Query Answering with Guarded Rules and Expressive Constraints

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- We are given:
  - Relational instance I (ground facts)
  - Logical constraints Σ
  - Boolean conjunctive query q (existentially quantified conjunction of atoms)

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- We ask:
  - Consider all possible completions $J \supseteq I$
  - Restrict to those that satisfy the constraints Σ

  $\rightarrow$ Is q certain among them?

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Relational instance I

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$\rightarrow$ The query is certain under the instance and constraints

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$\rightarrow$ Depends on the language used for logical constraints

$\rightarrow$ E.g., for arbitrary first-order constraints, QA is undecidable (because satisfiability is undecidable)

$\rightarrow$ Find fragments of first-order logic with decidable QA

Tuple-generating dependencies (or existential rules)

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$$\forall x\, y_1\, y_2\ S(x, y_1) \land S(x, y_2) \land U(x, y_1, y_2) \to \exists z\ S(y_2, z) \land T(y_1)$$

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→ QA for FGTGDs is decidable [Baget et al., 2011]

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This talk

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2. *Query Answering with Transitive and Linear-Ordered Data*
   A. A., Michael Benedikt, P. Bourhis, M. Vanden Boom, IJCAI'16
   $\rightarrow$ Extend FGTGDs with transitive relations and order relations that cannot be used as guards

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Rich Description Logics

Rich description logics **(DLs)**	FGTGDs

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Our problem

Can we have the best of both worlds?

- QA is decidable for frontier-guarded TGDs
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We show:

- QA is undecidable for rich DLs and FGTGDs
- QA with rich DLs is decidable for some new FGTGD classes
- Functional dependencies can be added under some conditions even to higher-arity relations

Undecidability of frontier-guarded plus DLs

Theorem

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- FGTGDs can express inclusion dependencies (IDs)
  $\forall \mathbf{x}\,\mathbf{y}\ A(\mathbf{x}, \mathbf{y}) \to \exists \mathbf{z}\ B(\mathbf{y}, \mathbf{z})$ with no variable repetitions
- Implication of IDs and FDs is undecidable [Mitchell, 1983]
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QA is undecidable for rich DLs and frontier-one TGDs

Problem:

- Rule heads and bodies may contain cycles
- We have Funct assertions

$\rightarrow$ We can build a grid and encode tiling problems

Undecidability of frontier-one plus DLs: proof

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- finite set of colors: ■, ■, ■

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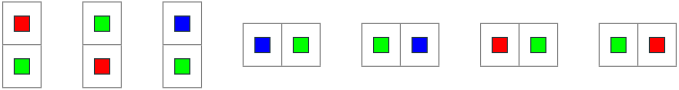

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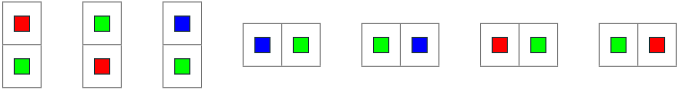

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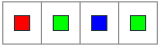
- input: initial configuration: 

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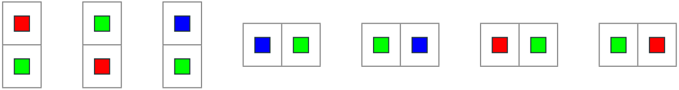

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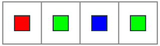
- input: initial configuration: 
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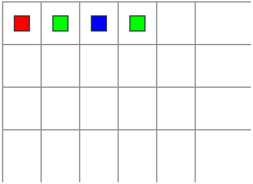

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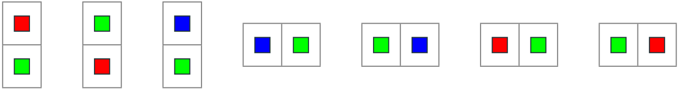

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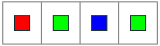
- input: initial configuration: 
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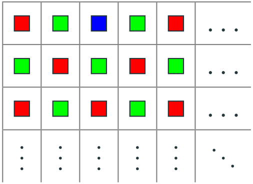

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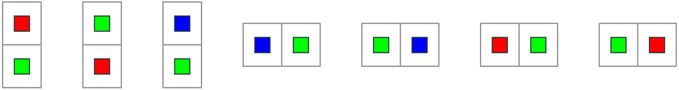

