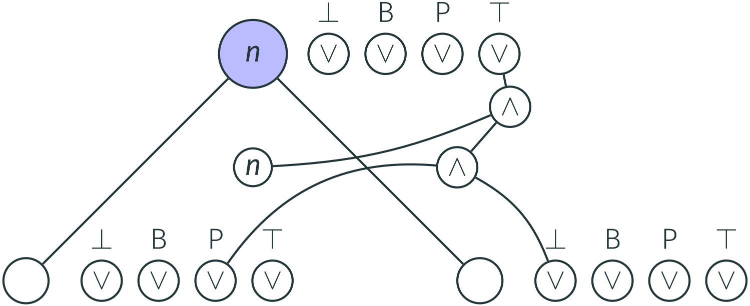

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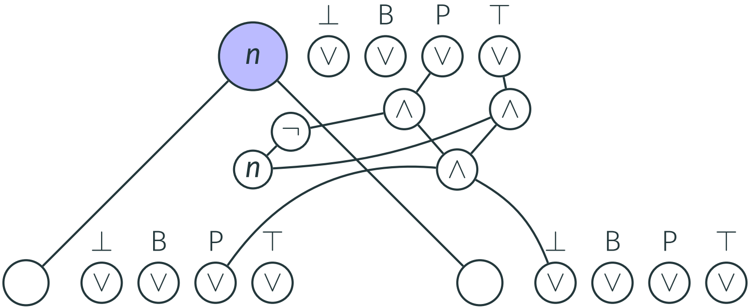

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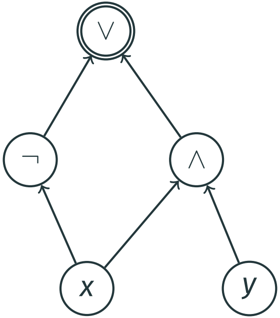

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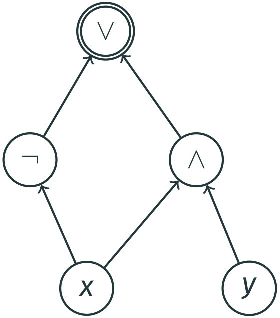

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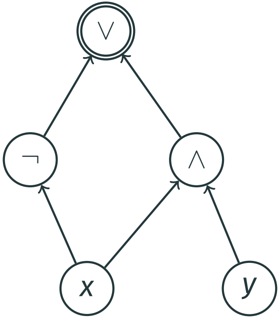

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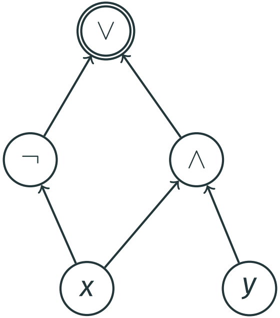

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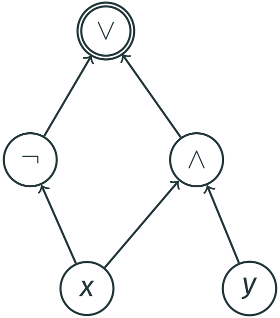

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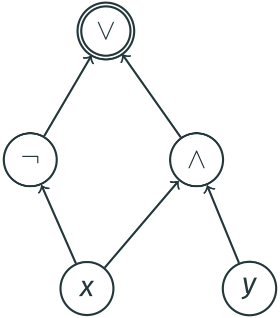

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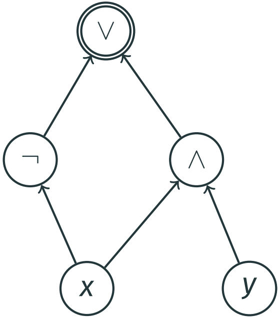

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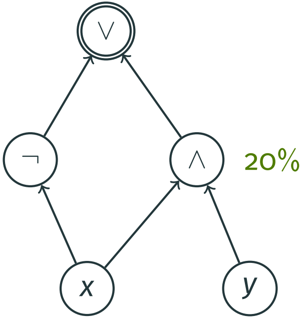

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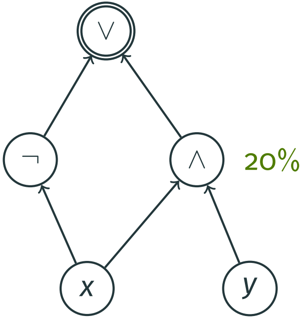

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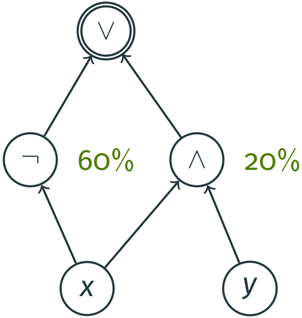

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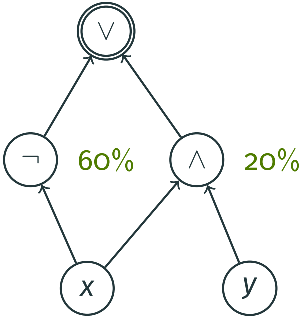