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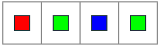
- input: initial configuration: 
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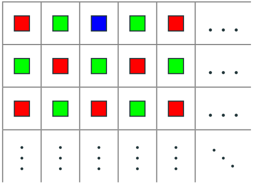

→ Undecidable for some sets of colors and configurations

Undecidability of frontier-one plus DLs: proof, cont'd

- Functional relations D for down and R for right
- Unary predicate T for tiles and $C_{\blacksquare}$ for each color

Undecidability of frontier-one plus DLs: proof, cont'd

- Functional relations D for down and R for right
- Unary predicate T for tiles and $C_{■}$ for each color

Initial instance: $C_{■} \xrightarrow{R} C_{■} \xrightarrow{R} C_{■} \xrightarrow{R} C_{■}$

Undecidability of frontier-one plus DLs: proof, cont'd

- Functional relations D for down and R for right
- Unary predicate T for tiles and $C_{\color{gray}\blacksquare}$ for each color

Initial instance: $C_{\color{red}\blacksquare} \xrightarrow{R} C_{\color{green}\blacksquare} \xrightarrow{R} C_{\color{blue}\blacksquare} \xrightarrow{R} C_{\color{green}\blacksquare}$

Constraints:

- DL Disjunction to color tiles: $T \sqsubseteq C_{\color{red}\blacksquare} \sqcup C_{\color{green}\blacksquare} \sqcup C_{\color{blue}\blacksquare}$

Undecidability of frontier-one plus DLs: proof, cont'd

- Functional relations D for down and R for right
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Constraints:

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- Frontier-one TGD: $\forall y\ T(y) \Rightarrow \exists tzw$
$$\begin{tikzcd} T(y) \arrow[r, "R"] \arrow[d, "D"] & T(t) \arrow[d, "D"] \\ T(z) \arrow[r, "R"] & T(w) \end{tikzcd}$$

Undecidability of frontier-one plus DLs: proof, cont'd

- Functional relations D for down and R for right
- Unary predicate T for tiles and $C_{\blacksquare}$ for each color

Initial instance: $C_{\blacksquare} \xrightarrow{R} C_{\blacksquare} \xrightarrow{R} C_{\blacksquare} \xrightarrow{R} C_{\blacksquare}$

Constraints:

- DL Disjunction to color tiles: $T \sqsubseteq C_{\blacksquare} \sqcup C_{\blacksquare} \sqcup C_{\blacksquare}$
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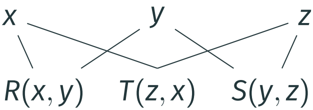
Query: $\exists x\, y\ C_{\blacksquare}(x) \xrightarrow{R} C_{\blacksquare}(y)$ for all forbidden pairs

$\rightarrow$ There is an extension of the instance iff there is a tiling

Decidability of non-looping frontier-one and DLs

Idea: prohibit cycles in FGTGDs

Decidability of non-looping frontier-one and DLs



Idea: prohibit cycles in FGTGDs

- $R(x, y)$ $S(y, z)$ $T(z, x)$ is a cycle

Decidability of non-looping frontier-one and DLs

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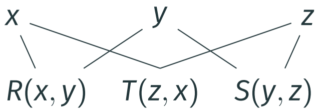

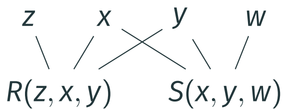

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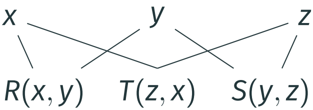
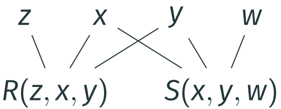
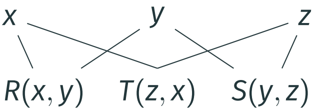




Formally:

- Berge cycle: cycle in the atom–variable incidence graph

Decidability of non-looping frontier-one and DLs



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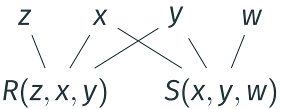

Formally:

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- Non-looping conjunction: no cycle except, e.g., $R(x, y)\ S(x, y)$
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Decidability of non-looping frontier-one and DLs

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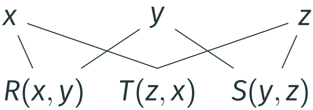

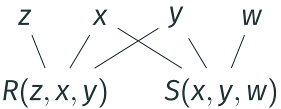

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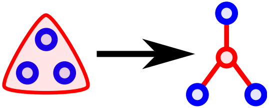
QA is decidable for non-looping frontier-one TGDs + rich DLs

Decidability of non-looping frontier-one and DLs (proof)

- Shred $R(a, b, c)$ to $R_1(f, a), R_2(f, b), R_3(f, c)$



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- Shred $R(a, b, c)$ to $R_1(f, a), R_2(f, b), R_3(f, c)$
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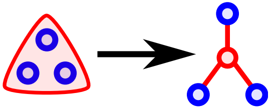

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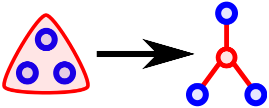

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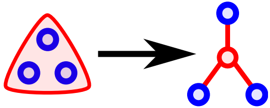

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- Rewrite shredded non-looping frontier-one TGDs to GC^2:
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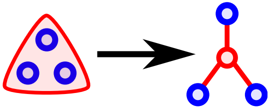

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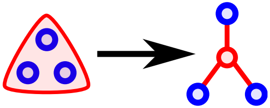

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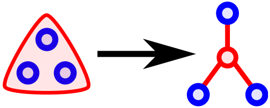

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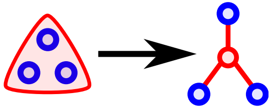

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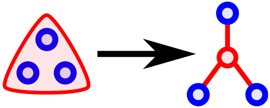

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    $\rightarrow\ \Big(\exists x_1\ T(y, x_1)\Big) \wedge \Big(\exists x_2\, f\ R_1(f, y) \wedge R_2(f, y)$

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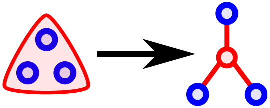

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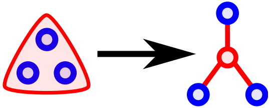

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    - $\rightarrow$ $\left(\exists x_1\ T(y, x_1)\right) \wedge \left(\exists x_2\, f\ R_1(f, y) \wedge R_2(f, y) \wedge (\exists y_2\ R_3(f, y_2) \wedge A(y_2))\right)$

  $\rightarrow$ Reduces to QA for $\mathbf{GC}^2$: decidable [Pratt-Hartmann, 2009]

Decidability of head-non-looping frontier-one and DLs

Head-non-looping frontier-one TGDs: no cycles in head

Decidability of head-non-looping frontier-one and DLs

Head-non-looping frontier-one TGDs: no cycles in head

Theorem

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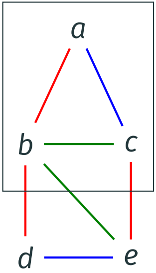
Basic idea:

- If there is a counterexample model to QA, we can unravel it
  - $\rightarrow$ It is still a counterexample
  - $\rightarrow$ It has no cycles (except in the instance part)

$\rightarrow$ Looping TGD bodies can only match on the instance part so non-looping frontier-one TGDs can be made head-non-looping

Head-non-looping frontier-one and DLs: unraveling



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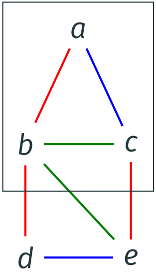

$\Rightarrow$

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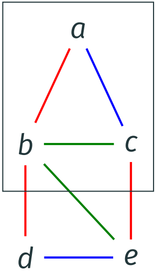
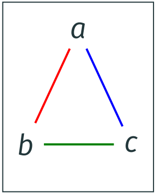


$\Longrightarrow$



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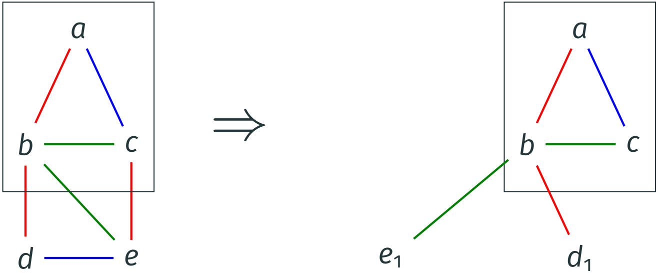

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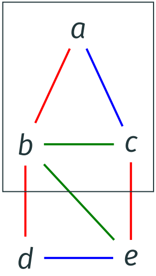