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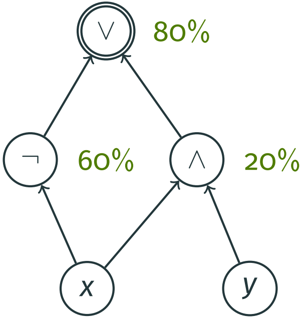

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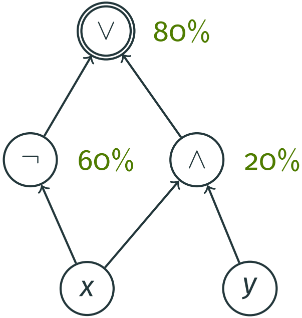

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$\rightarrow$ The circuit that we constructed falls in a **restricted class** satisfying such conditions

d-DNNFs

Lemma

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d-DNNF requirements

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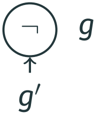

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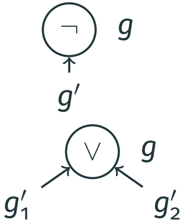

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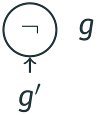

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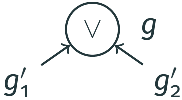

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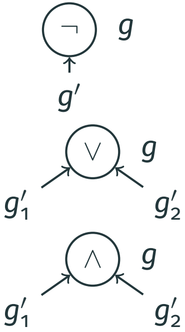

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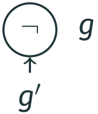

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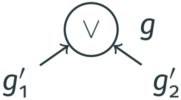

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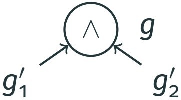

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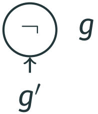

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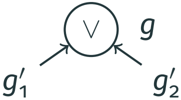

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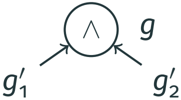

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$\rightarrow$ Connections to other circuit classes in the field of **knowledge compilation**

Treewidth

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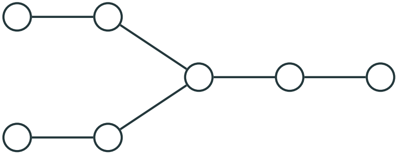
Treewidth by example:



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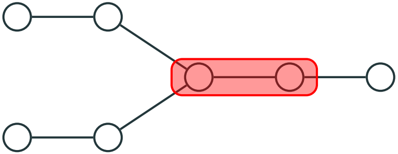

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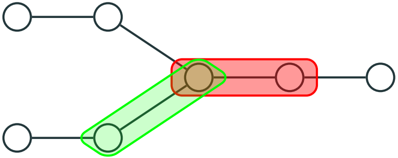

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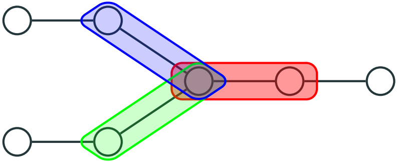

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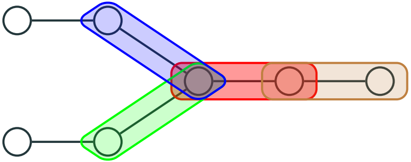

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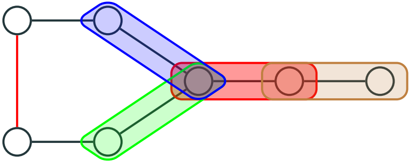

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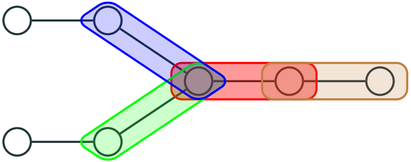

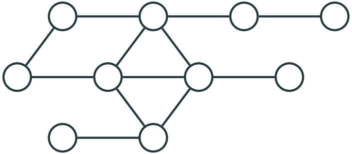

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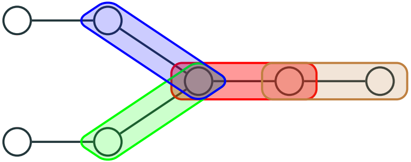

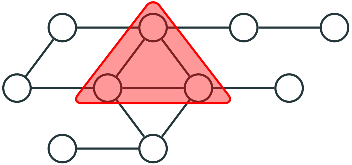

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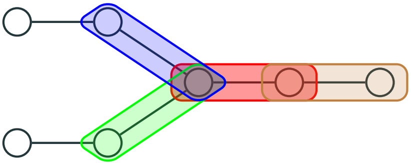