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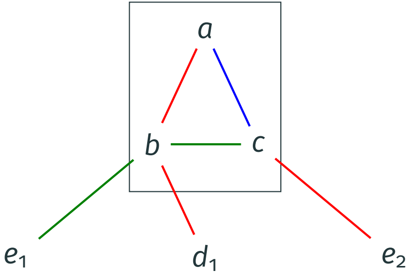

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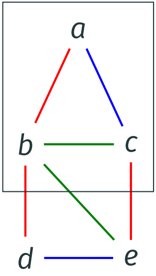
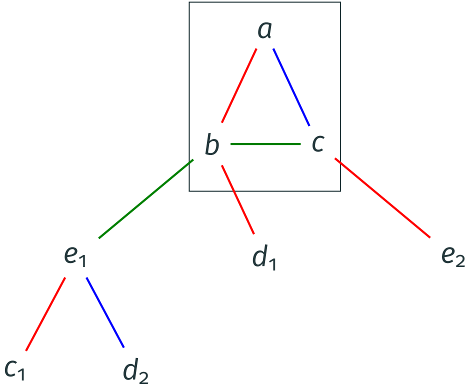
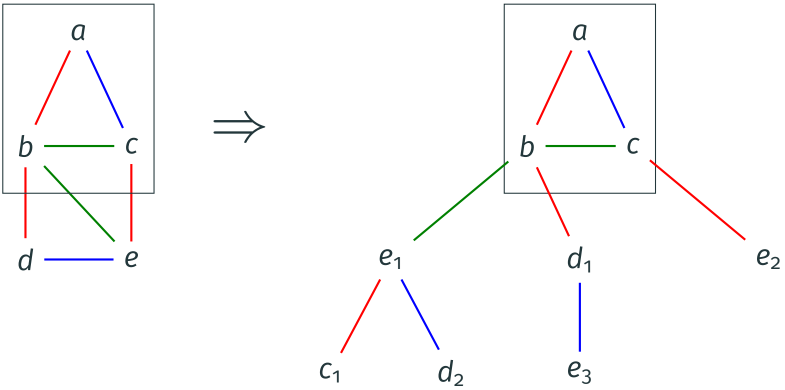


$\Longrightarrow$



Head-non-looping frontier-one and DLs: unraveling



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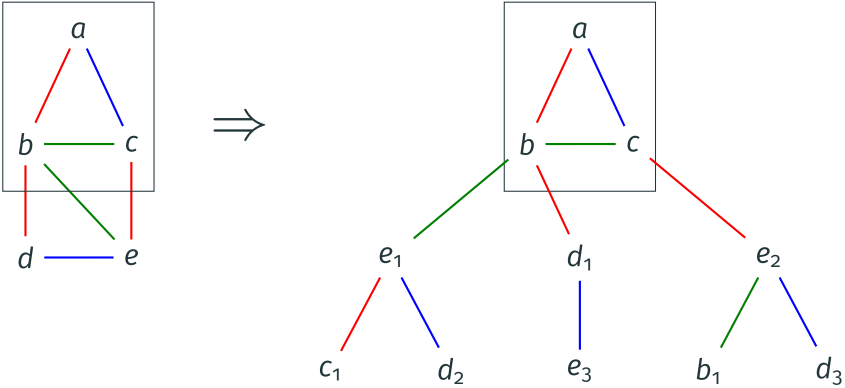

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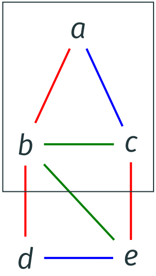

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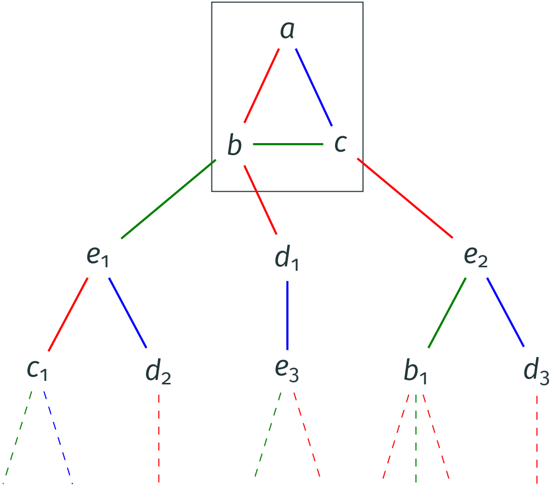

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QA for the shredded instance, treeified TGDs, query, and axioms is equivalent to QA for the original instance, TGDs, query

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We have shown:

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- We have functional dependencies $\mathrm{Funct}(R)$ on binary relations
- Could we also allow FDs on higher-arity relations?
  Ex.: $\mathrm{Talk}[\mathit{speaker}, \mathit{session}]$ determines $\mathrm{Talk}[\mathit{title}]$

Methods for TGDs + higher-arity FDs (no DLs)

- Consider QA under TGDs Σ and FDs Φ
- Σ and Φ are separable if $\mathrm{QA}(\Sigma, \Phi) \Leftrightarrow \mathrm{QA}(\Sigma)$ when $I \models \Phi$

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- Inclusion dependencies (IDs) and FDs generally not separable
- Frontier-one IDs and FDs are always separable
- [New:] Frontier-one TGDs with single-atom body and head (i.e., IDs with variable repetitions) and FDs are not separable and QA is undecidable for them (in our paper)

Non-conflicting TGDs and FDs [Calì et al., 2012]

Non-conflicting condition: sufficient condition for separability of single-head TGDs Σ and FDs Φ,

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We show that we can combine these two results:

Theorem

QA is decidable for:

- *Rich DL constraints (with Funct)*
- *Single-head (hence, head-non-looping) frontier-one TGDs*
- *Non-conflicting FDs (on higher-arity predicates)*

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Combining Existential Rules and Description Logics

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  - $\rightarrow$ We can also add non-conflicting FDs on higher-arity facts

Table of contents

Query answering with distinguished relations

We separate the **signature** σ (set of allowed relations) into:

- σ_{B}: the **base** relations (e.g., $\mathrm{Advisor}$)
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- Distinguished relations have specific built-in requirements
  (which implicitly adds unguarded logical constraints)

  QAtr: QA where each distinguished relation is transitive

  QAtc: QA where each distinguished relation is the transitive closure of another relation

Undecidability

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$\rightarrow$ **Base** Frontier-guarded TGDs (BaseFGTGDs):
body ϕ has an atom with all frontier variables **y**
and this atom is for a base relation $R \in \sigma_{\mathbf{B}}$

$$\forall x\, y_1\, y_2\; S(x, y_1) \wedge S(x, y_2) \wedge R(y_1, y_2) \rightarrow \exists z\; S(y_2, z) \wedge T(y_1)$$

Theorem

The QAtr and QAtc problems are decidable for BaseFGTGDs in 2EXPTIME combined complexity and PTIME data complexity.

Idea: Reduce QAtr to QA for FGTGDs by "axiomatizing" transitivity: change constraints Σ to Σ' enforcing transitivity within facts

$$\forall x\, y_1\, y_2\; F(x, y_1, y_2) \wedge T(y_1, x) \wedge T(x, y_2) \rightarrow T(y_1, y_2)$$

Lemma: Some superinstances of I_0 satisfying Σ' and violating Q can be extended by completing transitive relations to be transitive so that they still contain I_0, satisfy Σ and violate Q

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- Problem: QAlin is undecidable for BaseFGTGDs!

$\rightarrow$ Intuition:

  - $x < y \lor y < x$ codes inequality $x \neq y$
  - QA with inequalities in the query is often undecidable [Gutiérrez-Basulto et al., 2013]

Recovering decidability with order

$\rightarrow$ Solution: impose that guards are base relations
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Theorem

QAlin is decidable for BaseCovFGTGDs and base-covered queries

Complexity results
Query Answering with Transitive and Linear-Ordered Data

	QAtr		QAtc		QAlin	
	data	**combined**	**data**	**combined**	**data**	**combined**
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$\rightarrow$ Intuition: QAtc and QAlin can code disjunctive IDs

QAtc: In $T^+(a, b)$, is the T-path of length 1, 2, 3, ...?

QAlin: In $A(a, b)$, does $a < b$ or $b < a$ or $a = b$?

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Summary

We have studied QA with FGTGD constraints and:

- Number restrictions:
  - OK on low-arity when restricting FGTGD shapes
  - OK when further imposing non-conflicting condition
- Transitivity:
  - OK when not used as guards
- Orders:
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Ongoing work:

- Successor relations of linear orders:
  functional in both ways, acyclic, isomorphic to $\mathbb{Z}$
- General techniques for this type of problems?
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Thanks for your attention!

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