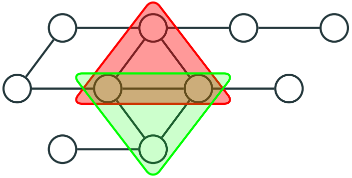

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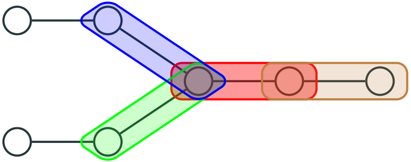

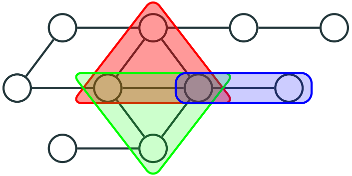

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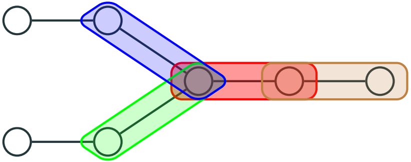

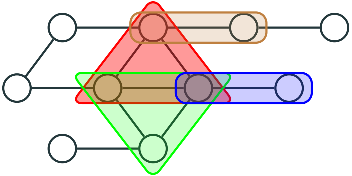

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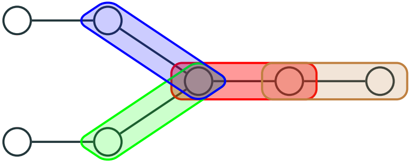

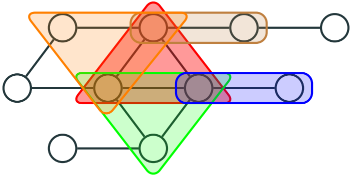

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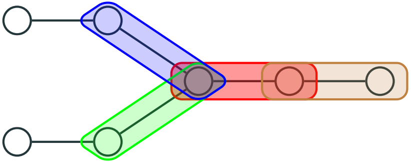

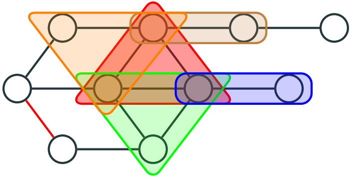

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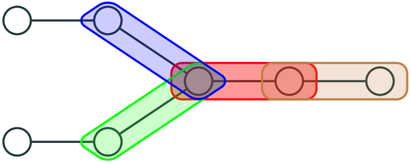

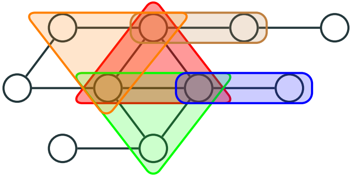

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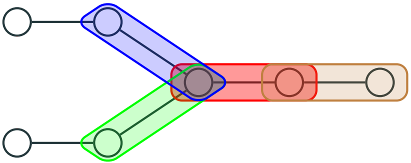

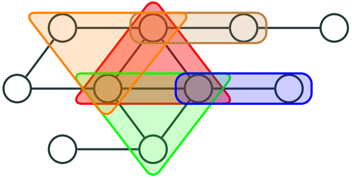

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- **Cycles** have treewidth 2
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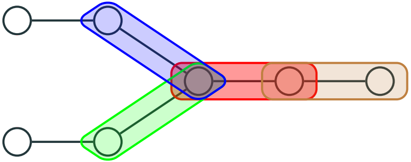

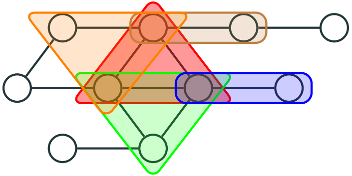

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$\rightarrow$ **Treelike**: the **treewidth** is bounded by a **constant**

Why is this a dichotomy? Where's the lower bound?

Theorem (Amarilli, Bourhis, and Senellart 2016)

For any set of edge colors, there is a first-order query **Q** *such that for any constructible unbounded-treewidth family* $\mathcal{I}$ *of probabilistic graphs, the PQE problem for* **Q** *and* $\mathcal{I}$ *is #P-hard under RP reductions*

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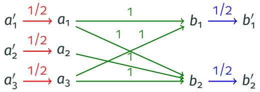
$\rightarrow$ Proof idea: **extract wall graphs as topological minors** (Chekuri and Chuzhoy 2014) and adapt a technique of Ganian, Hlineny, Langer, Obdrzalek, Rossmanith, and

Proof technique

Hard part: show hardness for (variants of) the query Q: $x \longrightarrow y \longrightarrow z \longrightarrow w$

We reduce from $\mathrm{PQE}(Q)$, on **probabilistic graphs** G of the following form:



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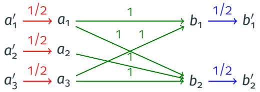

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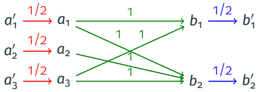

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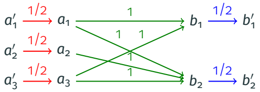

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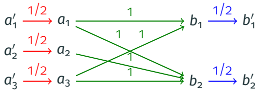

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- Show **invertibility** of this matrix to recover the X_i from the N_i

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